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Integrated impact model on interconnected critical infrastructures network cascading related to its operation process

Keywords

Impact model, operation process, cascading, critical infrastructure.

Abstract

In this paper we consider dependent Critical Infrastructures (CI) at variable operation conditions and present multistate approach to their safety analysis. The safety of dependent Critical Infrastructures at variable operation conditions is described. The local, equal and mixed load sharing models of dependency subjected to the operation process are introduced. The conditional safety functions at the particular operation states and the unconditional safety functions of a multistate series network with dependent assets, with dependent subnetworks and with dependent assets of its subnetworks according to LLS, ELS and MLS rules are determined. Furthermore, the all models of dependency are applied to safety analysis of a multistate parallel and "*m* out of *n*" network with dependent assets, parallel-series and "*m* out of *l*"-series network with dependent assets of their subnetworks at variable operation conditions.

1. Introduction

The paper is devoted to safety analysis of multistate critical infrastructure networks taking into account their operation processes and dependencies between their subnetworks and assets. The multistate approach to modelling safety of dependent critical infrastructure networks is linked with their operation processes' models, considering variable at the different operation states their safety structures, their assets' safety parameters and dependencies. These joint models of the critical infrastructure safety are constructed for multistate series, parallel, "*m* out of *n*", series-parallel, series-"*m* out of *k*", parallel-series and "*m* out of *l*"-series networks. The conditional safety functions at the particular operation states and the unconditional safety functions of such multistate critical infrastructure networks are determined at variable in time operation conditions for different models of dependency.

In this paper we consider dependent Critical Infrastructures (CI) at variable operation conditions and present multistate approach to their safety analysis. Describing cascading effects in CI networks both the dependencies between subnetworks of this network and between their assets are considered. Then, after changing the safety state subset by some of CI assets to the worse safety state subset, the lifetimes of remaining assets in the safety state subsets decrease. Models of dependency and behaviour of assets can differ depending on the structural and material properties of the CI network and its assets. Taking into account operating environment of critical infrastructures that can have significant influence on the CI safety and CI safety structure we assume that the changes of the CI operation states can also affect the model of dependency between CI assets.

We consider the local load sharing (LLS) model of dependency, in which after departure from the safety state subset by one of CI assets the safety parameters of remaining assets are changing dependently of the coefficients of the network load growth. These coefficients are concerned with the distance from the asset that has got out of the safety state subset and can be defined differently in various operation condions. In the equal load sharing (ELS) model of dependency, after changing the safety state subset by some of CI assets to the worse safety state subset, the lifetimes of remaining assets in this subnetwork in the safety state subsets decrease equally depending, inter alia, on the number of these assets that have left the safety state subset. In CI networks with more complex structures we consider a mixed load sharing (MLS) model subjected to operation process. Then, we take into account the dependencies between subnetworks of CI network and dependencies between assets of these subnetworks. All these dependency models are described in Section 3 of this report Part1 and [EU-CIRCLE Report D3.3-GMU6, 2016].

In the paper the conditional safety functions at the particular operation states and the unconditional safety functions of a multistate series network with dependent assets, with dependent subnetworks and with dependent assets of its subnetworks according to LLS rule are determined. Further, ELS model of dependency is applied to safety analysis of a multistate parallel and "*m* out of *n*" network with dependent assets, parallel-series and "*m* out of *l*"-series network with dependent assets of their subnetworks at variable operation conditions. Finally, the mixed load sharing model of dependency is considered for a multistate parallel-series and "*m* out of *l*"-series network with dependent subnetworks and dependent assets of these subnetworks related to their operation processes. Basic safety characteristics of these CI networks related to their varying in time safety structures and their assets safety parameters can be determined using definitions presented in Section 2 and in reports [EU-CIRCLE Report D2.1-GMU2, 2016], [EU-CIRCLE Report D3.3-GMU3, 2016].

2. Safety of dependent critical infrastructures at variable operation Ccnditions

Safety of critical infrastructures (CI) is concerned with their safety structure, safety of particular assets, their age and different models of dependencies occurring between assets. The safety of CI and CI networks may be also affected by many other factors, such as operating condition, actual demand on CI, weather condition, environment and its threats. Considering operation condition of CI we can distinguish and define CI operation states.

Further we assume, as in Section 2, that a critical infrastructure during its operation at the fixed moment t, $t \in <0,+\infty)$, may be at one of ν, $\nu \in N$, different operations states z_b, $b = 1,2,...,\nu$. Next, we mark by $Z(t)$, $t \in <0,+\infty)$, the critical infrastructure operation process, that is a function of a continuous variable t, taking discrete values in the set $\{z_1, z_2, ..., z_\nu\}$ of the critical infrastructure operation states. These states may be concerned for example with actual load and demand. Then, CI operation condition can have influence on model of failure dependency between assets and subnetworks of CI network. More generally in multistate CI networks, operation states can influence on lifetimes of CI assets in the safety state subsets and the way they decrease.

3. Local load sharing model of dependency subjected to operation process

3.1. Approach description

In the local load sharing (LLS) model of dependency for a multistate CI series network, described in [EU-CIRCLE Report D3.3-GMU6, 2016] and in Section 3 of this report Part1, the coefficients of the network load growth may be defined differently in various operation conditions. In LLS rule, we assume that after the departure of asset E_j, $j = 1,\dots,n$, in the network from the safety state subset $\{u,u+1,\dots,z\}$, $u = 1,2,\dots,z$, the lifetimes of remaining assets E_i, $i = 1,\dots,n$, $i \neq j$, in the safety state subsets decrease dependently on the coefficients of the network load growth concerned with the distance from the asset E_j. For example, if we denote the load on CI at operation state z_b, $b = 1,2,...,\nu$, by L_b, and maximal load on CI by L_{MAX}, the coefficients of the network load growth in LLS rule can be determined from following formula

$$q^{(b)}(\upsilon, d_{ij}) = [1 - \frac{L_b}{L_{MAX}}] \cdot q(\upsilon, d_{ij}), i = 1,\dots,n,$$
$$j = 1,\dots,n,\ \upsilon = u, u-1,\dots,1,\ b = 1,2,...,\nu, \qquad (1)$$

