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Numerical Study of the Natural Oscillations of Perforated Vibrating Surfaces with Holes of Complex Geometry

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ABSTRACT

The widespread use of perforated vibrating surfaces in various industries requires maximum productivity and construction reliability. The research task is to determine the significant factors and their degree of influence on the natural oscillations of vibrating surfaces with multiple holes of complex geometry. For this purpose, studies were carried out for three samples of plates: non-perforated, with basic round holes and holes of complex geometry in the form of a five-petal epicycloid. Studies of the natural oscillations of perforated vibrating surfaces have been conducted using the finite element method in Abaqus, which has proved sufficient accuracy of calculations. The dependencies of the natural oscillation frequency of perforated surface samples on their thickness, partition width between the holes, material type, and fixing method have been obtained. In addition, the analysis involved the study of eight modes of oscillation common in practice. The dependencies of the natural oscillation frequency of perforated surface on the relative parameters of ligament efficiency and stiffness coefficient have also been obtained. These parameters take into account the ratios of the partition width between the holes to the plate thickness and the dimensions of the holes. The research results allow to obtain levels of influence of the perforated vibrating surface parameters on their natural oscillations frequency. The obtained research results make it possible to further determine the absence of damage between the holes and predict the durability of perforated vibration surfaces in the presence of holes of complex geometry.

Keywords: finite element method, natural frequencies, perforated surface, holes of complex geometry, design parameters.

INTRODUCTION

Perforated vibrating surfaces are working elements of machines for separation of components by size in various industries, such as agriculture, chemical, food, construction, mining. In most cases, perforated surfaces are made by cold stamping. Mechanical stamping of steel sheet by means of a matrix and a punch leads to the appearance of deformations (small notches) on the surface of the hole edges, which contribute to further fractures and fatigue cracking [1]. The industrial process of producing perforated surfaces itself is accompanied by a decrease in reliability due to the presence of holes and a violation of the integrity of the structure, the formation of edges with a certain degree of damage (internal cracks) [2]. Due to the high contact stresses that occur when stamping sheet metal, there are significant contact stresses that cause the tool to wear out intensively, reduce its service life and degrade the finished perforated part [3]. The use of perforated surfaces in technological processes is due to the separation of loose medium components by dimensional parameters (width, thickness or length of particles). Different types of holes are used for this purpose. The industrial use of perforated surfaces in separating equipment is typical for technological processes in the following industries: mining and construction [4, 5], agricultural and food [6, 7], chemical and pharmaceutical [8].

Most perforated surfaces work with vibration, which significantly increases the productivity and separation quality of components by size: in mining machinery industries on vibrating screen machine [4]; on grain cleaning machines with flat vibrating sieves with sifting activators in the form of epicycloidal holes [9] or optimal kinematic parameters of sieve oscillations [6]; rational parameters of segregation and sifting of medium particles through sieve holes [10].

The negative factor of using vibration is the reduction of reliability indicators. The presence of impacts of loose material particles on the sieve surface causes stresses and its deformation [11]. In addition, vibration causes the natural oscillations of perforated surface, which requires the justification of design parameters and/or the use of stiffness elements [12]. The presence of the natural oscillations of perforated plate in relation to the forced oscillations can cause resonance, with subsequent damage to the structure [13]. The deformation process of perforated vibration plate is also affected by the conditions of plate fixation, parameters of holes and their location on the surface, vibration parameters, and material properties [14].

As a result of operation and due to the technological features of the equipment (inclination, suspension on flexible elements, presence of distributed load, frequency and amplitude of oscillations), perforated surfaces experience differentiated stress [15]. Presence of natural oscillations of perforated surfaces together with technological deviations during production, external loads of abrasive separated medium leads to deformations – cracks between holes [16]. The appearance of a crack in the plates is caused by a number of factors: material properties, load and boundary conditions, geometric and kinematic parameters [17]. Presence of cracks violates quality of technological process and reduces the term of exploitation of vibrosurface.

The use of perforated vibrating surfaces with holes of complex geometry has proven a significant intensification of productivity and quality of technological indicators compared to basic holes: round, triangular and rectangular (longitudinal) [18, 19]. Thus, the use of perforated vibrating surfaces with holes of a five-petal epicycloid shape showed a significant increase in productivity up to 100% compared to basic round holes in the separation of biological objects (loose grain mixtures of peas and chickpeas) [18]. The efficiency of sifting processes of loose medium particles on a perforated vibrating surface, as well as the sieve productivity also depends on its kinematic parameters of vibration [9].

Introduction of perforated vibration surfaces with holes of complex geometry into technological processes requires their scientific multi-criteria evaluation. It is necessary to optimize the parameters of such perforated surfaces according to the criteria: technological efficiency, reliability of design and damage to separating medium particles. Determination of reliability of perforated vibration plates with holes of complex geometry requires, at the initial stage, determination of the natural oscillation frequency.

To ensure the reliability of structures during operation of which vibration is present or occurs, it is necessary to have data on the natural oscillation frequency. In addition, it is important to understand the trends in the change (dependencies) of the natural oscillation frequency from the parameters of perforated surfaces of various types.

For the study, perforated vibrating surfaces will be considered as plates with a thickness significantly smaller than the other two dimensions. We also take the following conditions: rectangular configuration, homogeneity by material properties. The well-known methodologies for measuring the natural oscillation frequency consist of analytical methods, for example, Rayleigh's method [20] or based on the Rayleigh quotient and the Rayleigh-Ritz method [21]; experimental methods such as, using high resolution camera and by utilization of the acoustic emission technique [22], use of accelerometer under different conditions of plate fixation [23]; numerical methods, for example, using finite element package ANSYS with varying sizes and shapes of single holes [24], for square or rectangular shape of plates [25]. Most of the methods are based on the study of the natural oscillation frequency of research objects – plates with a single hole of regular geometric shape [25], isotropic and orthotropic properties [20], various boundary conditions in the form of relative coordinates of the location of the holes [26] or changes in the size of the central hole and the points of fixing the plate [27]. This does not allow their full use and requires additional study. The aim of the research was to determine significant factors and their degree of influence on the values of the natural oscillation frequency of perforated vibrating surfaces with holes of complex geometry by means of simulation FE-modeling.

The novelty of the work is the analysis and determination of significant factors of oscillation of perforated surfaces with holes of complex geometry, their evaluation and transformation into complex (relative) parameters. The developed methodology will allow to identify the values of relative parameters of perforated surfaces with holes of complex geometry, which will be used for optimization by criteria: technological efficiency of separation of loose material particles and reliability of perforated surfaces.

RESEARCH METHODS

The study of the perforated vibrating surface oscillations, in this formulation of the problem, refers to the oscillations of a flat rectangular plate with holes. The research plan consists of:

1. Analyzing and selecting methods for determining the natural oscillation frequency of the plate.
2. Acceptance of boundary and initial conditions of the field of research.
3. Carrying out calculations and analyzing the results.
4. Justification of scientific and technical recommendations.

The use of numerical methods for the analysis of oscillations of various complex structures of composite laminates [22], damaged cantilever beams [28], including perforated plates [24, 25] showed the necessary accuracy of the results. Preliminary studies with experimental and numerical methods produced an adequate numerical FE-model at Abaqus [29]. This model was the basis for this study.

The effectiveness of numerical simulations using FE-methods depends on the correspondence between boundary and initial conditions. Among the ways to obtain maximum accuracy of numerical modeling results are methodologies that involve comparison with analytical data, refinement with experimental results, and comparison with already known data. To implement the research tasks, we adopt a method that involves numerical modeling of the FEM of various rectangular plates: (1) non-perforated (without holes) (Fig. 1a); (2) perforated with round holes, the centers of which are located along the hexagon (Fig. 1b); (3) perforated with holes of complex geometry – in the form of a five-petal epicycloid, the centers of which are also located along the hexagon (Fig. 1c).

The hole shape is defined by the epicycloid equation as the trajectory of a circle point of radius r rolling without sliding on a stationary circle of radius R (Fig. 1c). For research the hole shape that has epicycloid modulus k = 5 (number of petals) is chosen. Changing the modulus of the epicycloid allows us to create different shapes of holes, for example, k = 2–9, which allows us to apply the term of holes of complex geometry and requires appropriate research.

