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COMPARATIVE ANALYSIS FOR KINEMATICS OF 5-DOF INDUSTRIAL ROBOTIC MANIPULATOR

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Abstract: This paper gives the kinematic analysis of a 5-DOF industrial robotic manipulator while considering wrist in motion. Analytical solutions have been obtained for forward kinematics and inverse kinematics to accurately position the end-effector of robotic manipulator in three dimensional spaces. For the first time, a hybrid neuro-fuzzy intelligent technique with two different membership functions has been studied and their performances are comparatively evaluated with analytical solutions. An experiment has been performed for a desired trajectory. It is seen that the results for the intelligent technique are reasonably in agreement with experiment. Also, the results obtained highlight the importance of selection of a particular membership function for robotic manipulators of industrial use.

Keywords: Kinematic Analysis, Wrist, ANFIS, Membership Function, Degree of Freedom

1. INTRODUCTION

Robotic manipulators exhibit an important role in industrial automation and applications. A number of industrial tasks such as pick and place operation, assembly, welding, spray painting etc. is performed by several complex robotic systems. Since any task is performed within the pre-defined work space of a robotic manipulator, the position of end-effector plays an important role in the quality of the final product. To put it another way, the point of interest is the accuracy of reaching desired coordinates by the end-effector which leads to the successful manufacturing of a product. The end-effector positioning of any robotic manipulator can be very well understood with the help of kinematic analysis. The kinematic analysis of any robotic manipulator can be carried out by forward and inverse kinematics. While the presence of Denavit-Hartenberg convention makes the forward kinematics an easy task; calculation of inverse kinematic solutions is complex and time consuming due to non-existence of unique solution.

In terms of literature available on kinematic analysis, the well-known Denavit-Hartenberg convention for position analysis was proposed (Denavit, 1955) and has been widely adopted. The kinematic solutions for industrial manipulator PUMA 560 has been presented (Elgazzar, 1985). Based upon the convention, software programs for five or six degrees-of-freedom (DOF) robotic manipulators of general geometry have been developed (Manseur, 1996; Koyuncu, 2007) along with their theoretical analysis. The performance of different robotic manipulators has been studied for pick and place and assembly operations (Kim, 1987; Azadivar 1987). The calculation of home position of a robotic manipulator has been given (Shah et al., 2013). The theoretical background for the calculations of forward and inverse kinematics has been described exhaustively (Niku, 2009; Mittal, 2003; Saha, 2008). A geometrical approach for inverse kinematics of hyper redundant manipulators has been proposed (Yahya et al., 2011).

For complex structures of robotic manipulators, traditional methods are inadequate, highly iterative and time consuming. The difficulties of traditional methods to calculate inverse kinematic solutions can be avoided by using artificial intelligent techniques, which gives an advantage of fast computation. Most of the work presented in literature has used different artificial intelligent techniques: fuzzy logic (Agarwal et al., 2005; Homaifar et al. 1994; Bingul, 2011) and neural network (Tejomurtula, 1999; Karlik, 2000) to calculate inverse kinematic solutions of planar two-DOF and three-DOF robotic manipulators.

In recent times, the complexities of research are tried to be solved with the use of adaptive and hybrid artificial intelligent techniques. The widely used artificial intelligent techniques are fuzzy logic and neural networks. Fuzzy logic approach provides quantitative value to verbal communication. Neural networks approach provides mathematical computations of a brain. It is interesting to work on certain applications like robotic manipulators involving the proper combination of these two approaches resulting in a hybrid system. ANFIS (Adaptive Neuro-Fuzzy Intelligent System) is a hybrid combination of fuzzy logic and neural networks, mostly used to find inverse kinematic solutions of robotic manipulators (Jang, 1993). The literature search shows that ANFIS has been used for two and three DOF planar robotic manipulator (Alavander, 2008). For ANFIS implementation, even the industrial manipulators like PUMA 560 and PUMA 600 have been reduced to three DOF link movement only (Bachir, 2012; Aghajarian, 2011). A comparative study on the development and application of three main artificial intelligent techniques, namely neural networks, fuzzy logic and combination of neural networks and fuzzy logic on different robotic manipulators has been presented (Er et al., 1997; Mohan, 2007; Efe, 2000). The kinematic study for 2-DOF, 3-DOF and 5-DOF planar robotic systems using artificial intelligence techniques and analytical approach have been performed (Manjaree et al., 2010). A comparative analysis for inverse kinematic solutions obtained using

ANFIS method, geometrical approach and experimental validation has been carried out for a 3-DOF robotic manipulator (Manjaree, 2013). The inverse kinematic solutions for 3-DOF robotic manipulator using ANFIS method moving in three dimensional spaces have been presented (Manjaree et al., 2013). In this paper, experimental validation has been carried out by plotting a desired trajectory. One of the fundamental problems of robotic manipulators is in trajectory planning. The issues of trajectory planning have been discussed and resolved by various methods (Gasparetto, 2007; Kuo, 1991; Chen, 2010; Conkur, 2003). After reviewing the available literature, it can very well be concluded that inverse kinematic analysis used for multi-DOF robotic manipulators have considered restricted wrist motion and have applied ANFIS method on links movement only. The issue of accurate end-effector positioning arises when wrist in motion is also considered.

This paper focuses on three important aspects, namely analytical analysis, use of ANFIS method and experimental validation of 5-DOF pick and place type industrial robotic manipulator while considering wrist in motion. For the very first time, ANFIS method has been used on an industrial robotic manipulator involving two different membership functions. A comparative analysis for better performance by used membership functions for all possible multiple solutions of 5-DOF robotic manipulator have been presented. The research work presented in this paper have very first time incorporated wrist movement while applying ANFIS method on 5-DOF robotic manipulator which is duly validated with experimental results as well. Along with this, the paper also acts as a single platform for analyzing the 5-DOF robotic manipulator using analytical methods, intelligent methods and experiments.

2. DESCRIPTION OF 5-DOF ROBOTIC MANIPULATOR

In this paper, a 5-DOF pick and place type robotic manipulator (Pravak make) of industrial use has been considered. The robotic manipulator under study consists of 3-DOF at joints and 2-DOF at wrist. The available degree of freedom in links is sufficient to bring the end effector to the required position; however, the wrist movement provides additional flexibility to reach a particular position by the end effector. The extra degrees of freedom made available at the wrist provide greater flexibility and applicability to the complete robotic system. It also enhances the accuracy of experiments performed.

