CYCLIC LINEAR RANDOM PROCESS AS A MATHEMATICAL MODEL OF CYCLIC SIGNALS

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received 21 April 2015, revised 11 December 2015, accepted 14 December 2015

Abstract: In this study the cyclic linear random process is defined, that combines the properties of linear random process and cyclic random process. This expands the possibility describing cyclic signals and processes within the framework of linear random processes theory and generalizes their known mathematical model as a linear periodic random process. The conditions for the kernel are given and the probabilistic characteristics of generated process of linear random process in order to be a cyclic random process. The advantages of the cyclic linear random process are presented. It can be used as the mathematical model of the cyclic stochastic signals and processes in various fields of science and technology.

Keywords: Linear Cyclic Random Process, Mathematical Model, Rhythm Function, Factorized Kernel

1. INTRODUCTION

Linear stochastic processes are widely studied in different fields of science, particularly in radio signals modeling, in technical and medical diagnostics, hydroacoustics, geophysics etc. (Yurekli et al., 2005; Blake and Thomas, 1968; Bartlett, 1955; Medvegyev, 2007; Bhansali, 1993; Giraitis, 1985; Olanrewaju and Al-Arfaj, 2005). A characteristic feature of linear random process (LRP) is its constructiveness, namely LRP is specified as a structure − stochastic integral Stieltjes of the process of independent (or not correlated) increments. Furthermore, it is possible to implement the description and analysis of signals using the multivariate distribution functions on the basis of LRP and to perform simulations using computers. The problem of identifying the elements of its structure from the known (given) probability characteristics of the signals and processes is somewhat difficult in applying LRP simulation to model real signals and processes.

In the scientific literature there exist two important subclasses of linear stochastic processes. These are stationary linear random processes (Bartlett, 1950) and linear periodic random processes (Martchenko, 1998; Zvarich and Marchenko, 2011; Pagano, 1978). Stationary linear random processes are used as appropriate models of stochastic signals with time-invariant probability characteristics. The linear periodic random processes allow to take into account randomness and recurrence of the studied cyclic signals that is provided by the periodicity of the process probabilistic characteristics.

In many cases, when the rhythm of a cyclic signal varies, the hypothesis about the periodicity of its probability characteristics is inadequate to the structure of the real signal. Therefore, it is not quite correct to apply a linear periodic random process. In this case, it is more correct to apply the class of cyclic random processes (CRP), defined in the work (Lupenko, 2006). This class includes class of periodic random processes (PRP) as its particular cases and accounts for the variability of cyclic signals rate (Naseri et al.,2013).

Since the class of cyclic random processes is wider class of processes than the class of periodic random processes, it is useful to define a class of random processes which is the intersection of cyclic classes of random processes and linear random processes. In this work a linear cyclic random process (LCRP) is defined. It combines the properties of linear random process and cyclic random process, in order to broaden the possibility of using a constructive approach to the description of cyclic signals and processes in the theory of linear stochastic processes. The conditions are given to be met by the kernel and probabilistic characteristics of generated process of linear random process to be a cyclic random process. An example of linear cyclic random process and its probabilistic characteristics are given.

2. DEFINITIONS AND BASIC PROPERTIES OF LINEAR RANDOM PROCESS

Let's present briefly some main equations for the LRP and CRP. LRP can be presented in the form of a stochastic Riemann integral (Berkes and Horváth, 2006):

$$\xi(\omega,t)=\int_{-\infty}^{\infty}\varphi(t,\tau)\zeta(\omega,\tau)d\tau,\omega\in\Omega,\ t\in R, \quad (1)$$

or as an Stieltjes integral over the stochastic measure (Protter, 2005):

$$\xi(\omega,t)=\int_{-\infty}^{\infty}\varphi(t,\tau)d\eta(\omega,\tau),\omega\in\Omega,\ t\in R, \quad (2)$$

where $\varphi(t,\tau)$ is a square-integrable deterministic function with respect to variable τ, which is called the kernel of LRP $\xi(\omega,t)$, the random process $\eta(\omega,t),\ \omega\in\Omega,\ t\in R$ is a process with independent (uncorrelated) increments, which is named as a generated process, the generalized derivative of which is a white noise $\zeta(\omega,t),\ \omega\in\Omega,\ t\in R$ in the narrow (broad) sense (Ω is a set of elementary events). From the application

DOI 10.1515/ama-2015-0035

point of view, LRP is a process from the output of a linear system with impulse response $\varphi(t,\tau)$, if white noise $\zeta(\omega,t),\ \omega\in\Omega,\ t\in R$ acts on its input (Fig. 1).

