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The Beam Pattern PDF In Target Strength Estimation - Theoretical And Practical Approach

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ABSTRACT

The knowledge of beam pattern probability density function (PDF) plays a crucial role in target strength estimation when using indirect methods. The paper describes the theoretical and practical aspects of approximations applied in calculating the PDF of this deterministic function from a probabilistic point of view. The solution of differential equation presented in the paper leads to a unified approach to approximations and has a clear interpretation.

INTRODUCTION

The beam pattern $b(\theta,\varphi)$ function - as it is well known - is the deterministic function of angular variables of acoustic pressure or intensity in the far field. However, in the problem of target strength estimation, the position of the target (fish) is treated as a random variable and from the point of the transducer it represents the function of beam pattern. This apparent contradiction in association with the random variable (or function of the random variable) with deterministic function can be easily clarified if one considers that the value of the beam pattern function for each echo is a random variable (function) in question. That's why the value of the beam pattern function is a random variable of unknown angular variables θ and φ. Below the concept of beam pattern PDF in target strength estimation is presented.

FORMULATION OF THE PROBLEM

In the case of an indirect method of target strength estimation the beam pattern PDF plays a role of the kernel of integral equation representing a solution to the inverse problem. This problem is expressed as a single-target, single-beam integral equation and in terms of echo amplitude PDF, beam pattern and square root of back scattering cross-section can be presented as a:

$$p_u(u)=\int_0^1 \frac{1}{b} p_b(b) p_{\sqrt{\sigma_{BS}}}\left(\frac{u}{b}\right) db \qquad (1)$$

Alternatively in terms of echo level PDF, logarithmic beam pattern and target strength, the PDF of the echo level may be written as:

$$p_E(E)=\int_{-\infty}^{0} p_B(B) p_{TS}(E-B) dB \qquad (2)$$

It is very important to precisely evaluate the beam pattern PDF as inverse problems are particularly sensitive to any errors.

As a starting point of our consideration let's imagine an ideally omnidirectional transducer having

$$b(\theta,\varphi)=1$$

for all values of angles. It is intuitively easy to guess that PDF of this pattern represents the delta distribution function:

$$p_b(b)=\delta(b-1) \qquad (3)$$

or for logarithmic measure:

$$p_B(B)=\delta\ (B) \qquad (4)$$

as only one value of b is possible.

Substituting (3) and (4) to the so called Mellin convolution integral (1) and to convolution integral (2) respectively and remembering that the delta distribution function is invariant for the above convolutions, we obtain $p_u = p_{\sqrt{\sigma_{BS}}}$ and $p_E = p_{TS}$. In that case the beam pattern does not affect the measurement so one may say that we directly measure back scattering cross-section PDF or target strength PDF.

In some acoustic problems the conical beam pattern gives a better approximation of the actual beam pattern than the omnidirectional one. For each of these cases the ideal beam patterns are equal to 1 in their entire angular variable range. The cases in question are depicted in Figure 1 a) and b).

This specific property of ideal beam patterns implies that their PDFs take the form of the delta function as shown in Fig. 1 c). The same PDF in logarithmic variables is shown in Fig. 1d).

