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REAL-TIME PARAMETER ESTIMATION STUDY FOR INERTIA PROPERTIES OF GROUND VEHICLES

Vehicle parameters have a significant impact on handling, stability, and rollover propensity. This study demonstrates two methods that estimate the inertia values of a ground vehicle in real-time.

Through the use of the Generalized Polynomial Chaos (gPC) technique for propagating the uncertainties, the uncertain vehicle model outputs a probability density function for each of the variables. These probability density functions (PDFs) can be used to estimate the values of the parameters through several statistical methods. The method used here is the Maximum A-Posteriori (MAP) estimate. The MAP estimate maximizes the distribution of P(β |z) where β is the vector of the PDFs of the parameters and z is the measurable sensor comparison.

An alternative method is the application of an adaptive filtering method. The Kalman Filter is an example of an adaptive filter. This method, when blended with the gPC theory is capable at each time step of updating the PDFs of the parameter distributions. These PDF's have their median values shifted by the filter to approximate the actual values.

1. Introduction

Vehicle control systems are designed to be robust, capable of dealing with inaccurate parameter values. These inaccurate parameter values are caused by the loading (objects, etc.) and unloading (fuel, etc.) of the vehicle. For most systems this is not very detrimental to their function, however vehicle rollover prevention is not one of these. Owing to the fact that vehicle rollovers are highly discontinuous events, the more accurate the measurements of the parameters, the more effectively the control systems can operate.

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The goal of this study is to estimate these changes and to provide updates for the onboard systems. There are several problems to be considered. The first is to choose the type and method of data acquisition. The second is to choose a model that requires as little information as possible while still maintaining the ability to estimate the parameters of interest.

This paper presents two methods by which one can extract the mass and inertia properties for a two dimensional vehicle model (pitch and roll) in operation, without requiring a terrain profile. The methods employed are an application of Bayesian statistics, and blending of the Extended Kalman Filter to the mathematical technique of Generalized Polynomial Chaos (gPC). The Generalized Polynomial Chaos technique gives a computationally efficient method for quantifying the uncertainty in the parameters [4, 12, 23, 24].

2. Review of Literature

The range of methods for estimating parameter values are as varied as the fields that they span. The parameters in question could be properties of electrical devices [11]. The method of estimation could be Kalman Filtering, Least Square Error, Lyapunov Stability, Genetic Algorithms, and many others [1, 13, 14, 17, 21].

Estimation of parameters, in general, is a difficult task, and it is no different in vehicle dynamics [6]. The uncertain parameters can be the mass of the vehicle, the inertia, the aerodynamic drag coefficient, the spring stiffness, the damping of the suspension, and many others.

There are several methods employed in estimating the vehicle mass and inertia. An example for mass estimation employs the vehicle engine torque, drive train inertia, wind resistance, rolling resistance and road grade [7, 13, 21]. The problem as addressed by [21], is that the parameter estimation is highly sensitive to the estimation of the rolling resistance of the vehicle, a parameter which changes non-trivially over time.

A method by which several parameters of a vehicle can be estimated is demonstrated in [17]. In this study, the authors solve for the vehicle CG in the horizontal plane, the mass and the inertia of the vehicle in pitch and roll. The problem with this estimation technique is that it requires the road noise to be Gaussian white noise, which is not always the case. A terrain profile is needed, and due to the method by which the estimations are made, errors in the assumed parameters can cause non-trivial estimation errors.

There are several methods that employ the gPC theory as a technique to propagate the uncertainty in the parameters [3, 9, 20]. Unfortunately, because they only include sprung mass dynamics [9, 16] they have to run the system over relatively flat terrain where the pitch and roll dynamics are not heavily excited. Expansions to these methods are shown in [2, 15] that add in either pitch or roll plane dynamics for the parameter estimation. These studies form the foundation of for this work, and motivate the expansion to include the roll dynamics of the vehicle for estimation of the roll inertia in an effort to be as thorough as possible.