where $L_b \le L_{MAX}$, $b = 1,2,...,\nu$, and the coefficients of the network load growth $q(\upsilon, d_{ij})$, $0 < q(\upsilon, d_{ij}) \le 1$, $i = 1,\dots,n$, $j = 1,\dots,n$, and $q(\upsilon,0) = 1$ for $\upsilon = u,u-1,\dots,1$, $u = 1,2,\dots,z-1$, are non-increasing functions of assets' distance $d_{ij} = |i-j|$ from the asset that has got out of the safety state subset $\{u,u+1,\dots,z\}$, $u = 1,2,\dots,z$. The distance between network assets can be interpreted in the metric sense as well as in the sense of relationships in the functioning of the network assets. We denote by $E[T_i^{(b)}(u)]$ and $E[T_{i/j}^{(b)}(u)]$, $i = 1,2,...,n$, $j = 1,2,...,n$, $u = 1,2,\dots,z$, the mean values of assets' conditional lifetimes $T_i^{(b)}(u)$ and $T_{i/j}^{(b)}(u)$, respectively, before and after departure of one fixed asset E_j, j = 1,…,n, from the safety state subset {u,u+1,…,z}, u = 1,2,…,z, at operation state z_b, $b = 1,2,...,\nu$. With this notation, in considered local load sharing rule, the mean values of assets conditional lifetimes in the safety state subset {υ,υ+1,…,z}, υ = u,u-1,…,1, u = 1,2,…,z, at particular operation state z_b, $b = 1,2,...,\nu$, are decreasing according to the following formula:

$$T_{i/j}^{(b)}(\upsilon) = q^{(b)}(\upsilon, d_{ij}) \cdot T_i^{(b)}(\upsilon),$$
$$E[T_{i/j}^{(b)}(\upsilon)] = q^{(b)}(\upsilon, d_{ij}) \cdot E[T_i^{(b)}(\upsilon)],$$
$$i = 1,\ldots,n, j = 1,\ldots,n. \quad (2)$$

In different operational states not only different values of coefficients of the network load growth can be assumed, but in special cases the different models of dependency between assets and subnetworks can be adopted. We can also assume that in some CI operation states cascading effect can be observed, while in others no dependency between assets or subnetworks are assumed. Then, $q^{(b)}(\upsilon, d_{ij}) = 1$, for some operation states $b \in \{1,2,...,\nu\}$, and for the other coefficients are given by (1).

3.2. Safety of a multistate series network with dependent assets at variable operation conditions

We define the conditional safety function of a asset E_i, i = 1,…,n, after departure of the asset E_j, j = 1,2,…,n, from the safety state subset {u,u+1,…,z}, u = 1,2,…,z, assuming that network is at operation state z_b, $b = 1,2,...,\nu$,

$$[S_{i/j}(t,\cdot)]^{(b)} = [1, [S_{i/j}(t,1)]^{(b)}, \ldots, [S_{i/j}(t,z)]^{(b)}],$$
$$t \geq 0,\ i = 1,\ldots,n, j = 1,\ldots,n,$$
$$b = 1,2,...,\nu, \quad (3)$$

with the coordinates given by

$$[S_{i/j}(t,\upsilon)]^{(b)} = P(T_{i/j}^{(b)}(\upsilon) > t),\ t \geq 0,$$
$$\upsilon = u, u-1, \ldots, 1,\ u = 1,2,\ldots,z-1,$$

$$[S_{i/j}(t,\upsilon)]^{(b)} = P(T_{i/j}^{(b)}(\upsilon) > t)$$
$$= P(T_i^{(b)}(\upsilon) > t) = [S_i(t,\upsilon)]^{(b)},$$
$$\upsilon = u+1, \ldots, z,\ u = 1,2,\ldots,z-1. \quad (4)$$

Then, the conditional safety function of a multistate series network at operation state z_b, $b = 1,2,...,\nu$, with assets dependent according to LLS rule is given by the vector

$$[\boldsymbol{S}_{LLS}(t,\cdot)]^{(b)} = [1, [\boldsymbol{S}_{LLS}(t,1)]^{(b)}, \ldots, [\boldsymbol{S}_{LLS}(t,z)]^{(b)}],$$
$$t \geq 0, \quad (5)$$

with the coordinates,

$$[\boldsymbol{S}_{LLS}(t,u)]^{(b)} = \prod_{i=1}^{n}[S_i(t,u+1)]^{(b)}$$
$$+ \int_0^t \sum_{j=1}^{n} [[\widetilde{f}_j(a,u+1)]]^{(b)} \cdot \prod_{\substack{i=1 \\ i \neq j}}^{n} [S_i(a,u+1)]^{(b)}$$
$$\cdot [S_j(a,u)]^{(b)} \cdot \prod_{i=1}^{n} [S_{i/j}(t-a,u)]^{(b)}]da,$$
$$u = 1,2,\ldots,z-1, \quad (6)$$

$$[\boldsymbol{S}_{LLS}(t,z)]^{(b)} = \prod_{i=1}^{n}[S_i(t,z)]^{(b)}, \quad (7)$$

where:

$[S_i(t,u+1)]^{(b)}$ – the conditional safety function coordinate of a asset E_i, i = 1,…,n, at operation state z_b, $b = 1,2,...,\nu$,

$[\widetilde{f}_j(t,u+1)]^{(b)}$ – the conditional density function coordinate of a asset E_j, j = 1,…,n, at operation state z_b, $b = 1,2,...,\nu$, corresponding to the distribution function $[\widetilde{F}_j(t,u+1)]^{(b)}$, given by

$$[\widetilde{F}_j(t,u+1)]^{(b)} = 1 - \frac{[S_j(t,u+1)]^{(b)}}{[S_j(t,u)]^{(b)}},$$
$$u = 1,2,\ldots,z-1,\ t \geq 0, \quad (8)$$

$[S_j(t,u)]^{(b)}$ – the conditional safety function coordinate of a asset E_j, j = 1,…,n, at operation state z_b, $b = 1,2,...,\nu$,

$[S_{i/j}(t,u)]^{(b)}$ – the conditional safety function coordinate of a asset E_i, i = 1,…,n, at operation state z_b, $b = 1,2,...,\nu$, after departure from the safety state subset {u+1,…,z}, u = 1,2,…,z-1, by the asset E_j, j = 1,…,n, such that

$$[S_{i/j}(t-a,u)]^{(b)} = \frac{[S_{i/j}(t,u)]^{(b)}}{[S_i(a,u)]^{(b)}},$$
$$u = 1,2,\ldots,z-1,\ 0 < a < t,\ t \geq 0. \quad (9)$$

Next, the unconditional safety function of a multistate series network with assets dependent according to LLS rule is given by the vector

$$\boldsymbol{S}_{LLS}(t,\cdot) = [1, \boldsymbol{S}_{LLS}(t,1), \ldots, \boldsymbol{S}_{LLS}(t,z)],\ t \geq 0, \quad (10)$$

where its coordinates can be determined from following formula

$$\boldsymbol{S}_{LLS}(t,u) \cong \sum_{b=1}^{\nu} p_b [\boldsymbol{S}_{LLS}(t,u)]^{(b)} \text{ for } t \ge 0,$$
$$u = 1,2,...,z, \tag{11}$$

and $[\boldsymbol{S}_{LLS}(t,u)]^{(b)}$, $u = 1,2,...,z$, $b = 1,2,...,\nu$, are the coordinates of the network conditional safety functions defined by (6)-(7) and p_b, $b = 1,2,...,\nu$, are the CI network operation process limit transient probabilities given by (2.4) in Section 2.
Next, the mean value $\mu_{LLS}(u)$ and the variance $\sigma^2_{LLS}(u)$ of the unconditional lifetime in the safety state subset $\{u, u+1,...,z\}$ of a multistate series network with assets dependent according to LLS rule can be determined respectively from (2.20)-(2.21) and (2.22). Then, the mean values of the unconditional lifetimes of the network in particular safety states can be determined from (2.23).

