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A condition-based maintenance for complex systems consisting of two different types of components

Keywords

complex systems, condition-based maintenance, opportunistic maintenance, gamma process

Abstract

A complex system consisting of monitored and non-monitored components is analyzed. Monitored components are subject to a degradation gamma process. Non-monitored components are subject to external failures. A Condition-Based Maintenance and an inspection policy are applied to reduce the impact of the failures in the monitored components. When a failure occurs, maintenance team performs a corrective replacement after a certain delay time. An opportunistic maintenance strategy is also implemented, meaning that a maintenance intervention can be used as an opportunity for preventive maintenance of monitored components. Each maintenance task implies a certain cost and each monitored component is assumed to provide a reward. The expected cost of the whole system is minimized through the optimization of the preventive thresholds and the time between inspections. Numerical examples are obtained from applying a blend of Genetic Algorithm and Monte-Carlo simulation.

1. Introduction

Maintenance is the set of actions carried out to keep a system into a condition where it can perform its function. It plays an important role in the areas of industry and engineering in order to reduce costs and avoid downs of the system. Maintenance tasks are usually divided in corrective maintenance and preventive maintenance. Corrective maintenance is performed when a system has failed, while preventive maintenance prevents the occurrence of failures by periodical inspections of the deterioration state of the system. Some maintenance can be done during production and other during regular scheduled stops of the production process.

In the 1960s, some authors as Barlow, Hunter and McCall [4], [19] developed mathematical models to quantify the cost and find the optimal maintenance strategy for a certain system. Deterioration is modelled in terms of a time-dependent stochastic process, such as Markov processes, which include stochastic processes with independent increments like the Brownian motion, the compound Poisson process and the gamma process. A gamma process model for gradual damage accumulating over time, such as wear, corrosion, crack growth or degrading health was initially proposed by Abdel-Hameed [1].

Gamma process is characterized by independent and non-negative increments distributed following gamma distributions with identical scale parameters. Because of these properties, it is considered as one of the most appropriate processes for the stochastic modelling of degradation. A wide survey about gamma process was perform by Noortwijk [23].

Çinlar [11] showed how a non-stationary gamma process can be transformed into a stationary gamma process, and how the parameters of a stationary gamma process can be estimated using the method of maximum likelihood and the method of moments.

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Since the development of stochastic deterioration models with discrete or continuous space states, Condition-Based Maintenance (CBM) has become one of the most popular maintenance techniques in the literature [21]–[22]. This maintenance program recommends performing maintenance actions based on the information collected through a condition monitoring process, so that many of the failures of the system are preceded by certain signs that one of these failures is going to occur. Early work on CBM is focused on single-component systems [7], [14]. Nowadays, systems are more complex, which affects the analysis since different dependencies inter-components can affect the system availability. References [15] and [16] gives a view of the application of CBM in multi-component systems and the dependencies associated with it.

Monitoring process in CBM can be of different types: continuous or periodic monitoring [8]–[9]. In continuous monitoring, it is usual to install sensors in monitored components, in order to know the degradation level in every moment. In the case of periodic monitoring, the deterioration level of monitored components is checked periodically through inspections.

Additionally to CBM, opportunistic maintenance is implemented in this maintenance strategy. Opportunistic maintenance reduces considerably maintenance costs since it takes advantage of breakdowns of the system [12]. These downs are mainly due to external factors (represented by the non-monitored components) which are unavoidable, like weather conditions, and internal factors of the system. The latter are produced when any of the monitored components fails.

Over the last few years, the maintenance of systems in general has become more and more complex, because most systems consist of many components which depend on each other [14]. The modelling and optimization of maintenance is complicated due to this interaction between components. Planning the maintenance strategy for complex systems is a big challenge. Cho and Parlar gave in 1991 [10] the following definition of multi-component maintenance systems: *"Multi-component maintenance models are concerned with optimal maintenance policies for a system consisting of several units of machines or many pieces of equipment, which may or may not depend on each other, economically, stochastically or structurally"*.

For multi-component systems, opportunistic maintenance has spawned much research in the field of optimization [17]. For example, it is considered one of the main maintenance strategies for offshore wind farms [13]. This combination of the two types of maintenance (opportunistic maintenance and maintenance based in the system condition) is usually referred to as OCBM. A pioneering work in the application of this policy application on OCBM was carried out by Castanier [7].