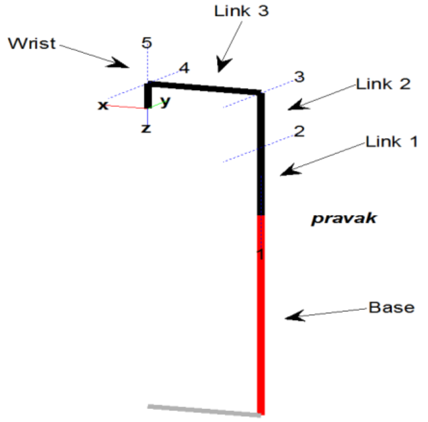


Fig. 1. Representation of 5-DOF (Pravak make) robotic manipulator

Tab. 1. Description of movement of robotic manipulator

S. No.	Type	Part of Manipulator	Movement	Rotation
1	Link 1	Waist	Left/Right	$-90^o - 90^o$
2	Link 2	Shoulder	Forward/Backward	$0^o - 180^o$
3	Link 3	Elbow	Up/Down	$0^o - 180^o$
4	Wrist	Wrist pitch	Sky-turn/Earth-turn	$0^o - 180^o$
5	Wrist	Wrist Roll	Clock-wise/Anti-clock-wise	$0^o - 360^o$

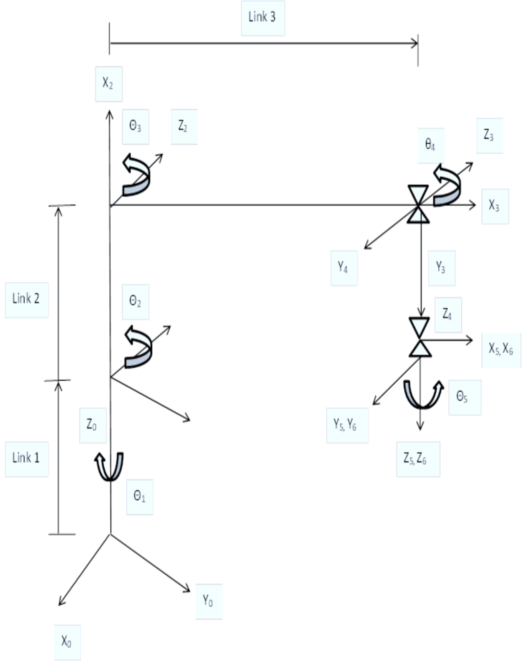


Fig. 2 Schematic representation of Denavit-Hartenberg convention for 5-DOF robotic manipulator

The 5-DOF robotic manipulator considered in present work is shown in Fig. 1. The robotic manipulator has been plotted using Peter Corke Robotics Toolbox for MATLAB (release 9.8) (Corke 2011). The complete description about the movement of each link of robotic manipulator is as quoted in Tab. 1. The schematic representation of Denavit-Hartenberg convention is as shown in Fig. 2.

3. ANALYTICAL ANALYSIS FOR FORWARD AND INVERSE KINEMATICS

The kinematic analysis of any robotic system is performed in two ways i.e. forward kinematics and inverse kinematics. The forward kinematics problem is to find the position and orientation as a function of joint variables, achieved by end-effector of robotic manipulator, as given in equation (1). The forward kinematics of multi-DOF robotic manipulators is an easy task due to the availability of Denavit-Hartenberg convention.

$$x(t) = f(\theta(t)) \qquad (1)$$

The calculation of joint variables to bring the end-effector of robotic manipulator to the required position and orientation is defined by inverse kinematics problem, as given in equation (2).

$$\theta(t) = f'(x(t)) \tag{2}$$

As compared to forward kinematics, calculation of inverse kinematic solutions is a complex task since there is no possible unique solution due to non-linear and time-varying nature of its governing equation. The inverse kinematics of multi-DOF robotic manipulator can be obtained using three different techniques, viz. algebraic approach, geometric approach and iterative approach. In this paper, analytical kinematic analysis for 5-DOF pick and place type robotic manipulator has been performed using algebraic approach. The Denavit-Hartenberg convention has been used to obtain the forward kinematic equations, as given in Tab. 2.

Tab. 2. Denavit-Hartenberg convention table for Pravak manipulator

Joint	$\boldsymbol{\theta_i}$ (°)	$\boldsymbol{\alpha_i}$ (°)	$\boldsymbol{a_i}$	$\boldsymbol{d_i}$
1	θ1	-90	0	L0
2	θ2	0	L1	0
3	θ3	0	L2	0
4	θ4 - 90	-90	0	0
5	θ5	0	0	L3

By substituting the Denavit-Hartenberg parameters (θ_i, d_i, a_i, α_i) in the general matrix given in equation (3); the transformation matrices A_1 to A_5 are obtained as below:

$$A_{n+1} = \begin{bmatrix} C\theta_{n+1} & S\theta_{n+1}C\alpha_{n+1} & S\theta_{n+1}S\alpha_{n+1} & a_{n+1}C\theta_{n+1} \\ S\theta_{n+1} & C\theta_{n+1}C\alpha_{n+1} & -C\theta_{n+1}S\alpha_{n+1} & a_{n+1}S\theta_{n+1} \\ 0 & S\alpha_{n+1} & C\alpha_{n+1} & d_{n+1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & L_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & L_1C_2 \\ S_2 & C_2 & 0 & L_1S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & L_2C_3 \\ S_3 & C_3 & 0 & L_2S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_4 = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & L_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Mathematically, the forward kinematics equations can be obtained by multiplying A1 to A5 matrices as given in equation (4):

$$A_5^0 = A_1 \ldots\ldots A_5 \tag{4}$$

which results to, $A_5^0 = \begin{bmatrix} R_{3\times3} & & & p_{1\times3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

After applying the above steps, the forward kinematic equations for 5-degree of freedom robotic manipulator under study has been obtained as given in equations (5-7):

$$p_x = -L_3 \times C_1 \times S_{234} + L_2 \times C_1 \times C_{23} + L_1 \times C_1 \times C_2 \tag{5}$$

$$p_y = -L_3 \times S_1 \times S_{234} + L_2 \times S_1 \times C_{23} + L_1 \times S_1 \times C_2 \tag{6}$$

$$p_z = -L_3 \times C_{234} - L_2 \times S_{23} - L_1 \times S_2 + L_0 \tag{7}$$

where $S_{23} = sin(\theta_2 + \theta_3) = S_2C_3 + C_2S_3$ *and* $C_{23} = cos(\theta_2 + \theta_3) = C_2C_3 - S_2S_3$.

The orientation of end-effector of 5-DOF robotic manipulator has been given below in equations (8-16):

$$n_x = C_1S_{234}C_5 + S_1S_5 \tag{8}$$

$$n_y = S_1C_{234}C_5 - C_1S_5 \tag{9}$$

$$n_z = -C_{234}C_5 \tag{10}$$

$$o_x = -C_1S_{234}S_5 + S_1C_5 \tag{11}$$

$$o_y = -S_1S_{234}S_5 - C_1C_5 \tag{12}$$

$$o_z = C_{234}S_5 \tag{13}$$

$$a_x = C_1C_{234} \tag{14}$$

$$a_y = S_1C_{234} \tag{15}$$

$$a_z = -S_{234} \tag{16}$$

The 5-DOF robotic manipulator used in this study comprises of a 2-DOF wrist motion. The sufficient condition to solve inverse kinematics is that it has two intersecting axes. For these types of manipulators it is possible to separate inverse kinematic problem into two sub-problems: position and orientation. To put it in another way, the 5-DOF robotic manipulator has 3-DOF available at links to find the end position of wrist and 2-DOF available at wrist to find the orientation of the wrist. It implicates that the robotic manipulator under study has closed form solutions. Thus, the wrist position p_W can be calculated as:

$$p_W = p_e - L_3a_e \tag{17}$$

where $\mathrm{p_e}$ denotes the end-effector position and orientation is specified in terms of $R_e = [n_e \quad o_e \quad a_e]$, respectively.