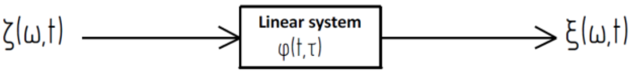

Fig. 1. The schematic presentation of LRP generation

Note that a random process $\eta(\omega,t),\ \omega\in\Omega,\ t\in R$ is the process with independent (uncorrelated) increments if for some fixed t_0 for all $t_{-m}<t_{-m+1}<\ldots<t_{-1}<t_0<t_1<\ldots<t_{n-1}<t_n$ $(m,n\in \mathrm{Z})$ from R the random values:

$$\eta(\omega,t_{-m})-\eta(\omega,t_{-m+1}),\ldots,\eta(\omega,t_0)-\eta(\omega,t_{-1}),\\ \eta(\omega,t_0),\eta(\omega,t_1)-\eta(\omega,t_0),\ldots,\eta(\omega,t_n)-\eta(\omega,t_{n-1}) \tag{3}$$

are independent (uncorrelated).

In case of LRP, one can write the logarithm of its multidimensional characteristic function $f_{k_\xi}(u_1,\ldots,u_k;t_1,\ldots,t_k)$ in the form of Levy:

$$\begin{aligned} &ln f_{k_\xi}(u_1,\ldots,u_k;t_1,\ldots,t_k)=\\ &= i\textstyle\sum_{j=1}^{k}u_j\int_{-\infty}^{\infty}\varphi(\tau,t_j)d\mu(\tau)-\\ &-\textstyle\sum_{i,j=1}^{k}u_iu_j\int_{-\infty}^{\infty}\varphi(\tau,t_i)\varphi(\tau,t_j)d\sigma(\tau)+\\ &+\textstyle\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[e^{ix\sum u_j\varphi(\tau,t_j)}-1-\\ &-\tfrac{ix}{1+x^2}\textstyle\sum_{k=1}^{n}u_j\varphi(\tau,t_j)]d_xd_\tau L(x,\tau)\end{aligned} \tag{4}$$

where $L(x,\tau)$ is a function, undefined at zero that is called Poisson spectrum jumps in the form of Levi. This function is defined as:

$$L(x,\tau)=\begin{cases}M(x,\tau), & x<0,\\ N(x,\tau), & x>0,\end{cases} \tag{5}$$

where $M(x,\tau)$ and $N(x,\tau)$ ($M(-\infty,\tau)$=$N(\infty,\tau)$=0) are non-decreasing functions that set negative and positive jumps (increments) of generating process, respectively.

Functions $\mu(\tau)$ and $\sigma(\tau)$ are defined as follows:

$$d\mu(\tau)=d\chi_1(\tau)-d\tau\cdot\int_{-\infty}^{\infty}x d_xL(x,\tau) \tag{6}$$

$$d\sigma(\tau)=d\chi_2(\tau)-d\tau\cdot\int_{-\infty}^{\infty}(1+x^2)d_xL(x,\tau) \tag{7}$$

where $\chi_1(\tau)$ and $\chi_2(\tau)$ are first and second cumulant functions of generated process $\eta(\omega,\tau)$.

3. DEFINITIONS AND BASIC PROPERTIES OF CYCLIC RANDOM PROCESS

The cyclic random process of a continuous argument is characterized by the fact that its family of distribution functions satisfies the following equation:

$$\begin{aligned}&F_{k_\xi}(x_1,\ldots,x_k,t_1,\ldots,t_k)=\\ &=F_{k_\xi}(x_1,\ldots,x_k,t_1+T(t_1,n),\ldots,t_k+T(t_k,n)),\\ &x_1,\ldots,x_k,t_1,\ldots,t_k\in R, n\in Z, k\in N.\end{aligned} \tag{8}$$

The rhythm function $\mathrm{T(t,n)}$ has the following properties:

1. $T(t,n)>0$, if $n>0$ $(T(t,1)<\infty)$;
   $T(t,n)=0$, if $n=0$; (9)
   $T(t,n)<0$, if $n<0$, $t\in \mathrm{R}$.
2. For any $t_1\in R$ and $t_2\in R$ for which $t_1<t_2$, the strict inequality holds for the function $T(t,n)$:
   $$T(t_1,n)+t_1<T(t_2,n)+t_2,\ \forall n\in Z. \tag{10}$$
3. Function $T(\mathrm{t,n})$ is the smallest by absolute value ($|T(t,n)|\le|T_\gamma(t,n)|$) among all such functions $\{T_\gamma(t,n),\ \gamma\in\Gamma\}$ satisfying the conditions (9) and (10).

