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Some remarks on the tolerance averaging of heat conduction in chessboard palisade-type periodic composites
Uśrednianie tolerancyjne przewodnictwa ciepła w kompozytach palisadowych o przekroju typu szachownicy

Key words: heat transfer, biperiodic conductors, tolerance averaging, boundary effects, dividing wall

Słowa kluczowe: przewodnictwo ciepła, przewodniki dwukierunkowo-periodyczne, uśrednianie tolerancyjne, efekt brzegowy, przegroda budowlana

Introduction

Throughout this note we deal with the microheterogeneous periodic chessboard palisade-type periodic conductor made of the perfectly bonded constituents. The behavior of these solids will be restricted to the heat conduction problem based on the Fourier heat conduction law and will be investigated in the framework of the well known parabolic equation which, under denotations $\nabla = [\partial_1, \partial_2, 0]^T$, $\partial = [0, 0, \partial_3]^T$, $z = x_3$, $x = [x_1, x_2, 0]^T$, will be rewritten in the form:

$$c\partial_t w - (\nabla + \partial)^T [\mathbf{A}(\nabla + \partial) w] + f = 0 \quad (1)$$

Symbol $w = w(\cdot)$ stands here for the temperature field defined in $\Omega \subset R^2$, $f = f(\cdot)$ is the known density of heat sources. In the above equation $c = c(x, z)$ is the specific heat and

$$\mathbf{K}(x, z) = \begin{bmatrix} \mathbf{A}(x,z) & \mathbf{h} \\ \mathbf{h}^T & a(z) \end{bmatrix} \quad (2)$$

is the conductivity matrix, both in $(x, z) \in \Omega$. We shall also assume the heat flux continuity conditions in normal directions onto the interfaces Γ, $\Gamma \subset \Omega$, between the constituents of the considered composite. Since considerations will

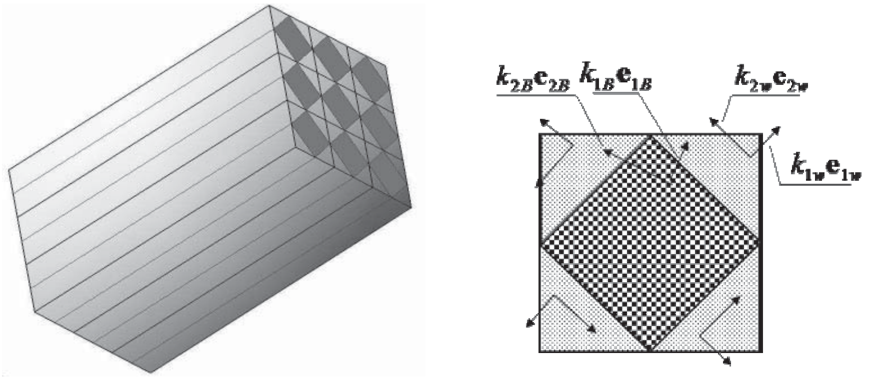

FIGURE 1. Palisade-type conductor with the basic cell of periodic chessboard cross-section
RYSUNEK 1. Przewodnik palisadowy o szachownicowym przekroju poprzecznym wraz z jego komórką szachownicową

be restricted to the palisade-type composites with the chessboard periodic cross-section, cf. Figure 1, we shall introduce the following periodicity conditions:

$$\begin{aligned} \mathbf{K}(\mathbf{x},z) &= \mathbf{K}(\mathbf{x}+\mathbf{e},z) = \mathbf{K}(\mathbf{x}+\mathbf{Qe},z) \\ c(\mathbf{x},z) &= c(\mathbf{x}+\mathbf{e},z) = c(\mathbf{x}+\mathbf{Qe},z) \end{aligned} \tag{3}$$