Equation (17) gives the generalized form of expression for calculation of wrist position. The components of above equation in x, y and z directions is given in equation (18).

$$\begin{bmatrix} p_{wx} \\ p_{wy} \\ p_{wz} \end{bmatrix} = \begin{bmatrix} p_{ex} - L_3a_{ex} \\ p_{ey} - L_3a_{ey} \\ p_{ez} - L_3a_{ez} \end{bmatrix} \tag{18}$$

The inverse kinematics solution for the complete 5-DOF robotic manipulator has been obtained by a closed solution of the above equation (18). Thus, the general solutions for the joint angles have been given below in equations (19-23) as:

$$\theta_1 = a\tan2(p_y, p_x) \tag{19}$$

$$\theta_2 = a\tan2\left(\frac{L_0 - p_z}{p_xC_1 + p_yS_1}\right) - a\tan2\left(\frac{L_2S_3}{\sqrt{\left(p_xC_1 + p_yS_1\right)^2 + (L_0 - p_z)^2 - (L_2S_3)^2}}\right) \tag{20}$$

where $S_1 = \pm\sqrt{1 - C_1^2}$

$$\theta_3 = a\tan2(S_3, C_3) \tag{21}$$

where

$$C_3 = \frac{\left(p_xC_1 + p_yS_1\right)^2 + (L_0 - p_z)^2 - L_1^2 - L_2^2}{2L_1L_2}, S_3 = \pm\sqrt{1 - C_3^2}$$

Also, the general solutions for wrist rotations are obtained as:

$$\theta_4 = a\tan2\left(\left(a_x C_1 S_{23} + a_y S_1 S_{23} + a_z C_{23}\right), \left(a_z S_{23} - a_x C_1 C_{23} - a_y S_1 C_{23}\right)\right) \tag{22}$$

$$\theta_5 = a\tan2\left(\left(n_y C_1 - n_x S_1\right), \left(o_y C_1 - o_x S_1\right)\right) \tag{23}$$

The above inverse kinematic solution gives one value of θ_1 and two values of θ_2 and θ_3 each as per the rotations of link 1, link 2 and link 3 given in Tab. 1. Two solutions are possible for θ_4 and one solution exist for θ_5 as per wrist rotations. It is clear from the obtained solutions that a number of multiple solutions are possible. It can be seen from equations (5-7) that a total of 8 multiple solutions exist for all possible combinations of joint angles with wrist rotation for the robotic manipulator under study. In this case, traditional mathematical computations for multiple data sets are almost impossible to perform due to their highly iterative and time-consuming nature. This disadvantage opens up way for the use of artificial intelligence techniques in the field of robotics. This paper has used a hybrid neuro-fuzzy intelligence technique known as ANFIS method to obtain end-effector position of 5-DOF (Pravak make) robotic manipulator.

4. ANFIS ARCHITECTURE WITH TWO DIFFERENT MEMBERSHIP FUNCTIONS

ANFIS method is a hybrid neuro-fuzzy intelligent technique which is functionally equivalent to Sugeno fuzzy inference system. It is a hybrid combination of artificial neural networks and fuzzy logic which basically exploits the individual advantages. It constructs a fuzzy inference system whose membership functions are tuned either by back propagation method alone or by a combination of least square method. The learning capabilities of artificial neural networks are brought into fuzzy inference system.

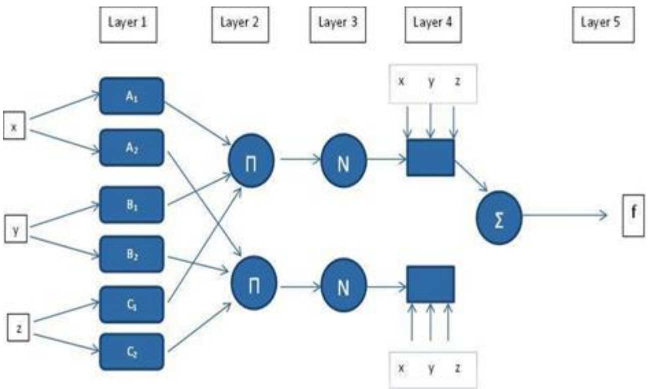


Fig. 3. ANFIS architecture

ANFIS method works in five layers (as shown in Fig. 3). The role of each layer is described as follows:

Layer 1: Calculation of membership function value for input parameter.

Each node in the first layer is an adaptive node where the node function O_i^k (for i^{th} position of k^{th} layer) is calculated as:

$$O_i^1 = u_{Ai}(x);\ O_i^1 = u_{Bi}(y);\ ;\ O_i^1 = u_{Ci}(z) \tag{24}$$

where (x, y, z) is input vector and $(u_{Ai},\ u_{Bi},\ \mathrm{u}u_{Ci})$ is the membership function for that particular input.

Layer 2: Firing strength of rule or output of every node is the product of all incoming signals.

Here, the output of each node is the product of membership functions.

$$O_i^2 = W_i = u_{Ai}(x) u_{Bi}(y) u_{Ci}(z), i = 1,2,3 \tag{25}$$

Layer 3: Normalize firing strength. In other words, each of firing strengths of rules is compared with sum of all firing strengths.

It contains fixed nodes which calculate the ratio of the firing strengths of the rules:

$$\overline{O_i^3} = W_i = (W_i\ /W_1 + W_2 + W_3) \tag{26}$$

Layer 4: Consequent parameter or linear combination of input variables of ANFIS with constant terms to form the output of each IF-THEN rule.

The nodes in this layer are adaptive and perform the consequent of the rules:

$$\overline{O_i^4} = W_i f_i \tag{27}$$

Layer 5: It performs the defuzzification process.

There is a single node here that computes the overall output:

$$\overline{O_i^5} = \sum W_i f_i \tag{28}$$

The reported literature consists of ANFIS method being applied on planar robotic manipulators only. In this paper, for the first time, ANFIS method has been used to find the position of end-effector of 5-DOF robotic manipulator including 2-DOF wrist motion and moving in three dimensional spaces. For the first time, two ANFIS architectures of first order Sugeno fuzzy inference system based upon two different membership functions (MF's) have been considered. An attempt has been made for the very first time, to clearly demonstrate the effect of different MF's on the functioning of robotic manipulator. The paper also highlights the significance of selection of a particular MF. Here, the selection of MF's is primarily based on their feature of being smooth and non-zero at all points. Based on this criterion, generalized bell (gbellmf), Gaussian (gaussmf and gauss2mf), sigmoidal (dsigmf and psigmf) and spline based curve (pimf) MF's qualify for selection. Out of these six, only two namely Gaussian MF and generalized bell MF are showing acceptable results. The details related to used MF's has been given below:

Case a: The generalized bell MF's with product inference rule have been used in fuzzification level while defuzzification has been performed using weighted average method. A bell MF is given by equation (29), where parameter 'c' gives distance from origin, parameter 'a' shows curve width and parameter 'b' is normally positive. Its representation is shown in Fig. 4 (a).

$$\mu(x; a, b, c) = \frac{1}{1 + \left|\frac{x - c}{a}\right|^{2b}} \tag{29}$$

Case b: In this case Gaussian MF's with product inference rule have been used for fuzzification while defuzzification has been performed using weighted average method. A symmetric Gaussian MF is given by equation (30), where parameter 'c' gives the distance from origin and 'σ' shows curve width. Its representation is shown in Fig. 4 (b).