Based on the analysis of preliminary research and existing studies, significant factors that influence the natural oscillation frequency of the plate have been identified: perforation level (permeability) of the plate [27], parameters of the hole itself [30], parameters of the partitions (pitch arrangement) between the holes [31], plate thickness [15], material properties [17], and the shape of the plate fixation [32].

In terms of rigidity, the following absolute parameters are accepted for analysis: R – hole radius, which is a technological parameter and determines size of the fraction component during material separation; r – radius of the circle that forms the contour of an epicycloidal-shaped hole; a – minimum distance between holes (partitions); s – surface thickness; p – pitch between hole centers.

During the research, the values of some factors were fixed from the conditions of maximum technological productivity of sifting: the radii R = 3.5 mm and r = 1.5 mm forming the holes (Fig. 1b, 1c). The pitch between the round holes of a perforated surface is denoted p = 2R + a, and for a perforated surface with holes of complex geometry p = 2R + r + a.

Considering machine and equipment designs, the results of the known research [17, 33] apply: two types of perforated surface fixing (rigid CCCC and loose (supported) on two opposite sides CSCS), as well as two types of material (steel and aluminium). For the convenience of comparing with the results of the conducted preliminary experimental studies, the following dimensions of the plates are adopted: length – 640 mm and width – 260 mm.

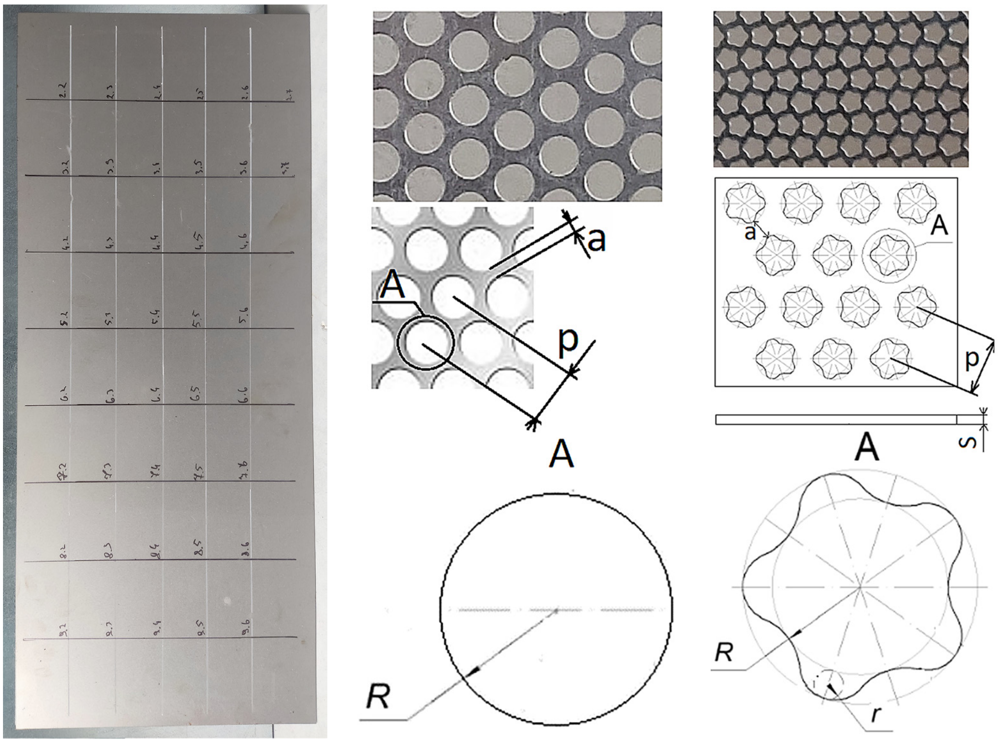


Fig. 1. Diagrams and general view of the test samples-plates: (a) non-perforated; (b) perforated surfaces with basic round holes that are arranged along a hexagon; (c) perforated surfaces with holes of complex geometry

Studies were conducted for the remaining factors with the following levels of variation (Table 1). Ranges of variation of partition width and surface thickness are selected from the conditions of economic costs minimization, maximization the technological sifting of loose medium particles and the results of known studies [17, 34].

EQUIPMENT AND MATERIALS

For FE-modeling and analysis of frequency characteristics, the Simulia Abaqus software complex was used, designed for multi-purpose interdisciplinary analysis. The work of the software complex is based on the finite element method, which is one of the effective numerical methods used to solve the problems of mechanics of deformable solid. The main stages of modeling are: – constructing a geometric three-dimensional model of a plate with holes; – assigning material properties and section properties to the geometry; – assembling the model; – setting analysis steps; – setting boundary conditions; – construction of a finite element grid; – creating and initializing a task; – visualization of the calculation.

The investigated vibrosurfaces have a rectangular shape and are naturally divided into rectangles, so initially, the quadrangular shell elements of the first-order S4 were used for modeling (Fig. 2). The following conditions and parameters were adopted for numerical modeling in Abaqus software: the linear model of elastic material, material characteristics in the form of density, Young's modulus, and Poisson's ratio (Table 2); finite element mesh: shell elements (S4 – without reduced integration); the size of the final element – 1 mm; the total number of model elements – 166573.

Table 1. Significant factors and ranges of their variation

Level and range of factors variation	Factors			
	Partition width a, mm	Surface thickness *s*, mm	Material	Fastening type*
+1 0 –1	3 2 1	1.2 1 0.8	Steel Aluminium	CCCC CSCS

Note: *- C – pinched edge of the plate; S – supported edge of the plate

For numerical calculations we accept the following materials from which the perforated surfaces are made: steel S235JR, aluminium AL99.5 EN AW1050 H24 (Table 2). These materials are some of the popular engineering metals [33, 35]. The physical properties of the solid aluminum plate material are as follows: modulus of elasticity 69.0 GPa, Poisson's ratio 0.33 and mass density 2.700 kg/m^3 [17, 33]. Such materials are the most common due to the optimal reliability of operation and the cost of manufacture.

It should also be noted that the perforated surfaces have a galvanised external layer for protection against corrosion and partial wear. Among industrial protective coatings, these coatings are among the most common. One of the reasons for this is the economical and environmentally friendly side of protecting steel from corrosion. Galvanizing of sheet steel is carried out by hot method in continuous operation units according to the standard PN-EN ISO 1461:2023-02. Among the coating parameters, it is necessary to highlight the crystallization pattern and the coating class, according to which we have information about the thickness and density of the coating. For research, we use perforated plates with the following coating parameters: the mass of the coating layer on both sides is 240–250 g/m^2, the thickness is 32–35 microns. Considering the insignificance of the coating thickness in relation to the thickness of the steel layer (3.2–3.5%) and the relatively similar properties, this factor has been excluded from further research.

To analyze the natural oscillation frequency of perforated plates, identical separating diameters of holes were accepted, ensuring the separation of loose material particles. It is possible to obtain similar forms of oscillation for the studied