3. Vehicle Dynamics

A common model used in vehicle dynamics is a seven degree of freedom (DOF) base excitation model, such as the one presented in Figure 1. This model consists of a chassis (denoted as the sprung mass) and the suspension systems and wheels on the four corners (denoted as unsprung masses). The model uses the terrain profile to excite the unsprung masses through the tire dynamics; it then propagates the forces up through the suspension elements to excite the sprung mass dynamics. In the present study the terrain profile requirement has been removed; in addition to vertical bounce and pitch DOFs considered in [15], the model in this study includes the roll DOF. This is very important since without including the roll motion of the vehicle, the roll inertia of the vehicle would be ignored, and thus not possible to estimate it.

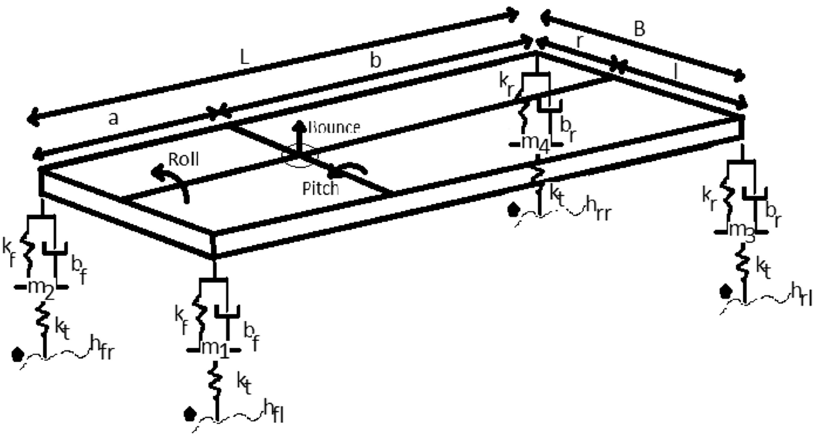

Fig. 1. The dynamics of the seven degree of freedom vehicle model

The parameters m_1, m_2, m_3, m_4 are the mass values of the four unsprung masses. The parameter k_t is the tire stiffness value. The parameters a, b, r, l, L, B are the geometric properties of the sprung mass (a is the distance from the front axle to the center of mass of the sprung mass, b is the distance from the rear axle to the center of gravity of the sprung mass, r is the distance from the right side of the vehicle to the center of gravity of the sprung mass, l is the distance from the left side of the vehicle to the center of gravity of the sprung mass, L is the wheelbase and B is the trackwidth of the vehicle). The parameters k_f, b_f, k_r, b_r are the stiffness and damping of the front and rear wheels.

The modified model of the seven DOF system uses the four vertical acceleration motions of the wheels as inputs. This reduces the computational complexity, as well as removes the need for knowledge about the unsprung masses' stiffnesses, weights, and damping and knowledge about the terrain profile. Thus, the modified model, illustrated in Figure 2, has three DOF: vertical bounce, pitch rotation and roll rotation of the sprung mass.

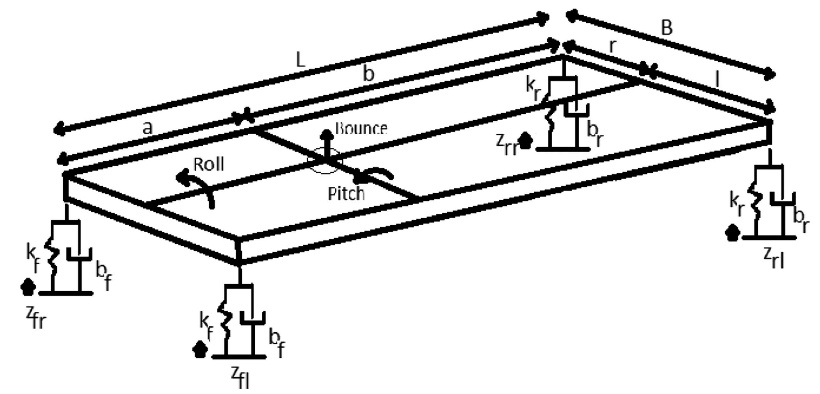

Fig. 2. The dynamics of the three degree of freedom model

This model has been developed based on the following assumptions: small lateral velocity, small yaw velocity, small longitudinal acceleration, small lateral acceleration, small roll angle, small pitch angle, linear suspension elements, symmetry of front suspension elements, as well as rear elements: $k_{fr} = k_{fl} = k_f$.