$$\mu(x;\sigma,c) = e^{\frac{-(x-c)^2}{2\sigma^2}} \tag{30}$$

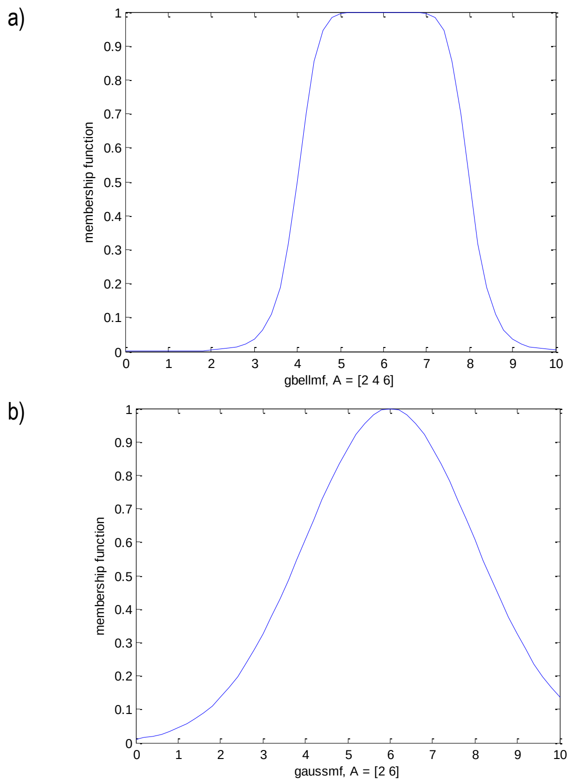


Fig. 4 Representation of generalized bell MF and Gaussian MF

Two ANFIS architectures with two different MF's (as discussed earlier) have been used individually on 5-DOF robotic manipulator. A comparative analysis of these two MF's of ANFIS method based on positioning error for end-effector coordinates has been presented in later paragraphs.

5. IMPLEMENTATION OF ANFIS METHOD ON 5-DOF ROBOTIC MANIPULATOR

In this section of paper, the complete process of implementing ANFIS method on used 5-DOF robotic manipulator has been discussed. The basic procedure of ANFIS method is defined in four steps:

Step (1): Initialization of fuzzy inference system using *genfis1* or *genfis2* command.
Step (2): Define learning parameters such as membership functions, number of epochs and so on
Step (3): Start the learning process using *anfis* command and
Step (4): Validation of individual data set, respectively.

ANFIS method works in two phase viz. training phase and testing phase. In trained ANFIS data, the (x, y, z) coordinates of end-effector of 5-DOF robotic manipulator and joint angles act as the input. Here, five training data sets comprising of coordinates and joint angles has been considered as (x, y, z, θ_1), (x, y, z, θ_2), (x, y, z, θ_3), (x, y, z, θ_4) and (x, y, z, θ_5), respectively.

The respective MF's and number of rules have been assigned for each training data set. Data set 1 consists of seven MF for each end-effector coordinate leading to a total of 343 rules. Data set 2 consists of six MF for each coordinate leading to a total of 216 rules. Data set 3 have five MF each and total 125 rules. Four MF for each coordinate and 64 rule in total has been assigned to data set 4. Dataset 5 has three MF for each coordinate and a total of 27 rules. In this paper, these five data sets have been used on two ANFIS architectures concerning two different MF. The number of epochs used is 10.

The adaptive nature of ANFIS method means that both the premise and consequent parameters are adjustable. The complete adaptive process of ANFIS method is divided into two steps. Consequent parameter training is the first step which uses least square method because the ANFIS output is a linear combination of consequent parameters. During this step, the premise parameters are fixed. In the second step, approximation error is back propagated through every layer to update premise parameters. This part of learning procedure is based on gradient descent principle which is equivalent to training of back propagation neural network. The consequence parameters identified by least square method are optimal as the premise parameters are fixed. This hybrid learning algorithm is more effective than gradient descent method as it reduces the search space dimensions of original back propagation neural network. With this hybrid learning algorithm, ANFIS converges with smaller number of iterations.

The output of ANFIS method has been obtained in the form of variations in $(\mathrm{x}, \mathrm{y}, \mathrm{z})$ coordinates for 5-DOF robotic manipulator. The variations in coordinates help us to understand the necessity of proper positioning of end-effector of a robotic manipulator. The difference between the $(\mathrm{x}, \mathrm{y}, \mathrm{z})$ coordinates calculated using forward kinematic equations and $(\mathrm{x}, \mathrm{y}, \mathrm{z})$ coordinates calculated using ANFIS acts as the individual data set for validation of proper functioning of ANFIS. Out of 0.1 million points generated as coordinate, a total of 500 observation points have been considered to plot the error of end-effector in achieving x-coordinate, y-coordinate and z-coordinate, respectively.

5.1. Membership functions analysis of ANFIS method for positioning error

Here, ANFIS method has been used to find the error obtained in achieving defined coordinates by the end-effector of robotic manipulator. For the very first time, two different MF's, generalized bell MF and Gaussian MF have been used with ANFIS method on 5-DOF robotic manipulator. The accurate positioning of end-effector of robotic manipulator (with wrist in motion) plays an important role in any industrial application. With this point of interest, dataset involving all possible combinations of multiple solutions with respect to joint angles and wrist rotations have been formed. To reach x and y coordinates by end-effector of robotic manipulator, eight set of multiple solutions {(θ_{11}, θ_{21}, θ_{31}, θ_{41})........, (θ_{12}, θ_{22}, θ_{32}, θ_{42})..........and so on} are possible. To reach z coordinate, eight set of multiple solutions {(θ_{21}, θ_{31}, θ_{41})........, (θ_{22}, θ_{32}, $\theta\theta_{42}$)..........and so on} are possible.

In the training phase of ANFIS method, the end-effector coordinates have been calculated for each data set using forward kinematic equations, better known as 'deduced' coordinates. The obtained values of end-effector coordinates in training phase have been taken as reference values. Then the joint angles have been predicted by ANFIS method which in turn has been used to predict end-effector coordinates of 5-DOF robotic manipulator. In this case, the coordinates obtained using ANFIS method is known as 'predicted' coordinates. The above process has been used for both generalized bell MF and Gaussian MF.