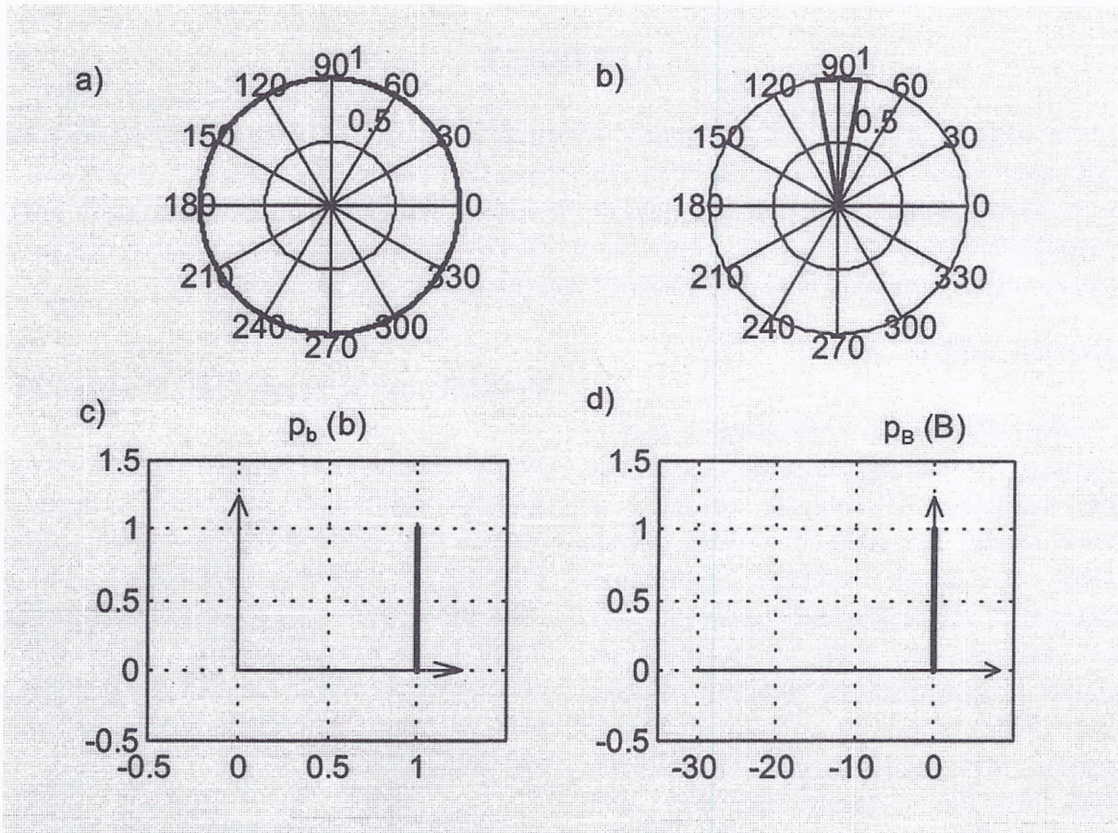

Figure 1. Ideall isotropic (a) and conical (b) beam patterns and their PDFs.

For a circular piston transducer in an infinite baffle the two-way (transmitting-receiving) beam pattern has the following form:

$$b(\theta)=\left(\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right)^2 \qquad (5)$$

To calculate the PDF function of such a beam pattern we need to know the PDF function of a random position of the target expressed as a function of angle θ. It can be shown that for uniform spatial distribution of targets the PDF of angular position of targets can be written as:

$$p_\theta(\theta) = \sin\theta \quad , \quad \theta \in \left(0, \frac{\pi}{2}\right) \qquad (6)$$

In practical cases, when the angular variable is limited to XXX (angular detection zone) this PDF is given by:

$$p_\theta(\theta) = \frac{1}{1-\cos\theta_{MAX}} \sin\theta \qquad (7)$$

$$\theta \in \left(0, \theta_{MAX}\right)$$

In equation (7) the maximum angle θ_{MAX} is related to the minimum value of the beam pattern b_{MIN} originated from the level of sidelobes. In such a case the PDF is expressed as a parametric function of angle θ:

$$p_b(b) = (b(\theta), p_b(\theta)) = \left(\left(\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right)^2 , \frac{\frac{1}{1-\cos\theta_{MAX}}\sin\theta}{\left|\frac{8J_1(ka\sin\theta)J_2(ka\sin\theta)}{(ka\sin\theta)^2} ka\cos\theta\right|} \right) \qquad (8)$$

