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MATHEMATICAL MODELLING OF DYNAMIC PROCESSES IN GAS TRANSMISSION

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Abstract. The problem of finding the parameters of the gas transmission system in terms of its operation under unsteady operating modes are considered. We show mathematical models of gas flows in the basic facilities in the system with a complicated piping diagram. The solution is built to solve a system of partial differential equations with the finite element method of large dimension with boundary conditions, some of which are designed in the process of solving the system of equations. This is due to the provision of technical and technological constraints on the pressure of the controlled points of the system. On the resistance to finding a solution significantly affects a step of the time coordinate. The proposed algorithm in the editing process of flow diagrams provides an increase in stability of solving systems of equations, and significantly reduces the time for the simulation.

Key words: transmission system, compressor station, optimal operating mode, gas flow model, network model, piping diagram, finite element method.

INTRODUCTION

The calculation of the parameters of the gas pipeline system is a complex mathematical problem. Main gas pipelines with compressor stations and other facilities are a single facility, all parameters which are hydraulically interconnected. Changing the mode of operation of an individual object is to change the mode of the entire system. The gas transmission system (GTS) belongs to a class of nonlinear systems with distributed parameters, which are characterized by a network layered structure, a large space and time dimension and distribution, the presence of continuous and discrete control actions, a high level of uncertainty of objectives, structure, properties and states, as well as influences from the environment.

Transmission system, in which objects are hydro thermodynamic, filtration and other physical processes, is the subject of control. The physical nature of the gas allows a wide range to change its parameters - pressure, temperature and volume. In this regard, the gas-dynamic processes that take place in the facilities of the system are described by complex mathematical dependencies. The nonlinearity of the dynamic processes that evolve over large spatial and time dimensions, the weak predictability of input parameters, a significant influence of external factors and insufficient metrological support greatly complicate both the formulation and solution of problems of the analysis, optimization and finding control parameters of gas flows. Not yet fully solved the problem of creating modelling software for optimization of unsteady operating modes of gas transmission systems. The effectiveness solving unsteady flows of gas transmission will improve the operative dispatch control of gas flows

The work for finding solutions of the corresponding nonlinear systems of partial differential equations that describes the processes that take place in the facilities of the GTS used iterative approaches, which are based on the linearization of the original system with a further refinement of the solution by calculating the respective discrepancies.

THE ANALYSIS OF RECENT RESEARCHES AND PUBLICATIONS

In general, the simulation of the transmission of gas through pipelines is reduced to the analysis of the complete system of equations of gas dynamics, which contains the equation of continuity, change of momentum, energy, and the equation of gas state. The complete system of partial differential equations of gas dynamics is generally a non-linear, and its solution is connected with considerable difficulties. At this time, thay built a significant number of ways to solve problems of this type [1-6]. All of them can be divided into several classes: analytical, numerical, approximate and iterative. Each has its advantages and disadvantages. As practice shows, the choice of models and methods dictated mainly problems that need to be solved on the basis of the constructed mathematical model. The one of effective solution approaches have linearization of the initial equations with the following refinement of solutions based on the building of iterative procedures. Most modelling techniques that have practical use, lead to the necessity of solving systems of differential or algebraic equations of high dimensionality. This raises issues of resistance and the minimum time for solving systems with guaranteed accuracy.

Publications on the subject of the work carried out by many groups [7-11]. Attention should be given only to those works that have been tested on real data and can be used for the development of modern systems of dispatching management. Requirements for such works are well-known - they have to work steadfastly in the entire range of gas-dynamic processes that produce results in the modeling of gas-dynamic processes in systems with complex piping diagram does not require simplification or equivalenting parts of the facilities, to be adaptable to the different characteristics of the actual operation of gas transmission systems. At this time, the models of gas flows for individual facilities are sufficiently developed. The main problems associated with the development of methods and algorithms for the implementation of models of individual and interrelated systems of facilities. This requires a minimum simplified models of systems, providing stability of simulation process in the whole range of flow parameters and meet many of the technical and technological limitations, to minimize the time of obtaining results and maximizing the automated process of formulating and solving problems. A certain part of the works that related to the development of commercial products, they are type of advertising and advertise only functional part of the developed systems, rather than mathematical apparatus implementation. This applies to software systems: Astra (Russia), SIMONE (designer SIMONE Research Group sro) and developments, which are operated by Schlumberger. The most informative conference proceedings are, in particular, conferences which are held in "Gazprom VNIIGAZ" [22-24].