Determining the optimal maintenance policy for a system implies to determine the preventive thresholds for all monitored components and the time between inspections that minimize the objective cost function [6]. To deal with the large number of combinations for multi-component systems, different solutions, such as sweep space reduction and simulated annealing, were adopted to find the optimal maintenance strategy. In this paper, semi-regenerative techniques are used to evaluate the expected cost rate. The use of this technique is not a novelty in the reliability literature, and they have already been used for unitary systems.

Meta-heuristic algorithms such as Genetic Algorithm, Colony Algorithm or Pattern Search are used to find the optimal values for the periodic inspection intervals and the preventive thresholds, that is, those values which minimize the objective cost function [18]. Combining them with traditional Monte-Carlo simulation, a new method to find the optimal maintenance strategy is developed. The aim is to refine these methods, achieving a shorter execution time of the algorithms, especially when adding a large number of components to the system.

The problem of analysing maintenance hybrid strategies in multi-component systems with two or more components is an open problem today, which we will try to explain and solve in this paper.

2. System description

A system consisting of heterogeneous components requiring different maintenance strategies is analysed. The components of this system are divided into two groups: m monitored (or critical) components and n non-monitored (also called non-critical) components, being $m, n > 0$. Monitored components are subject to a continuous degradation, which is modelled using a gamma process. This process is widely used to model stochastic deterioration in maintenance, especially gradual damage monotonically accumulated [23].

Non-monitored components fail immediately and without evidence of degradation. An exponential distribution models the time between failures of the non-monitored components. Obviously, both groups of components (monitored and non-monitored) require different types of maintenance.

Monitored components are subject to a condition-based maintenance and non-monitored components to a corrective maintenance. When a component fails, a signal is sent to the maintenance team and it starts the maintenance with a delay of $\tau > 0$ time units. These maintenance times due to corrective failures give the maintenance team an opportunity to replace those monitored components which are found too degraded or failed. Inspections are also performed at periodical times to check the state of the system. These inspection times are opportunities of maintenance for the monitored components, therefore some monitored components can be preventively replaced if they are too degraded, correctively replaced if they are failed, or simply left as they are if the degradation level does not exceed the so-called preventive threshold. In short, a preventive maintenance strategy is performed during inspection times and breakdowns of the system. The degradation level of the monitored components is checked, and the necessary replacements are made. At each maintenance time (corrective or preventive), a partial renewal policy is applied on the monitored components, and a group of monitored components are replaced while others are left as they are, depending on its degradation state.

Regular maintenance is usually costly: it requires the breakdown of the system, but it is controllable. On the contrary, opportunistic maintenance is relatively cheaper, but it is not possible to control its frequency, as it depends on the rest of the system. With a combination of both strategies, a reduction in the maintenance cost is expected. Specific costs are assigned to the different maintenance actions. The objective is to determine the values of the time between inspections and the preventive thresholds that minimize the objective cost function.

2.1. General assumptions of the model

Some assumptions are implemented in the model:

- The m monitored components are subject to a continuous degradation process, following a gamma process [18]. Let $X_i(t)$ be the degradation level of component i at time t, for $i = 1,2,\dots,m$.
- A monitored component suffers a degradation failure when its degradation level exceeds a failure or corrective threshold. A signal is immediately sent to the maintenance team and it takes τ units of time to start the maintenance.
- Corrective maintenance implies the replacement of the component by a new one. This maintenance time is used as an opportunity to check the degradation levels of the rest of the monitored components.
- Preventive replacement is performed when the degradation of a monitored component exceeds the preventive threshold in an inspection time. It also implies the total replacement of the component, even it is not a serious failure which causes the system's down.
- The time between failures of the non-monitored components is denoted by Y, and follows an exponential distribution with parameter λ.
- Periodic inspections are performed on the system in order to check the deterioration state of the system and perform a preventive or corrective maintenance
- A sequence of costs and a certain reward provided by the monitored components when they are working is considered. Given the deterioration level $X_i(t)$ of the monitored component i, the reward function g_i is:

$$g_i(X_i(t)) = \theta_0 + g \exp(-\gamma_i X_i(t)),$$

with $g, \theta_0, \gamma_i > 0$ and $i = 1,\dots,m$.

Parameters g and θ_0 are constant, which are chosen arbitrarily, and γ_i depends on the i-th monitored component. It is a classical exponential reward function, but there are more reward functions in the literature that could be used. The condition to be fulfilled is that the reward provided by the monitored components decreases as the deterioration of the corresponding component increases.