The system uses two "centers", one for the sprung mass, and one for the ensemble of the unsprung masses. For the sprung mass, the center is defined as the height, pitch, and roll of the center of mass. For the unsprung masses, the center is defined as the geometric average height, $z_{u,cg}$, roll, $\theta_{u,cg}$, and pitch, $\phi_{u,cg}$, for each body, making this an adaptation of a quarter car model; the center for the ensemble of the unsprung masses in vertical bounce, pitch, and roll are thus described as:

$$z_{u,cg} = (L-a)\,\frac{B-l}{LB} z_{fl} + (L-a)\,\frac{l}{LB} z_{fr} + a\frac{B-l}{LB} z_{rl} + a\frac{l}{LB} z_{rr} \tag{1}$$

$$\theta_{u,cg} = \frac{\left[-\left(r\,z_{fl} + l\,z_{fr}\right) + \left(r\,z_{rl} + l\,z_{rr}\right)\right]}{LB} \tag{2}$$

$$\phi_{u,cg} = \frac{\left[\left(b\,z_{fl} + a\,z_{rl}\right) - \left(a\,z_{rr} + b\,z_{fr}\right)\right]}{LB} \tag{3}$$

Similar relations to Equations (1), (2), and (3) can be written in terms of accelerations, corresponding directly to the measurement done using accelerometers on the instrumented vehicle. The accelerations thus computed are then fed into Equations (4), (5), and (6). (The parameters z_{fl}, z_{fr}, z_{rl}, z_{rr} are the vertical displacements of the wheels (front left, front right, real left, rear right).)

Using the parameters to be estimated M_u, J_{pitch}, J_{roll} as the mass, pitch inertia, and roll inertia of the sprung mass, the dynamic equations of motion of the sprung mass are defined as:

$$M_u \ddot{Z} = \left(\sum_{i=fl,\ fr,rl,rr} F_i \right) - \ddot{z}_{u,cg} \tag{4}$$

$$J_{pitch} \ddot{\theta} = T_{pitch} - J_{pitch} \ddot{\theta}_{u,cg} \tag{5}$$

$$J_{roll} \ddot{\phi} = T_{roll} - J_{roll} \ddot{\phi}_{u,cg} \tag{6}$$

Where the relative displacements in vertical bounce (Z), pitch (θ) and roll (ϕ) between the centers of the unsprung and sprung mass bodies are:

$$Z = z_{s,cg} - z_{u,cg} \tag{7}$$

$$\theta = \theta_{s,cg} - \theta_{u,cg} \tag{8}$$

$$\phi = \phi_{s,cg} - \phi_{u,cg} \tag{9}$$

The forces and moments for the sprung mass system using the relative displacements are:

$$F_{fl} = -k_f Z - b_f \dot{Z} + a k_f \theta + a b_f \dot{\theta} - l k_f \phi - l b_f \dot{\phi} \tag{10}$$

$$F_{fr} = -k_f Z - b_f \dot{Z} + a k_f \theta + a b_f \dot{\theta} + r k_f \phi + r b_f \dot{\phi} \tag{11}$$

$$F_{rl} = -k_r Z - b_r \dot{Z} - b k_r \theta - b b_r \dot{\theta} - l k_r \phi - l b_r \dot{\phi} \tag{12}$$

$$F_{rl} = -k_r Z - b_r \dot{Z} - b k_r \theta - b b_r \dot{\theta} + r k_r \phi + r b_r \dot{\phi} \tag{13}$$

$$T_{pitch} = -a \left(F_{fl} + F_{fr} \right) + b(F_{rl} + F_{rr}) \tag{14}$$

$$T_{roll} = -r \left(F_{fr} + F_{rr} \right) + l \left(F_{fl} + F_{rl} \right) \tag{15}$$

4. Methodology of Study

The sensors and instrumentation used for this purpose are described in this section. Further, the mathematical methods employed in developing the estimators are also presented here.

4.1. Sensors and Data Collected

Artificial sensor readings are produced from a seven DOF vehicle model operating on a synthetic road profile. The synthetic road profile is created from a combination of sine and cosine functions, operating between 0.77 Hz and 8.3 Hz with magnitudes of 3 cm and 0.3 cm, and step functions.