3.3. Safety of a multistate exponential series network with dependent assets at variable operation conditions

Further, we assume that assets E_i, $i = 1,\dots,n$, of the network at the operation states z_b, $b = 1,2,...,\nu$, have the exponential safety functions given by

$$[S_i(t,\cdot)]^{(b)} = [1, [S_i(t,1)]^{(b)}, \dots, [S_i(t,z)]^{(b)}],$$
$$t \in <0,\infty), \; b = 1,2,...,\nu, \; i = 1,2,...,n, \tag{12}$$

with the coordinates

$$[S_i(t,u)]^{(b)} = P(T_i^{(b)}(u) > t \,|\, Z(t) = z_b)$$
$$= \exp[-[\lambda_i(u)]^{(b)} t], \; t \in <0,\infty), \; b = 1,2,...,\nu,$$
$$i = 1,2,...,n, \tag{13}$$

and the intensities of ageing of the network assets E_i, $i = 1,\dots,n$, (the intensities of their departure from the safety state subset $\{u,u+1,\dots,z\}$, $u = 1,2,\dots,z$,) related to operation impact, existing in 13), are given by

$$[\lambda_i(u)]^{(b)} = \rho_i^{(b)}(u) \cdot \lambda_i(u), \; u = 1,2,...,z,$$
$$b = 1,2,...,\nu, \; i = 1,2,...,n, \tag{14}$$

where $\lambda_i(u)$ are the intensities of ageing of assets E_i, $i = 1,\dots,n$, without operation impact and

$$[\rho_i(u)]^{(b)}, \; u = 1,2,...,z,$$
$$b = 1,2,...,\nu, \; i = 1,2,...,n, \tag{15}$$

are the coefficients of operation impact on the network assets intensities of ageing (the coefficients of operation impact on critical infrastructure asset E, $i = 1,2,...,n$, intensities of departure from the safety state subset $\{u,u+1,\dots,z\}$, $u = 1,2,\dots,z$,) without operation impact.

Then, according to the well known relationship between the lifetime mean value in this safety state subset and the intensity of departure from this safety state subset we get the formula for the intensities $[\lambda_{i/j}(\upsilon)]^{(b)}$, $i = 1,\dots,n$, $j = 1,\dots,n$, of assets' departure from the safety state subset $\{\upsilon,\upsilon+1,\dots,z\}$, $\upsilon = u,u-1,\dots,1$, $u = 1,2,\dots,z$, after the departure of the jth asset E_j, $j = 1,\dots,n$, from that safety state subset, at particular operation state z_b, $b = 1,2,...,\nu$. Namely, from (2), we obtain

$$[\lambda_{i/j}(\upsilon)]^{(b)} = \frac{[\lambda_i(\upsilon)]^{(b)}}{q^{(b)}(\upsilon, d_{ij})}, \; i = 1,\dots,n, j = 1,\dots,n,$$
$$\upsilon = u, u-1,\dots,1, \; b = 1,2,...,\nu. \tag{16}$$

And further, substituting (14) into (16), these intensities take form

$$[\lambda_{i/j}(\upsilon)]^{(b)} = \frac{\rho_i^{(b)}(\upsilon) \cdot \lambda_i(\upsilon)}{q^{(b)}(\upsilon, d_{ij})}, \; i = 1,\dots,n,$$
$$j = 1,\dots,n, \; \upsilon = u, u-1,\dots,1, \; b = 1,2,...,\nu, \tag{17}$$

where the coefficients of the network load growth $q^{(b)}(\upsilon, d_{ij})$ in LLS rule can be given for instance by (1).
Thus, considering (12)-(13) and (17), the assets E_i, $i = 1,\dots,n$, after the departure of the jth asset E_j, $j = 1,\dots,n$, from that safety state subset $\{u,u+1,\dots,z\}$, $u = 1,2,\dots,z$, have the conditional safety functions at operation state z_b, $b = 1,2,...,\nu$, given by (3) with the coordinates

$$[S_{i/j}(t,\upsilon)]^{(b)} = \exp[-\frac{\rho_i^{(b)}(\upsilon) \cdot \lambda_i(\upsilon)}{q^{(b)}(\upsilon, d_{ij})} t],$$
$$\upsilon = u, u-1,\dots,1, \; u = 1,2,\dots,z-1, \tag{18}$$

$$[S_{i/j}(t,\upsilon)]^{(b)} = \exp[-\rho_i^{(b)}(\upsilon) \cdot \lambda_i(\upsilon) t],$$
$$i = 1,\dots,n, j = 1,\dots,n,$$
$$\upsilon = u+1,\dots,z, \; u = 1,2,\dots,z-1. \tag{19}$$

Further for the exponential multistate series CI network with dependent assets, the distribution function corresponding to the CI network asset E_j, given by (8), takes form

$$[\tilde{F}_j(t,u+1)]^{(b)} = 1 - \frac{\exp[-[\lambda_j(u+1)]^{(b)}t]}{\exp[-[\lambda_j(u)]^{(b)}t]} = 1$$
$$-\exp[-(\rho_j^{(b)}(u+1)\cdot\lambda_j(u+1) - \rho_j^{(b)}(u)\cdot\lambda_j(u))t], \quad (20)$$

and its corresponding density function is

$$[\tilde{f}_j(t,u+1)]^{(b)} = (\rho_j^{(b)}(u+1)\cdot\lambda_j(u+1)$$
$$-\rho_j^{(b)}(u)\cdot\lambda_j(u))$$
$$\cdot\exp[-(\rho_j^{(b)}(u+1)\cdot\lambda_j(u+1) - \rho_j^{(b)}(u)\cdot\lambda_j(u))t], \quad (21)$$

for $u = 1,2,\ldots,z-1,\ b = 1,2,\ldots,\nu$.