In this case, the reward function is decreasing in $(X_i(t))$. Its maximum is reached in $\theta_0 + g$ and the minimum is reached in θ_0.

2.2. Maintenance actions

Different maintenance actions (corrective replacement of a monitored component, preventive replacement of a monitored component or corrective replacement of the non-monitored components) are performed in a maintenance time, which could be an inspection time or τ units of time after a failure. In short, the different maintenance actions in a maintenance time are the following:

Monitored components

1. The monitored component i is correctively repaired (it is replaced for a new one) if the degradation level exceeds the corrective threshold L_i.
2. A preventive replacement of a monitored component i (consisting on the replacement of the component by a completely new one) is performed if its deterioration level exceeds its preventive threshold M_i, but it is less than its corrective threshold L_i.

3. The monitored component is left as it is if its degradation level is lower than its preventive threshold M_i.

Non-monitored components

1. Non-monitored components are always correctively replaced after τ time units (or during an inspection time) when they fail.

A corrective maintenance of the component i implies a cost of C_{c_i} monetary units, and similarly, a preventive maintenance of the component i implies a cost of C_{p_i} monetary units. Related to the non-monitored components, their corrective maintenance implies a cost of C_f monetary units. The costs for downs of the system are c_i monetary units per time unit for monitored components and c_{nm} monetary units per time unit for non-monitored components.

Two approaches are envisioned to analyse the behaviour of the maintained system based on the time to the next inspection.

- Rescheduled policy

The inspection policy is rescheduled every time a maintenance intervention is performed, that is, after a maintenance time; the next inspection is scheduled T units of time after.

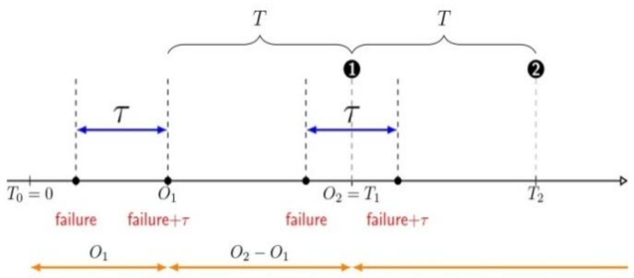

Figure 1. Rescheduled policy

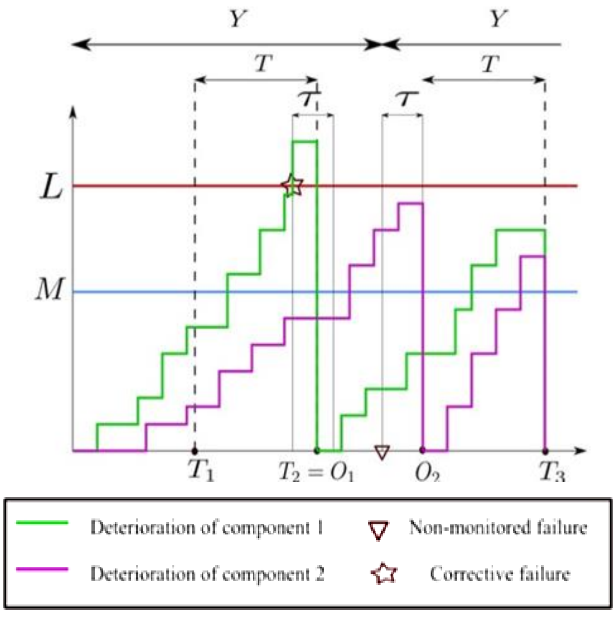

Figure 2. Rescheduled policy

Figures 1 and *2* show an example of a system with two monitored components and an arbitrary number of non-monitored components, following a rescheduled policy.

The numbered circles in *Figure 1* correspond to inspection times $T_1, T_2, \ldots$. Maintenance times are represented by $O_1, O_2, \ldots$. Time between maintenance actions is represented with an orange arrow. In this case, with the rescheduled policy, when a maintenance action is performed (corrective failures of the system or periodic inspections), the time to the next inspection is rescheduled exactly T units of time after. In *Figure 2*, we have two monitored components represented in green and purple, the preventive threshold in blue and the corrective threshold represented in red. Also, there is an arbitrary number of non-monitored components, which produce sudden failures of the system and they cannot be prevent.