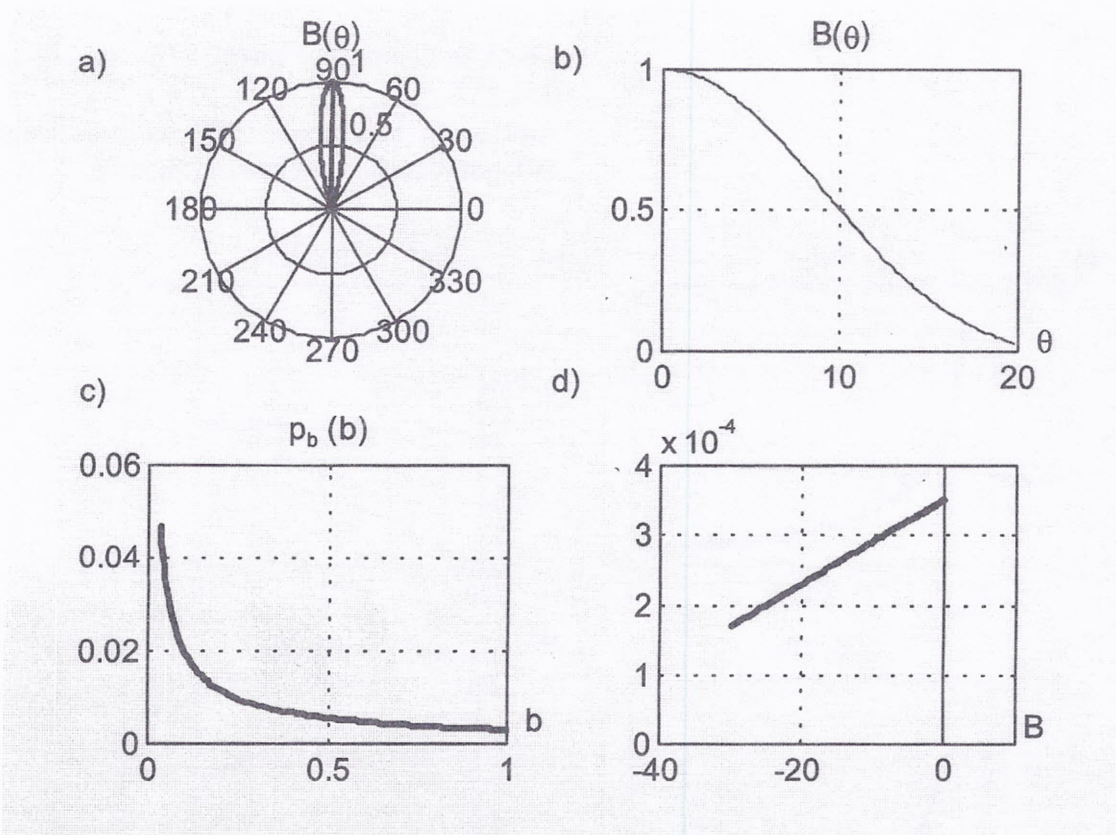

Figure 2. Two-way circular piston beam pattern and its PDFs.

The logarithmic version of the beam pattern PDF could be expressed as a function of a random variable $B = 20\log b$ which gives:

$$p_B(B(b)) = \frac{p_b(b)}{\left|\frac{dB}{db}\right|} = \frac{\ln 10}{20} p_b(b)\, b \qquad (9)$$

Ehrenberg derived a general expression for approximation of the above beam pattern PDF (8), as [1]:

$$p_b(b) = k_0 \frac{1}{b^{2C_0-1}} \ , \qquad (10)$$

$$C_o = 0.895 \ , \quad b \in (0.0175, 1)$$

and for the logarithmic version:

$$p_B(B) = \frac{\ln 10}{20} k_0 e^{\frac{\ln 10}{10}(1-C_0)B} \qquad (11)$$

$$B \in (-35,0)$$

It is clear that if C_0 is a constant , then k_0 should also be a constant value as PDF should integrate to 1. Thus PDF does not depend on beam width.

Another approximation of the main lobe presented by Lozow [2] derived from exponential approximation of Bessel function and a small angle approximation $\sin\theta = \theta$ gives the following:

$$p_b(b) = k_1 \frac{1}{b} \quad , \; k_1 = \frac{2}{\theta^2_{MAX}(ka)^2} \qquad (12)$$

$$b \in (e^{-\frac{1}{2}(ka\sin\theta)^2}, 1)$$

The approximations are quite similar at the beam pattern and beam pattern PDF but in logarithmic version of the beam pattern PDF the difference is significant!