MAIN RESULTS

Unsteady non-isothermal model of gas flow.

Unsteady, nonisothermal gas flow in pipelines is described by the system of equations [5,6,9]:

$$\begin{aligned}
&\frac{\partial(\rho v)}{\partial t}+\frac{\partial}{\partial x}\left(p+\rho v^2\right)=-\rho\left(\frac{\lambda v|v|}{2D}+g\frac{dh}{dx}\right),\\
&\frac{\partial\rho}{\partial t}+\frac{\partial}{\partial x}(\rho v)=0,\\
&\frac{\partial}{\partial t}(\rho E)+\frac{\partial}{\partial x}\rho v\left(E+\frac{P}{\rho}\right)=\frac{4k(T_{гр}-T)}{D}-\rho vg\frac{dh}{dx}.
\end{aligned} \quad , (1)$$

where: ρ - the density of the gas; p - gas pressure; v - gas velocity; λ - coefficient of hydraulic resistance; k - coefficient of heat transfer from the pipeline to the ground; T and $T_{гр}$ - the gas temperature and the soil temperature, respectively; h - the depth of the pipeline; E - total energy per unit of gas mass; g - acceleration of gravity; D - the diameter of the pipeline; $x,\ x\in[0,l]$ - the coordinate of the pipeline; l - the length of the pipeline.

To close this system of equations using the equation of gas state:

$$p=\rho zRT\ , \quad (2)$$

where: z - compressibility factor that characterizes the difference between a real gas from the ideal and is determined on the basis of empirical relationships built [1], R - gas constant.

Gas compressibility factor is calculated for the well-known formula [12-13]:

$$z=\frac{1}{1+f(a-bp)p}\ , \quad (3)$$

where: a and b - approximation coefficients calculated z for the known procedures - procedures for calculating gas compressibility factor, such as Hall-Yamburg, Redlich-Kwong and others $f=\left(24-0.21t^{\circ}C\right)\cdot 10^4$, here $p(x)$ - measured in atmospheres.

When steady-state conditions of gas flow the temperature depends only on the coordinates and calculated by the formula:

$$T(x)=T_{01}+T_{02}e^{-ax}\ ,$$

where:

$$T_{01}=T_г-T_{00},\ T_{02}=T_0-T_г+T_{00},\ a=\frac{k\pi D}{C_pM}\ ,$$

$$\Delta p=p_0-p_k\ ,\ T_{00}=\frac{1}{aL}\left(\Delta p\left(D_i-\frac{1}{c_p\rho_0}\right)+\frac{g\Delta h}{c_p}\right).$$

The Reynolds number is calculated by the formula:

$$\mathrm{Re}=\frac{Dvp}{\mu_0RT}\ \frac{T+C}{273+C}\ \left(\frac{273}{T}\right)^{3/2}.$$

where: $M=\rho_0Q_0$ - the mass flow rate; T_0 - temperature of the inflow gas to the pipeline; T_{gr} - temperature of the soil; D_h - Joule–Thomson coefficient; C_p - heat transfer coefficient from the gas to the soil; Δh - height difference between the end and the beginning of the pipeline; p_0 - the value of the pressure at the beginning of the pipeline; $p=p(x)$ - the distribution of the pressure along the pipeline; ρ_0 - the gas density under normal conditions; x - running coordinate. Gas constant and compressibility factor, usually taken constant. (3)

The linearized system by velocity occurs for horizontal pipelines in certain ranges of velocity change along the pipeline. The large-diameter pipes with significant volumes of gas transmission must take into account the change in its kinetic energy, to which little attention in publications for the subject. Linearization method, which allows you to specify the linearized model and iterative process to build solutions of the system is as follows.