A corrective failure occurs after the first inspection time T_1 (in the first inspection the system is left as it is). Component 2 fails in (T_1, T_2) and, since the time to the repair τ exceeds $T_2 = T_1 + T$, the first maintenance action O_1 is performed at T_2. Non-monitored components fail in (T_2, T_3) it is repaired after τ units of time. This maintenance time O_2 is an opportunity to maintain the components 1 or 2 if necessary: component 2 is replaced since its degradation level exceeds the preventive threshold. Next inspection is rescheduled at time $T_3 = O_2 + T$. Finally, at inspection time T_3, the two monitored components are preventively replaced. Hence, T_3 is a regeneration point of the system.

- Non-rescheduled policy

In this case, inspections at periodic times remain fixed over time, with time between inspections

$$T_k - T_{k-1} = T > 0.$$

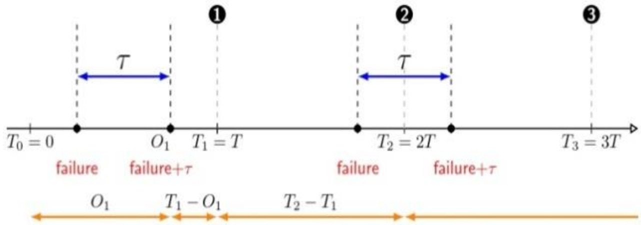

Figure 3. Non-rescheduled policy

Figures 3 and *4* represent a non-rescheduled inspection policy for two monitored components and an arbitrary number of non-monitored components.

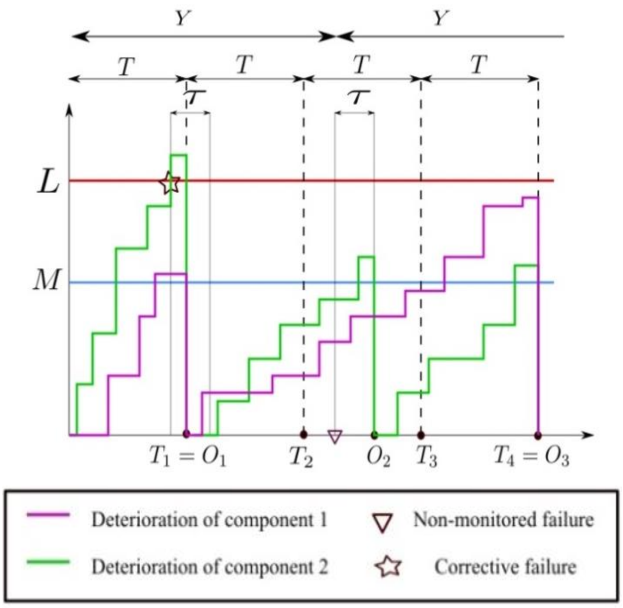

Figure 4. Non-rescheduled policy

The numbers in *Figure 3* correspond to inspection times $T_1, T_2, \ldots$. Maintenance times are represented by $O_1, O_2, \ldots$. In *Figure 4*, the process is very similar to *Figure 2*. The main difference is that inspection times remain fixed over time and they are performed with period T, independently if there are more maintenance times of the system. Component 2 reaches its corrective threshold before the first inspection time T_1, but the time to T_1 is less than τ, so the first maintenance action is performed in the first inspection time T_1. There is a fail of the non-monitored components between (T_2, T_3) and, since the time to the repair τ is less than the time to the next inspection time T_3, the maintenance action O_2 is performed before T_3. The two monitored components are preventively replaced in T_4, which is the third maintenance time and a regeneration point of the system, due the degradation state is restored to 0.

For both inspection policies, if a component (monitored or non-monitored) fails at time t, with $T_{k-1} < t < t + \tau < T_k$, then the maintenance action is performed at time $t + \tau$.

However, if a component fails at time t, with $T_{k-1} < t < T_k < t + \tau$, the maintenance action is performed at time T_k. To sum up, if a component fails at time t with $T_{k-1} < t < T_k$, then the maintenance action is performed at time $\min(t + \tau, T_k)$. It is assumed that T is large enough with respect to τ, that is, $\tau \leq T - \tau$.

3. System modeling

The asymptotic behaviour of the system is studied using semi-regenerative properties of the system. This allows us to study the asymptotic behaviour of the expected cost. A semi-regenerative cycle, which is the time between two maintenance actions, is analysed instead of a complete renewal cycle. With renewal techniques, the evaluation of the expected cost rate is time-consuming, especially for a large number of monitored components. However, using semi-regenerative techniques the computation time is shortened.