4.2 Theory used in quantifying unknown parameters

The parameters to be estimated are defined in the model dynamics as having uncertain values. To propagate the uncertainty in the parameters through the dynamics of the model, the mathematical technique of Generalized Polynomial Chaos is employed (gPC).

The model is initiated by specifying a mean and variance for the parameters, as shown in Equations (16) to (18):

$$Mass = \overline{mass} + \Delta_{mass} \tag{16}$$

$$J_{Pitch} = \overline{J_{Pitch}} + \Delta_{J_{Pitch}} \tag{17}$$

$$J_{Roll} = \overline{J_{Roll}} + \Delta_{J_{Roll}} \tag{18}$$

Because these parameters are uncertain, the solution of the differential equations will also be uncertain. The state space is represented in gPC as:

$$\mathbf{x} = \left[\sum_{\mathrm{i=1}}^{\mathrm{S}} x_1^i \Psi^{\mathrm{i}}(\xi) \quad \cdots \quad \sum_{\mathrm{i=1}}^{\mathrm{S}} x_n^i \Psi^{\mathrm{i}}(\xi) \quad \sum_{\mathrm{i=1}}^{\mathrm{S}} v_1^i \Psi^{\mathrm{i}}(\xi) \quad \cdots \quad \sum_{\mathrm{i=1}}^{\mathrm{S}} v_n^i \Psi^{\mathrm{i}}(\xi) \right]^T \tag{19}$$

The state space vector is formally expanded to contain the parameter values:

$$\mathbf{x} = \left[\sum_{\mathrm{i=1}}^{\mathrm{S}} x_1^i \Psi^{\mathrm{i}}(\xi) \cdots \sum_{\mathrm{i=1}}^{\mathrm{S}} x_n^i \Psi^{\mathrm{i}}(\xi) \sum_{\mathrm{i=1}}^{\mathrm{S}} v_1^i \Psi^{\mathrm{i}}(\xi) \cdots \sum_{\mathrm{i=1}}^{\mathrm{S}} v_n^i \Psi^{\mathrm{i}}(\xi) \sum_{\mathrm{i=1}}^{\mathrm{S}} p_1^i \Psi^{\mathrm{i}}(\xi) \cdots \sum_{\mathrm{i=1}}^{\mathrm{S}} p_d^i \Psi^{\mathrm{i}}(\xi) \right]^T \tag{20}$$

with x_j^i representing the i^{th} term of the power series, and the j^{th} state variable. Similarly v_j^i represents the j^{th} state space variable velocity and the i^{th} term in its power series expansion. The parameters are described as p_d^i with d indexing the parameter of interest, and i indexing the power series coefficient. For a gPC series, the term $\Psi^{\mathrm{i}}(\xi)$ is a tensor product of the basis functions and random variables, ξ which are used to span the space of the uncertain parameters. The basis functions are orthogonal or orthonormal polynomials (such as Legendre Polynomials). More explicit detailing of this can be found in [8, 18, 19, 23, 25].

The coefficients of these power series are solved through the collocation technique. The collocation technique functions much like Monte Carlo simulations, but with two major differences. The first is that we choose specific points. The second is that the set of solutions are combined through the collocation matrix. The collocation matrix is defined as:

$$A_{j,i} = \Psi^{\mathrm{i}}\left(\xi_{\mathrm{j}}\right) \tag{21}$$

where the i^{th} index tracks the tensor project of the basis functions and the j^{th} index is the choice of points from the set of collocation points. The collocation points are given in vector form:

$$\xi^j = \left[\xi_1^j \dots \xi_d^j\right] \tag{22}$$

Where j represents the rows of points, occupying the subset $1 \leq j \leq S$, and d indexes the points chosen for each uncertain parameter. In general, a stable solution requires $3S \leq Q \leq 4S$ number of collocation [5]. The coefficients are thus solved for as:

$$x^j\,(T) = \sum_{i=1}^{Q} (A^{\#})_{j,i}\, X^i(T) \tag{23}$$

Where $A^{\#}$ is the Moore-Penrose pseudo-inverse. X^i is the i^{th} solution of the state space parameters from the dynamics using the i^{th} row vector of collocation points and x^j denotes the j^{th} power series coefficient.