In other words, the cyclic random process is a random process, the distribution functions of which are invariant to cyclic countable discontinuous group of transformations $\Gamma=\{\mathrm{T_n(t)=t+T(t,n),\ n\in Z}\}$ for a set of time arguments.

Note that the family of characteristic functions and moment functions (if they exist) of cyclic random process also satisfy the condition of invariance, which is similar to the condition (8). Thus, they are invariant to cyclic countable discontinuous group of transformations $\Gamma=\{\mathrm{T_n(t)=t+T(t,n),\ n\in Z}\}$.

In particular, according to reference (Giraitis, 1985), we will give the definition of the process with the independent cyclical increments.

The stochastically continuous process with independent increments $\eta(\omega,\mathrm{t})$ will be called the process with independent cyclic increments if there is a function $\mathrm{T(t,n)}$ that satisfies the rhythm functions conditions, that is, for a fixed $\mathrm{h}>0$ the distributions of increments (differentials) $\Delta_\mathrm{h}\eta(\omega,\mathrm{t})(d\eta(\omega,\mathrm{t}))$ and $\Delta_\mathrm{h}\eta(\omega,\mathrm{t+T(t,n)})\ \ (d\eta(\omega,\mathrm{t+T(t,n)}))$ are the same for any $\mathrm{n\in Z}$ and for any $\mathrm{t\in R}$.

4. LINEAR CYCLIC RANDOM PROCESS

To take into account the cyclicity of the probabilistic characteristics of LRP, we can impose relevant conditions on its kernel $\varphi(t,\tau)$ and probabilistic characteristics of the generated process $\eta(\omega,t),\ \omega\in\Omega,\ t\in R$. Namely, a linear stochastic process $\xi(\omega,t)$ is cyclic in the following cases:

1. When $\varphi(t,\tau)=\varphi(t-\tau)$, that is, a linear system is stationary (time-invariant) with the same parameters, and the generated random process $\eta(\omega,t),\ \omega\in\Omega,\ t\in R$ is a process with independent (uncorrelated) cyclic increments, with the function of rhythm $\mathrm{T(t,n)}$;
2. When $\varphi(t,\tau)=\varphi(t+T(t,n),\tau)$, that is a linear system is described by a cyclic linear operator, and the generated process is a homogeneous random process with independent (uncorrelated) increments;
3. When $\varphi(t,\tau)=\varphi(t+T(t,n),\tau)$, that is a linear system is described by a cyclic linear operator, and the generated random process $\eta(\omega,t),\ \omega\in\Omega,\ t\in R$ is a process with independent (uncorrelated) cyclic increments of the rhythm function $T(t,n)$.

If the condition of generated process $\eta(\omega,t),\ \omega\in\Omega,\ t\in R$ increments non-correlatedness holds true, we have a linear cyclic random process in a broad sense. This means that its expectation and correlation function are cyclic on the set of its two arguments. If you require an independence of generated process increments, we obtain a linear cyclic random process in the narrow sense, for which its multidimensional distribution functions and characteristic functions are cyclic on the set of all time arguments.

We will show the correctness of formulated conditions of the cyclical linear random process only for the second case when the generated process is homogeneous. In this case $L(x,\tau)=L(x)$,$\mu(\tau)=m\sigma(\tau)=\sigma$, and the kernel of a linear process is a cyclic function for argument τ. Then, for the characteristic function (4) we have:

$$\begin{aligned} & ln f_k(u_1,\dots,u_k;t_1,\dots,t_k)= \\ & = im\sum_{j=1}^{k} u_j \int_{-\infty}^{\infty}\varphi(\tau,t_j)d\tau - \\ & -\sigma^2\sum_{i,j=1}^{k} u_i u_j \int_{-\infty}^{\infty}\varphi(\tau,t_i)\varphi(\tau,t_j)d\tau + \\ & +\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[e^{ix\sum u_j\varphi(\tau,t_j)}-1- \\ & -\frac{ix}{1+x^2}\sum_{k=1}^{n} u_j\varphi(\tau,t_j]dL(x)d\tau = im\sum_{j=1}^{k} u_j \times \\ & \times\int_{-\infty}^{\infty}\varphi\left(\tau,t_j+T(t_j,n)\right)d\tau - \sigma^2\sum_{i,j=1}^{k} u_i u_j \times \\ & \times\int_{-\infty}^{\infty}\varphi(\tau,t_i+(T(t_i,n))\varphi(\tau,t_j+T(t_j,n))d\tau + \\ & +\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[e^{ix\sum u_j\varphi(\tau,t_j+T(t_j,n))}-1-\frac{ix}{1+x^2}\sum_{k=1}^{n} u_j \times \\ & \times\varphi\left(\tau,t_j+T(t_j,n)\right)dL(x)d\tau = ln f_k(u_1,\dots,u_k;t_1+ \\ & +T(t_1,n),\dots,t_k+T(t_k,n)) \end{aligned} \tag{11}$$