Let $T_1, T_2, \ldots$ be the inspection times and T_k^+ the instant of time just after an inspection time T_k.

To model the degradation process, a Markov chain with continuous state space

$$[0, M_1) \cdot [0, M_2) \cdot \ldots \cdot [0, M_m)$$

is defined as

$$Z_k = \left(X_1(T_k^+),\ X_2(T_k^+), \ldots, X_m(T_k^+)\right). \quad (1)$$

The multiple process $(X_1(t), X_2(t), \ldots, X_m(t))$ is a semi-regenerative process with embedded Markov chain Z_k. Due to gamma process properties and the assumptions of the model, the future evolution of the system after T_k^+ only depends on the state at time T_k.

From [8], we have the following proposition related to semi-regenerative processes.

Proposition 4.1. Let us denote S the first replacement time, $\tau = inf\{n \geq 1 : Y_n = 0\}$ and suppose that:

- $E(\tau) < +\infty$
- $E(\Phi_S) < +\infty$
- the probability law of S is not arithmetic.

We have:

$$\lim_{t\to\infty} \frac{E[\Phi_t]}{t} = \frac{E_\pi[\Phi_{T_1}]}{E_\pi[T_1]},$$

where the random process Φ_t is an additive function of the semi-regenerative process $(X_1(t), X_2(t), \ldots, X_m(t))$, for example, a counting process.

4. Objective function

The regenerative property allows us to use the so-called renewal theorems and we know in particular that the expected cost of the system per unit of time is equal to the ratio of the expected cost incurred in a renewal cycle divided by the expected length of a renewal cycle.

Proposition 1. Let $C(t)$ be the cost of the system at time t. Using *Proposition 4.1.* in [8], for a semi-regenerative process with a unique stationary probability distribution π, it is true that the asymptotic cost C_∞ (that is, the expected cost per time unit when $t \to \infty$) fulfills:

$$C_\infty = \lim_{t\to\infty} \frac{E[C(t)]}{t} = \frac{E_\pi[C(O_1)]}{E_\pi[O_1]}, \quad (2)$$

where O_1 is the time to the first maintenance action.

Proof: It is trivial that the expected time to the first replacement is finite. In the same way, the expected cost is assumed to be finite. We are able to apply *Proposition 4.1.* from [8]. The successive replacements of the system are considered as semi-regeneration points of the semi-regenerative process $(X_1(t), X_2(t), \dots, X_m(t)$.
Developing (2) we have:

$$C_\infty = \frac{E_\pi[C_p\,(O_1)]}{E_\pi[O_1]} + \frac{E_\pi[C_c\,(O_1)]}{E_\pi[O_1]} + \frac{E_\pi[C_{nm}(O_1)]}{E_\pi[O_1]} + \frac{E_\pi[I(O_1)]}{E_\pi[O_1]} + \frac{E_\pi[D(O_1)]}{E_\pi[O_1]} - \frac{E_\pi[R(O_1)]}{E_\pi[O_1]}, \quad (3)$$

being
$E_\pi[C_p\,(O_1)]$: expected cost due to preventive maintenance of the monitored components in a semi-regenerative cycle O_1.
$E_\pi[C_c\,(O_1)]$: expected cost due to corrective maintenance of the monitored components in a semi-regenerative cycle O_1.
$E_\pi[C_{nm}\,(O_1)]$: expected cost due to the corrective replacements of the non-monitored components in a semi regenerative cycle O_1.
$E_\pi[I(O_1)]$: expected number of inspections in a semi regenerative cycle O_1.
$E_\pi[D(O_1)]$: expected downtimes of the system in a semi-regenerative cycle O_1.
$E_\pi[R(O_1)]$: expected reward obtained in a semi-regenerative cycle O_1.