Yet another approximation presented by Hedgepeth [3] based on two parametric exponential approximation of the beam pattern of a transducer used by him $b(\theta) = \exp(-\alpha\theta^\beta)$ gives

$$p_b(b) = \left| \frac{1}{\alpha\beta} \frac{1}{b} \left(\frac{\ln(b)}{\alpha} \right)^{\frac{1}{\beta}-1} \right| \sin\left(\frac{\ln(b)}{\alpha} \right)^{\frac{1}{\beta}} \qquad (13)$$

and for the logarithmic version

$$p_B(B) = \left| \frac{1}{\alpha\beta} \left(\frac{\ln 10}{20} B \right)^{\frac{1}{\beta}-1} \right| \sin\left(\frac{\ln 10}{20} B \right)^{\frac{1}{\beta}} \qquad (14)$$

which is shown in Figures X for α=145 and β=1.837.

And again logarithmic PDF is significantly different which is depicted in figure 3.

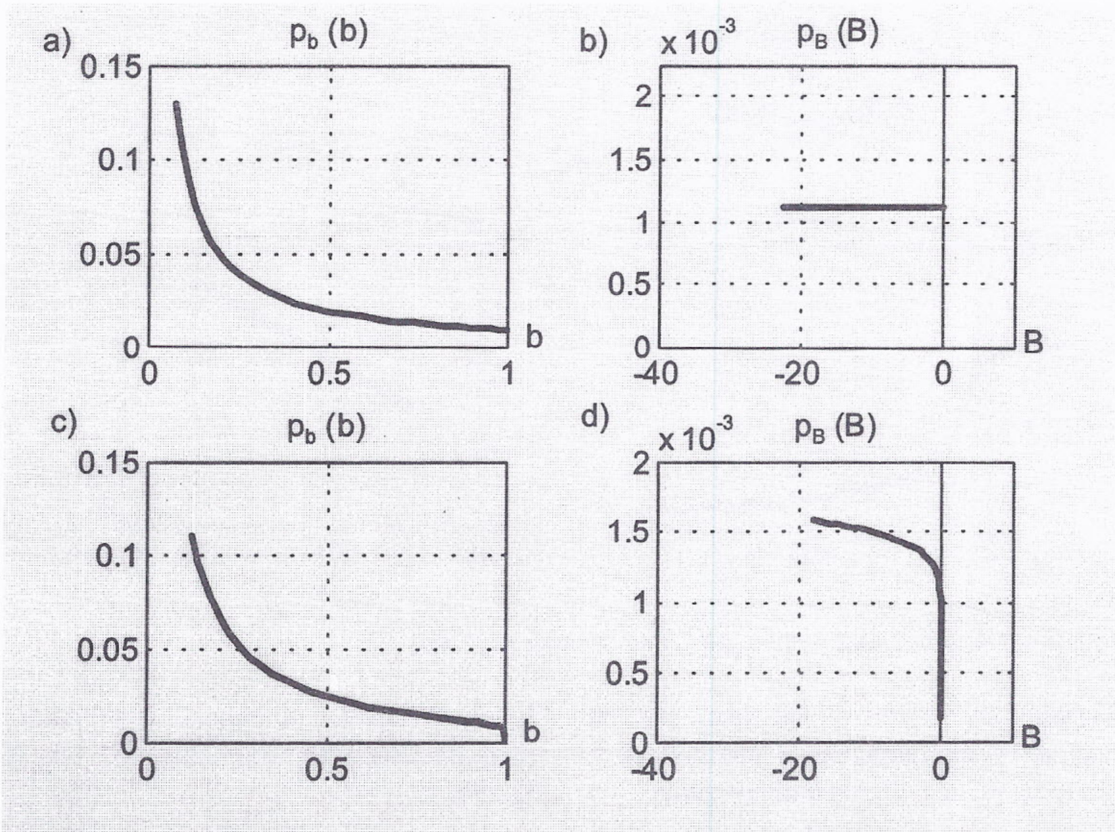

Figure 3. Hyperbolic approximation of PDF (upper figures) and PDF's for exponential approximation of the beam pattern.

SOLUTION

The problem in question can be formulated in the following way: if Ehrenberg PDF approximation is so similar to the PDF of a theoretical beam pattern for a circular transducer and Hedgepeth approximation holds also very well for the same beam pattern, what is the distinctive mark of the beam pattern that changes the PDF so significantly?