The curve $f\left(v\right)=v^2$ will replace the chord and tangent. Since the transmission of gas velocity changes from v_1 to v_2, then the equation of the chord and the tangent that passes through the point:

$$\left(v_1, f\left(v_1\right)\right),\ \left(v_2, f\left(v_2\right)\right),$$

of view $\varphi_i\left(v\right)=a_{0i}v-b_{0i}$,

where: $i=1$ are for chord, and $i=2$ - a tangent. And wherein:

$$a_{01}=a_{02}=v_1+v_2,\ b_{01}=v_1v_2 \text{ та } b_{02}=0.5\left(v_1+v_2\right)^2.$$

Given the above, the system (1) in an isothermal mode of flow can be written as:

$$\begin{cases} \dfrac{\partial\left(\rho v\right)}{\partial t}+\dfrac{\partial p}{\partial x}+\dfrac{\lambda a_{0i}}{2D}\rho v-\dfrac{\lambda b_{0i}}{2D}\rho=\dfrac{\lambda\rho}{2D}\left[v^2-\varphi_i\left(v\right)\right] \\ -\dfrac{\partial\left(\rho v\right)}{\partial x}=\dfrac{1}{c^2}\dfrac{\partial p}{\partial t} \end{cases},$$

or

$$\begin{cases} \dfrac{\partial\omega}{\partial t}+\dfrac{\partial p}{\partial x}+a_i\ \omega-b_i p=\dfrac{\lambda\rho}{2D}\left[v^2-\varphi_i\left(v\right)\right] \\ \dfrac{1}{c^2}\dfrac{\partial p}{\partial t}+\dfrac{\partial\omega}{\partial x}=0 \end{cases},\quad(4)$$

where indicated:

$$b_i=\frac{\lambda b_{0i}}{2DgzRT},\ \omega=\rho v,$$

$$a_i=\frac{\lambda a_{0i}}{2D}.$$

To determine the hydraulic resistance of the gas flow there is empirical formula:

$$\lambda=\alpha_1\left(\frac{\alpha_2}{\mathrm{Re}}+\frac{\alpha_3 k_{ш}}{D}\right)^{\alpha_4},\quad(5)$$

where: $\alpha_i,\ i=\overline{1,4}$ - are some known constants, $k_{ш}$ - roughness of pipelines ($k_{ш}$ and D in mm), μ_0 - the dynamic viscosity under normal conditions, C - the Sutherland's constant. Formula (5) gives the greatest error under unsteady operating modes (laminar to turbulent flow and vice versa).

Present system (2) is a key system of equations of unsteady model of non-isothermal gas flow in the pipeline, taking into account the profile of the pipeline.

Unsteady flow model of gas transmission networks [19-21]. Mathematical model of the gas transmission system is based on its piping diagram. Structural properties of the piping diagram affect both the dimension of the system (system model) and the complexity of its solutions. Conducted numerical experiments showed that some simplification of the piping diagram provides greater stability and reduce the time for solving the corresponding systems. The basic graph operations such is the union of edges and contraction of the edges into the vertex. Important is the sequence of operations.

Building and editing piping diagrams. GTS piping diagram presented as a graph $G=\left(V,E\right)$, where V - the set of vertices (nodes), E - a set of edges. Edges represented objects that have extension in space, vertices - all other objects. In the case of mathematical modeling of processes that take place in the edges, each of which is divided into a certain amount (which depends on the length of an edge) segments. The end result is the calculation of unsteady flow, which take place in the GTS, is reduced to solving the system of equations. An important performance criterion is the choosing of the minimum number of segments of edges, to receive the smaller dimension of the system of equations, and thereafter, it will be solved. On the other hand - for higher accuracy simulation of unsteady processes need more of these segments. It should therefore be a balanced approach to reduce the dimensionality of the system of equations.