5. Numerical experiments

The analysis of the maintenance strategy is completed through the numerical search of the optimal maintenance strategy. We considered a certain number of similar monitored components, that is, with the same parameter alpha and beta of the corresponding gamma processes.
The number of non-monitored components does not have influence on the final result, because they are considered as one group. The time between failures of non-monitored components follows an exponential distribution, regardless of the number of components.
A two-stage optimization method is proposed to find the optimal maintenance policy in the case of similar components. First, Monte-Carlo simulation is used to search the initial points that are potential solutions for the problem (see *Table 1*). Then, using these points, a Pattern Search algorithm is applied to look for the minimum expected cost, that is, the optimal maintenance policy. Results are shown in *Table 2*.
Simulations are performed using the software MATLAB, and, in particular, the commercially available *optimtool* toolbox implemented in it.
We consider m identical monitored components whose associated gamma process have parameters $\alpha_i = 0.5$ and $\beta_i = 1$, with $i = 1,2,\dots,m$. An arbitrary number of non-monitored is considered, and they have a failure rate of $\lambda = 0.025$ failures per unit time.
The following sequence of costs is imposed:

$$C_{c_i} = 80, C_{p_i} = 30, C_f = 80, c_i = 5, c_{nm} = 5.$$

Notice that it is logical to consider that the cost due to corrective repairs (80) is higher than the cost due to preventive repairs (30).
For the reward function, the following parameters are used:

$$g = 2,\ \theta_0 = 2,\ \gamma_i = 20.$$

All the costs are expressed in monetary units, and the costs due to a corrective failure or a reward, in monetary units per time unit.
We assume that a component fails when its degradation exceeds $L = 6$. Finally, when a failure occurs, maintenance team takes $\tau = 0.5$ units of time to start the repair. Each inspection involves a cost of 10 monetary units. Since the monitored components are identical, the search of the optimal maintenance policy corresponds to find the values T and M that minimize the expected cost C_∞. In other words, finding the values (T_{opt}, M_{opt}) such that:

$$C_\infty(T_{opt}, M_{opt}) = \inf\{C_\infty(T, M); \; T > 2\tau, \quad M \le L\}. \quad (4)$$

The use of typical Monte-Carlo simulation and Genetic Algorithm to find the optimal expected cost was explained by Marseguerra and Zio in [18].
Genetic algorithms are numerical search tools whose objective is finding the global maximum (or minimum) of a given objective. They are inspired by the rules of natural selection. The initial population, generated by randomly sampling, (which are the initial points) will evolve over successive generations until the fitness of the average individual in each generation increases towards the global optimum. The individuals of each population are the children of the previous population and the parents of the successive population. At each step, the new population is obtained by manipulating the

old population with mutation and recombination in order to arrive to a new population characterized by an increased mean fitness. This process continues until a termination criterion is reached.
The diagrams of the three main algorithms used to compute the expected cost C_∞ of the total maintenance of the system are below.

Monte-Carlo Simulation

1. Discretize the interval [1,10] corresponding to T into 20 equal parts.
2. Get T_i with $i = 1, \dots, 20$.
3. Discretize the interval [1,6] corresponding to M into 20 equal parts.
4. Get M_i with $i = 1, \dots, 20$.
5. For each combination (T_i, M_i), 15000 simulations of C_∞ are performed.

Genetic Algorithm

1. Selection of the initial population.
2. Crossover and mutation.
3. Replacement of the population.
4. Create new generations
5. Evaluation of the cost.
6. Repeat until a stop condition.

Pattern Search Algorithm

1. Start with the initial points.
2. Look for neighboring solutions.
3. Calculate the cost.
4. Compare and redefine the minimum cost.
5. Repeat until a stop condition.

The Colony Ant algorithm is a probabilistic and computational technique based on the actions of an ant colony. Ants locate optimal solutions by moving through a parameter space representing all possible solutions, so Colony Ant algorithm (CA) is used to solve problems which involve paths through graphs.
The artificial 'ants' record their positions and the quality of the solutions helped by their pheromones, so that in later iterations they locate better solutions.

Table 1. Initial points obtained with Monte-Carlo simulation

m	T_0	M_0
2	6.2749	2.0667
3	4.8333	2.8947
4	3.5012	3.2145
5	3.5134	2.333

Table 2. Optimal values obtained with Pattern Search

m	(T_{opt}, M_{opt})	$C_\infty(T_{opt}, M_{opt})$
2	$(5.02, 2.02)$	8.94
3	$(3.86, 2.40)$	11.02
4	$(3.195, 2.88)$	13.05
5	$(3.198, 2.385)$	13.10

Some comparisons between different well-known meta-heuristic algorithms are made. Meta-heuristic algorithm such as Genetic Algorithm and Colony Algorithm, apart from typical Monte-Carlo simulation, are used.
Figure 5 and *Figure 6* show the optimal values for T and M, respectively, obtained with the three different algorithms (Monte-Carlo simulation, Genetic Algorithm and Colony Algorithm) and adding more components in each step.