Considering (18)-(21), in case the assets have exponential safety functions (12)-(13), from (5)-(7) we can obtain the conditional safety function of a multistate series network with assets dependent according to the local load sharing rule

$$[\boldsymbol{S}_{LLS}(t,\cdot)]^{(b)} = [1,[\boldsymbol{S}_{LLS}(t,1)]^{(b)},\ldots,[\boldsymbol{S}_{LLS}(t,z)]^{(b)}],$$
$$t \geq 0, \quad (22)$$

where

$$[\boldsymbol{S}_{LLS}(t,u)]^{(b)} = \exp[-\sum_{i=1}^{n}\rho_i^{(b)}(u+1)\cdot\lambda_i(u+1)t]$$
$$+\sum_{j=1}^{n}\frac{\rho_j^{(b)}(u+1)\cdot\lambda_j(u+1) - \rho_j^{(b)}(u)\cdot\lambda_j(u)}{\sum_{i=1}^{n}\rho_i^{(b)}(u+1)\cdot\lambda_i(u+1) - \sum_{i=1}^{n}\rho_i^{(b)}(u)\cdot\lambda_i(u)}$$
$$\cdot[\exp[-\sum_{i=1}^{n}\frac{\rho_i^{(b)}(u)\cdot\lambda_i(u)}{q(u,d_{ij})}t]$$
$$-\exp[-(\sum_{i=1}^{n}\rho_i^{(b)}(u+1)\cdot\lambda_i(u+1) - \sum_{i=1}^{n}\rho_i^{(b)}(u)\cdot\lambda_i(u)$$
$$+\sum_{i=1}^{n}\frac{\rho_i^{(b)}(u)\lambda_i(u)}{q^{(b)}(u,d_{ij})})t]],\ u = 1,2,\ldots,z-1, \quad (23)$$

$$[\boldsymbol{S}_{LLS}(t,z)]^{(b)} = \exp[-\sum_{i=1}^{n}\rho_i^{(b)}(z)\cdot\lambda_i(z)t]. \quad (24)$$

Next, applying (10)-(11), we can determine the unconditional safety function of a multistate series network with assets dependent according to LLS rule, where the coordinates of the network conditional safety functions are given by (22)-(24).

3.4. Safety of a multistate series network with dependent subnetworks at variable operation conditions

If we consider a series network composed of k subnetworks, and assume the local load sharing model of dependency between subnetworks described in [EU-CIRCLE Report D3.3-GMU6, 2016] and in Section 3 of this report Part1, then after departure from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, by the subnetwork N_g, $g = 1,2,\ldots,k$, the safety parameters of assets of remaining subnetworks are changing dependently of the coefficients of the network load growth concerned with the distance from the subnetwork that has got out of the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$.

We assume that in the i-th subnetwork N_i, $i = 1,2,...,k$, there are l_i assets, denoted by E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l_i$ with exponential safety functions of the form

$$[S_{ij}(t,\cdot)]^{(b)} = [1,[S_{ij}(t,1)]^{(b)},\ldots,[S_{ij}(t,z)]^{(b)}],$$
$$t \in <0,\infty), \quad (25)$$

with the coordinates

$$[S_{ij}(t,u)]^{(b)} = \exp[-[\lambda_{ij}(u)]^{(b)}t],\ b = 1,2,\ldots,\nu,$$
$$i = 1,2,...,k, j = 1,2,...,l_i,\ u = 1,2,\ldots,z, \quad (26)$$

and the intensities of ageing of the network assets E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l_i$, (the intensities of theirdeparture from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$,) related to operation impact, existing in (26), are given by

$$[\lambda_{ij}(u)]^{(b)} = \rho_{ij}^{(b)}(u)\cdot\lambda_{ij}(u),\ u = 1,2,\ldots,z,$$
$$b = 1,2,\ldots,\nu,\ i = 1,2,...,k, j = 1,2,...,l_i, \quad (27)$$

where $\lambda_{ij}(u)$ are the intensities of ageing of the assets E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l_i$, without operation impact and

$$[\rho_{ij}(u)]^{(b)},\ u = 1,2,\ldots,z,\ b = 1,2,\ldots,\nu,$$
$$i = 1,2,...,k, j = 1,2,...,l_i, \quad (28)$$

are the coefficients of operation impact on the assets E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l_i$, intensities of ageing (the coefficients of operation impact on critical infrastructure asset E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l_i$, intensities of departure from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$,) without climate-weather change impact.

We denote by $E[T_{i,j}^{(b)}(u)]$ and $E[T_{i/g,j}^{(b)}(u)]$, $i = 1,2,...,k$, $g = 1,2,...,k$, $j = 1,2,...,l_i$, $u = 1,2,\ldots,z$, the mean values of the lifetimes of ith subnetwork assets $T_{i,j}^{(b)}(u)$ and $T_{i/g,j}^{(b)}(u)$, respectively, before and after departure of one fixed subnetwork S_g, $g = 1,\ldots,k$, from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at operation state z_b, $b = 1,2,\ldots,\nu$. With this notation, in LLS model used between subnetworks, the mean values of their assets lifetimes in the safety state subset $\{\upsilon,\upsilon+1,\ldots,z\}$, $\upsilon = u,u-1,\ldots,1$, $u = 1,2,\ldots,z$, at particular operation state z_b, $b=1,2,\ldots,\nu$, are decreasing according to the following formula:

$$E[T^{(b)}_{i/g,j}(\upsilon)] = q^{(b)}(\upsilon,d_{ig})\cdot E[T^{(b)}_{i,j}(\upsilon)],\ i = 1,2,\ldots,k,$$
$$g = 1,2,\ldots,k,\ j = 1,2,\ldots,l_i,\ \upsilon = u,u-1,\ldots,1, \quad (29)$$

where the coefficients of the network load growth $q^{(b)}(\upsilon,d_{ig})$ can differ at particular operation state z_b, $b=1,2,\ldots,\nu$.

Then, the intensities of departure from the safety state subset $\{\upsilon,\upsilon+1,\ldots,z\}$, $\upsilon = u,u-1,\ldots,1$, $u = 1,2,\ldots,z$, of ith subnetwork assets after the departure of the subnetwork S_g, $g = 1,\ldots,k$, from (27) and (29), are given by

$$[\lambda_{i/g,j}(\upsilon)]^{(b)} = \frac{[\lambda_{ij}(\upsilon)]^{(b)}}{q^{(b)}(\upsilon,d_{ig})} = \frac{\rho^{(b)}_{ij}(\upsilon)\cdot\lambda_{ij}(\upsilon)}{q^{(b)}(\upsilon,d_{ig})},$$
$$i = 1,2,\ldots,k,\ g = 1,2,\ldots,k,\ j = 1,2,\ldots,l_i,$$
$$\upsilon = u,u-1,\ldots,1. \quad (30)$$

Similarly as in Section 3.3 using results presented in Section 2, we can determine the conditional and unconditional safety function of a multistate series network composed of k dependent according to LLS rule subnetworks and other safety characteristics.