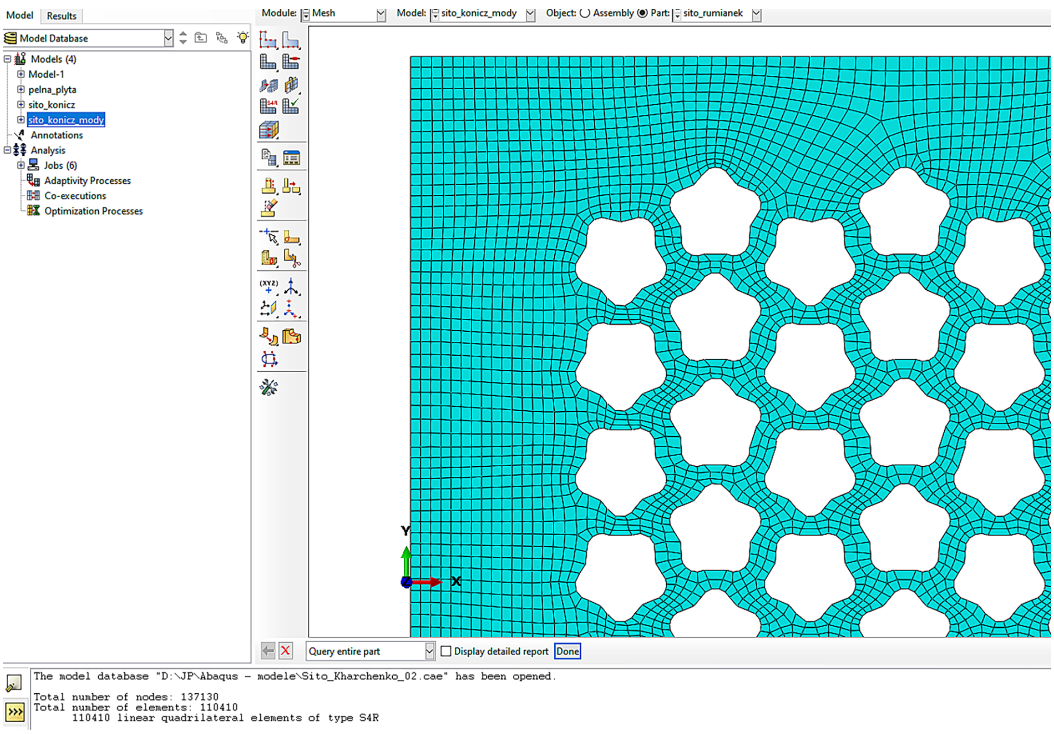


Fig. 2. FE-model grid

Table 2. Materials of perforated surfaces

Indicators	Steel S235JR	Aluminium EN AW-1050A
Yield strength, MPa	235	85
Tensile strength, MPa	345	145
Young's modulus, GPa	210	69
Poisson's ratio	0.3	0.33
Density, g/cm^3	7.847	2.7

perforated plates, which have different frequency values. Increasing the frequency range revealed differences in the vibration modes between plates with round holes and holes of complex geometry. We focused on low-frequency plate vibrations due to their potential practical applications.

RESEARCH RESULTS

As a result of the modeling, the natural oscillations frequencies of the studied plates were obtained depending on the type of perforation, material, thickness and type of their fixing (Tables 3–5, Fig. 3). When the thickness of the studied plates increases, their natural oscillation frequency increases by 31.85–33.35% (when fixing the CSCS) and by 32.17–33.32% (CCCC), which is explained by an increase in cylindrical stiffness. However, the plate mass itself is also increased. It should be remembered from analytical expressions that when the plate thickness increases, the mass increases linearly, and the cylindrical stiffness increases proportionally to h^3.

The plate thickness provides structural rigidity while simultaneously increasing its metal consumption and the cost of the component during manufacturing. Therefore, a similar range of plate thickness variation was chosen. The analysis shows that modes with lower values of the natural oscillation frequency (mode 1) respond less to thickness variation (e.g., mode 7). The use of different materials, in the ranges of investigated parameters, showed insignificant influence on the natural oscillation frequency of the plates. The frequency of oscillation of the studied steel plates exceeded the frequency of aluminium plates by 0.42–1.66% (at CSCS) and 0.11–1.24% (at CCCC). When comparing the fixing methods of plates, the change in structural stiffness should also be considered. Replacing two free-supported opposite sides (CSCS) with a rigid pinch (CCCC) increases its natural oscillation frequency by 1.74–9.37 times. To analyze the influence of the holes shape of the plate on its values of the natural oscillation frequency, the corresponding comparisons for eight modes were carried out (Fig. 4, Table 6).

The comparative analysis showed that the natural oscillation of plates with holes of complex geometry are lower by 0.46–7.25% (at CSCS) and higher by 0.1–3.23% (at CCCC) compared to plates with round holes; lower by 15.68–31.95% (at CSCS) and by 13.76–19.8% (at CCCC) compared to non-perforated (without holes) plate. The change in plate thickness, within the scope of the comparison with respect to the type of holes, has a negligible influence of 0.05–0.22% (at CSCS) and 0.11–0.28% (at CCCC). To check the adequacy of the obtained results of numerical

Table 3. Modes of plate vibrations and their visualization

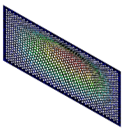
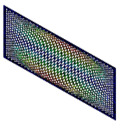
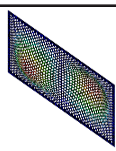
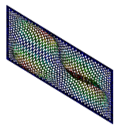
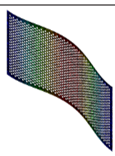
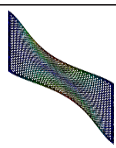
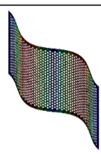
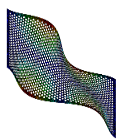
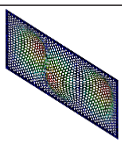
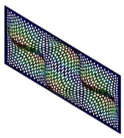
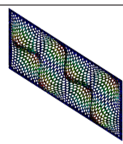
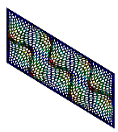
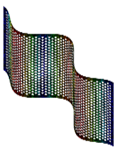
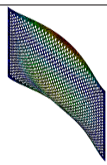
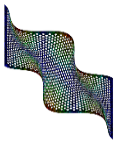
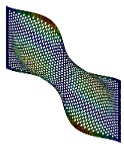
Moda		1	2	3	4
Visual images	Fixing type CCCC				
	Fixing type CSCS				
Moda		5	6	7	8
Visual images	Fixing type CCCC				
	Fixing type CSCS				

Table 4. Dependence of the natural oscillation frequency on the plate thickness rigid fixing of all 4 sides of the plate type CCCC

Moda	Sieve with holes of complex geometry (a = 2 mm, R = 2.5 mm, k = 5)			Sieve with round holes (a = 2 mm, R = 2.5 mm)			Solid plate		
	Thickness s, mm			Thickness s, mm			Thickness s, mm		
	0.8	1.0	1.2	0.8	1.0	1.2	0.8	1.0	1.2
1	55.86* / 55.20	69.32 / 68.51	82.57 / 81.61	54.11 / 53.47	67.20 / 66.42	80.11 / 79.19	64.77 / 64.69	80.95 / 80.86	97.14 / 97.02
2	147.94 / 146.21	183.62 / 181.48	218.76 / 216.22	143.86 / 142.17	178.65 / 176.57	212.96 / 210.52	172.54 / 172.34	215.66 / 215.40	258.76 / 258.45
3	65.44 / 64.65	81.17 / 80.19	96.62 / 95.47	64.21 / 63.44	79.73 / 78.79	95.04 / 93.94	77.21 / 77.12	96.51 / 96.39	115.80 / 115.66
4	157.79 / 155.93	195.78 / 193.47	233.13 / 230.41	154.16 / 152.34	191.43 / 189.20	228.19 / 225.56	185.28 / 185.06	231.57 / 231.30	277.85 / 277.52
5	82.96 / 81.95	102.86 / 101.62	122.41 / 120.94	82.62 / 81.64	102.58 / 101.38	122.27 / 120.85	99.92 / 99.81	124.90 / 124.75	149.87 / 149.69
6	174.73 / 172.63	216.68 / 214.10	257.90 / 254.85	171.89 / 169.86	213.43 / 210.93	254.39 / 251.45	207.15 / 206.91	258.92 / 258.61	310.66 / 310.29
7	199.11 / 196.69	243.83 / 243.85	293.66 / 290.15	197.43 / 195.08	245.12 / 242.24	292.15 / 288.76	238.73 / 238.45	298.38 / 298.03	358.01 / 357.58
8	231.24 / 228.41	286.59 / 283.11	340.90 / 336.78	231.04 / 228.28	286.83 / 283.45	341.83 / 337.86	280.35 / 280.02	350.39 / 349.97	420.41 / 419.90