where $\mathbf{e} = [1, 0, 0]^T$, $\mathbf{Qe} = [0, 1, 0]^T$ are unit vectors determining the basic chessboard cell and $\mathbf{Q}$ is the $\pi/2$-rotation matrix with Oz-axis as the rotation axis, i.e. $\mathbf{Q} = \mathbf{Q}(\alpha)$ for:

$$\mathbf{Q}(\alpha) = \begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

Figure 1 illustrates an example of the considered palisade-type conductor: this conductor is a composite material made of two constituents: matrix material and fiber material. Eigenvectors of the conductivity matrix of white and black materials are $\mathbf{e}_{1w}$, $\mathbf{e}_{2w}$, $\mathbf{e}_{3w}$ and $\mathbf{e}_{1B}$, $\mathbf{e}_{2B}$, $\mathbf{e}_{3B}$ respectively. The corresponding eigenvalues are denoted by k_{1w}, k_{2w}, k_{3w} and k_{1B}, k_{2B}, k_{3B}. It must be emphasized that the directions of the above eigenvectors not necessarily should coincide with the directions of coordinate axes directions determined by unit vectors $\mathbf{e}_1 = \mathbf{e} = [1, 0, 0]$, $\mathbf{e}_2 = \mathbf{Qe} = [0, 1, 0]^T$, and $\mathbf{e}_3 = [0, 1, 0]^T$. Figure 1 illustrates situation in which $\mathbf{e}_{3w} = \mathbf{e}_{3B} = [0, 0, 1]^T$ and hence $\mathbf{h} = \mathbf{0}$.

The well-known fact is that, due to the discontinuous and highly oscillating form of functional coefficients $c(\cdot)$, $\mathbf{K}(\cdot)$, the direct application of (1) to the analysis of special problems in most cases is difficult. That is why heat conduction description based on equation (1) is usually replaced by other mathematical descriptions which take into account mathematical models with more regular coefficients. The most of these descriptions is based on the assumption that microstructure of considered conductor is characterized by a certain scalar microstructure parameter $\lambda > 0$. It means that in the aforementioned case conductivity matrix of the considered palisade-type conductor also should depend on λ, $\mathbf{K} = \mathbf{K}_\lambda$. This remark deals also specific heat. Hence $c = c_\lambda$. Since the aim of this paper is to discuss the macroproperties of heat conduction of the considered composite without the need of summons for due fulfilment of asymptotic limit passage $\lambda \to 0$, the tolerance averaging approach will be taken into account to the description of the heat conduction problems. Obtained on this way tolerance averaged model consists of the system of differential equations with constant coefficients for averaged temperature field $u = u(\cdot) \in SV_0^1(\Omega)$ and fluctuation amplitudes fields $w^A = w^A(\cdot, t) \in SV_0^1(\Omega)$, which are new basic unknowns. Introduced here functional space $SV_0^1(\Omega)$ consists of slowly-varying functions. It must be emphasized that consideration of this paper is focused on the tolerance approach based on the orthogonalization method. For particulars the reader is referred to Woźniak and Wierzbicki (2000). Other approaches to the formulation of tolerance averaged models can be found in Jikov at al. (1994), Thermomechanics... (2009), Developments... (2010).

It must be emphasized that, from among averaging approaches, the tolerance averaging approach seems to be the most familiar method to the investigations of various behaviors in complicate periodic material structures. It is a consequence of the fact that the tolerance effective modulus for structures with two-directional periodicity can be usually determined on the algebraic way and they are good approximations of the related effective modulus investigated for example in the asymptotic homogenization which are possible to be determined exclusively in the simplest cases. 'Such situations can be evidently imagined in the case of chess-board type periodic conductors for which homogenized effective modulus is known for isotropic coefficients, cf. Jikov at al. (1994). In this case homogenized effective modulus have been determined by the investigation of the known *heuristic hypothesis*, cf. Jikov at al. (1994), but not in the direct way given in classical homogenization by investigating the well-known periodic cell problem. That is why the references of this paper include papers in which tolerance averaging technique has been applied to the modeling of macrodynamics of chess-board structures in Wierzbicki and Woźniak (1998) and to the modeling of various problems dealing to hexagonal-type material structures in Cielecka (1995, 1999), Cielecka i Woźniak (1999), Cielecka i Jędrysiak (2002), Wierzbicki i Siedlecka (2004a, b).