3.5. Safety of a multistate series network with dependent assets of its subnetworks at variable operation conditions

Next, we consider a series network composed of k subnetworks with assets dependent according to LLS rule. We assume that in the i-th series subnetwork N_i, $i = 1,2,\ldots,k$, there are l_i dependent assets denoted by E_{ij}, $i = 1,2,\ldots,k$, $j = 1,2,\ldots,l_i$. Then, $T^{(b)}_{ij}(u)$, $i = 1,2,\ldots,k$, $j = 1,2,\ldots,l$, k, $l \in N$, are random variables representing lifetimes of assets E_{ij} in the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, at particular operation state z_b, $b=1,2,\ldots,\nu$. We denote by $E[T^{(b)}_{i,j}(u)]$ and $E[T^{(b)}_{i/j,g_i}(u)]$, $i = 1,2,\ldots,k$, $j = 1,2,\ldots,l_i$, $g_i = 1,2,\ldots,l_i$, $u = 1,2,\ldots,z$, the mean values of assets' lifetimes respectively, before and after departure of one fixed asset E_{ig_i}, $g_i = 1,2,\ldots,l_i$, from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, in the i-th subnetwork N_i, $i = 1,2,\ldots,k$, at operation state z_b, $b=1,2,\ldots,\nu$. We assume that the safety parameters of remaining assets E_{ij}, $j = 1,2,\ldots,l_i$, $j \neq g_i$, in this subnetwork are changing dependently of the distance from the asset E_{ig_i}. Then, the mean values of these assets lifetimes in the safety state subset $\{\upsilon,\upsilon+1,\ldots,z\}$, $\upsilon = u,u-1,\ldots,1$, $u = 1,2,\ldots,z$, are decreasing according to the following formula:

$$E[T^{(b)}_{i,j/g_i}(\upsilon)] = q^{(b)}(\upsilon,d_{jg_i})\cdot E[T^{(b)}_{ij}(\upsilon)],\ i = 1,2,\ldots,k,$$
$$j = 1,2,\ldots,l_i,\ g_i = 1,2,\ldots,l_i,\ \upsilon = u,u-1,\ldots,1, \quad (31)$$

where the coefficients of the network load growth $q^{(b)}(\upsilon,d_{jg_i})$ can differ at particular operation state z_b, $b=1,2,\ldots,\nu$.

If assets have exponential safety functions given by (25)-(26), then after departure of the asset E_{ig_i}, $g_i = 1,2,\ldots,l_i$, from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, in the i-th subnetwork N_i, $i = 1,2,\ldots,k$, the intensities $[\lambda_{i,j/g_i}(\upsilon)]^{(b)}$, $i = 1,2,\ldots,k$, $j = 1,2,\ldots,l_i$, $g_i = 1,2,\ldots,l_i$, of departure from the safety state subset $\{\upsilon,\upsilon+1,\ldots,z\}$, $\upsilon = u,u-1,\ldots,1$, $u = 1,2,\ldots,z$, of remaining assets E_{ij}, $j = 1,2,\ldots,l_i$, $j \neq g_i$, in this subnetwork at operation state z_b, $b=1,2,\ldots,\nu$, from (27) and (31), are given by

$$[\lambda_{i,j/g_i}(\upsilon)]^{(b)} = \frac{[\lambda_{ij}(\upsilon)]^{(b)}}{q^{(b)}(\upsilon,d_{jg_i})} = \frac{\rho^{(b)}_{ij}(\upsilon)\cdot\lambda_{ij}(\upsilon)}{q^{(b)}(\upsilon,d_{jg_i})},$$
$$i = 1,2,\ldots,k,\ j = 1,2,\ldots,l_i,\ g_i = 1,2,\ldots,l_i,$$
$$\upsilon = u,u-1,\ldots,1. \quad (32)$$

Similary as in Section 3.3 using results presented in Section 2, we can determine the conditional and unconditional safety function of a multistate series network composed of k subnetworks with dependent according to LLS rule assets and other safety characteristics.

We can also determine, the conditional and unconditional safety functions for a multistate series-parallel and series-"m out of k" network with dependent according to LLS rule assets of its subnetworks.

4. Equal load sharing model of dependency subjected to operation process

4.1. Approach description

In equal load sharing model (ELS) of dependency for a parallel network composed of subnetworks/assets, described in [EU-CIRCLE Report D3.3-GMU6, 2016] and in Section 3 of this report Part1, we assume that after decreasing the safety state by some of subnetworks/assets the increased load can be shared equally among the remaining subnetworks/assets. Considering CI network in changing operation conditions, we assume that asset stress proportionality correction coefficient can take different values at particular network operation states. Then, if $\omega, \omega = 0,1,2,\ldots,n-1$, subnetworks/assets are out of the safety state subset $\{u,u+1,...,z\}$, the mean values of the lifetimes in the safety state subset $\{u,u+1,...,z\}$, $u = 1,2,\ldots,z$, of the remaining subnetworks/assets at operation state z_b, $b = 1,2,...,\nu$, are decreasing according to the formula

$$E[T_i^{(b)}{}'(u)] = c^{(b)}(u)\frac{n-\omega}{n}E[T_i^{(b)}(u)],$$
$$i = 1,2,\ldots,n,\ u = 1,2,\ldots,z, \tag{33}$$

where $c^{(b)}(u)$ is the asset stress proportionality correction coefficient for each u, $u = 1,2,\ldots,z$, at particular network operation state z_b, $b = 1,2,...,\nu$.