Note: *plate material: numerator – steel; denominator – aluminium

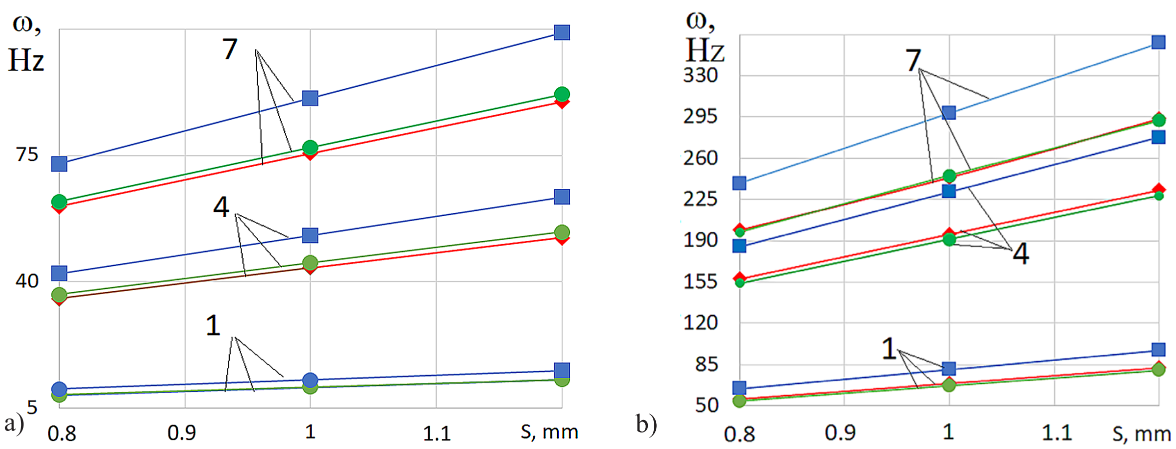


Fig. 3. Dependence of the natural oscillation frequency of perforated steel surface on its thickness: a – rigid fixing of the CSCS; b – CCCC; – non-perforated surface; – perforated surface with basic round holes; – perforated surface with holes of five-petal epicycloid (R = 3.5 mm; a = 2 mm)

modeling, we will compare them with analytical methods. We will conduct analytical determination of the natural oscillation frequencies of the non-perforated (without holes) plate according to the procedures [17]. For calculations, we take the following conditions: homogeneity of the material and its characteristics (Table 2), invariable specified thickness, rectangular shape.

The problem of determining the oscillation frequency of a rectangular plate pinched along the contour in an analytical form is solved by approximate methods [36, 37]. The application of the Rayleigh-Ritz formula gives an approximate value for the natural oscillation frequency:

$$\omega^2 = \frac{D}{\rho s}\,\frac{\int_0^a\int_0^b \left\{(\Delta\omega)^2 + 2(1-\mu)\left[\left(\frac{\partial^2\omega}{\partial x\partial y}\right)^2 - \frac{\partial^2\omega}{\partial x^2}\frac{\partial^2\omega}{\partial y^2}\right]\right\}dxdy}{\int_0^a\int_0^b \omega^2\,dxdy} \qquad (1)$$

where: $D = \frac{Es^3}{12(1-\mu^2)}$ – cylindrical stiffness; ρ – material density; E – modulus of elasticity; μ – Poisson's ratio; s – plate thickness; a, b – length and width of the plate.

After integration, we obtain the final expression for determining the natural oscillation frequency of the plate pinched along the contour (CCCC):

Table 5. Dependence of the natural oscillation frequency on the plate thickness rigid fixing of all 2 sides of the plate type CSCS

Moda	Sieve with holes of complex geometry (a = 2 mm, R = 2.5 mm, k = 5)			Sieve with round holes (a = 2 mm, R = 2.5 mm)			Solid plate		
	thickness s, mm			thickness s, mm			thickness s, mm		
	0.8	1.0	1.2	0.8	1.0	1.2	0.8	1.0	1.2
1	8.71* 8.61	10.83 10.71	12.94 12.78	8.77 8.67	10.90 10.77	13.00 12.85	10.33 10.28	12.91 12.85	15.50 15.42
2	15.96 15.71	19.75 19.44	23.45 23.08	16.58 16.31	20.50 20.17	24.33 23.94	19.02 18.71	23.77 23.39	28.52 28.06
3	23.98 23.69	29.82 29.47	35.60 35.19	24.17 23.88	30.03 29.68	35.82 35.40	28.50 28.35	35.62 35.43	42.74 42.52
4	35.43 34.92	43.90 43.28	52.22 51.49	36.52 35.99	45.25 44.59	53.82 53.04	42.36 41.79	52.94 52.23	63.52 62.67
5	46.90 46.35	58.32 57.64	69.63 68.82	47.33 46.77	58.82 58.12	70.16 69.33	56.03 55.77	70.04 69.70	84.04 83.64
6	49.06 48.35	60.80 59.92	72.32 71.28	52.81 52.07	65.47 64.57	77.91 76.85	70.13 69.84	87.66 87.29	105.18 104.74
7	60.81 60.02	75.48 74.50	89.82 88.76	62.18 61.34	77.15 76.12	91.89 90.67	72.63 71.85	90.78 89.81	108.92 107.75
8	71.20 70.12	88.20 86.87	104.85 103.27	74.84 73.74	92.73 91.37	110.28 108.68	92.88 92.47	116.09 115.58	139.30 138.69

Note: *plate material: numerator – steel; denominator – aluminium.

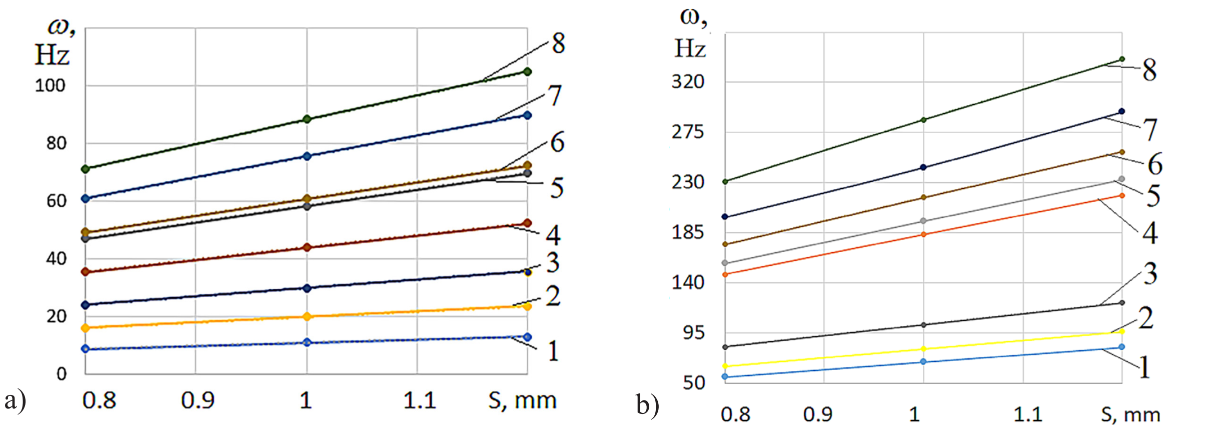


Fig. 4. Dependences of the natural oscillation frequency of the steel plate with holes of complex geometry on its thickness for 8 modes at the fixation method: (a) rigid fixing of the CSCS; (b) – CCCC (R = 3.5 mm; a = 2 mm)

$$\omega=\pi^2\left[\frac{D}{\rho s}\left(\left(\frac{A_m}{a}\right)^4+\left(\frac{A_n}{b}\right)^4+2\frac{B_mB_n}{a^2b^2}\right)\right]^{\frac{1}{2}} \qquad (2)$$

where: $A_m=\begin{cases}1,506\ (m=1)\\ m+0,5\ (m\ge 2)\end{cases}$ $B_m=\begin{cases}1,248\ (m=1)\\ A_m\left(A_m-\frac{2}{\pi}\right)(m\ge 2)\end{cases}$, m,n – number of half-waves in the direction of axes x and y, respectively (m,n = 1,2,3…). The constants $\grave{A}_n, B_n$ are defined similarly by the boundary conditions at y = 0, b.