4.3 Estimation Techniques

In section 4.2 the method by which the uncertainties are propagated through the dynamics of the model are shown. Once the system is constructed, the output is a stochastic solution that only propagates the uncertainties but does nothing to estimate them. In the next two sub-sections, methods for estimating these parameter values are described.

4.3.1 Extended Kalman Filter

A differential equation system can be described in state space form as:

$$\dot{x} = f\,(\mathbf{x}) + \mathbf{w} \tag{24}$$

Where:

$$\mathbf{x} = [x_1 \ldots x_n,\ v_1 \ldots v_n]^T \tag{25}$$

And **w** is the vector of process noise. The system measurement equation is defined as:

$$\mathbf{z} = h(\mathbf{x}) + \mathbf{v} \tag{26}$$

h is the observation matrix, that incorporates the state vector into an output solution. **v** is the vector of the sensor noise. The Kalman Filter is designed for linear systems. The Extended Kalman Filter (EKF) linearizes the system mechanics in an attempt to produce an approximately linear system. This is done through linearizing the system dynamics and observation matrices and evaluating them at each time step, k:

$$F_k = \frac{\delta f(x)}{\delta x}|_{x=x_k} \tag{27}$$

$$H_k = \frac{\delta h(x)}{\delta x}|_{x=x_k} \tag{28}$$

The EKF equation is:

$$\mathbf{x}_k^u = \mathbf{x}_k^f + K_k(\mathbf{z_k} - H_k * \mathbf{x_k}) \tag{29}$$

The system takes the initial forecast (or model solution), $\mathbf{x}_k^f$, and updates it through the Kalman Update equations, $\mathbf{K_k}$, and the residual, $(\mathbf{z_k} - H_k * \mathbf{x_k})$, to update the state variables, $\mathbf{x}_k^u$. The Kalman Update equation is defined as:

$$K_k = M_k H_k^T \left(H_k M_k H_k^T + R_k\right)^{-1} \tag{30}$$

The covariance matrices, M_k, and, P_k are thus obtained as:

$$M_k = \Phi_k P_{k-1} \Phi_k + Q_k \tag{31}$$

$$P_k = (I - K_k H_k)\ M_k \tag{32}$$

The system covariance matrix, M_k, is created through the functional matrix, Φ_k, and the forecasted system covariance, P_{k-1}. The R_k matrix is the measurement noise matrix, defined as:

$$R_k = E(\mathbf{v}\mathbf{v}^T) \tag{33}$$

E is the mathematical expectation operator. Q_k is the matrix that describes the discrete process noise matrix, through the process noise matrix, Q.

$$\mathbf{\Phi_k} = \mathbf{e}^{F_k T_s} \tag{34}$$

$$Q = E(\mathbf{w}\mathbf{w}^T) \tag{35}$$

$$\mathbf{Q}_k = \int_0^{T_s} \mathbf{\Phi_k}\,\mathbf{Q}\,\mathbf{\Phi_k}\,\mathbf{dt} \tag{36}$$

More explicit detailing and implementation of the EKF can be found in [10, 22, 26].

4.3.2 The Generalized Polynomial Chaos – Extended Kalman Filter

The EKF equations are modified to accept the gPC power series solutions of the state variables. The gPC method calculates the covariances of the variables through multiplication of the power series coefficients as defined by Equation (37), for normalized basis functions:

$$cov\left(x_{d,k}, x_{j,k}\right) = \sum_{i=2}^{Q} x^i_{d,k}\, x^i_{j,k} \tag{37}$$

For gPC-EKF the Kalman Update equation is defined as:

$$\mathbf{x}^{u,i}_k = \mathbf{x}^{f,i}_k + K_k\left(\mathbf{z}_k\delta\,(i-1) - H_k\mathbf{x}^{f,i}_k\right) \tag{38}$$

$$K_k = cov\left(\mathbf{x^f_k},\, \mathbf{p_k}\right) H^T_k \left(R_k + H_k cov\,(\mathbf{x}_{1\ldots 2n},\, \mathbf{x}_{1\ldots 2n})\, H^T_k\right)^{-1} \tag{39}$$

More explicit derivation of this equation can be found in reference [3]. The indexes are defined as: The subscript k indexes time. The u and f superscripts denote the updated and forecasted state space vectors. The superscript i indexes the term of the power series. The $1\ldots 2n$ subscript denotes that only the state variables, and not the parameters are to be used here. T is the matrix transpose operator. The variables are defined as: $\mathbf{z}$ is the vector of the sensor signals. R is the sensor signal noise matrix δ is the dirac delta function H is the linearized observation matrix $\mathbf{p}$ is the vector of parameters.