That is, the linear random process $\xi(\omega,t)$ is a cyclic random process with a rhythm function $T(t,n)$. The moment functions of such a linear random process are also cyclic. Consider this only for its two first moment functions − mathematical expectation and correlation function. These moment functions fully exhaust the description of a LRP in the broad sense, when only the non-correlatedness of the generated process increments is required, rather than their independence. In this case, we can write the following expression for the expectation function LCRP:

$$\begin{aligned} & m_\xi(t)=M\{\xi(\omega,t)\}=m\int_{-\infty}^{\infty}\varphi(t,\tau)d\tau= \\ & = m\int_{-\infty}^{\infty}\varphi(t+T(t,n),\tau)d\tau = M\{\xi(\omega,t+T(t,n))\}, \\ & t\in R, n\in Z, \end{aligned} \tag{12}$$

where m is the homogeneous process $\eta(\omega,t),\ \omega\in\Omega,\ t\in R$ increments expectation with uncorrelated increments.

For the correlation function LCRP, we have:

$$\begin{aligned} & R_\xi(t_1,t_2)=M\{\dot{\xi}(\omega,t_1)\dot{\xi}(\omega,t_2)\}= \\ & \sigma^2\int_{-\infty}^{\infty}\varphi(t_1,\tau)\varphi(t_2,\tau)d\tau= \\ & =\sigma^2\int_{-\infty}^{\infty}\varphi(t_1+T(t_1,n),\tau)\varphi(t_2+T(t_2,n),\tau)d\tau= \\ & = R_\xi(t_1+T(t_1,n),t_2+T(t_2,n)), \qquad t_1,t_2\in R, n\in Z, \end{aligned} \tag{13}$$

where σ is standard deviation of increments homogeneous process $\eta(\omega,t),\ \omega\in\Omega,\ t\in R$ with uncorrelated increments.

That is, LCRP expectation is a cyclic deterministic function with the rhythm function $T(t,n)$, and its correlation function is cyclic with the rhythm function $T(t,n)$.

5. PARTICULAR CASE OF CYCLIC LINEAR RANDOM PROCESS WITH FACTORIZED KERNEL

In a particular case when the LRP kernel $\varphi(t,\tau)$ can be factorized, it is represented as a product of two functions:

$$\varphi(t,\tau)=\varphi_1(t)\varphi_2(\tau),\ t,\ \tau\in R, \tag{14}$$

then a LRP looks like:

$$\begin{aligned} & \xi(\omega,t)=\int_{-\infty}^{\infty}\varphi_1(t)\varphi_2(\tau)d\eta(\omega,\tau)= \\ & =\varphi_1(t)\int_{-\infty}^{\infty}\varphi_2(\tau)d\eta(\omega,\tau),\ \omega\in\Omega,\ t\in R \end{aligned} \tag{15}$$

This presentation is equivalent to the presentation of a random process as a product of deterministic function $\varphi_1(t)$ and random variable $A(\omega)$, namely:

$$\xi(\omega,t)=\varphi_1(t)\cdot A(\omega),\omega\in\Omega,\ t\in R \tag{16}$$

where $A(\omega)=\int_{-\infty}^{\infty}\varphi_2(\tau)d\eta(\omega,\tau)$ is a random functional.

If, in this case, the function $\varphi_1(t)=\varphi_1(t+T(t,n))$ is a cyclic numerical function, then the formulae (12) and (13) take the following form:

$$\begin{aligned} & m_\xi(t)=M\{\xi(\omega,t)\}=m\int_{-\infty}^{\infty}\varphi(t,\tau)d\tau= \\ & = m\int_{-\infty}^{\infty}\varphi_1(t)\varphi_2(\tau)d\tau = m\varphi_1(t)\int_{-\infty}^{\infty}\varphi_2(\tau)d\tau, \\ & t\in R, n\in Z, \end{aligned} \tag{17}$$

$$\begin{aligned} & R_\xi(t_1,t_2)=\sigma^2\int_{-\infty}^{\infty}\varphi(t_1,\tau)\varphi(t_2,\tau)d\tau= \\ & =\sigma^2\int_{-\infty}^{\infty}\varphi_1(t_1)\varphi_1(t_2)\varphi_2(\tau)\varphi_2(\tau)d\tau= \\ & =\sigma^2\varphi_1(t_1)\varphi_1(t_2)\int_{-\infty}^{\infty}\left(\varphi_2(\tau)\right)^2d\tau, \\ & t_1,t_2\in R, n\in Z. \end{aligned} \tag{18}$$