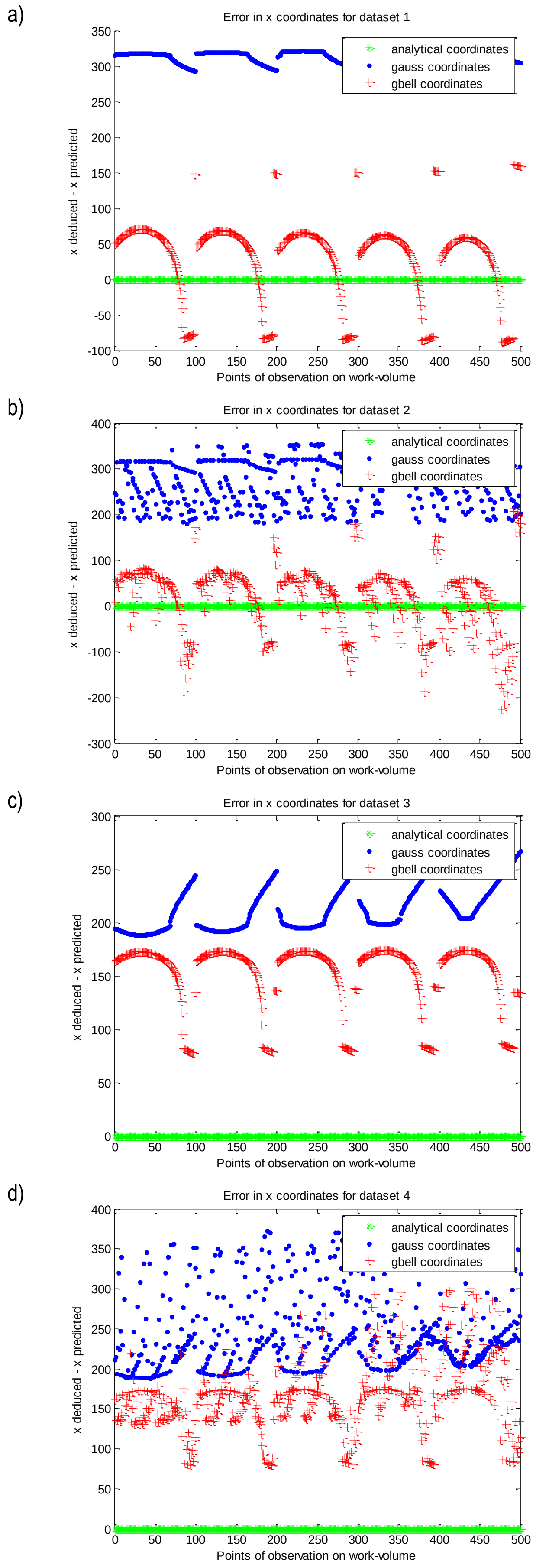
The difference between deduced coordinates and predicted coordinates gives the error in proper positioning of end-effector of robotic manipulator. In the end, the average % errors have been calculated by dividing the error in positioning of end-effector by deduced coordinates of robotic manipulator. For each dataset, average percentage error has been obtained in x, y, z axes using two MF's individually, has been given in Tab. 3.

Tab. 3. Average % error obtained in coordinates using ANFIS method with two different MF's

Dataset	% error using Generalized bell MF			% error using Gaussian MF		
	x	y	z	x	y	z
1	0.41	0.67	2.40	0.4	0.13	0.52
2	0.3	0.35	2.84	0.28	0.44	0.29
3	0.30	1.01	1.57	0.29	0.20	1.05
4	0.35	1.13	0.96	0.34	0.33	1.36
5	0.36	1.16	0.90	0.35	0.36	1.32
6	0.37	0.27	2.46	0.36	0.52	0.43
7	0.26	0.57	3.00	0.25	0.23	0.12
8	0.33	1.24	1.58	0.32	0.44	0.97

As quite evident from Tab. 3, average % error varies from (maximum – 0.4104, minimum – 0.2690) in x axis, (maximum – 1.1672, minimum – 0.2789) in y axis and (maximum – 3.0098, minimum – 0.9073) in z axis using generalized bell MF. The average % error varies from (maximum – 0.4, minimum – 0.2586) in x axis, (maximum – 0.5234, minimum – 0.1323) in y axis and (maximum – 1.3635, minimum – 0.1286) in z axis using Gaussian MF. It can be clearly seen that generalized bell MF gives least average % error for dataset 4 combination. Similarly, Gaussian MF gives least average % error for dataset 7 combination.

The error in all eight set of multiple solutions for x-axis coordinates have been plotted in Fig. 5 (a to h). Similarly, the errors in y-coordinates and z-coordinates have been plotted in Fig. 6 (a to h) and Fig. 7 (a to h), respectively. Fig. 5 (d) and Fig. 7 (d) shows that for dataset 4, generalized bell MF gives better results than Gaussian MF for error in x -coordinates and z-coordinates, respectively. Fig. 5 (g) and Fig. 7 (g) shows that for dataset 7, Gaussian MF gives better results than generalized bell MF for x and z coordinates errors. Fig. 6 (d) and Fig. 6 (g) shows almost similar variations for y-coordinates in dataset 4 and dataset 7, respectively. The selection of dataset has been done on the basis of least average % error obtained in coordinates using ANFIS method with two different MF's. It can be clearly seen that overall Gaussian MF provide better results as compared to generalized bell MF of ANFIS method in end-effector positioning. Since the results during the present study have been obtained in 3D space while considering wrist in motion for the very first time using ANFIS method, the range of error is quite acceptable.