Model equations

Following tolerance averaging of heat conduction equation (1) we look for the temperature field in the form:

$$w(x, z, t) = u(x, z, t) + \lambda g^A(x)\psi^A(x, z, t) \tag{5}$$

where: $u(x, z, t) = \langle c \rangle^{-1} \langle cw \rangle (x, z, t)$ is referred to as *the averaged temperature field* and $\psi^A(x, t)$, $A = 1, ..., N$, are extra unknowns which are usually referred to as *the fluctuation amplitudes*. Here and in the sequel $\langle \cdot \rangle$ stands for the integral averaged operator over the basic cell, cf. Woźniak and Wierzbicki (2000). Superscripts denoted by latin capitals A, B, ... run over 1, 2, ..., N, where N is a number of fluctuation amplitudes. *Shape* *functions* $\lambda g^A(x)$, caused by the periodic structure of the composite, should be periodic and should satisfy some additional conditions like $\langle cg^A \rangle = 0$ and $\lambda g^A \in O(\lambda)$, $\lambda^2 \nabla g^A \in O(\lambda)$, cf. Woźniak and Wierzbicki (2000). Following Woźniak and Wierzbicki (2000) the system of tolerance averaged equations (based on the orthogonalization approach) can be written in the form:

$$
\begin{aligned}
&(\nabla + \partial)^T [\langle \mathbf{K} \rangle (\nabla + \partial) u] - \langle c \rangle \partial_t u + \\
&+ \nabla^T [\langle \mathbf{K} \nabla g^A \rangle \psi^A] = \langle f \rangle \\
&\lambda^2 [\langle cg^A g^B \rangle \partial_t \psi^B - \langle ag^A g^B \rangle \partial^T \partial \psi^B] + \\
&+ \langle \nabla^T g^A \mathbf{A} \nabla g^B \rangle \psi^B + \lambda (\langle \nabla g^A \mathbf{h} g^B \rangle - \\
&- \langle g^A \nabla g^B \mathbf{h} \rangle) \partial \psi^B + \langle \nabla^T g^A \mathbf{A} \rangle \nabla u = \\
&= -\langle fg^A \rangle
\end{aligned}
\qquad (6)
$$

Under additional assumption $\mathbf{h} = 0$ tolerance model equations (6) take form which can be found in Thermomechanics... (2009). In this paper we shall deal with the full anisotropy properties of the conductor, i.e. $\mathbf{h} \neq 0$. Hence, considerations of this paper can be treated as an extension of those realized in Woźniak i Wierzbicki (2000) in the framework of the partial anisotropic determined by condition $\mathbf{h} = 0$.

It must be emphasized that so far in special problems model equations (6) have been applied usually in the case of one shape functions, i.e. for $N = 1$, when model equations (6) takes the form:

$$
\begin{aligned}
&(\nabla + \partial)^T [\langle \mathbf{K} \rangle (\nabla + \partial) u] - \langle c \rangle \partial_t u + \\
&+ \nabla^T [\langle \mathbf{A} \nabla g \rangle \psi] = \langle f \rangle \\
&\lambda^2 [\langle cgg \rangle \partial_t \psi - \langle agg \rangle \partial^T \partial \psi] + \\
&+ \langle \nabla^T g \mathbf{A} \nabla g \rangle \psi + \\
&+ \langle \nabla^T g \mathbf{A} \rangle \nabla u = -\langle fg \rangle
\end{aligned}
\qquad (7)
$$

in which we deal with only one shape function g and hence with only one fluctuation amplitude ψ. In this case since $\langle \nabla^T g \mathbf{A} \nabla g \rangle \neq 0$ after applying limit passage $\lambda \to 0$ the concept of effective modulus is possible to introduce, cf. Woźniak i Wierzbicki (2000).