The obvious fact is that the graph must not contain zero length edges or diameters, therefore these edges are identified with one of the vertices (this includes open valves, bypass valves, and etc.). It is also appropriate to consider a sequence of edges that have the same diameter as one edge. That is, if certain adjacent edges $e_j=\left(v_{i-1},v_i\right)$, $e_{j+1}=\left(v_i,v_{i+1}\right)$ have the same diameter $\left|D_{e_j}-D_{e_{j+1}}\right|<\varepsilon_D$, where ε_D - the tolerance value for the diameter of edges e_j and e_{j+1} and assign a value to the length of a new edge equal to the sum of the two combined $L_{e_j^*}=L_{e_j}+L_{e_{j+1}}$ and $D_{e_j^*}=D_{e_j}$. Another parameter that allows you to simplify the system of equations is to set a minimum length of edges in the graph L_{sh}. If an edge is shorter than L_{sh}, it is identified with a vertex. This reduces the number of edges, and therefore the number of equations. By choosing the value L_{sh} necessary to take care, given that the volume of the geometric edges of the modified graph was not significantly different from the original graph of the GTS, as well as the topology was not changed. Considering it is also contemplated not to conduct shrinkage of the edge, despite the fact that $L<L_{sh}$.

It should also be noted that certain parameters of the vertex v of any incident to the joint edges (pressure or inflow or outflow of gas), which is absent in the modified graph, should be considered at the vertex of the start $v_{поч}$ or end of the resulting edge e_R. The algorithm is implemented choice of the vertex with regard to the distance corresponding to the vertices, that is, if $L\left(\overrightarrow{v,v_{поч}}\right)\le L\left(\overrightarrow{v,v_{пкін}}\right)$, then the change will occur to the vertex $v_{поч}$.

Network model of unsteady flow [14-15]. Since when unsteady gas flow Kirchhoff's second law is not fulfilled, then the design of the corresponding mathematical model should be carried out on other principles - conservation equations.

For ease description, consider a system with a single vertex that contains M pipeline sections. Suppose that the system consists of the M_{in} input and M_{out} output sections that are indexed in the appropriate order. Denote x_J the point of connection. Denote the length and the diameter of the k - section through L_k and S_k respectively and ($k=\overline{1,M}$). On Each site we choose a point $\{x_{J\mp}\}_k$, which is fairly close to x_J (the "+" or "-" in the index depends on the direction of gas flow). Then for. k section system (1) takes the form:

$$\begin{cases} \left\{ \dfrac{\partial\omega(t,x)}{\partial t} + \dfrac{\partial p(t,x)}{\partial x} + C_\omega \omega(t,x) + C_P\, p(t,x) \right\}_k = 0, \\ \left\{ \dfrac{\partial p(t,x)}{\partial t} + c^2 \dfrac{\partial \omega(t,x)}{\partial x} \right\}_k = 0, \ k=\overline{1,M}, \end{cases} \quad (6)$$

where: c - the speed of sound in the gas, $\omega = \rho v$,

$C_\omega = \dfrac{\lambda v_c}{2S}$, $C_P = \dfrac{g}{zRT}\dfrac{dh}{dx}$,

v_c - average speed,

$x \in \left[0, \{x_{J-}\}_k\right]$, when $k \le M_{in}$ or

$x \in \left[\{x_{J+}\}_k,\ L_k\right]$, when $k > M_{in}$.

Given the equality of pressures for all sections at the point of connection and the Kirchhoff's first law we will have model of the gas flow in the vicinity of the connection vertex:

$$\begin{cases} \left\{ \dfrac{\partial\omega(t,x)}{\partial t} + \dfrac{\partial p(t,x)}{\partial x} + C_\omega \omega(t,x) + C_P p(t,x) \right\}_k = 0, \ k=\overline{1,M}, \\ \left\{ \dfrac{\partial p(t,x)}{\partial t} + c^2 \dfrac{\partial \omega(t,x)}{\partial x} \right\}_k = 0, \\ \{p(t,x_J)\}_i = p(t,x_J)\}_j, \qquad \forall i,j=\overline{1,M}, \\ \sum\limits_{i=1}^{M_{in}} \{S\omega(t,x_J)\}_i - \sum\limits_{j=M_{in}+1}^{M} \{S\omega(t,x_J)\}_j = 0, \end{cases} \quad (7)$$

where: $x \in \left[0, \{x_{J-}\}_k\right]$, when $k \le M_{in}$ or

$x \in \left[\{x_{J+}\}_k,\ L_k\right]$, when. $k > M_{in}$.