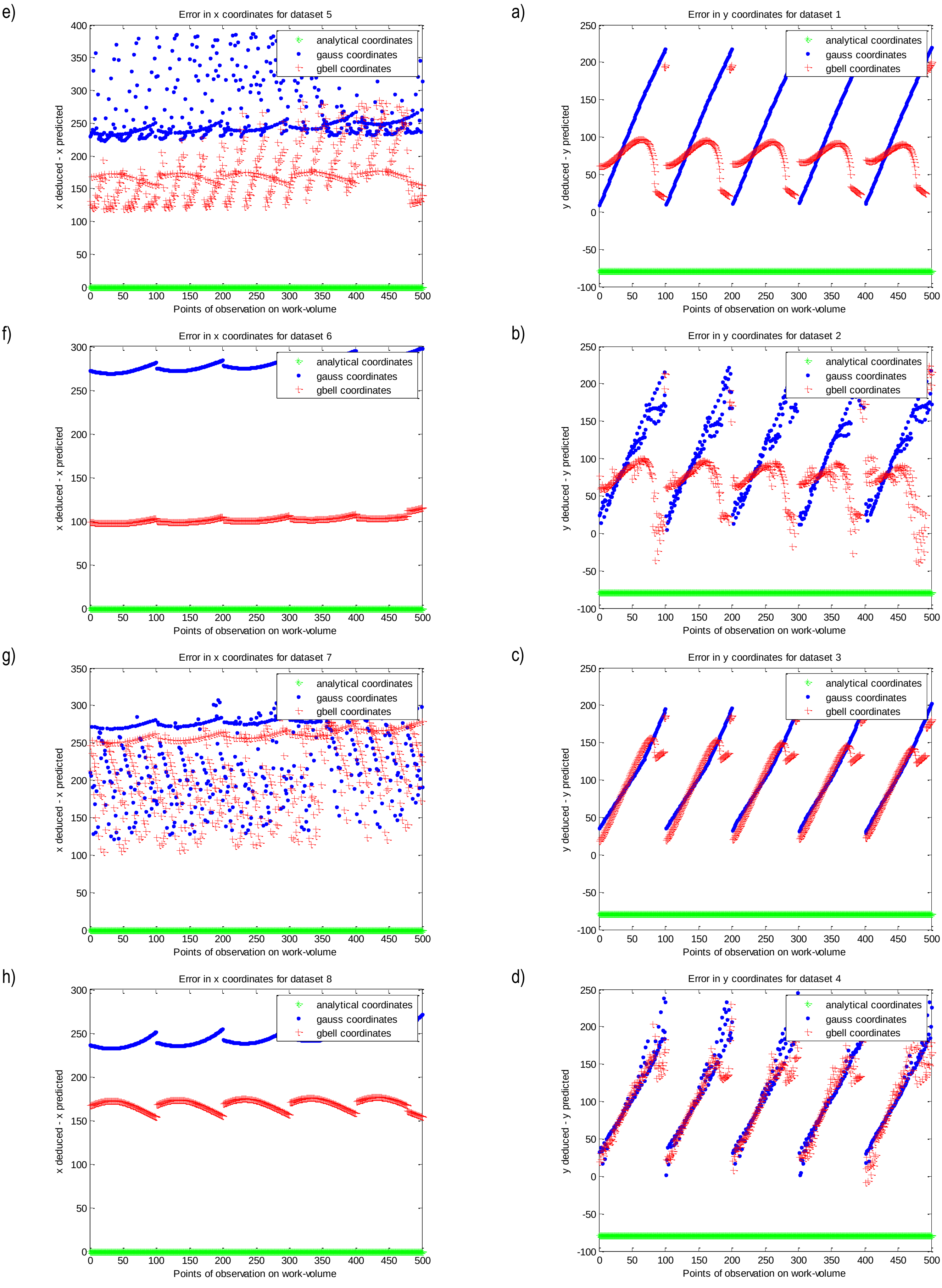


Fig. 5. Comparison of error in x coordinates for eight set of solutions using analytical method and ANFIS method with two MF's

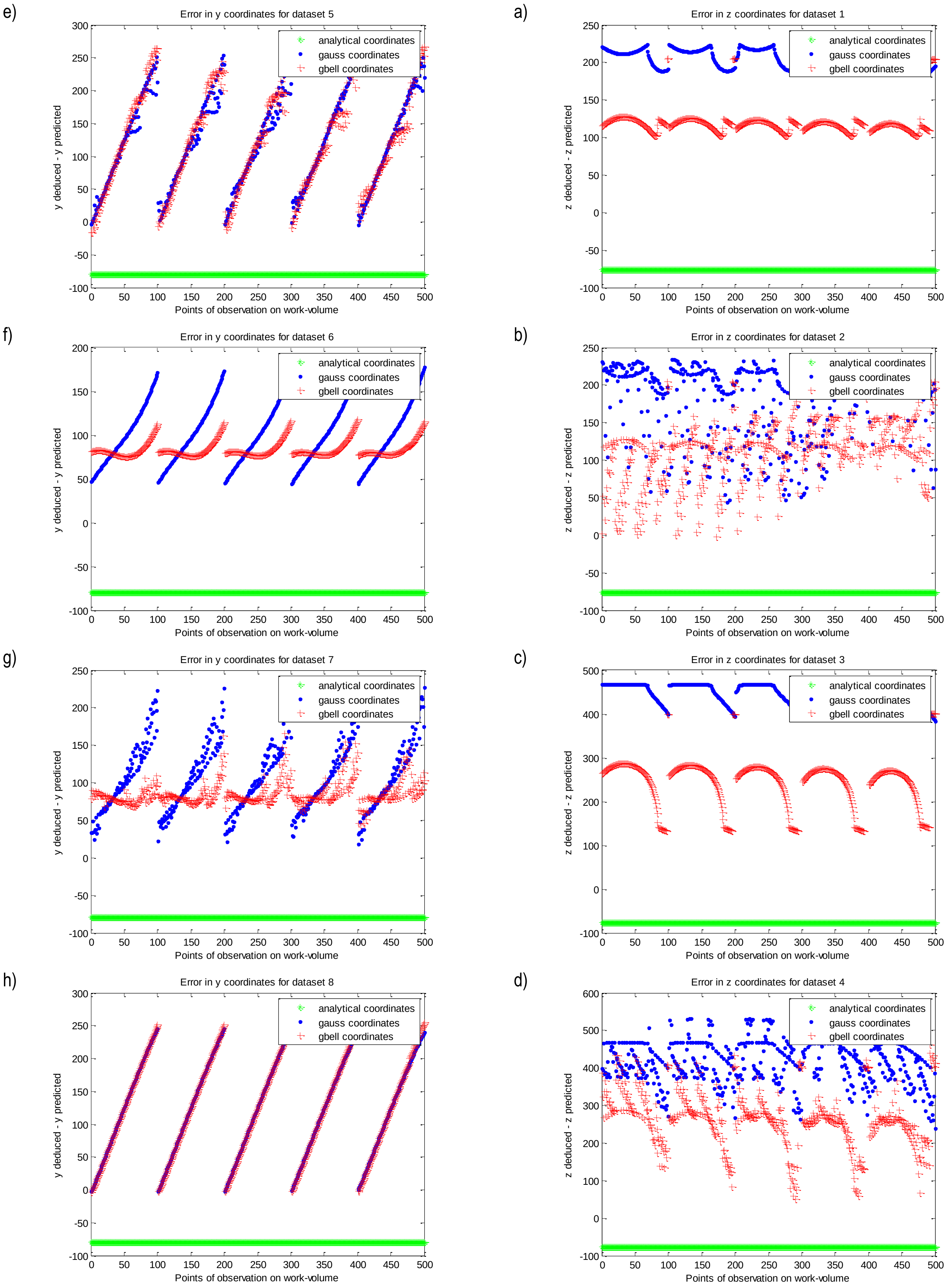


Fig. 6. Comparison of error in y coordinates for eight set of solutions using analytical method and ANFIS method with two MF's

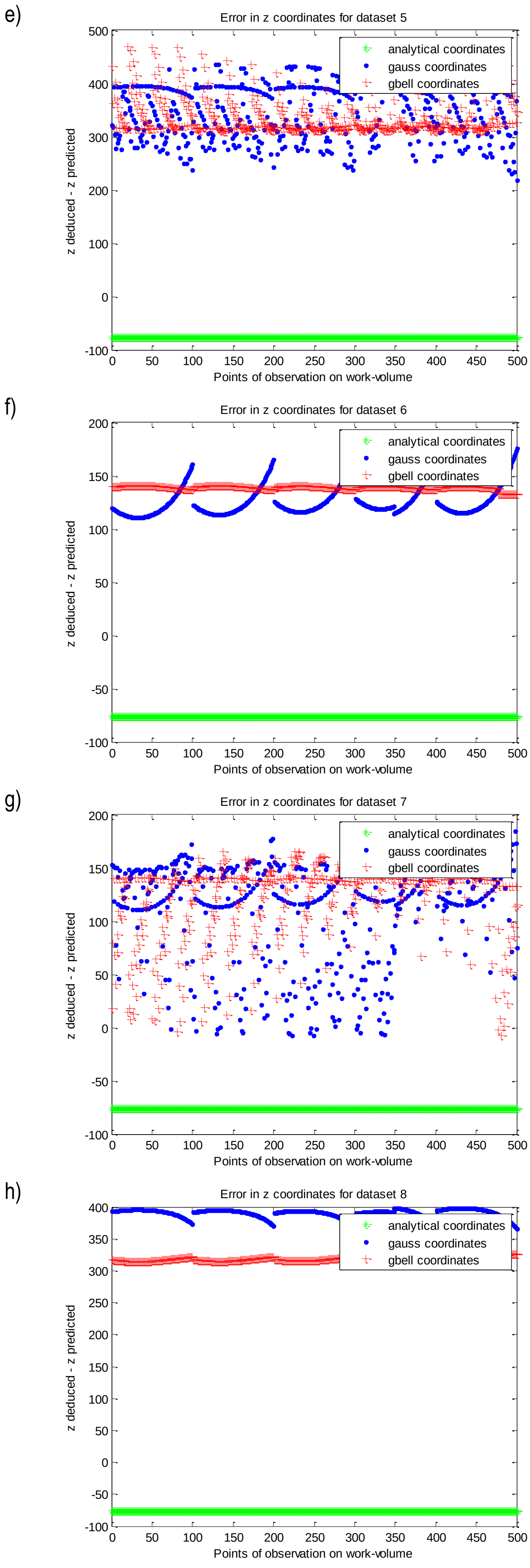


Fig. 7 Comparison of error in z coordinates for eight set of solutions using analytical method and ANFIS method with two MF's