Remark. In the most cases, in which $N > 1$, shape functions $g^1(x), g^2(x), ..., g^N(x)$ are not independent. It means that in many cases matrix $\langle \nabla^T g^A \mathbf{A} \nabla g^B \rangle$ being the coefficient in term $\langle \nabla^T g^A \mathbf{A} \nabla g^B \rangle W^B$ in equations (6) is not invertible.

In the asymptotic case, i.e. when limit passage $\lambda \to 0$ should be applied, the very important typical procedure of determination from the model equations (6) the effective modulus is possible to be realized provided that matrix $\langle \nabla^T g \mathbf{A} \nabla g^B \rangle$ is invertible. It is mean that model equations (6) are practically useless when matrix $\langle \nabla^T g^A \mathbf{A} \nabla g^B \rangle$ is not invertible. That is why in such situations we shall transform model equations (6) to the form in which this inconvenience does not take place.

In the subsequent considerations we are to describe example of such transformation. To this end let us consider the chessboard palisade-type conductor satisfying two following assumptions:

Assumption 1. The material structure of the anisotropic conductor is invariant under π/2-rotations with respect to the axis of symmetry of any chessboard-type palisade as the axis of rotations.

Assumption 2. The sequence $g^1, ..., g^N$ of the shape functions is independent on z variable and invariant over π/2-rotations with the center of a chessboard cell as the origin of the rotation.

In the next section, similarly as in the similar considerations dealing hexagonal-type periodic conductors explained in Wierzbicki and Woźniak (1998), we are to reformulate tolerance equations system (6) to an alternative form.

Isotropic properties of model equations

To transform tolerance equations (6) to the form familiar to the investigation of isotropic properties of the considered conductor we are to outline the line of approach similar to that which has been presented in Wierzbicki (2010). Firstly, we shall represent decomposition (5) in the form:

$$w(x,z,t) = u(x,z,t) + \lambda g_r^a(x)\psi_r^a(x,z,t) \qquad (8)$$

where indices a, r run over the sequences 1, 2, ..., n and 0, 1, 2, 3, respectively. Taking into account (8) model equations (6) yield

$$\begin{aligned}
&(\nabla+\partial)^T[\langle \mathbf{K}\rangle(\nabla+\partial)u] - \langle c\rangle \partial_t u + \\
&+\nabla^T\langle \mathbf{K}\nabla g_r^a\rangle \psi_r^a = \langle f\rangle \\
&\langle c g_s^a g_r^b\rangle \partial_t \psi_r^b - \langle a g_s^a g_r^b\rangle \partial^T\partial \psi_r^b + \\
&\langle \nabla^T g_s^a \mathbf{A}\nabla g_r^b\rangle \psi_r^b + (\langle \nabla g_s^a \mathbf{h} g_r^b\rangle - \\
&-\langle g_s^a \nabla g_r^b \mathbf{h}\rangle)\partial^T \psi_r^b + \langle \nabla^T g_s^a \mathbf{A}\rangle \nabla u = -\langle f g_s^a\rangle
\end{aligned} \qquad (9)$$

According to the *Assumption* 2 we shall assume that

$$g_{r+1}^a(x) = g_r^a(rot(x)) \qquad (10)$$

where operation $rot(x)$ is defined on Figure 2, and the set consisting of fluctuation amplitudes ψ_r^a, $a = 1, ..., n$, $r = 0, 1, 2, 3,$