4.3.3 Bayesian Statistics

Bayesian Statistics can be used to estimate the parameter values. This is done by assuming that the error between the signal and the model is a statistical distribution. The Bayesian framework for parameter estimation is defined as:

$$P\left[p\,|\mathbf{z}\right] = \frac{P\left[\mathbf{z}\,|p\right]P\left[p\right]}{P\left[\mathbf{z}\right]} \tag{40}$$

For the purposes of estimation, the term $P\left[z\right]$ can be ignored as a constant scaling factor. This reduces Equation (41) to:

$$P\left[p\,|\mathbf{z}\right] \propto P\left[\mathbf{z}\,|p\right]P\left[p\right] \tag{41}$$

$P[p|z]$ is the posterior probability density function of the parameter values given the data. The term $P[z|p]$ is the statistical distribution of the error between the signal and the model. This is defined for a normal distribution as:

$$P\left[\mathbf{z}\,|p\right] = e^{-\frac{1}{2}\sum_{t=T_i}^{T_f}(\mathbf{z}_t - h_t(\mathbf{x}))^T R_t^{-1}(\mathbf{z}_t - h_t(\mathbf{x}))} \tag{42}$$

Where z_t is the signal vector at time t, and h_t is the model output vector at the same time, t. The term R_t is the signal noise matrix. The term $P[p]$ is the prior distribution of the parameters. x is the state space vector. This is one of the powerful tools of the Bayesian framework, as it incorporates previous knowledge about the distributions of the parameters. This term is used to allow the estimator to learn.

A sequence of measurements is collected over a time span, $[T_i \ldots T_f]$, and the distribution $P\left[p\,|\mathbf{z}\right]$ is calculated. The Maximum A Posteriori (MAP) estimation finds the values of the parameters that maximize this function $P\left[p\,|\mathbf{z}\right]$ The probability density function of $P[p]$ is then fed into the next estimation as the distribution for $P[p]$.

The values being estimated here are not the values of the parameters, p, but the values of the random variables, ξ. This redefines Equation (29) as:

$$P[\mathbf{z}\,|\xi\,] = e^{-\frac{1}{2}\sum_{t=T_i}^{T_f}(\mathbf{z}_t - h_t(\mathbf{x},\xi))^T R_t^{-1}(\mathbf{z}_t - h_t(\mathbf{x},\xi))} \tag{43}$$

$$P\left[\xi\,|\mathbf{z}\right] \propto P\left[\mathbf{z}\,|\xi\,\right] P\left[\xi\right] \tag{44}$$

The MAP estimate of the random variables from Equation (31) is used in the collocation matrix to return the values of the state space variables (positions and velocities) and the parameter values (mass, pitch inertia, and roll inertia) as:

$$A\left(\xi_{Est}\right)\mathbf{x}\left(t, \xi_{Est}\right) \tag{45}$$

5. Simulation Results and Discussion

5.1 Extended Kalman Filter Results

Four different simulations are performed using the Extended Kalman Filter. Table 1 shows the variation in the parameters for each simulation, and figures (3-5) show the parameter estimations for each simulation.

The values for the Mass, Pitch Inertia and Roll Inertia are set as initial estimations for the parameters. The variance for the parameters is defined as 600 kg, 700 kgm^2 and 400 kgm^2 for Mass, Pitch Inertia and Roll Inertia

Table 1.
Variations in the parameters used as inputs for the EKF estimation simulations

Run	Mass	Pitch Inertia	Roll Inertia	Poly Order
1	2250	3500	1100	2
2	2250	3500	1100	4
3	1500	2700	600	2
4	1500	2700	600	4