System (7) will describe the gas flow in the M sections which are connected at one point x_J. This approach can be easily generalized to the case of more complex network structure (with lots of vertices). To do this, we design a directed graph whose edges correspond to sections of the pipelines. Then, for every edge and vertex (vertices degree which is greater than 1) we can work out the set of equations (6) and (7) respectively.

To find a numerical approximation of the solution (6), (7) it is advisable to perform a spatial and temporal sampling of the model. On each time step we take the number of iterations to solve the linearized system of equations with sufficient accuracy, and have solutions of nonlinear equations (1).

Simulation of compressor stations [16-18]. The model of a compressor station (CS) is based on the model of the structure and the models of its facilities. The structure model is represented as a graph in which the objects that have extension represented as edges and all others as vertices. The main object is the gas compressor unit (GCU), and consist of the engine and the centrifugal compressor (CC). It is known [16] that the parameters of the gas inflow and outflow of CC are associated by a set of empirical equations:

$$\varepsilon = \varphi_1\left([q]_{пр}, \left[\frac{n}{n_H}\right]_{пр} \right),\ \eta_{пол} = \varphi_2\left([q]_{пр}\right),$$

$$\frac{N_i}{\gamma_n}\left(\frac{n_n}{n}\right)^3 = \varphi_3\left([q]_{пр}\right).$$

$$T_{вих} z_{вих} = T_{вх} z_{вх} \varepsilon^{\frac{k-1}{k\cdot\eta_{пол}}},$$

$$N_e^p = N_e^n K_{Ne}\left(1 - K_t\frac{t_0 - t_0^n}{t_0 + 273}\right)\frac{p_a}{0{,}1033},$$

Other operating parameters of the GCU folow the equations:

$$q_{pg} = q_{pg}^n K_t (0,75\frac{N_e}{N_e^n} + 0,25\sqrt{\frac{t_0+273}{t_0^n+273}\frac{p_a}{0{,}1033}}),$$

$$q_{pg}^n = \frac{860 N_e^n}{\eta_e^n Q_n 10^3}, \quad N_e = N_i : (\eta_m K_N).$$

where: n - speed of the CC, q - the gas flow rate through the CC, $\eta_{пол}$ - polytropic efficiency, q_{pg}^n - nominal flow rate of fuel gas, ε - pressure ration, N_e^n - rated power of the the gas turbine; K_{Ne} - coefficient of technical state of the gas turbine; K_t - coefficient, which takes into account the effect of air temperaturer; t_0 - temperature of the air at the inlet of the gas turbine; t_0^n - nominal temperature of the air at the inlet of the gas turbine; p_a - the absolute pressure of the air depending on the height above sea level H; Q_n - the nominal lower heating value; η_e^n - rated polytropic efficiency; η_m - mechanical efficiency, K_N - technical condition according to power.

The developed algorithm of CS operation for a set of input data $(\rho_c, P_1, P_2, T_1, q, \{M_i^k\})$ (the density of gas at standard conditions, the gas pressure at the inlet, outlet gas pressure, gas flow rate, count of GCU in each i workshop) calculates the operating mode of the CS $(T_2, s_{ij}^k, n_{ij}, \varepsilon_{ij}, q_{pg}^{ij}, N_{ij})$ (the outlet temperature; scheme of GCU connections; i - stage number; j - number of GCU in the stage; k - the type of GCU; CC speed; pressure ratio; the amount of fuel gas and gas turbine power). Indices $(i,j) \in \{N_i^k, N_j^k\}$ (N_i^k - set of stages of CC; N_j^k - set of CC at i - stage).