The problem of identifying the elements of its structure from the known probability characteristics of the studied signals and processes is somewhat difficult in applying LRP simulation to model real signals and processes. In this case, the probability characteristics cyclicity of a LRP is due to cyclical numerical function $\varphi_1(t)$.

Consider an example of LCRP and its probability characteristics in the case of LCRP kernel factorizing. Let LCRP kernel has the form:

$$\varphi(t,\tau)=sin(2t^2)e^{-0.3\tau}cos(\tau),\ t\in(0,\infty),\ \tau\in R, \tag{19}$$

and the generated stochastic process $\eta(\omega,t),\ \omega\in\Omega,\ t\in R$ is a homogeneous process with independent increments with expectation m and standard deviation σ of its increments. Then, the linear cyclic random process will look like:

$$\begin{aligned} & \xi(\omega,t)=sin(2t^2)\int_{-\infty}^{\infty}e^{-0.3\tau}cos(\tau)d\eta(\omega,\tau), \\ & \omega\in\Omega, \qquad t\in(0,\infty) \end{aligned} \tag{20}$$

and its expectation and correlation function will be:

$$\begin{aligned} & m_\xi(t)=msin(2t^2)\int_{-\infty}^{\infty}e^{-0.3\tau}cos(\tau)d\tau, \\ & t\in(0,\infty),\quad n\in Z, \end{aligned} \tag{21}$$

$$\begin{aligned} & R_\xi(t_1,t_2)= \\ & \sigma^2 sin(2{t_1}^2)sin(2{t_2}^2)\int_{-\infty}^{\infty}\left(e^{-0.3\tau}cos(\tau)\right)^2d\tau, \\ & t_1,t_2\in(0,\infty), \qquad n\in Z. \end{aligned} \tag{22}$$

Fig. 2 and 3 show plots of kernel sections $\varphi(t,\tau)$ (19) for fixed values of t and τ.

Fig. 4 presents a plot of the expectation function, and Fig. 5 represents the plot of section correlation function LCRP.

It can be seen from Fig. 4 and 5, that the respective probabilistic characteristics of LRP are cyclic.

DOI 10.1515/ama-2015-0035

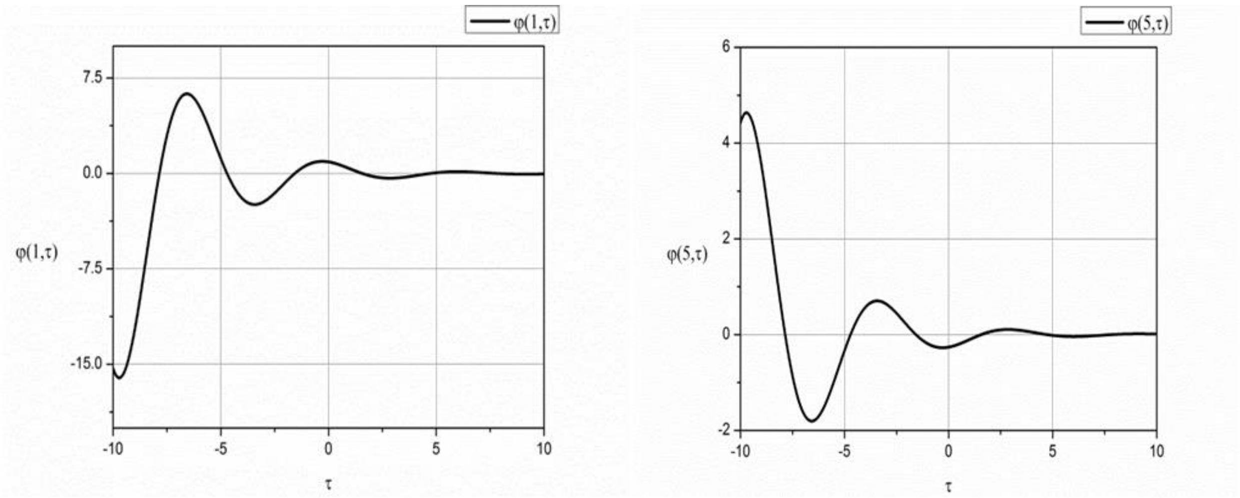

Fig. 2. Plots of kernel sections $\varphi(t,\tau)$ for some fixed values of t (t =1 left and t =5 right)