Hence, for case of network with dependent subnetworks/assets having identical exponential safety functions at the operation states z_b, $b = 1,2,...,\nu$, of the form

$$[S(t,\cdot)]^{(b)} = [1,[S(t,1)]^{(b)},\ldots,[S(t,z)]^{(b)}],$$
$$t \in <0,\infty),\ b = 1,2,...,\nu,\ i = 1,2,...,n, \tag{34}$$

with the coordinates

$$[S(t,u)]^{(b)} = \exp[-[\lambda(u)]^{(b)}t],$$
$$t \ge 0,\ b = 1,2,...,\nu,\ i = 1,2,\ldots,n,\ u = 1,2,\ldots,z, \tag{35}$$

using (33) and (14), we get following formula for intensities of departure from the safety state subset $\{u,u+1,...,z\}$, of remaining subnetworks/assets

$$[\lambda^{(\omega)}(u)]^{(b)} = \frac{n}{n-\omega}\frac{[\lambda(u)]^{(b)}}{c^{(b)}(u)} = \frac{n}{n-\omega}\frac{\rho^{(b)}(u)\cdot\lambda(u)}{c^{(b)}(u)},$$
$$b = 1,2,...,\nu,\ \omega = 0,1,\ldots,n-1,\ u = 1,2,\ldots,z. \tag{36}$$

4.2. Safety of a multistate parallel network with dependent assets at variable operation conditions

The conditional safety function of a multistate parallel network at operation state z_b, $b = 1,2,...,\nu$, with assets dependent according to ELS rule and having identical exponential safety functions (34)-(35), is given by the vector

$$[\boldsymbol{S}_{ELS}(t,\cdot)]^{(b)} = [1,[\boldsymbol{S}_{ELS}(t,1)]^{(b)},\ldots,[\boldsymbol{S}_{ELS}(t,z)]^{(b)}],$$
$$t \ge 0, \tag{37}$$

with the coordinates

$$[\boldsymbol{S}_{ELS}(t,u)]^{(b)} = \sum_{\omega=0}^{n-1}\frac{\left[\frac{n\rho^{(b)}(u)\cdot\lambda(u)}{c^{(b)}(u)}t\right]^{\omega}}{\omega!}$$
$$\exp[-\frac{n\rho^{(b)}(u)\cdot\lambda(u)}{c^{(b)}(u)}t],$$
$$b = 1,2,...,\nu,\ u = 1,2,\ldots,z. \tag{38}$$

Next, the unconditional safety function of a multistate parallel network with assets dependent according to ELS rule is given by the vector

$$\boldsymbol{S}_{ELS}(t,\cdot) = [1,\boldsymbol{S}_{ELS}(t,1),\ldots,\boldsymbol{S}_{ELS}(t,z)],\ t \ge 0, \tag{39}$$

where its coordinates can be determined from following formula

$$\boldsymbol{S}_{ELS}(t,u) \cong \sum_{b=1}^{\nu} p_b[\boldsymbol{S}_{ELS}(t,u)]^{(b)}$$
$$\text{for } t \ge 0,\ u = 1,2,...,z, \tag{40}$$

and $[\boldsymbol{S}_{ELS}(t,u)]^{(b)}$, $u = 1,2,...,z$, $b = 1,2,...,\nu$, are the coordinates of the network conditional safety functions given by (37)-(38) and p_b, $b = 1,2,...,\nu$, are the CI network operation process limit transient probabilities given by (2.4) in Section 2.

Next, the mean value $\mu_{ELS}(u)$ and the variance $\sigma^2_{ELS}(u)$ of the unconditional lifetime in the safety state subset $\{u,u+1,...,z\}$ of a multistate parallel network with assets dependent according to ELS rule can be determined respectively from (2.20)-(2.21) and (2.22). Then, the mean values of the unconditional lifetimes of the network in particular safety states can be determined from (2.23).

4.3. Safety of a multistate "*m* out of *n*" network with dependent assets at variable operation conditions

Similary as in Section 4.2 we can determine conditional and unconditional safety functions and other safety characteristics of a multistate "*m* out of *n*" network with subnetworks/assets dependent according to the equal load sharing rule. If subnetworks/assets in a multistate "*m* out of *n*" network have identical exponential safety functions at the operation states z_b, $b = 1,2,...,\nu$, given by (34)-(35), then its conditional safety function is given by the vector

$$[\boldsymbol{S}_{ELS}(t,\cdot)]^{(b)} = [1,[\boldsymbol{S}_{ELS}(t,1)]^{(b)},\ldots,[\boldsymbol{S}_{ELS}(t,\cdot z]^{(b)}], \tag{41}$$

with the coordinates

$$[\boldsymbol{S}_{ELS}(t,u)]^{(b)} = \sum_{\omega=0}^{n-m} \frac{\left[\frac{n\rho^{(b)}(u)\cdot\lambda(u)}{c^{(b)}(u)}t\right]^{\omega}}{\omega!}$$

$$\exp[-\frac{n\rho^{(b)}(u)\cdot\lambda(u)}{c^{(b)}(u)}t],\ u = 1,2,\ldots,z. \quad (42)$$

Further, the unconditional safety function of a a multistate "*m* out of *n*" network with dependent subnetworks/assets can be determined from (39)-(40).

4.4. Safety of a parallel-series network with dependent assets of its subnetworks at variable operation conditions

In this section we consider a multistate parallel-series network composed of *k* parallel subnetworks and in *i*th subnetowork we assume there are l_i, $i = 1,2,...,k$, assets dependent according to the equal load sharing rule, described in Section 3 of this report Part1 and in [EU-CIRCLE Report D3.3-GMU6, 2016]. Then, we assume that after leaving the safety state subset by some of assets in a subnetwork, the lifetimes of remaining assets in this subnetwork decrease equally depending on the number of these assets that have left the safety state subset. Additionally these changes are influenced by the asset stress proportionality correction coefficient $c_i^{(b)}(u)$, $i = 1,2,...,k$, $u = 1,2,\ldots,z$, concerned with features of *i*th subnetowork and its assets, changing in different operation states z_b, $b = 1,2,...,\nu$. We denote by E_{ij}, $i = 1,2,...,k$, $j = 1,2,...,l_i$, assets of a network and assume that all assets E_{ij} have the same safety state set as before $\{0,1,...,z\}$. Then, $T_{ij}(u)$, $i = 1,2,...,k$, $j = 1,2,..., l_i$, are random variables representing lifetimes of assets E_{ij} in the safety state subset $\{u,u+1,...,z\}$, $u = 1,2,\ldots,z$.