To calculate operating mode parameters at time $t_{j+1} \cdot \left(t_{j+1} = t_j + \Delta t\right)$ is necessary to solve a system of equations for the unknowns q (volume flow rate) and P (gas pressure). In this system of equations, among other conditions are met pairing. If we consider the CS as the edge of the GTS, it is necessary to set the compression ratio ε, which reached by CC, working for a given power $W = (W^1, ..., W^n)$, where n - the count of workshops of the CS, and this edge will provide the fulfill of the equation $P_2 = P_1 \cdot \varepsilon$. To calculate ε we have realized function that calculates the operating mode of the CS $(T_2, s_{ij}^k, n_{ij}, \varepsilon_{ij}, q_{pg}^{ij}, N_{ij})$ using data $(\rho_c, P_1, P_2, T_1, q, \{M_i^k\})$.

Transition of the CS to the non operating mode, occurs using compression ration $\varepsilon = 1$. Turning on CS is performed within N_{On} steps of approching compression ratio ε from $\varepsilon_{\min}$ to $\varepsilon_{default}$. Then there is the operating mode of the CS in the situation that has occured and we fix the power of the CS.

Changing power of the CS (increase or decrease) is proportional to all the operating workshops by a certain percentage κ. In this case, κ defined as the percentage deviation of the monitored parameters from the optimum value

The boundary conditions. At the inputs and outputs of the network of gas pipelines are set boundary conditions on the pressure and flow rate. Some boundary conditions are calculated during the simulation of gas-dynamic processes. On calculation of the boundary conditions have a significant impact technical and technological restrictions: the work points on the characteristics of CC, surge area; CC maximum volume flow rate; CC shaft speed $n_{\min} \le n \le n_{\max}$; maximum power of the gas turbine; CC maximum initial pressure, which is determined by the strength of pipelines at the inlet of CC; the maximum temperature at the outlet of CC is defined by the insulation coating pipelines; minimum value of pressure at the outlet of each CC; conditions of the consistency of connection scheme with inlet and outlet pipelines.

The numerical experiment. The numerical experiment was conducted on a real gas transmission system, which belongs to one of the depatments of the PJSC "Ukrtransgaz" (see Fig.1). The main objective was to analyze the impact of topology changes on the dynamics of changes in the flow parameters, the distribution over time. The change of topology is the opening of three taps at a specified time. Before the simulation was carried out to identify models of gas flows in the facilities of the system. Opening taps lasted 30 minutes. Step-by-time variable was equal $\Delta t = 10 x в$.

After simulating parameters of the gas flows we got values for each facility type vertex - pressure and temperature, and type of edge – volume flow rate over a simulation time. The simulation results, as an example, are shown for one of the valves in Fig. 2.

The current system of finding of the boundary conditions and simulation of unsteady modes with variable step provides simulation and control valves - pressure reducers and valve systems for hydraulic control functions and protection. (control valves MOKVELD). The ratio of simulation time with a complex piping diagram to the real-time transmission of gas dynamic processes is in the range 1: 15 - 1: 20, which is completely acceptable for practical use. The main factor of influence on the stability of the simulation - change states of a few taps simultaneously. In this case, stability of the method is enshured by a speed of model gas dynamic processes in the vicinity of taps, as well as the reduction of the time step in the numerical analysis.