In the case of boundary conditions for two opposite supported edges of the plate, choosing the product of beam functions as approximating functions, the Rayleigh-Ritz formula takes the following form:

$$\omega=\pi^2\left[\frac{D}{\rho s}\left(\left(\frac{A_m}{a}\right)^4+\left(\frac{A_n}{b}\right)^4+2\frac{(\mu B_mB_n+(1-\mu))C_mC_n}{a^2b^2}\right)\right]^{\frac{1}{2}} \qquad (3)$$

where: $\grave{A}_{m(n)}, B_{m(n)}, \tilde{N}_{m(n)}$ – coefficients that take into account the given boundary conditions:

$$A_{m(n)}=\begin{cases}1,25\ (m(n)=1)\\ m(n)+0,75\ (m(n)\ge 2)\end{cases}$$

$$B_{m(n)}=\begin{cases}A_{m(n)}\left(A_{m(n)}-\frac{1}{\pi}\right)\ (m(n)=1)\\ A_{m(n)}\left(A_{m(n)}-\frac{1}{\pi}\right)\ (m(n)\ge 2)\end{cases}$$

$$C_{m(n)}=\begin{cases}B_{m(n)}\ \ (m(n)=1)\\ B_{m(n)}\ (m(n)\ge 2)\end{cases} \qquad (4)$$

Table 6. Degree of influence (%) of holes shape on the values of the natural oscillation frequency of the plate

Moda	ω_G/ω_R, %			ω_G/ω_S, %			ω_G/ω_R, %			ω_G/ω_S, %		
	Plate thickness s, mm			Plate thickness s, mm			Plate thickness s, mm			Plate thickness s, mm		
	0.8	1.0	1.2	0.8	1.0	1.2	0.8	1.0	1.2	0.8	1.0	1.2
	Fixing type CSCS						Fixing type CCCC					
1	-0.68** -0.69	-0.64 -0.56	-0.46 -0.54	-15.68 -16.25	-16.11 -16.65	-16.52 -17.12	3.23 3.24	3.15 3.15	3.07 3.06	-13.76 -14.67	-14.37 -15.27	-15 -15.88
2	-3.74 -3.68	-3.66 -3.62	-3.62 -3.59	-16.09 -16.03	-16.91 -16.89	-17.78 -17.75	2.84 2.84	2.78 2.78	2.72 2.71	-14.26 -15.16	-14.86 -15.75	-15.46 -16.34
3	-0.79 -0.80	-0.70 -0.71	-0.61 -0.59	-15.86 -16.44	-16.28 -16.82	-16.71 -17.24	1.92 1.91	1.81 1.78	1.66 1.63	-15.24 -16.17	-15.89 -16.81	-16.56 -17.46
4	-2.98 -2.97	-2.98 -2.94	-2.97 -2.92	-16.36 -16.44	-17.08 -17.14	-17.79 -17.84	2.35 2.36	2.27 2.26	2.16 2.15	-14.84 -15.74	-15.46 -16.36	-16.10 -16.98
5	-0.91 -0.90	-0.85 -0.83	-0.76 -0.74	-16.29 -16.89	-16.73 -17.30	-17.15 -17.72	0.41 0.38	0.27 0.24	0.11 0.07	-16.97 -17.89	-17.65 -18.54	-18.32 -19.21
6	-7.10 -7.14	-7.13 -7.20	-7.17 -7.25	-30.04 -30.77	-30.64 -31.36	-31.24 -31.95	1.65 1.63	1.52 1.5	1.38 1.35	-15.65 -16.57	-16.31 -17.21	-16.98 -17.87
7	-2.20 -2.15	-2.16 -2.13	-2.25 -2.11	-16.27 -16.46	-16.85 -17.05	-17.54 -17.62	0.85 0.83	0.53 3.05	0.52 0.48	-16.60 -17.51	-18.28 -21.20	-17.97 -18.86
8	-4.86 -4.91	-4.89 -4.93	-4.92 -4.98	-23.34 -24.17	-24.02 -24.84	-24.73 -25.54	0.09 0.06	0.08 0.12	0.27 0.32	-17.52 -18.43	-18.21 -19.10	-18.91 -19.80

Note: * ω_G – the natural frequency of the plate with holes of complex geometry; ω_R – plates with round holes; ω_S – plate without holes; ** plate material: numerator – steel; denominator – aluminium.

Using expressions (2), (3) numerical calculations for steel and aluminium plates without holes were carried out. As a result of comparing the results of analytical determination and the results of FE-modeling (Tables 4, 5), it was found that the discrepancy in the natural oscillation frequency of the plates was up to 2.1%. This corresponds to the results of known researches [38] and confirms the adequacy of the data and the possibility of using the Simulia Abaqus software complex for further analysis. Also, as a result of modeling the natural oscillation frequencies of the studied plates depending on the type of perforation, material, width of partitions and type of their fixing were obtained (Tables 7, Fig. 5, 6). Degree of influence (%) of holes shape on the values of the natural oscillation frequency of the plate (steel/aluminium) are presented in Table 8. The obtained dependences of the natural frequencies of the investigated perforated surfaces take into account the presence of holes of complex geometrical shape and allow to make changes in the design to suppress vibration, change the character of excitation to avoid entering into resonance. At increasing the distance between holes, in the investigated ranges (Table 8), the stiffness of plates and, accordingly, their natural oscillation frequency increases by 6.2–24.57%. Thus, for rigidly fixed plates (CCCC) with holes of complex geometry the increase of natural oscillation frequency by 8.32–10.91% (steel) and by 8.57–11.11% (aluminium) is observed; for plates with round holes by 8.78–10.26% (steel) and by 8.95–10.43% (aluminium). For plates with CSCS fixation: by 6.2–24.35% (steel) and 6.27–24.57% (aluminium); for plates with round holes by 8.11–21.15% (steel) and 8.22–21.31% (aluminium).

The use of different materials (Table 8) also showed an insignificant influence on the natural oscillation frequency of the plates. Oscillation frequency of the studied steel plates exceeded the frequency of aluminium plates: for plates with holes of complex geometry by 1.12–1.63% (at CSCS) and 0.01–1.32% (at CCCC); for plates with round holes by 1.08–1.64% (at CSCS) and 1.09–1.26% (at CCCC). Analysis of the influence of holes type on the natural oscillation frequency of plates of the plates showed its variation in the range of 0.52–3.15% (at CCCC fixing), 1.37–9.13% (at CSCS). The analysis of the influence of varying the distance between the holes showed that the natural oscillations of plates with holes of complex geometry are smaller by 0.46–7.25% (at CSCS) and larger by 0.1–3.23% (at CCCC) compared to plates with round holes.

ANALYSIS AND DISCUSSION OF RESULTS

For generalized analysis of the influence of the hole shape on the values of the natural

Table 7. Dependence of the natural oscillation frequency on the partition width between the holes (thickness S = 1 mm; R = 3.5 mm)

Moda	Sieve with holes of complex geometry (k = 5)			Sieve with round holes (d = 7 mm)			Sieve with holes of complex geometry (k = 5)			Sieve with round holes		
	Partition width between holes a, mm			Partition width between holes a, mm			Partition width between holes a, mm			Partition width between holes a, mm		
	1	2	3	1	2	3	1	2	3	1	2	3
	Fixing type CCCC						Fixing type CSCS					
1	65.05* 64.20	69.32 68.51	70.70 69.94	63.11 62.33	67.20 66.42	68.72 67.97	10.49 10.36	10.83 10.71	11.14 11.01	10.34 10.22	10.90 10.77	11.18 11.06
2	172.66 170.41	183.62 181.48	187.03 185.02	167.85 165.79	178.65 176.57	182.59 180.62	17.95 17.68	19.75 19.44	20.63 20.30	18.74 18.45	20.50 20.17	21.26 20.91
3	75.82 74.83	81.17 80.19	83.20 82.26	74.64 73.72	79.73 78.79	81.76 80.87	28.84 28.49	29.82 29.47	30.65 30.30	28.49 28.14	30.03 29.68	30.80 30.45
4	183.70 181.32	195.78 193.47	199.81 197.64	179.69 177.47	191.43 189.20	195.93 193.81	40.48 39.94	43.90 43.28	45.68 45.04	41.86 41.27	45.25 44.59	46.79 46.10
5	95.63 94.39	102.86 101.62	106.06 104.88	95.71 94.52	102.58 101.38	105.53 104.38	56.28 55.59	58.32 57.64	59.99 59.32	55.77 55.08	58.82 58.12	60.35 59.68
6	202.65 200.01	216.68 214.10	221.78 219.35	199.99 197.52	213.43 210.93	218.81 216.44	52.49 51.68	60.80 59.92	65.27 64.38	57.69 56.87	65.47 64.57	69.89 68.99
7	230.19 227.19	243.83 243.85	253.49 250.70	229.27 226.43	245.12 242.24	251.77 249.03	70.78 69.90	75.48 74.50	78.14 77.14	72.20 71.25	77.15 76.12	79.51 78.45
8	266.60 263.13	286.59 283.11	295.30 292.01	267.83 264.50	286.83 283.45	295.11 291.89	78.94 77.73	88.20 86.87	93.10 91.75	83.54 82.30	92.73 91.37	97.57 96.18

Note: *plate material: numerator – steel; denominator – aluminium.