We assume similarly as in formula (33) for a multistate parallel network that if $\omega_i, \omega_i = 0,1,2,\ldots,l_i - 1$, assets in *i*th parallel subnetwork $i = 1,2,...,k$, are out of the safety state subset $\{u,u+1,...,z\}$, $u = 1,2,\ldots,z$, the mean values of lifetimes in the safety state subset $\{u,u+1,...,z\}$ of this subnetwork remaining assets at the operation state z_b, $b = 1,2,...,\nu$, are given by

$$E[T_{ji}^{(b)}{}'(u)] = c_i^{(b)}(u)\frac{l_i-\omega_i}{l_i}E[T_{ij}^{(b)}(u)],$$
$$i = 1,2,\ldots,k,\ j = 1,2,\ldots,l_i,\ \omega_i = 0,1,2,\ldots,l_i - 1,$$
$$u = 1,2,\ldots,z. \quad (43)$$

We assume that in *i*-th parallel subnetwork N_i, $i = 1,2,...,k$, assets are dependent according to ELS rule and have identical exponential safety functions at the operation state z_b, $b = 1,2,...,\nu$, of the form

$$[S_i(t,\cdot)]^{(b)} = [1,[S_i(t,1)]^{(b)},\ldots,[S_i(t,z)]^{(b)}],$$
$$t \ge 0,\ b = 1,2,...,\nu,\ i = 1,2,\ldots,k, \quad (44)$$

where

$$S_i(t,u)]^{(b)} = \exp[-[\lambda_i(u)]^{(b)}t],\ t \ge 0,$$
$$b = 1,2,...,\nu,\ i = 1,2,\ldots,k,\ u = 1,2,\ldots,z. \quad (45)$$

Then, after the departure of $\omega_i, \omega_i = 0,1,2,\ldots,l_i - 1$, assets from this safety state subset in the *i*th subnetwork $i = 1,2,...,k$, we get following formula for the intensities of departure from this subset of remaining assets in the *i*th subnetowork at the operation state z_b, $b = 1,2,...,\nu$,

$$[\lambda_i^{(\omega)}(u)]^{(b)} = \frac{l_i}{l_i-\omega_i}\cdot\frac{[\lambda_i(u)]^{(b)}}{c_i^{(b)}(u)}$$
$$= \frac{l_i}{l_i-\omega_i}\cdot\frac{\rho_i^{(b)}(u)\cdot\lambda_i(u)}{c_i^{(b)}(u)},\ b = 1,2,...,\nu,\ i = 1,2,\ldots,k,$$
$$\omega_i = 0,1,2,\ldots,l_i - 1,\ u = 1,2,\ldots,z. \quad (46)$$

Using (46) and results presented in Section 3 of this report Part1, we can determine the conditional safety function of a multistate parallel-series network, composed of *k* parallel subnetworks N_i, $i = 1,2,\ldots,k,$ with assets dependent according to ELS rule and having exponential safety functions (44)-(45)

$$[\boldsymbol{S}_{ELS}(t,\cdot)]^{(b)} = [1,[\boldsymbol{S}_{ELS}(t,1)]^{(b)},\ldots,[\boldsymbol{S}_{ELS}(t,\cdot z]^{(b)}],$$
$$t \ge 0, \quad (47)$$

where

$$[\boldsymbol{S}_{ELS}(t,u)]^{(b)} = \prod_{i=1}^{k}[\sum_{\omega_i=0}^{l_i-1}\frac{[\frac{l_i\rho_i^{(b)}(u)\cdot\lambda_i(u)}{c_i^{(b)}(u)}t]^{\omega_i}}{\omega_i!}$$
$$\exp[-\frac{l_i\rho_i^{(b)}(u)\cdot\lambda_i(u)}{c_i^{(b)}(u)}t]]\ b = 1,2,...,\nu,$$
$$u = 1,2,\ldots,z. \quad (48)$$

Further, the unconditional safety function of a a multistate parallel-series network with dependent assets of its subnetworks can be determined from (39)-(40).

4.5. Safety of a "*m* out of *l*"-series network with dependent assets of its subnetworks at variable operation conditions

Similarly, we can also determine, the conditional and unconditional safety functions for a multistate "m_i out of l_i"-series network with dependent according to ELS rule assets of its subnetworks.

We consider a multistate "m_i out of l_i"-series network composed of k linked in series "m_i out of l_i", i = 1,2,...,k, subnetworks and we assume that assets in each "m_i out of l_i", i = 1,2,...,k, subnetwork are dependent according to ELS rule. We assume that if $\omega_i, \omega_i = 0,1,2,\ldots,l_i - m_i$, assets in ith "m_i out of l_i", i = 1,2,...,k, subnetwork are out of the safety state subset {u,u+1,...,z}, u = 1,2,…,z, the mean values of lifetimes in the safety state subset {u,u+1,...,z} of this subnetwork remaining assets at the operation state z_b, $b = 1,2,...,\nu$, are given by

$$E[T_{ji}^{(b)}{}'(u)] = c_i^{(b)}(u)\frac{l_i - \omega_i}{l_i} E[T_{ij}^{(b)}(u)],$$
$$i = 1,2,\ldots,k,\ j = 1,2,\ldots,l_i,$$
$$\omega_i = 0,1,2,\ldots,l_i - m_i,\ u = 1,2,\ldots,z. \quad (49)$$

We assume that in i-th subnetwork N_i, i = 1,2,...,k, assets are dependent according to ELS rule and have identical exponential safety functions at the operation state z_b, $b = 1,2,...,\nu$, given by (44)-(45). Then, the intensities of departure from the safety state subset {u,u+1,...,z}, u = 1,2,…,z, of remaining assets in the ith, i = 1,2,...,k, subnetwork at the operation state z_b, $b = 1,2,...,\nu$, are

$$[\lambda_i^{(\omega)}(u)]^{(b)} = \frac{l_i}{l_i - \omega_i} \cdot \frac{[\lambda_i(u)]^{(b)}}{c_i^{(b)}(u)}$$
$$= \frac{l_i}{l_i - \omega_i} \cdot \frac{\rho_i^{(b)}(u) \cdot \lambda_i(u)}{c_i^{(b)}(u)},\ b = 1,2,...,\nu,\ i = 1,2,\ldots,k,$$
$$\omega_i = 0,1,2,\ldots,l_i - m_i,\ u = 1,2,\ldots,z. \quad (50)$$

Using (50) and results presented in Section 3 of this report Part1, we can determine the conditional safety function of a multistate "m_i out of l_i"-series network, composed of k "m_i out of l_i" subnetworks N_i, $i = 1,2,\ldots,k$, with assets dependent according to ELS rule and having exponential safety functions (44)-(45) as the vector

$$[\boldsymbol{S}_{ELS}(t,\cdot)]^{(b)} = [1,[\boldsymbol{S}_{ELS}(t,1)]^{(b)},\ldots,[\boldsymbol{S}_{ELS}(t,\cdot z]^{(b)}],$$
$$t \geq 0, \quad (51)$$

with the coordinates

$$[\boldsymbol{S}_{ELS}(t,u)]^{(b)} = \prod_{i=1}^{k}[\sum_{\omega_i=0}^{l_i-m_i} \frac{[\frac{l_i\rho_i^{(b)}(u)\cdot\lambda_i(u)}{c_i^{(b)}(u)}t]^{\omega_i}}{\omega_i!}$$
$$\exp[-\frac{l_i\rho_i^{(b)}(u)\cdot\lambda_i(u)}{c_i^{(b)}(u)}t]],\ b = 1,2,...,\nu,$$
$$u = 1,2,\ldots,z. \quad (52)$$

Further, the unconditional safety function of a a multistate "m_i out of l_i"-series network with dependent assets of its subnetworks can be determined from (39)-(40).