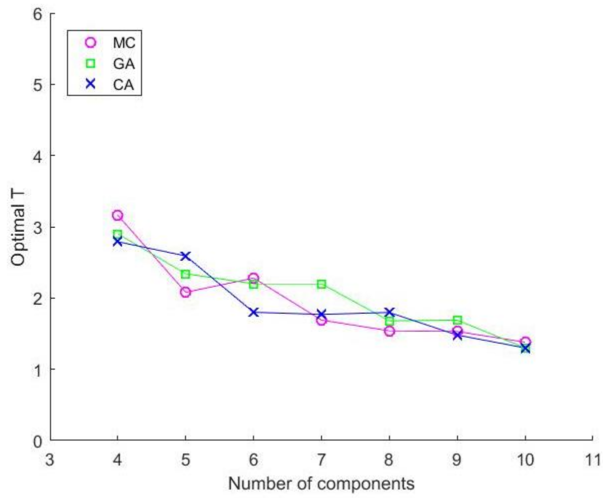

Figure 5. Optimal T obtained with MC, GA and CA

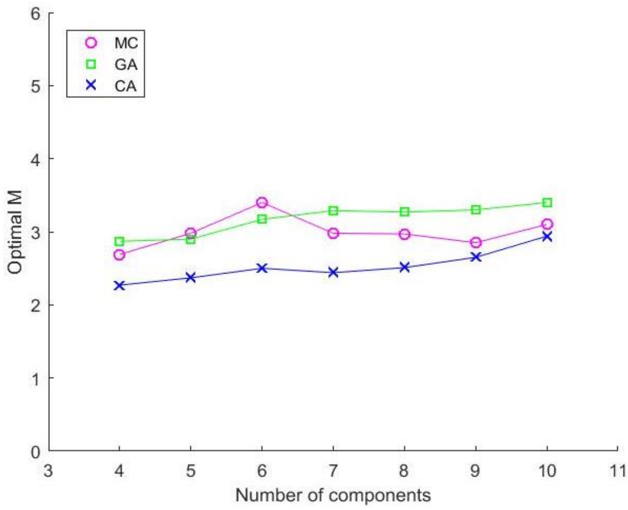

Figure 6. Optimal M obtained with MC, GA and CA

As a result, we have from *Table 2* that the optimal time between inspections, T_{opt}, decreases as the number of monitored components increases. This means that, with more monitored components, it would be necessary to perform inspections more often to reduce the expected cost. On the other hand, the value of the optimal preventive threshold, M_{opt}, in general, increases slowly, which means that it would be necessary to increase the value of the preventive threshold when the number of monitored components increases. The optimal expected cost calculated with the Pattern Search algorithm increases by adding more monitored components. It is logical that the expected cost will be higher with more components.

Related to the graphics, we can see a comparison between results obtained with Monte-Carlo simulation, Genetic Algorithm and Colony Ant algorithm. In the case of the optimal T, the results are very similar with the three algorithms. With the optimal preventive threshold M, the results have greater variability. Monte-Carlo simulation presents more fluctuation. However, the growing trend as in results with GA and CA can be observed.

In terms of computing times, Monte-Carlo method is the fastest, while meta-heuristic algorithms (Genetic Algorithm and Colony Ant) take longer.

6. Conclusion

A maintenance policy combining CBM, opportunistic maintenance and an inspection policy has been developed in this paper. Numerical examples are given combining Monte-Carlo simulation and meta-heuristic algorithms such as Genetic Algorithm and Pattern Search, in order to deal with the optimization of the objective cost function and find the optimal maintenance strategy.

It is considered that the monitored components work independently, however, in practice, components of a system are not independent and dependence relationships (structural, stochastic or economic) can be established between them.

Observing the results, the increase in the number of similar monitored components produces a rise in the optimal value of the objective cost function and the optimal preventive threshold. However, the optimal time between inspections is decreasing as the number of components increases. In the case of many different monitored components, with different parameters in the gamma processes, the results are more variable. The optimal values for the time between inspection, thresholds and the final cost differ depend on the number of components, so we cannot come to a definitive conclusion.

Other structures of complex systems, such as parallel or series systems can be analyzed as further works. The analysis of this paper could expand considering another distribution (different from the exponential distribution) for the failure times of the non-monitored components. Furthermore, different meta-heuristic algorithms could be considered for the optimization of the objective function.

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