To solve this problem let's present the beam pattern PDF as the following function:

$$p_b(b) = \frac{k}{b^{1-\gamma}} \tag{15}$$

with γ as a parameter having the sense of slope of logarithmic beam pattern PDF (from (9) $p_B \approx p_b b$). Considering (7) and putting it to

$$p_b(b) = \frac{p_\theta(\theta)}{\left|\frac{db}{d\theta}\right|}$$

the following differential equation can be combined:

$$\left|\frac{db}{d\theta}\right| \frac{1}{b^{1-\gamma}} = \frac{c_o}{k}\sin\theta$$
$$b(0) = 1 \;\;,\;\; c_0 = \frac{1}{1-\cos\theta_{MAX}} \tag{16}$$

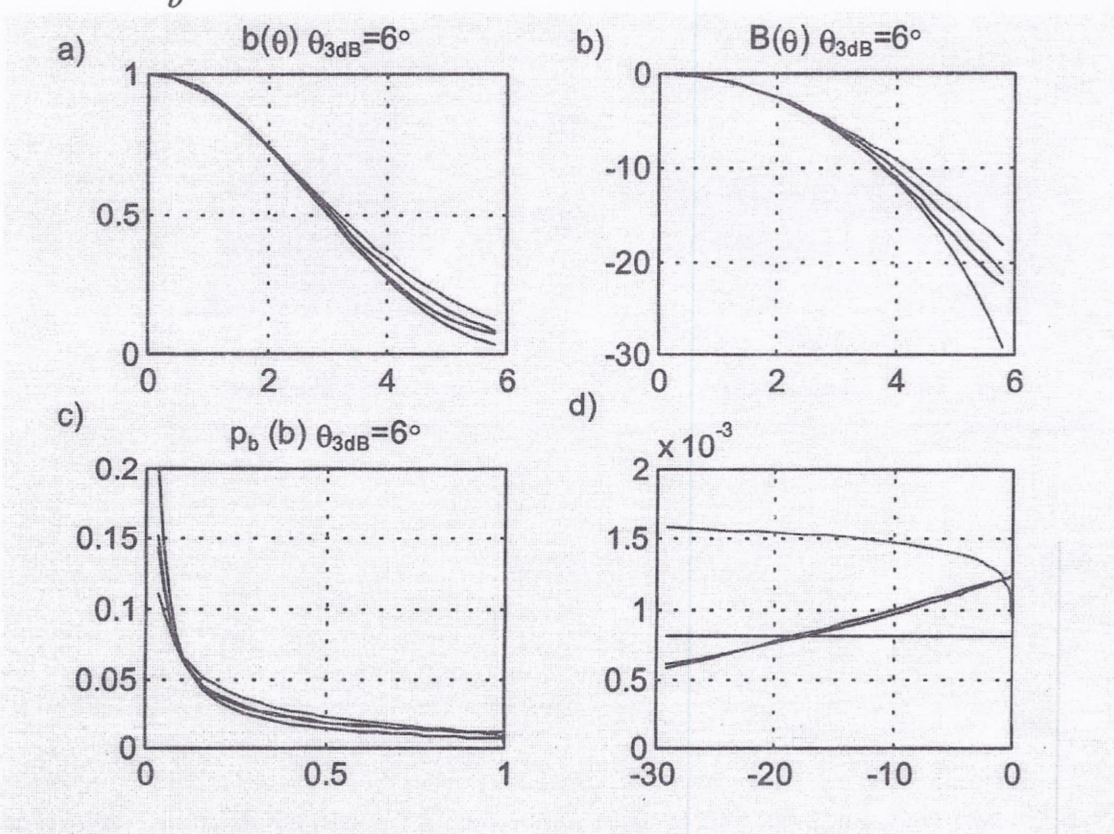

Figure 4. Selected approximations of the beam pattern and their PDFs.

From six different solutions two of them represent functions similar to the beam pattern PDF:

$$b(\theta) = \begin{cases} e^{-x} & ,\gamma = 0 \\ (1-\gamma x)^{\frac{1}{\gamma}} & ,\gamma \neq 0 \end{cases} \tag{17}$$

where $x = (c_0 / k)(1-\cos\theta)$.