5. Mixed load sharing model of dependency subjected to operation process

5.1. Approach description

Considering cascading effects in networks with more complex structures we can link the results of safety analysis for previously described dependency models. Then, apart from the dependency of subnetworks' departures from the safety states subsets we can take into account the dependencies between assets in subnetworks. This way we can proceed with parallel-series and "m out of l"-series networks assuming the dependence between their parallel, respectively "m out of l", subnetworks according to the local load sharing rule and the dependence between their assets in subnetworks according to the equal load sharing rule. Further, such model of dependency we will call a mixed load sharing (MLS) model.

5.2. Safety of a multistate parallel-series network with dependent subnetworks and dependent assets of these subnetworks at variable operation conditions

In this section, we consider a multistate parallel-series network composed of k parallel subnetworks N_i, i = 1,2,…,k, with mixed load sharing model of dependency between subnetworks and between assets in these subnetworks, described in Section 3 of this report Part1 and in [EU-CIRCLE Report D3.3-GMU6, 2016]. We denote the jth asset being in the ith subnetwork N_i, i = 1,2,…,k, by E_{ij}, i = 1,2,…,k, j = 1,2,…,l_i, and we assume that assets in the ith subnetwork have identical exponential safety functions, given by (44)-(45).

In the ith parallel subnetwork N_i, i = 1,2,…,k, we consider dependency of its l_i assets according to the equal load sharing model, presented in Section 10.4. Then, after departure from the safety state subset {u,u+1,…,z}, u = 1,2,…,z, by $\omega_i, \omega_i = 0,1,2,\ldots,l_i - 1$, assets of the subnetwork, the intensities of departure from this safety state subset of the remaining assets in the subnetwork at the operation state z_b, $b = 1,2,...,\nu$, are given by (10.43).

Further, between these subnetworks, linked in series, we assume the local load sharing model of dependency, presented in Section 3. Then, we assume that after departure from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, by the subnetwork N_g, $g = 1,2,\ldots,k$, the safety parameters of assets of remaining subnetworks are changing dependently of the distance from the subnetwork that has got out of the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, expressed by index d. In the local load sharing model used between subsnetworks, the mean values of their assets lifetimes in the safety state subset $\{\upsilon,\upsilon+1,\ldots,z\}$, $\upsilon = u,u-1,\ldots,1$, $u = 1,2,\ldots,z$, at particular operation state z_b, $b = 1,2,...,\nu$, are decreasing according to the formula (29).

Linking the results for these two dependency models we can determine the conditional and unconditional safety functions for a multistate parallel-series network with dependent according to ELS rule subnetworks and dependent according to LLS rule assets of these subnetworks.

5.3. Safety of a multistate "*m* out of *l*"-series network with dependent subnetworks and dependent assets of these subnetworks at variable operation conditions

Similary, we apply a mixed load sharing model of assets and subnetworks dependency to the safety analysis of a multistate "m_i out of l_i"-series network. We consider a multistate "m_i out of l_i"-series network composed of k "m_i out of l_i" subnetworks N_i, $i = 1,2,\ldots,k$, linked in series and dependent according to the local load sharing rule. We assume that assets in the ith subnetwork have identical exponential safety functions, given by (44)-(45).

In the ith "m_i out of l_i" subnetwork N_i we consider, similary as in previous section, dependency of its l_i assets according to the equal load sharing model, presented in Section 4. Then, after departure from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, by $\omega_i, \omega_i = 0,1,2,\ldots,l_i - m_i$, assets of the subnetwork, the intensities of departure from this safety state subset of the remaining assets in the subnetwork at the operation state z_b, $b = 1,2,...,\nu$, are given by (49).

Further, between these subnetworks, linked in series, we assume the local load sharing model of dependency, presented in Section 3. Then, we assume that after departure from the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$, by the subnetwork N_g, $g = 1,2,\ldots,k$, the safety parameters of assets of remaining subnetworks are changing dependently of the distance from the subnetwork that has got out of the safety state subset $\{u,u+1,\ldots,z\}$, $u = 1,2,\ldots,z$. The mean values of assets lifetimes of remaining subnetworks in the safety state subset $\{\upsilon,\upsilon+1,\ldots,z\}$, $\upsilon = u,u-1,\ldots,1$, $u = 1,2,\ldots,z$, at the operation state z_b, $b = 1,2,...,\nu$, are decreasing according to the formula (29).

Linking the results for these two dependency models we can determine the conditional and unconditional safety functions for a multistate "*m* out of *l*"-series network with dependent according to ELS rule subnetworks and dependent according to LLS rule assets of these subnetworks.

6. Conclusions

In this paper, linking the safety models for dependent critical infrastructures and their operation processes models, we analyse safety of CI networks at variable operation conditions taking into account dependencies and cascading effects occurring in them. The construction of the joint general safety model for dependent critical infrastructures related to their operation processes allowes to find the main an practically important safety characteristics of dependent critical infrastructures defined as complex technical systems at the variable operation conditions.

Most real critical infrastructures are strongly influenced by changing in time their operation conditions that initiate their degradation/ageing. Moreover, after changing the safety state subset by some of CI assets to the worse safety state subset, the lifetimes of remaining CI assets in the safety state subsets decrease. This way dependencies between CI assets may accelerate degradation of the critical infrastructure and have influence on the CI and CI network safety. Considering the network composed of multistate subnetworks, the influence of subnetworks' inside-dependences on their safety as well as the impact of subnetworks' degradation on other subnetworks safety have been analyzed. The operating environment of critical infrastructures can have also significant influence on the model of dependency between CI assets, that has been included in the report as well. For instance in local load sharing model of dependency, the coefficients of the network load growth can take different values at various operation states.

This approach, upon the sufficient accuracy of the critical infrastructures' operation processes and the critical infrastructures' assets safety parameters identification, makes their safety prediction much more precise than in the case of omitting their operation processes impacts. As a next step the operation and safety optimization for dependent critical infrastructures at the variable operation conditions will be proposed. More exacly, the method of the optimization of the critical infrastructures operation processes determining the optimal values of limit transient probabilities at the critical infrastructure operation states that maximize the critical infrastructure lifetimes in the safety state subsets will be proposed.

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