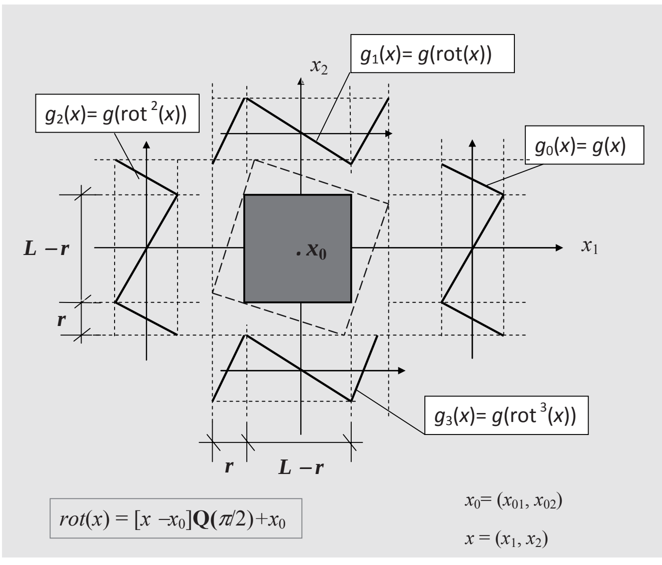

FIGURE 2. The four-tuple $(\gamma(x) = g_0(x), g_1(x), g_2(x), g_3(x))$ generated by single basic shape function $\omega(x) = \lambda g_0(x)$
RYSUNEK 2. Czwórka $(\omega(x) = g_0(x), g_1(x), g_2(x), g_3(x))$ funkcji generowanych przez pojedynczą tworzącą funkcję kształtu $\omega(x) = \lambda g_0(x)$

is the same as the set consisting of fluctuation amplitudes ψ^A but enumeration of elements of both sets are different. Hence $N = 4n$. Functions $\gamma^a(x), \quad a = 1, ..., n$, defined by

$$\gamma^a(x) \equiv g_0^a(x), \quad a = 1, ..., n \tag{11}$$

will be referred to as *a basic shape functions*.

Now, instead of fluctuation amplitude ψ_r^a we shall introduce new amplitudes

$$\mathbf{v}^a = \mathbf{t}^r \psi_r^a = \mathbf{e}(\psi_0^a + \mathbf{Q}\psi_1^a + \mathbf{Q}^2\psi_2^a + \mathbf{Q}^3\psi_3^a) \tag{12}$$

where $\mathbf{t}^r = \mathbf{Q}^r\mathbf{e}$, $r = 0, 1, 2, 3$, are four unit vectors determining the basic chess-board cell. It is easy to verify that for an arbitrary $a = 1, ..., n$, formula (10) does not represent any invertible linear transformation between four–tuple $(\psi_0^a, \psi_1^a, \psi_2^a, \psi_3^a)$ and $\mathbf{v}^a$. Although, formula (12) determines an invertible linear transformation between pair $(\psi_0^a - \psi_2^a,\ \psi_1^a - \psi_3^a)$ and $\mathbf{v}^a$. Indeed amplitudes

$$\frac{1}{2}\mathbf{t}_s\mathbf{v}^a = (\mathbf{t}^s)^T\mathbf{v}^a = \frac{1}{2}(\mathbf{t}^s)^T\mathbf{t}^r\psi_r^a =$$
$$= \frac{1}{2}\delta^{sr}(\psi_r^a - \psi_{(r-2)\bmod 2}^a) = \psi_s^a - \psi_{(s-2)\bmod 2}^a \tag{13}$$

where $\mathbf{t}_s = (\mathbf{t}^s)^T$, $s = 0, 1, 2, 3$, represents related transformation invertible to (12). Let us introduce the following averaged coefficients

$$\mathbf{A}_2^{ab} = \langle \nabla^T g_r^a \mathbf{A} \nabla g_s^b \rangle \mathbf{t}_r \otimes \mathbf{t}_s$$
$$\mathbf{B}^a = \langle \mathbf{A} \nabla g_r^a \rangle \otimes \mathbf{t}_r$$
$$\mathbf{C}_2^{ab} = \langle c g_r^a g_s^b \rangle \mathbf{t}_r \otimes \mathbf{t}_s$$
$$\mathbf{D}_2^{ab} = \langle a g_r^a g_s^b \rangle \mathbf{t}_r \otimes \mathbf{t}_s$$
$$\mathbf{s}^{ab} = \mathbf{t}_r \langle \nabla^T g_r^a \mathbf{K} g_s^b \rangle \mathbf{t}_s^T - \mathbf{t}_r \langle g_r^a \nabla g_p^b \mathbf{K} \rangle \mathbf{t}_p^T$$
$$\mathbf{f}^a = \langle f g_r^a \rangle \mathbf{t}_r \tag{14}$$