Table 8. Degree of influence (%) of holes shape on the values of the natural oscillation frequency of the plate

Moda	ω_G/ω_R, %					
	Partitions width (distance between holes) a, mm			Partitions width (distance between holes) a, mm		
	1	2	3	1	2	3
	Fixing type CSCS			Fixing type CCCC		
1	1.25** 1.17	1.12 1.21	1.18 1.08	1.32 1.25	1.18 1.17	1.09 1.1
2	1.53 1.57	1.59 1.64	1.63 1.67	1.32 1.24	1.18 1.09	1.18 1.09
3	1.23 1.24	1.19 1.18	1.16 1.15	1.32 1.25	1.22 1.19	1.14 1.1
4	1.35 1.43	1.43 1.48	1.42 1.5	1.31 1.25	1.19 1.18	1.1 1.09
5	1.24 1.25	1.18 1.2	1.13 1.12	1.31 1.26	1.22 1.18	1.13 1.1
6	1.57 1.44	1.47 1.39	1.38 1.3	1.32 1.25	1.21 1.19	1.11 1.09
7	1.26 1.33	1.32 1.35	1.3 1.35	1.32 1.25	0.01 1.23	1.11 1.1
8	1.56 1.51	1.53 1.49	1.47 1.45	1.32 1.26	1.23 1.19	1.13 1.1

Note: * ω_G - the natural frequency of the plate with holes of complex geometry; ω_R - plates with round holes
** plate material: numerator – steel; denominator – aluminium

oscillation frequency of the plates, the results are summarized in Table 9. Additionally, to achieve a full understanding and expand the possibilities of utilizing research results, we consider relative parameters. Among the known used relative parameters that characterize the level of plate perforation is ligament efficiency [33, 39]:

$$K_P = a/p \quad (5)$$

where: a – partition width between holes; p – pitch between hole centers (Fig. 1).

Taking into account the initial data and research conditions, we have the following ligament efficiency values for plates with holes in complex

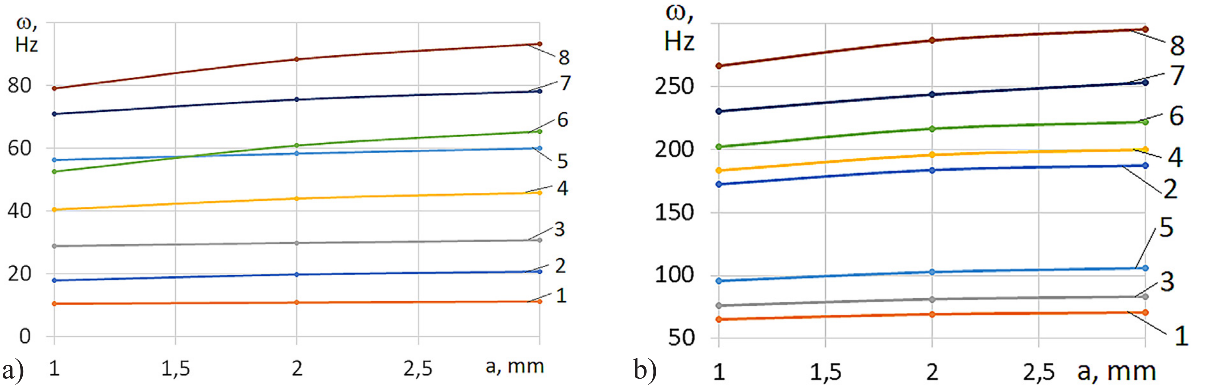


Fig. 5. Dependences of the natural oscillation frequency of the steel plate with holes of complex geometry on a partitions width (distance between holes) for 8 modes at the fixation method: (a) rigid fixing CSCS; (b) CCCC (R = 3.5 mm; s = 1 mm)

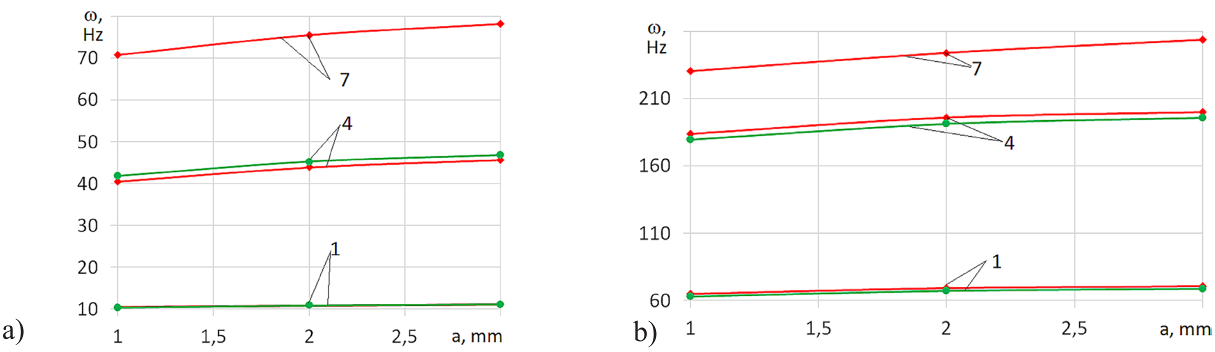


Fig. 6. Dependence of the natural oscillation frequency of perforated surface on a partitions width: a – rigid fixing CSCS; b – CCCC; (green line) – perforated surface with basic round holes; (red line) – perforated surface with holes of five-petal epicycloid (R = 3.5 mm; s = 1 mm)

Table 9. Degree of influence (%) of factors on the natural oscillation frequency of the studied perforated plates

Method of fixing the plate	Factors			
	Material	Plate thickness s	Partition width a	Presence and shape of holes
CSCS	0.42–1.66	31.85–33.35	6.2–24.57	0.46–31.95
CCCC	0.01–1.32	32.17–33.32	8.32–11.11	0.1–19.8

Table 10. Ligament efficiency of different perforated surfaces

Partition width between holes a, mm	Plate with holes of complex geometry		Plate with round holes	
	Pitch between hole centers p, mm	K_{PG}	Pitch between hole centers p, mm	K_{PR}
1	9.5	0.105	8	0.125
2	10.5	0.190	8	0.250
3	11.5	0.261	8	0.375
1	9.5	0.105	9	0.111
2	10.5	0.190	9	0.222
3	11.5	0.261	9	0.333
1	9.5	0.105	10	0.100
2	10.5	0.190	10	0.200
3	11.5	0.261	10	0.300

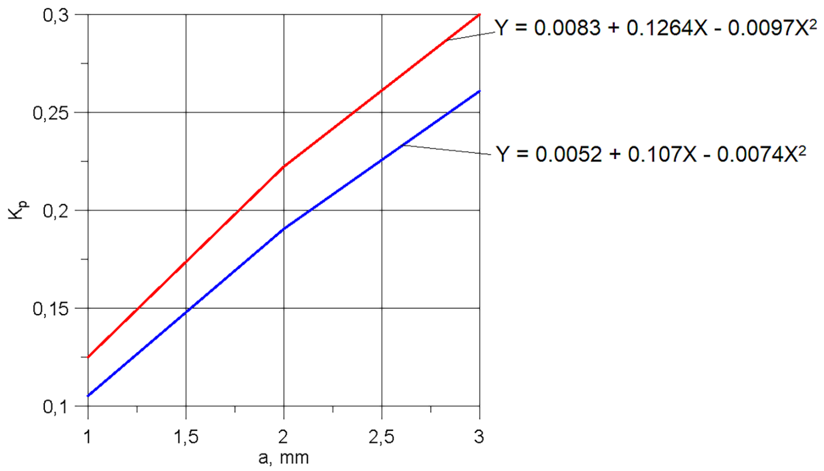


Fig. 7. Dependence of ligament efficiency of plates with holes of complex geometry (blue line) and with round holes (red line) on the width of the partition between the holes