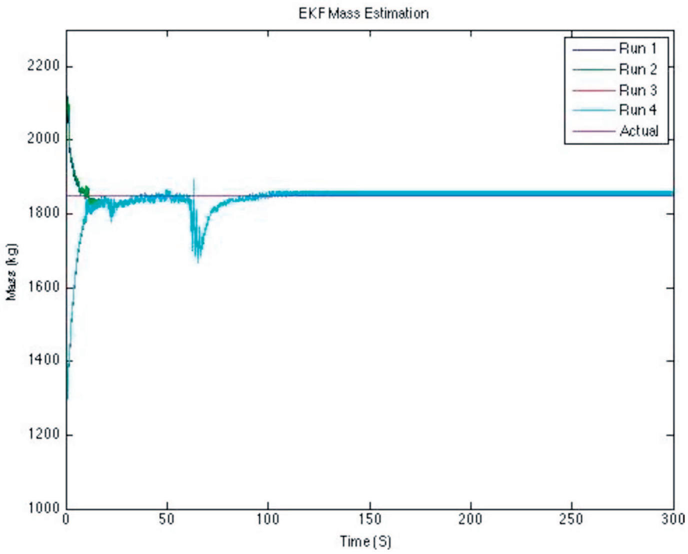

Fig. 3. EKF mass estimate versus time

respectively. The time step of integration is 0.005 seconds. Total time length is 300 seconds.

The EKF estimations are excellent when the models match well. As the sensor measurements leave the bounds imposed by the constraints used in the derivation of the model the EKF estimates diverge. This can easily be seen by the bump in the road at $t = 61s$ causing a shifting of the parameter values. The effect can be reduced through higher polynomial orders, modification of the sensor noise matrix, changes in the time step or any combination thereof. The steady state error percentages between the model parameters and the actual are listed in Table 2:

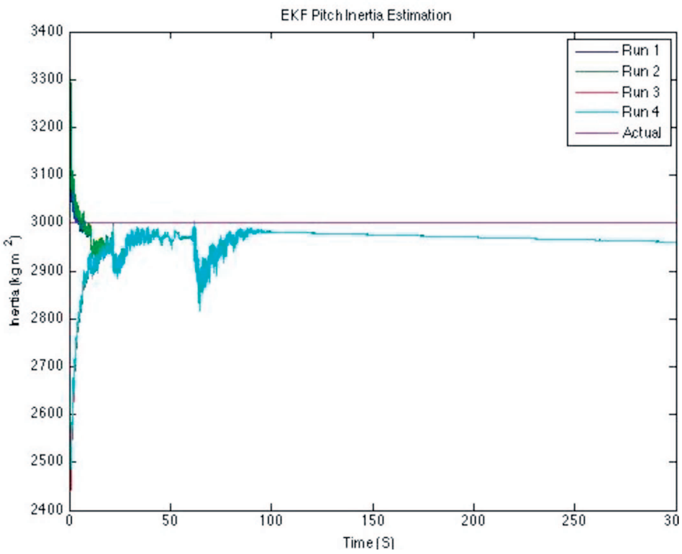

Fig. 4. EKF pitch inertia estimate versus time

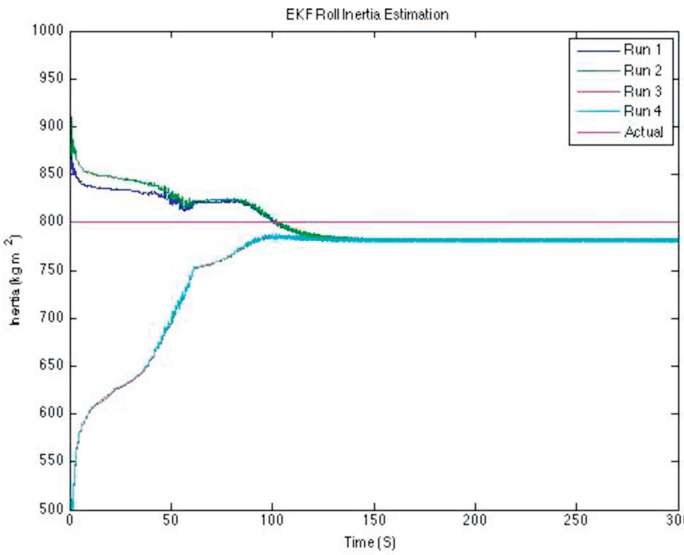

Fig. 5. EKF roll inertia versus time

Table 2.
EKF error percentages for final estimation

Mass Error %	Pitch Inertia Error %	Roll Inertia Error %
0.01	-1.31	-2.27

5.2 Bayesian Statistics

Eight experiments are detailed below in Tables 3 and 4. For each of the estimation experiments, several of the parameters are changed. These are listed below as the initial estimation of the mass, pitch inertia and roll inertia mean values, the polynomial order (Poly Order) of the gPC expansions, the length of each time interval used for estimation and the number of estimations performed.