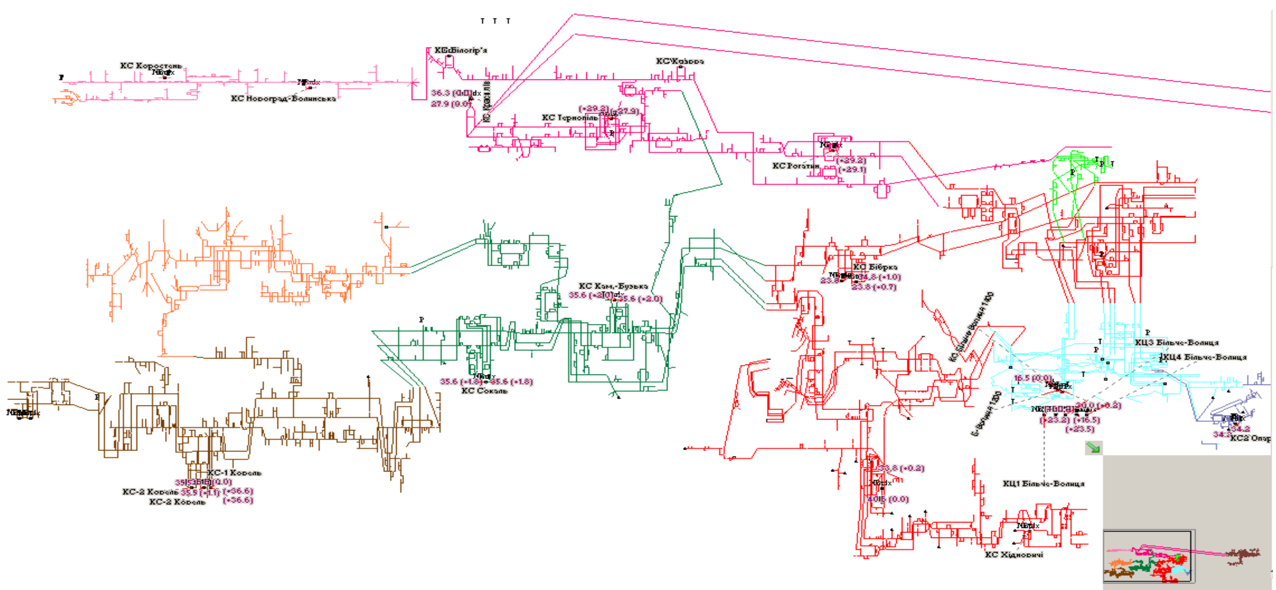

Fig.1. Piping diagram of gas transmission system of pipeline operator "Lvivtransgaz"

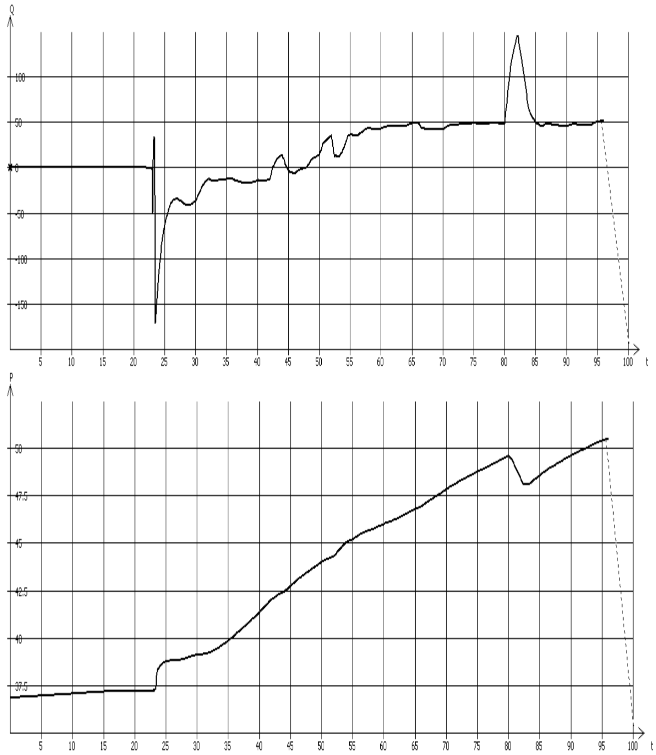

Fig. 2. The graph of the gas volume flow rate (above) and the graph of the gas inlet pressure of tap (bottom)

CONCLUSIONS

1. The proposed approach to the calculation of the parameters of gas transmission systems with complex piping diagram under the conditions of its operation in unsteady conditions has been tested on real data and demonstrated a high level of stability.
2. Stability of the method for solving systems of equations of large dimension is ensured by editing options of piping diagram and adaptive methods for passage speed of gas-dynamic processes.
3. It is remains the open problem of optimal control transition operating modes from the current to a certain optimal steady mode.

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