The use of artificial intelligence techniques may be considered as methods for conducting experiments on digital computers, as substitutes for experiments that are impossible in reality or as special case of experiments. However, the use of computer simulations with different methods in experimental activities related to robotic manipulators involves a number of problems: (a) to what extent can we 'trust' results coming from simulations? and (b) is it reasonable to simulate a real robot by operating it in a scale environment? The above raised problems can be solved by validating the results obtained using analytical method and artificial intelligence method with the help of experiments. Experiments form the essential part of science/engineering with a role to either confirm or decline a theory and to find out new theories. It is also reasonable to expect that it can be useful in engineering, especially when the behavior and performance are difficult to characterize analytically, as it has often been the case in robotics. The experiment performed also gives insight of the actual movements of links and joints of robotic manipulator which is of utmost importance for any industrial/medical application. In this paper, the results obtained using analytical method and ANFIS method have been experimentally validated by moving the end-effector of 5-DOF robotic manipulator on desired trajectory.

6. EXPERIMENTAL SET UP

The effectiveness of ANFIS method using two different membership functions along with analytical method has been experimentally verified by 5-DOF robotic manipulator. For experimentation, it is desired to move the end-effector of robotic manipulator on the circumference of a 'circle' trajectory. The trajectory has been selected in such a way that the end-effector of robotic manipulator is free to move on the desired coordinates without getting struck to any singular condition. It is evident that multiple solutions exist for the robotic manipulator under study. The end-effector coordinates obtained using analytical and ANFIS methods have been validated experimentally for all the eight set of possible multiple solutions.

The complete experimental set up for 5-DOF robotic manipulator is as shown in Fig. 8 (a). The robotic manipulator under study has three links where L_o = 226 mm, L_1 = 177 mm, L_2 = 179 mm are the respective link lengths and L_3 = 80 mm, is the distance between wrist center and end-effector center (as given in Tab. 2). The 'circle' trajectory as drawn by the 5-DOF robotic manipulator is shown in Fig. 8 (b).

The complete trajectory is divided into two parts namely, outer half and inner half with two quadrants each having 15 reading points in total. Here, the robotic manipulator is controlled by six stepper motors (Pravak 2008), where five motors are used to move the joints while the sixth motor opens and closes the gripper. The stepper motors move by an angle of 7.5° in each step, which subsequently moves the joints. Thus, the position of the joint can be calculated by counting the number of steps. The 5-DOF Pravak make of robotic manipulator has a dedicated micro-controller located in the base of the robot, which controls all the operations of the robot. The micro-controller communicates with a computer through serial port. It is a closed loop control system where, the feedback is sent by the robot every 100 ms.

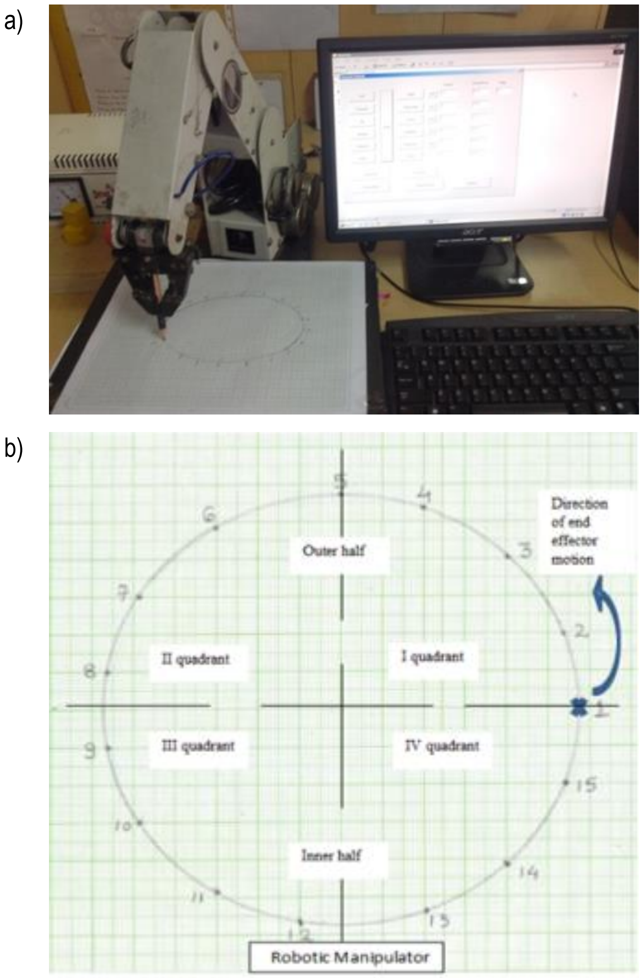


Fig. 8. a) Actual photograph of experimental set
b) Labelled diagram of ‘circle’ trajectory

6.1. Positioning error analysis using analytical and ANFIS methods with experiment validation

In this paper, the end effector position of robotic manipulator for a ‘circle’ trajectory has been obtained using analytical and ANFIS methods which are duly validated by experimental data obtained in the laboratory as per the set up discussed above. As discussed in section 6, the joints of robotic manipulator are operated by stepper motors which move by a precise amount of 7.5° with each step, thereby, counting the number of steps moved gives a very good idea of position of the joint. Then with reference to the home position of robotic manipulator, the coordinates of end-effector at each point on trajectory has been calculated. The forward kinematic equations have been used to obtain analytical solutions of coordinates of end-effector at each point on desired trajectory, where experimentally obtained joint angles for specific points on ‘circle’ trajectory act as the input.

ANFIS method has been used once with generalized bell MF and secondly with Gaussian MF. From the set of multiple solutions obtained (as given in Tab. 3), dataset 4 for generalized bell MF and dataset 7 for Gaussian MF have been used for experimental validation. The reason for dataset selection is based upon the criteria of least error to reach the desired coordinates by the end-effector of the robotic manipulator. Here, the analytical solutions for end-effector coordinates have been taken as reference values; using which the average % error for generalized bell MF, Gaussian MF and experiment has been calculated. Tab. 4 gives the average percentage error obtained in ANFIS method with generalized bell MF, ANFIS method with Gaussian method and experimental data for desired ‘circle’ trajectory. It clearly shows that Gaussian MF provide better results as compared to generalized bell MF for a 5-DOF robotic manipulator with wrist movement, moving in three-dimensional spaces.

The end-effector coordinates for all the specific points on ‘circle’ trajectory obtained using analytical method, ANFIS method with two different MF’s and experiment have been plotted along x-axis, y-axis and z-axis, as shown in Fig. 9. It lays the background in order to understand the difference between analytical movement and actual movement of industrial robotic manipulators.