Table 11. Values of the relative stiffness coefficient from perforated plate parameters

Partition width between holes a, mm	Plate thickness s, mm	Plate with holes of complex geometry		Plate with round holes	
		Pitch between hole centers p, mm	K_{SG}	Pitch between hole centers p, mm	K_{SR}
1	0.8	9.5	0.0089	8	0.0013
2	0.8	10.5	0.0145	8	0.0024
3	0.8	11.5	0.0181	8	0.0033
1	1	9.5	0.0111	9	0.0013
2	1	10.5	0.0181	9	0.0024
3	1	11.5	0.0227	9	0.0032
1	1.2	9.5	0.0133	10	0.0013
2	1.2	10.5	0.0218	10	0.0023
3	1.2	11.5	0.0272	10	0.0031

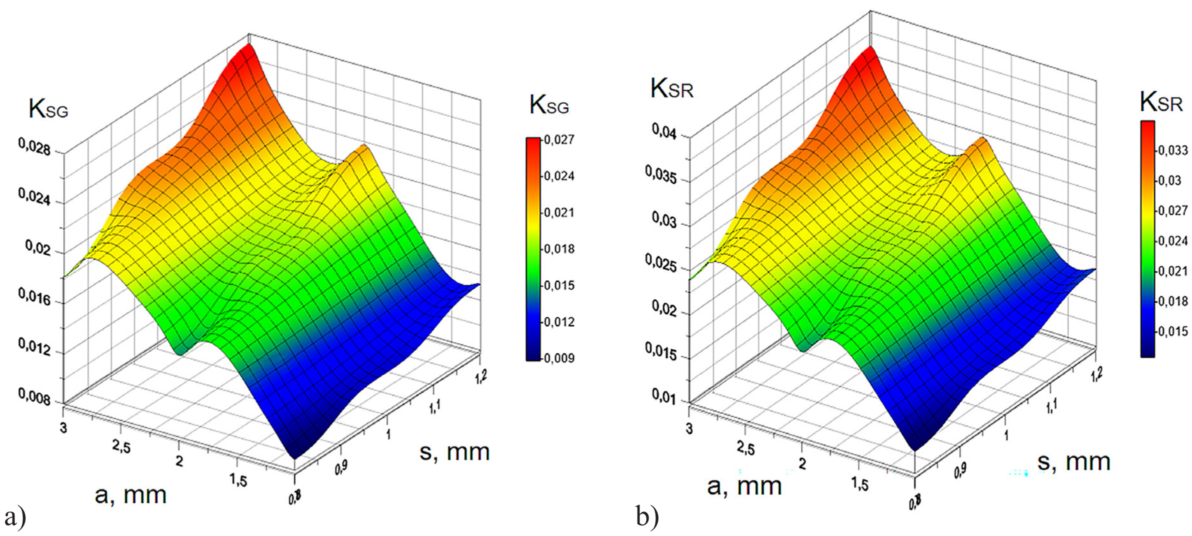


Fig. 8. Dependencies of the stiffness coefficient of the perforated surface K_{SG} on the structural parameters of the partition width (a) and thickness (s): (a) plates with holes of complex geometry (K_{SG}); (b) plates with round holes (K_{SR})

geometry (K_{PG}) and with round holes (K_{PR}) (Table 10, Figure 7). The second relative parameter for complex analysis is the surface stiffness coefficient, which determines the ratio of the partition width (non-perforated part) to the surface thickness (Fig.8, Table 11):

$$K_s = as/p^2 \tag{6}$$

This parameter reflects the ratio of the cross-sectional area of the periodic element to its surface area, which allows for accounting for changes in the plate's thickness. It should be noted that this expression is applicable for thin plates, according to the research conditions. Then, the maximum value of the stiffness coefficient is $K_s = s/p$, at which there are no holes. It should be noted that the further development of studies of the reliability of perforated plates with holes of complex geometry should be to take into account the design parameters of the hole itself. Among the design parameters, which are of scientific interest, it is possible to distinguish the number of epicyclic petals, radii forming the hole shape (R and r, Fig. 1). The obtained regularities of changes in oscillation frequency allow to find possibilities of increasing the durability of perforated vibrating surfaces.

CONCLUSIONS

The method of numerical studies of the natural oscillation frequencies of perforated surfaces with holes of complex geometry was developed and tested, which is based on the use of the Simulia Abaqus software complex. The method involved the analysis of oscillations of base plates: solid non-perforated and with round holes, as well as perforated plates with holes of complex geometry in the form of a five-petal epicycloid.

Dependence of the natural oscillation frequency of perforated plate on its thickness, width of partition between holes, material and method of their fixation are obtained. As a result of the studies, significant factors and their degrees of influence on the oscillations of perforated plates were determined. It was found that the plate thickness has the greatest influence (31.85–33.35%), the following are the hole parameters: partition width between holes; pitch between hole centers. Analysis of the results showed that the natural vibrations of plates with holes of complex geometry differ by up to 7% compared to plates with basic round holes. Taking into account the increased technological permeability (criterion 1) of the perforated plate with holes of complex geometry and the obtained dependencies, it became possible to ensure the reliability of this structure (criterion 2). The adequacy of the obtained values of the natural oscillation frequencies of a solid non-perforated plate is verified by comparing (up to 2%) them with the analytically obtained data.

It is also recommended to use relative parameters of ligament efficiency and stiffness coefficient to analyze the natural oscillation frequency of perforated surface with holes of complex geometry. These parameters take into account the ratios of significant design parameters of perforated surfaces and make it possible to comprehensively characterize changes in the oscillation frequency. Comparative analysis showed that the relative stiffness coefficient of the perforated plate with holes with complex geometry is 5.4–10.2 times greater with respect to the plate with basic round holes. It is also found that value of efficiency coefficient of binding of perforated surfaces with holes of complex geometry is less by 0.05–30% relative to plate with basic round holes.

The developed methodology has allowed for the identification of relative parameters of perforated surfaces with complex geometry holes, which can be used for optimization based on criteria such as technological efficiency of particle separation in loose materials and the reliability of perforated surfaces. Such research results will be useful for scientific research, engineering design of reliability of perforated vibrating surfaces.

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REFERENCES

1. Thomas D. Effect of Mechanical Cut-Edges on the Fatigue and Formability Performance of Advanced High-Strength Steels. J. Fail. Anal. and Preven. 2012; 12: 518–531, https://doi.org/10.1007/

s11668-012-9591-z.