Table 3.

Initial parameters fed into the Bayesian MAP estimation algorithm

Run	Mass	Pitch Inertia	Roll Inertia	Poly Order	Time Interval	# of Intervals
1	2250	3500	1000	6	24	1
2	2250	3500	1000	6	1	24
3	2250	3500	1000	4	24	1
4	2250	3500	1000	4	1	24
5	1650	2600	1000	6	24	1
6	2250	3500	1000	4	6	4
7	2250	3500	1000	4	60	1
8	2250	3500	1000	10	24	1

Table 3 details the results of the estimation algorithm. The table details what the final estimates are, and what their percent error is relative to the actual values of the synthetic data model.

Table 4.

Results of the Bayesian simulations

Run	Mass Est	Pitch Est	Roll Est	% Err Mass	% Err Pitch	% Err Roll
1	1905.685	3054.9	785.44	3.01%	1.83%	-1.82%
2	1897.915	3017.19	823.28	2.59%	0.57%	2.91%
3	1934.36	3159	780.08	4.56%	5.3%	-2.49%
4	1852.405	2877	833.44	0.13%	-4.1%	4.18%
5	1914	3069.4	766.7	3.46%	2.31%	-4.17%
6	1897.9	3048.3	858.2	2.59%	2.61%	7.28%
7	1802.5	2876.4	803.4	-1.59%	-4.12%	0.43%
8	1874.2	3047.3	777.7	1.31%	1.58%	-2.78%

It can be seen that the higher the polynomial order, the more accurate are the estimations. This is consistent with the proposed behavior of the gPC mathematics. The longer the time sequence fed into the Bayesian estimation algorithm the better the estimation, which is consistent with statistical theory. It can also be seen that the closer the initial estimate is, the more accurate the estimation.

6. Conclusions

This paper derives a vehicle model capable of estimating mass, pitch inertia, and roll inertia of a vehicle without the need for a terrain profile and using a reduced set of sensors. The techniques employed show acceptable agreement between the derived model and the seven DOF model. The Bayesian model is more robust in comparison to the EKF model. The EKF model can be designed to run much faster, while also providing updates to the parameters much faster.

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Metody estymacji parametrów w czasie rzeczywistym dla wyznaczania właściwości inercyjnych pojazdu terenowego

Streszczenie

Parametry pojazdu mają znaczny wpływ na jego właściwości, takie jak sterowalność, stabilność i odporność na wywrócenie. W pracy przedstawiono dwie metody estymacji parametrów inercyjnych pojazdu terenowego w czasie rzeczywistym.

W modelu pojazdu z niepewnościami wyznacza się funkcje gęstości prawdopodobieństwa (PDF) dla każdej wielkości opisując propagację niepewności przez zastosowanie techniki uogólnionego chaosu wielomianowego (gPC). Funkcje te mogą być użyte do estymacji wartości parametrów przy wykorzystaniu różnych metod statystycznych. W pracy zastosowano metodę maksymalnego estymatora a posteriori (MAP). Estymator MAP maksymalizuje funkcję rozkładu $P(\beta \mid z)$, gdzie β jest wektorem funkcji gęstości prawdopodobieństwa parametrów, a z jest wielkością mierzalną, otrzymaną z porównania wyjść czujników.

Metodą alternatywną jest zastosowanie filtru adaptacyjnego, którego przykładem jest filtr Kalmana. Metoda ta, w połączeniu z techniką uogólnionego chaosu wielomianowego (gPC), umożliwia, w każdym kroku adaptacji, uaktualnianie funkcji gęstości prawdopodobieństwa (PDF) parametrów systemu. Działanie filtru powoduje, że mediany tych funkcji zmieniają się dążąc do rzeczywistych wartości poszukiwanych parametrów.