Rather simple manipulations yield to

$$\nabla^T[\langle \mathbf{A} \rangle \nabla u] + \mathbf{B}^a \nabla \mathbf{v}^a - \langle c \rangle \partial_t u = \langle f \rangle$$
$$\mathbf{C}_2^{ab}\partial_t \mathbf{v}^b - \mathbf{D}_2^{ab}\partial^T\partial\mathbf{v}^b + \mathbf{s}^{ab}\partial\mathbf{v}^b +$$
$$+\mathbf{A}_2^{ab}\mathbf{v}^b + (\mathbf{B}^a)^T \cdot \nabla u = -\mathbf{f}^a \tag{15}$$

where

$$\langle \mathbf{A} \rangle = k\mathbf{1}, \quad \mathbf{A}_2^{ab} = \widehat{a}_2^{ab}\mathbf{1} + \breve{a}_2^{ab} \in$$
$$\mathbf{B}^a = \widehat{b}^a\mathbf{1} + \breve{b}^a \in$$
$$\mathbf{C}_2^{ab} = \widehat{c}_2^{ab}\mathbf{1} + \breve{c}_2^{ab} \in$$
$$\mathbf{D}_2^{ab} = \widehat{d}_2^{ab}\mathbf{1} + \breve{d}_2^{ab} \in \tag{16}$$

for

$$k = 0.5\langle tr\mathbf{A} \rangle, \quad \widehat{b}^a = \frac{3}{4}\langle (tr\mathbf{A})\nabla g_r^a \rangle \mathbf{t}^r$$
$$\breve{b}^a = \frac{3}{4}\langle (tr\mathbf{A})\nabla g_r^a \rangle \cdot \mathbf{t}_r$$
$$\widehat{a}_2^{ab} = \mathbf{t}^r \cdot \langle \nabla^T g_r^a (tr\mathbf{A}) \nabla g_s^b \rangle \mathbf{t}^s$$
$$\breve{a}_2^{ab} = \frac{3}{8}\mathbf{t}^r \langle \nabla^T g_r^a (tr\mathbf{A}) \nabla g_s^b \rangle \mathbf{t}_s$$
$$\widehat{c}_2^{ab} = \langle c g_r^a g_s^b \rangle \delta^{rs}, \ \breve{c}_2^{ab} = \langle c g_r^a g_s^b \rangle \in^{rs}$$
$$\widehat{d}_2^{ab} = \langle a g_r^a g_s^b \rangle \delta^{rs}, \ \breve{d}_2^{ab} = \langle a g_r^a g_s^b \rangle \in^{rs} \tag{17}$$

At the end of the paper we shall distinguish an important situation in which the term with coefficient $\mathbf{s}^{ab}\partial\mathbf{v}^b$ can be omitted and we deal with four shape functions determined by single basic shape function (11). In particular such situation takes place provided that parts $\mathbf{h}$ and $\mathbf{h}^T$ of the conductivity matrix $\mathbf{K}$ given by (2) are equal to zero, a.e where considerations reduces to the cases investigated in Woźniak and Wierzbicki (2000). In this case we have $n = 1$ and model equations (15) reduces to the form:

$$k\nabla^2 u + (\widehat{b}\mathbf{1} + \breve{b}\in^T)\nabla\mathbf{v} - \langle c\rangle\partial_t u = \langle f\rangle$$
$$(\widehat{c}_2\,\mathbf{1} + \breve{c}_2\in)\partial_t\mathbf{v} - (\widehat{d}_2\,\mathbf{1} + \breve{d}_2\in)\partial^T\partial\mathbf{v} +$$
$$+ (\widehat{b}\mathbf{1} + \breve{b}\in)\nabla u = -\mathbf{f} \qquad (18)$$

in which all coefficients have the form of scalar linear combination of unit matrix **1** and Ricci matrix $\in$ and hence are isotropic provided that $\mathbf{f} = \mathbf{0}$. This property has an important influence on mechanical behaviors described by the above model equations in which all coefficients have the form of scalar linear combination of unit matrix **1** and Ricci matrix $\in$ and hence are isotropic. This property has an important influence on mechanical behaviors described by the above model equations.

It must be emphasized that, if we deal in (16) with four shape functions determined by single basic shape function (9) being well known saw like function the tolerance heat flux vector $-\mathbf{K}\nabla^T w$ in which temperature field w is given by micro-macro decomposition (8) is continuous in all directions normal to the interfaces Γ, cf. Figure 2. This result as well as similar result dealing hexagonal palisade-type rigid conductor is has not been proved so far.

It must be emphasized that the results of this paper deal not only palisade-type periodic conductor, but also to the certain class of fiber reinforced conductors satisfying two assumptions formulated in Section 2. This remark is illustrated in Figure 2 where straight lines including opportunity apexes of the basic chessboard cell are not cell symmetry axes.

Possible engineering applicaations

To explain a certain application of the obtained model equations (15) we are to restrict considerations to the stationary case in which we deal with fourtuple $(\gamma(x) = g_0(x), g_1(x), g_2(x), g_3(x))$ given by exclusively one basic shape function $l\ \gamma(x)$. In this case latin indices a, b in (15) takes only one value and it will be omitted. Moreover, single one fluctuation amplitude **v** can be decompose onto two terms $\mathbf{v} = \mathbf{v}_0 + \mathbf{v}_1$ where:

– the first term $\mathbf{v}_0$ is a solution to the boundary value problem for the hexagonal layer, formulated independent for every $x \in \Xi$:

$$\mathbf{D}_2^{ab}\partial^T\partial\mathbf{v}_0 - \mathbf{s}^{ab}\partial\mathbf{v}_0 - \mathbf{A}_2^{ab}\mathbf{v}_0 = \mathbf{0}$$
$$\mathbf{v_0}(x, H_1) = \hat{\mathbf{v}}_{01}(x), \qquad (19)$$
$$\mathbf{v_0}(x, H_2) = \hat{\mathbf{v}}_{02}(x)$$

where $x \in [H_1, H_2]$,

– the second term $\mathbf{v}_1 = \mathbf{v}_1(\nabla u)$ is a certain partial solution to the differential equation

$$\mathbf{D}_2^{ab}\partial^T\partial\mathbf{v}_1 - \mathbf{s}^{ab}\partial\mathbf{v}_1 -$$
$$-\mathbf{A}_2^{ab}\mathbf{v}_1 + (\mathbf{B}^a)^T\nabla u = -\mathbf{f}^a \qquad (20)$$

The investigation of solutions $\mathbf{v}_1 = \mathbf{v}_1(\nabla u)$ is not analyzed in this paper and in its general form seems to be very difficult problem.

Boundary value problem (19) describes the thermal boundary layer behavior which can be observed in periodic