Tab. 4. Comparison of average % error obtained in coordinates of ‘circle’ trajectory

Coordinates	Generalized bell MF	Gaussian MF	Experiment
x	3.28	0.81	10.18
y	29.17	4.62	01.82
z	1.60	0.03	12.48

Fig. 9 (a, b and c) show a comparative analysis of all the used methods. The end effector positioning is highly influenced with the movement of each link and wrist depending upon the complexity of robotic manipulator. Here also, the obtained results can be well understood with the help of motion of each link and wrist of robotic manipulator, as shown in the experimental set up. This analysis also helps in identifying the importance of rigidity of links and end effector required for improved performance during industry use.

Fig. 9 (a) gives a comparison of end effector positioning along x -axis obtained using all three methods. It can be seen that the results obtained using analytical and ANFIS methods (with generalized bell MF and Gaussian MF) are matching quite accurately with each other throughout the ‘circle’ trajectory. Apart from this, these results are also showing good agreement with experimental results in the outer half of ‘circle’ trajectory. However, small deviations from experimental results can be seen in the inner half of the ‘circle’ as compared to other two methods.

Fig. 9 (b) shows a comparison of end effector positioning along y-axis obtained using all three methods. It can very well be concluded that the results obtained using analytical and ANFIS method (with Gaussian MF) are showing good agreement with the experimental results at all points along ‘circle’ trajectory. The results for ANFIS method with generalized bell MF are little deviated as compared with the results of other methods.

Fig. 9 (c) shows a comparison of end effector positioning along z-axis obtained using all three methods. It can be seen that the results obtained using analytical and ANFIS methods are showing deviations at some points along ‘circle’ trajectory which are not significant. The Gaussian MF of ANFIS method is matching quite accurately with analytical method as compared to generalized bell MF. However, these results are showing considerable deviations from experimental results at all points along ‘circle’ trajectory especially in the last three quadrants.

A good agreement between results obtained using analytical and ANFIS methods supports the equations and methodology presented in this paper. On the other hand, deviations in these results from experimental results are due to inherent irregular motion in the links and wrist by the virtue of physical constraint of experimental set up used in this research work. This physical constraint results in erroneous plotting of 'circle' trajectory coordinates on the graph. Whereas joint angles required as inputs for analytical and ANFIS methods are calculated using step count of stepper motors of robotic manipulator only which remains unaffected by these constraints observed at end effector. This limitation of experimental set up is showing its effect largely while plotting inner half of the 'circle' trajectory resulting due to end-effector being closer to the body of the robotic manipulator.

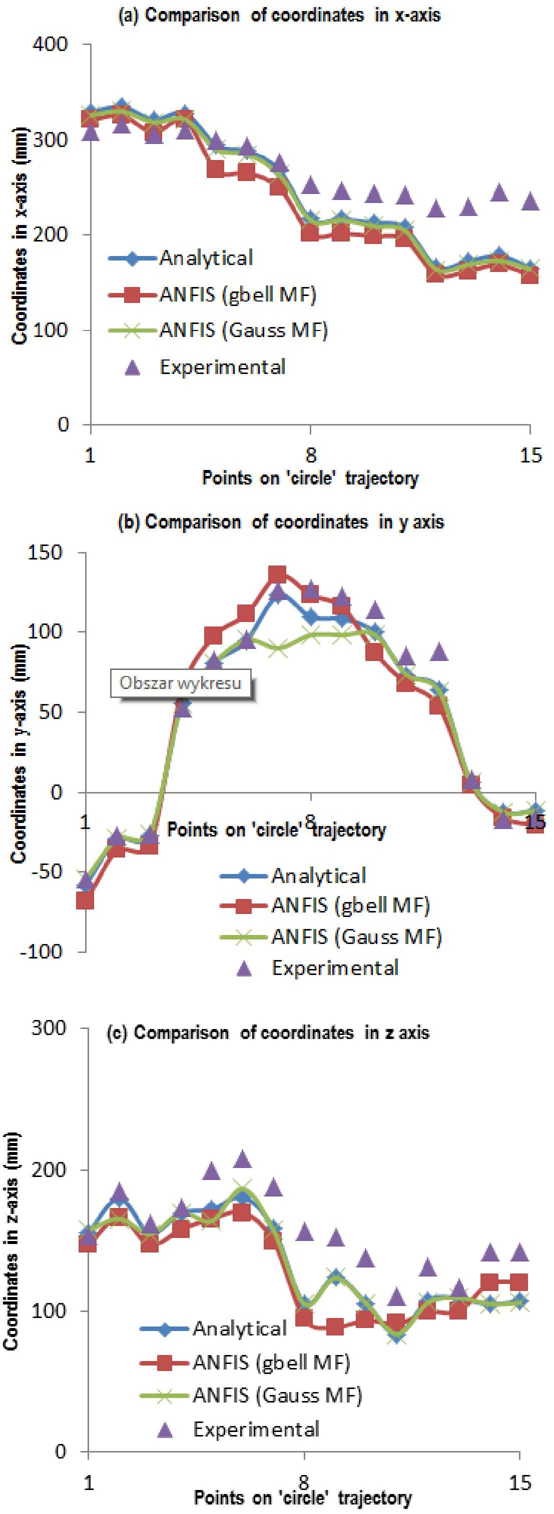


Fig. 9. Comparison of positioning errors of end-effector of robotic manipulator

7. CONCLUSIONS

The work presented in this paper has been tried to provide a single platform for analytical analysis, simulation and experimental methods for a 5-DOF robotic manipulator with wrist in movement. The experimental validation has been performed for a 'circle' trajectory.

The following specific conclusions are drawn from the research work:

i. Forward and inverse kinematic equations have been derived for analytical solution of a 5-DOF robotic manipulator with 2-DOF wrist movement in consideration.
ii. ANFIS method with two different MF's i.e. generalized bell MF and Gaussian MF have been applied for the very first time on a 5-DOF robotic manipulator considering wrist in movement.
iii. The average percentage error obtained to reach desired coordinate shows that Gaussian MF provides better results than generalized bell MF for a 5-DOF robotic manipulator moving in three-dimensional spaces.
iv. A good agreement between results obtained using analytical and ANFIS methods supports the equations and methodology presented in this paper.
v. Analytical and ANFIS results show good agreement with experimental results in first and fourth quadrants along 'circle' trajectory for x and z coordinates. However, deviations in these results are observed in y coordinates.

The future scope of the work can be as follows:

i. Further studies can be made to more complex architectures of robotic manipulators.
ii. Various other hybrid or non-hybrid artificial intelligent techniques can be applied which may produce far more accurate results.

Nomenclature: a_i, d_i, α_i, θ_i (i = 1,2,3,4,5) – the Denavit-Hartenberg parameters; L_i (i = 0,1,2,3) – respective link lengths; p_e – end-effector coordinates; p_w – coordinates of wrist; p_x, p_y, p_z – translation about x-axis, x -axis and x -axis; n_x, n_y, n_z, o_x, o_y, o_z, a_x, a_y, a_z – rotation about x -axis, x -axis and x -axis; C_i (i = 1, 2, 3, 4, 5) – cosine; S_i (i = 1, 2, 3, 4, 5) – sine; A_{i} – the transformation matrix; DOF – degree of freedom; MF – membership function.

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