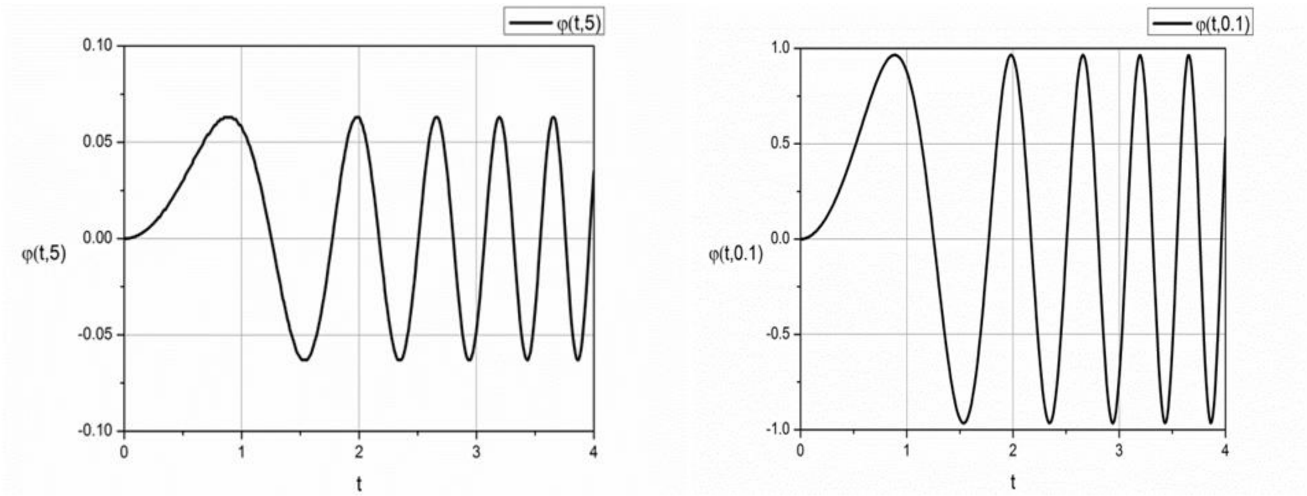

Fig. 3. Plots of kernel sections $\varphi(t,\tau)$ for fixed values of $\tau(\tau = 0.1$ left and $\tau = 5$ right)