Now it is clear that the Lozow approximation (12) does not exactly hold for the beam pattern of the circular piston described by equation(5) but it is rather adequate for the beam pattern of the form:

$$b(\theta) = e^{-\frac{c_0}{k}(1-\cos\theta)}$$

It prescribes zero value to parameter γ in equation (15). This function represents lower decay in the tail of the beam pattern than for the Ehrenberg approximation, which is equivalent to an ideal circular transducer (positive γ). The last presented Hedgepeth approximation of a real circular piston is related to the negative value of parameter γ. The comparison is illustrated in Figure 4.

CONCLUSIONS

The most important distinctive mark of the beam pattern from the point of view of its probability distribution function PDF is a change in slope in decay of the main-lobe. Practically constructed transducers have lower decay than theoretical ones which has a significant impact on the logarithmic beam pattern PDF. It was shown that this aspect of the beam pattern characteristic leads to a modification of the Ehrenberg approximation of the theoretical beam pattern PDF. The introduced parameter γ has a sense of slope of the logarithmic beam pattern PDF.

Figure 5 shows the implementation of the beam pattern PDF to target strength estimation.

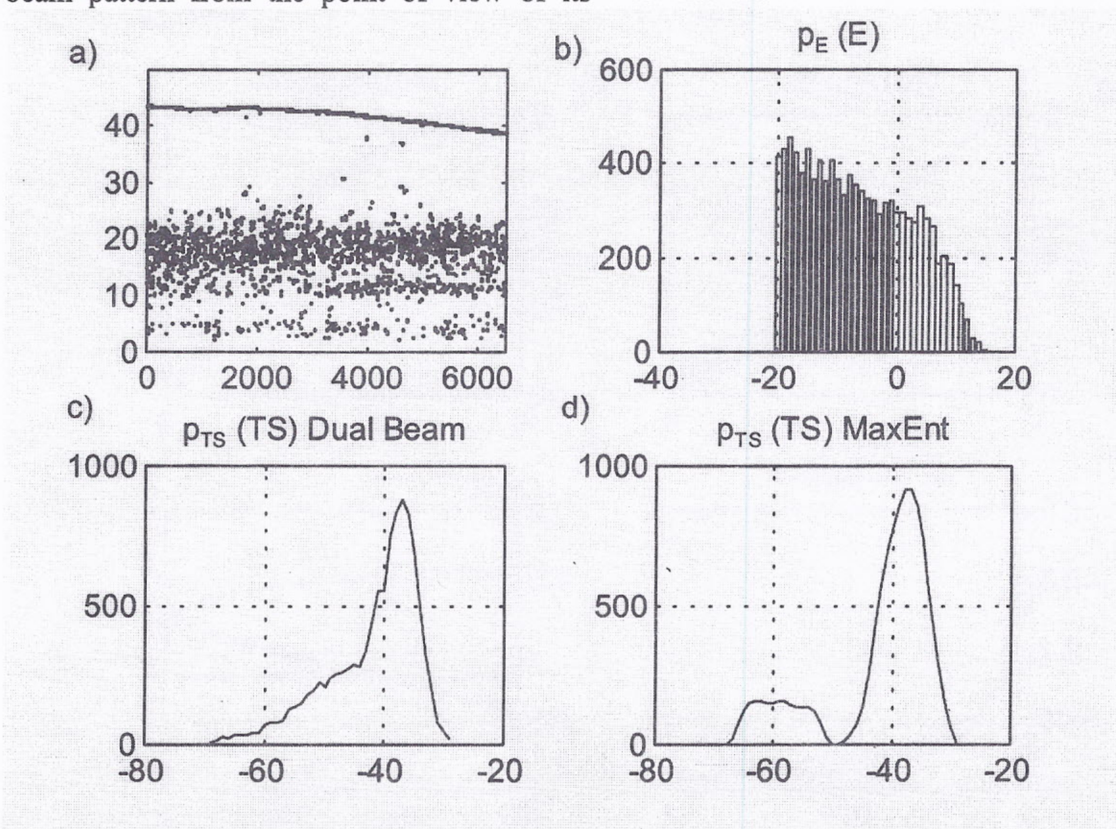

Figure 5. Sample data from the survey with dual-beam TS - estimation results compared to results of the maximum entropy deconvolution method.

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