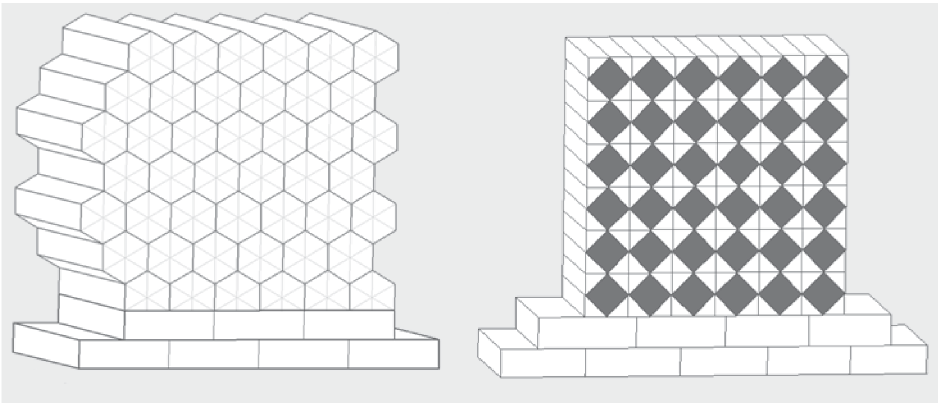

FIGURE 3. The dividing walls made of palisade-type periodic composites
RYSUNEK 3. Przegroda budowlana zbudowana z periodycznego kompozytu palisadowego

microstructured conductors. The aim of investigations of solutions to boundary value problem (19) for palisade-type periodic conductors with chessboard as well as hexagonal cross-section (with respect to different conductivity properties of constituents) yield to the examination of engineering validity of using in civil engineering dividing walls of the form presented in Figure 3. The existence of boundary effect behavior suggests that for a special choice of material properties of every constituent illustrated in Figure 3 dividing wall can properly protect the interior of building house from the temperature fluctuations.

Concluding remarks

In the paper an alternative form of the tolerance averaged model of heat conduction in the composite conductors with microperiodic palisade-type material structure with a chessboard cross-section has been proposed. It was assumed that chessboard periodic cell has material structure invariant with respect to the rotations over $\pi/2$ with origin of an arbitrary cell as the origin of the rotation. That same invariant property should have applied shape functions. Since typical tolerance averaged equations have certain inconveniences make impossible of its direct approach, In the paper the alternative form of the tolerance model is proposed. This new form is free under these inconvenience. Moreover, in many important cases this new form of model equations has isotropic coefficients. It means that in the macroscale geometrical properties of the microstructure has major influence on the material properties of considered composite in macroscale.

Considerations of this paper can be treated as a introduction to the investigation of the boundary effect behaviors in the case in which chessboard palisade-type periodic conductor is invariant under described above internal $\pi/2$-rotations.

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Summary

Some remarks on the tolerance averaging of heat conduction in chessboard palisade-type periodic composites. The paper deals with periodic composites which material structure is described by the anisotropic conductivity matrix invariant with respect to the pair of orthogonal translations such that the periodic cell determined by them coincides with the shape of chess-board cell. Moreover, it will be assumed that the material structure is invariant over the $\pi/2$-rotations with the symmetry axes of any chess-board cell and orthogonal to this cell as the axes of the rotation. The main aim of this paper is to discuss the problem of the macroisotropic properties of heat conduction of the considered conductor.

Streszczenie

Uśrednianie tolerancyjne przewodnictwa ciepła w kompozytach palisadowych o przekroju typu szachownicy. W pracy zaproponowano alternatywną postać tolerancyjnie uśrednionego modelu przewodnictwa ciepła kompozytu o mikroperiodycznej strukturze materialnej typu palisadowego o przekroju szachownicowym. Założono niezmienniczość tej mikrostruktury względem obrotów o kąt $\pi/2$ względem osi przechodzącej przez środek pojedynczej kostki szachownicy, a także taką samą niezmienniczość stosowanych funkcji kształtu. Ponieważ typowe równania tolerancyjne mają w takim przypadku matematyczne wady uniemożliwiające ich bezpośrednie stosowanie, zaproponowano więc nową postać

modelu, który jest już wolny od tych niedogodności. W niektórych ważnych przypadkach nowa postać modelu ma współczynniki izotropowe, co wskazuje na to, że w skali makro zdarza się, że geometryczna budowa kompozytu ma decydujący wpływ na jego makrowłasności materiałowe.

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