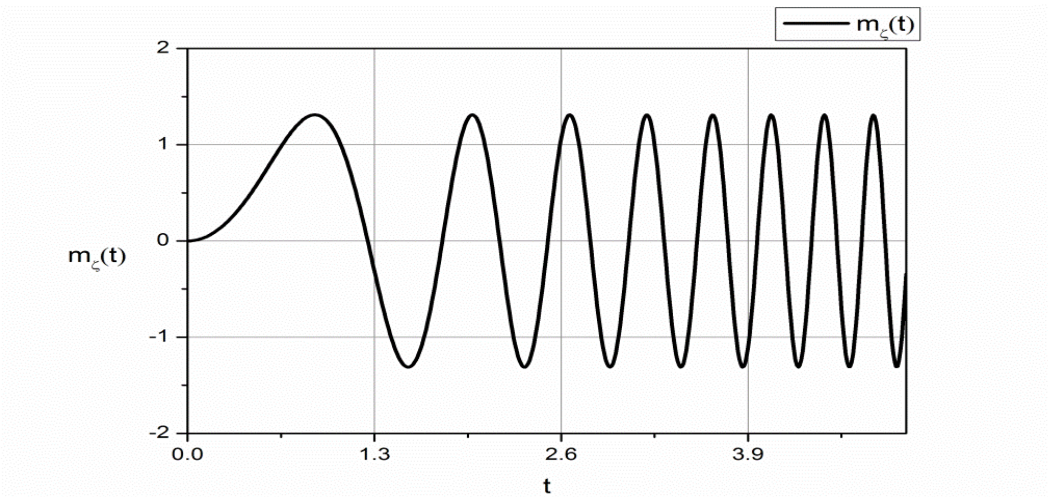

Fig. 4. Plot of linear cyclic random process expectation

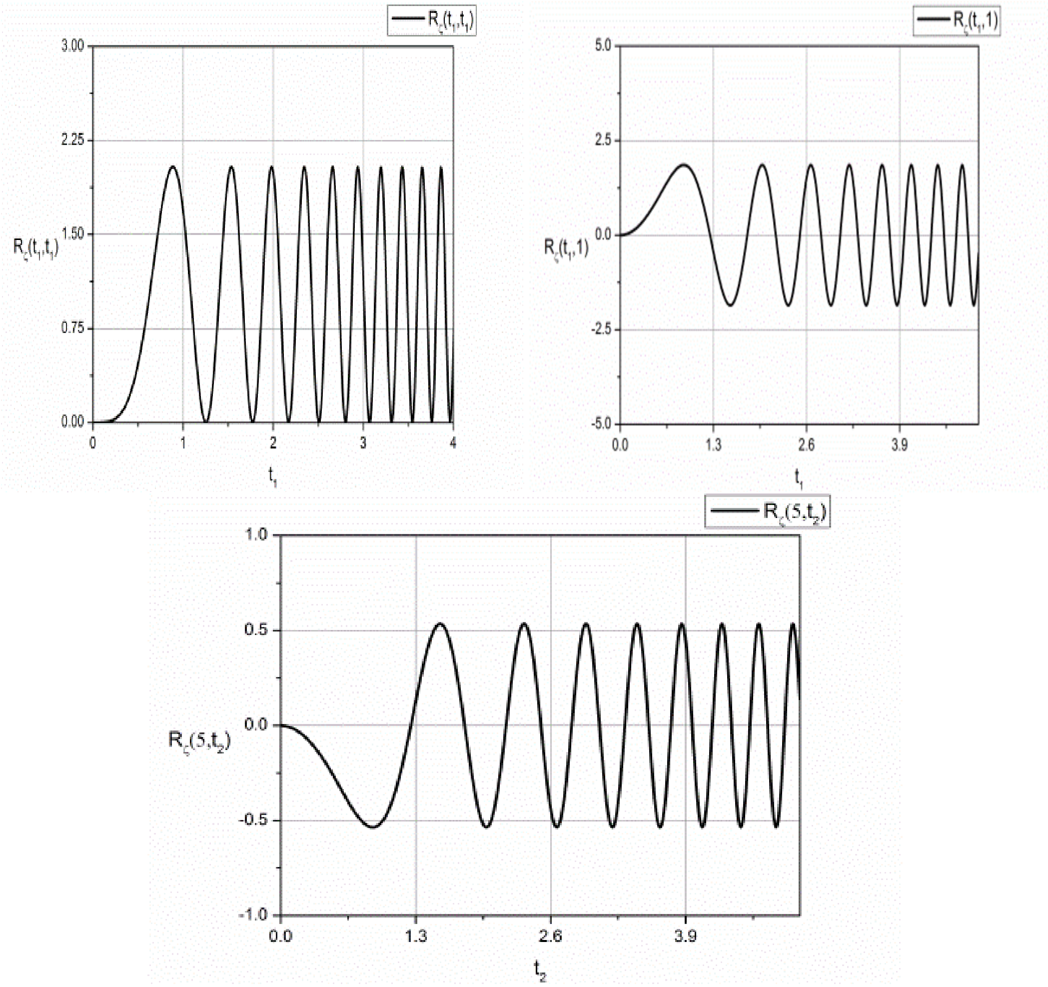

Fig. 5. Plots of correlation function $R_\xi(t_1,t_2)$ section of linear cyclic random process a) $R_\xi(t_1,t_1)$, *b)* $R_\xi(t_1,1)$, *c)* $R_\xi(5,t_2)$

6. SOME ADVANTAGES OF LINEAR CYCLIC RANDOM PROCESS

The LCRP that is proposed in this study has the advantage, that allows to take into account the variation of rhythm of the investigated signals and processes as compared with known mathematical models of random cyclic signals and processes such as periodically correlated random process (cyclic stationary random process) (Gardner et al., 2006; Hurd and Miamee, 2006), periodically distributed random process, linear periodic random process.

Namely the following advantages in modeling of cyclic signals and processes can be mentioned in comparison with the known cyclic random process.