2. Lange K. Umformtechnik – Handbuch für Industrie und Wissenschaft, Band 2: Massivumformung (2nd edition), Springer-Verlag, Berlin, Germany, 1988.
3. 3. Merklein M., Allwood J., Behrens B.-A., Brosius A., Hagenah H., Kuzman K., Mori K., Tekkaya A., Weckenmann A. Bulk forming of sheet metal. CIRP Annals 2012; 61(2): 725–745, https://doi.org/10.1016/j.cirp.2012.05.007.
4. Makinde O., Ramatsetse B., Mpofu K. Review of vibrating screen development trends: Linking the past and the future in mining machinery industries. International Journal of Mineral Processing 2015; 145: 17–22, https://doi.org/10.1016/j.minpro.2015.11.001.
5. Honghai Liu, Jie Jia, Nieyangzi Liu, Xiaojin Hu, Xiong Zhou. Effect of material feed rate on sieving performance of vibrating screen for batch mixing equipment. Powder Technology 2018; 338: 898-904, https://doi.org/10.1016/j.powtec.2018.07.046.
6. Tishchenko L., Ol'shanskii V., Ol'shanskii V. On velocity profiles of an inhomogeneous vibrofluidized grain bed on a shaker. Journal of Engineering Physics and Thermophysics 2011; 84(3): 509–514, https://doi.org/10.1007/s10891-011-0498-4.
7. Hou J., Liu X., Zhu H., Ma Z., Tang Z., Yu Y., Jin J., Wang W. Design and Motion Process of Air-Sieve Castor Cleaning Device Based on Discrete Element Method. Agriculture 2023; 13: 1130, https://doi.org/10.3390/agriculture13061130.
8. Jesny S., Prasobh G.R. A Review on Size Separation International Journal of Pharmaceutical Research and Applications 2022; 7(2): 286-296, https://doi.org/10.35629/7781-0702286296.
9. Tishchenko L., Kharchenko S., Kharchenko F., Bredykhin V., Tsurkan O. Identification of a mixture of grain particle velocity through the holes of the vibrating sieves grain separators. Eastern-European Journal of Enterprise Technologies 2016; 2(7(80)): 63–69, https://doi.org/10.15587/1729-4061.2016.65920.
10. Li Z., Tong X. A study of particles penetration in sieving process on a linear vibration screen. Int. J. Coal. Sci. Technol. 2015; 2: 299–305, doi.org/10.1007/s40789-015-0089-7.
11. Wang Z., Peng L., Zhang Ch., Qi L., Liu Ch., Zhao Yu. Research on impact characteristics of screening coals on vibrating screen based on discrete-finite element method. Energy Sources 2020; Part A: Recovery, Utilization, and Environmental Effects, 42; 16: 1963–1976, https://doi.org/10.1080/15567036.2019.1604905.
12. Lenggana B.W., Prabowo A.R., Ubaidillah U., Imaduddin F., Surojo E., Nubli H., Adiputra R. Effects of mechanical vibration on designed steel-based plate geometries: behavioral estimation subjected to applied material classes using finite-element method. Curvedand Layer. Struct. 2021; 8: 225–240, https://doi.org/10.1515/cls-2021-0021.
13. Leissa A.W.. Literature Review: Survey and Analysis of the Shock and Vibration Literature: Recent Studies in Plate Vibrations: 1981-85 Part I. Classical Theory. The Shock and Vibration Digest 1987; 19: 11-18. https://doi.org/10.1177/058310248701900204.
14. Nkounhawa P., Ndapeu D., Kenmeugne B., Beda T. Analysis of the Behavior of a Square Plate in Free Vibration by FEM in Ansys. World Journal of Mechanics 2020; 10: 11–25, https://doi.org/10.4236/wjm.2020.102002
15. Kharchenko S., Kharchenko F., Samborski S., Paśnik J., Kovalyshyn S., Sirovitskiy K. Influence of Physical and Constructive Parameters on Durability of Sieves of Grain Cleaning Machines. Advances in Science and Technology Research Journal 2022; 16(6): 156–165. https://doi.org/10.12913/22998624/156128.
16. Kalita, K., Haldar, S. Free Vibration Analysis of Rectangular Plates with Central Cutout. Cogent Engineering 2016; 3, 1163781. https://doi.org/10.1080/23311916.2016.1163781,
17. Israr A. Vibration analysis of cracked aluminium plates. PhD thesis, University of Glasgow 2008, 181 p.
18. Kharchenko S., Kovalyshyn S., Zavgorodniy A., Kharchenko F., Mikhaylov Y. Effective sifting of flat seeds through sieve. INMATEH-Agricultural Engineering 2019; 58(2): 17–26. https://doi.org/10.35633/INMATEH-58-02.
19. Kharchenko S. Intensification of grain sifting on flat sieves of vibration grain separators: monograph. Kharkiv: Dissa Plus, 2017, 217.
20. Gharaibeh M.A., Obeidat A.M. Vibrations Analysis of Rectangular Plates with Clamped Corners. Open Engineering 2018; 8(1): 275–283.
21. Senjanović I., Tomic M., Vladimir N., Hadžić N. An Analytical Solution to Free Rectangular Plate Natural Vibrations by Beam Modes – Ordinary and Missing Plate Modes. Transactions of FAMENA 2016; 40: 1–18. https://doi.org/10.21278/TOF.40301.
22. Rzeczkowski J., Samborski S. Experimental and Numerical Research of Delamination Process in CFRP Laminates with Bending-Twisting Elastic Couplings. Materials 2022; 15: 7745. https://doi.org/10.3390/ma15217745.
23. Kemparaju H, Samal P. Experimental Investigations on Free Vibration of Plates. Journal of Testing and Evaluation 2019; 47(4): 2750–64.
24. Jayavardhan H.K., Samal P.K. Effect of Shape of Cut-out on Natural Frequency of Square Plate. IOP Conf. Series: Materials Science and Engineering 2021; 1189 (2021): 012028. https://doi.org/10.1088/1757-899X/1189/1/012028.

25. Pavan Kishore M.L., Rajesh C.H., Komawar R. Free vibrational characteristics determination of plates with various cutouts. Vibroengineering Procedia 2019; 22(4): 2345–0533, https://doi.org/10.21595/vp.2018.20286.
26. Merneedi A., RaoNalluri M., Rao V.V.S. Free vibration analysis of a thin rectangular plate with multiple circular and rectangular cut-outs. Journal of Mechanical Science and Technology 2017; 31(11): 5185–5202.
27. Torabi K,. Azadi A.R. Vibration Analysis for Rectangular Plate Having a Circular Central Hole with Point Support by Rayleigh-Ritz Method. Journal of Solid Mechanics 2014; 6(1): 28-42.
28. Samborski S, Wieczorkiewicz J., Rusinek R., Dziedzic J. Methodology of structures damage estimation in case of cantilever isotropic beam. Journal of technology and exploitation in mechanical engineering 2015; 1: 1-2.
29. Kharchenko, S.; Samborski, S.; Kharchenko, F.; Mitura, A.; Paśnik, J.; Korzec, I. Identification of the Natural Frequencies of Oscillations of Perforated Vibrosurfaces with Holes of Complex Geometry. Materials 2023, 16, 5735. https://doi.org/10.3390/ma16175735.
30. Kharchenko S., Samborski S., Kharchenko F. Intensification of technological processes of equipment for post-harvest processing of grain. Study of reliability of sieves with complex shapes of holes. 1 st Workshop on Experimental and Computational Mechanics "WECM'22" + DIACMEC Lublin, Poland, June 1st 2022, 12.
31. Kharchenko S., Sumborski S.. Reliability studies of sieves with holes of complex geometric shape. Multidisciplinary conference for young researchers Sustainable Development in Wartime Ukraine and the World 25.11.2022 (Prague, Czech Republic), 20–21.
32. Ajay C., Muthuveerappan A., Gopalakrishnan V., et al. Vibration Analysis of Fully Perforated Rectangular Plates with Circular Perforations. SAE Technical Paper 2021-28-0195, 2021, https://doi.org/10.4271/2021-28-0195.
33. Jhung M.J., Jeong K.H. Free vibration analysis of perforated plate with square penetration pattern using equivalent material properties, Nuclear Engineering and Technology 2015; 47(4): 500–511, https://doi.org/10.1016/j.net.2015.01.012.
34. Xue C., Zhu B., Liu Xi, Nekomoto Y., Liu J., Liu D. Random vibration characteristics of perforated plates in parallel flow IOP Conf. Series: Earth and Environmental Science240 2019; 062005, IOP Publishing, https://doi.org/10.1088/1755-1315/240/6/062005.
35. Kowal M., Pietras D. Carbon Fibre Reinforced Polymer Fatigue Strengthening of Old Steel Material. Advances in Science and Technology Research Journal 2023; 17(1): 197–209. https://doi.org/10.12913/22998624/156216.
36. Cho D.S., Vladimir N., Choi T.M. Approximate natural vibration analysis of rectangular plates with openings using assumed mode method. International Journal of Naval Architecture and Ocean Eng. 2013; 5(3): 478–491. https://doi.org/10.2478/IJNAOE-2013-014.
37. Senjanović I., Vladimir N., Cho D.S., Choi T.M. Vibration Analysis of Thick Plates: Analytical and Numerical Approaches. Proceedings of the International Conference on Offshore Mechanics and Arctic Engineering – OMAE 2014. 4. https://doi.org/10.1115/OMAE2014-23273.
38. Senjanović I., Vladimir N., Tomic M. An advanced theory of moderately thick plate vibrations. Journal of Sound and Vibration 2013; 332: 1868–1880. https://doi.org/10.1016/j.jsv.2012.11.022.
39. Sadek K., Lueke J., Moussa W. A Coupled Field Multiphysics Modeling Approach to Investigate RF MEMS Switch Failure Modes under Various Operational Conditions. Sensors 2009; 9: 7988–8006. https://doi.org/10.3390/s91007988.