1. The entire probabilistic structure of LCRP is completely determined by the deterministic kernel $\varphi(t,\tau)$ and characteristics $L(x,\tau)$, $\mu(\tau)$, $\sigma(\tau)$ of its generated process, which can be parameterized in many applied problems. This allows a compact and economical description of the studied stochastic signals of the cyclic structure.
2. Using LCRP as the mathematical model, cyclic stochastic signals and processes can be explored in a broad range of probability characteristics, namely within the framework of the spectral correlation theory of random processes, within the framework of the higher orders moment distribution functions, within the framework of the multivariate distribution functions and characteristic functions.
3. LCRP remains LCRP after transformation by a linear dynamic system. This LCRP has unchanged characteristics of the generated process. It has the changed kernel only. This property simplifies the study of the transformations of cyclic stochastic signals in the linear systems that often arise in the problems of radio engineering, technical and medical diagnostics, geophysics and mechanics.
4. The result of the LCRP constructiveness is the ability to display the mechanisms of the studied signals formation in the LCRP structure that allows to study the influence of various parameters of generation process mechanism on its probabilistic characteristics.
5. The construction of the LRCP is directly suitable for generating and simulating cyclic stochastic signals and processes by modern software and hardware systems.

7. CONCLUSIONS

1. The linear cyclic random process is defined. It combines the properties of linear random process and cyclic random process. This enables the extension of a constructive approach to the description of cyclic signals within the linear theory of random processes and summarizes their mathematical model as a linear periodic random process.
2. The conditions are given for the kernel and the probabilistic characteristics of generated process of linear random process

DOI 10.1515/ama-2015-0035

in order to be a cyclic random process.

3. The advantages of the LCRP allow to use it as mathematical models of the cyclic stochastic signals and processes in various fields of science and technology, particularly as mathematical models of a wide class of human body heart and respiratory system cyclic signals, as models of the data transmission channels process, as models of vibrating processes in rotating mechanical structures of different types of generators, turbines, propellers, as models of the cyclical economic processes.
4. Presented results open some perspectives such as development of discrete analogues of cyclic linear stochastic processes, including definitions and study of classes of cyclic moving average (MA), autoregressive (AR), autoregressive moving average (ARMA). This will permit to create further effective models and methods of analysis, simulation and prediction of cyclic processes with the use of modern digital technology.

REFERENCES

1. **Yurekli K., Kurunc A., Ozturk F.**, (2005) Application of linear stochastic models to monthly flow data of Kelkit Stream, *Ecological Modelling*, 183 (1), 67–75.
2. **Blake I., Thomas J.** (1968), The Linear Random Process, *Proc. Of IEEE*, 56 (10) 1696–1703.
3. **Bartlett M.** (1955), *An introduction to stochastic processes with special reference to methods and applications*, Cambridge University Press.
4. **Medvegyev P.** (2007), *Stochastic Integration Theory*, Oxford University Press, New York.
5. **Bhansali R**. (1993), Estimation of the impulse response coefficients of a linear process with infinite variance, *Journal of Multivariate Analysis,* 45, 274-290.
6. **Giraitis L.** (1985), Central limit theorem for functionals of a linear process, *Lithuanian Mathematical Journa*l, 25, 25–35.
7. **Olanrewaju J., Al-Arfaj M.** (2005), Development and application of linear process model in estimation and control of reactive distillation, *Computers and Chemical Engineering*, 30, 147–157.
8. **Bartlett M.** (1950) Periodogram Analysis and Continuous Spectra, *Biometrika*, 37(1/2), 1–16.
9. **Martchenko B**. (1998) Concerning on a theorem for periodic in Slutskiy sense linear random processes, *International Congress of Mathematicians*, *Berlin*.
10. **Zvarich V., Marchenko B.** (2011), Linear autoregressive processes with periodic structures as models of information signals, *Radioelectronics and Communications Systems*, 54(7), 367–372.
11. **Pagano M.** (1978), On periodic and multiple autoregressions, *The Annals of Statistics*, 6, 1310–1317.
12. **Lupenko S.** (2006), Deterministic and random cyclic function as a model of oscillatory phenomena and signals: the definition and classification*, Electronic Modeling, Institute of modeling problems in power of H.E Pukhov NAS*, 28, 29–45.
13. **Naseri H., Homaeinezhad M.R., Pourkhajeh H**., (2013), Noise/spike detection in phonocardiogram signal as a cyclic random process with non-stationary period interval, *Computers in Biology and Medicine*, 43, 1205–1213.
14. **Berkes I., Horváth L**., (2006), Convergence of integral functionals of stochastic processes, *Econometric Theory*, 22, 304–322.
15. **Protter P.E.**, (2005), Stochastic integration and differential equations, *Second edition, New York.*
16. **Gardner W., Napolitano A., Paura L.,** (2006) Cyclostationarity: Half a century of research*, Signal Processing*, 86, 639–697.
17. **Hurd H., Miamee A.** (2006), *Periodically Correlated Random Sequences*, Spectral Theory and Practice, Wiley, New York.