Paper

A DFT-based Low Complexity LMMSE Channel Estimation Technique for OFDM Systems

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Abstract—The linear minimum mean square error (LMMSE) channel estimation technique is often employed in orthogonal frequency division multiplexing (OFDM) systems because of its optimal performance in the mean square error (MSE) performance. However, the LMMSE method requires cubic complexity of order $O(N_p^3)$, where N_p is the number of pilot subcarriers. To reduce the computational complexity, a discrete Fourier transform (DFT) based LMMSE method is proposed in this paper for OFDM systems in the frequency selective channel. To validate the proposed method, the closed form mean square error (MSE) expression is also derived. Finally, a computer simulation is carried out to compare the performance of the proposed LMMSE method with the classical LS and LMMSE methods in terms of bit error rate (BER) and computational complexity. Results of the simulation show that the proposed LMMSE method achieves exactly the same performance as the conventional LMMSE method, with much lower computational complexity.

Keywords—channel estimation, LMMSE, mean square error, OFDM.

1. Introduction

Orthogonal frequency division multiplexing (OFDM) has been attracting a lot of attention due to its high spectrum efficiency, as well as fast and easy implementation using the fast Fourier transformation (FFT) approach. It is also resilient to inter symbol interference (ISI) which occurs due to the frequency selective nature of the fading channel [1]. However, high peak-to-average power ratio (PAPR) [2] and channel estimation accuracy [3] are the major challenges for OFDM systems. The equalization of the OFDM system solely depends solely on the accuracy of channel estimation [4].

Generally, there are two schemes that may be relied upon to estimate the channel, namely non-blind or pilot-aided and blind schemes. When compared to pilot-aided channel estimation, the blind channel estimation approach is limited to slow time varying channels and also has a higher level of complexity and is characterized by poorer performance. Hence, pilot-aided channel estimation is preferred over blind channel estimation. Based on the comb-type pilot, the least square (LS) and the minimum mean square error (MMSE) based channel estimation methods have been investigated in [5]. LS estimation has low computational complexity but its mean square error (MSE) is high due to the noise enhancement problem. To obtain better performance of the LS based estimation method, several denoising strategies have been proposed in [6]–[9].

Eigen-select denoising threshold [6], linear filtering least square method [7], AdaBoost [8] and singular spectrum analysis (SSA) [9] based channel estimation techniques are proposed for channel estimation in OFDM systems.

In [10], the authors proposed an adaptive SSA based channel estimation method. In adaptive SSA, additional noise reduction is performed at a singular value level and, therefore, it provides better performance as compared to the SSA algorithm-based channel estimation method. However, all these channel estimation methods provide a trade-off between performance and computational complexity. If the power delay profile (PDP) of the channel is known to the receiver a priori, the LMMSE channel estimation method is typically implemented. However, it requires cubic complexity due to matrix inversion operation.

To reduce the complexity of LMMSE estimation, the low rank approximation-based singular value decomposition (SVD) approach is proposed in [11]. Based on the SVD method [11], the authors proposed two efficient channel estimation methods for OFDM/OQAM systems in [12]. However, these SVD-based estimation methods are still characterized by high computational complexity, as decomposing the R_{HH} matrix using the SVD method itself requires cubic complexity [13].

In [14], the authors approximate the LMMSE method using the law of large numbers to reduce the computational complexity of the channel to $O(N \log N)$. However, its performance is poor at high SNR values. In [15], the authors proposed a dual-diagonal LMMSE (DD-LMMSE) channel estimation method with $O(N \log N)$ and also derive the closed form expression of the asymptotic MSE of the DD-LMMSE.

In [16], the author proposed a low complexity LMMSE channel estimation method based on K terms Neumann series expansion method to avoid the matrix inversion. A joint low complexity channel estimation and symbol detection approach is proposed in [17], based on a message passing algorithm. In this method, the Sherman-Morrison formula is applied which transforms cubic matrix inversion into a series of diagonal matrix inversions, thus reducing computational complexity.

In [18], the authors proposed a conjugant gradient (CG)-based channel estimation method to achieve similar performance as that of the LMMSE method. This method performs the channel estimation process in an iterative manner and requires computational complexity of the order $O[(N_p \log N_p)G]$, where G is the number of iterations. Typically, a high value of G is required to achieve optimal performance. A structure-based LMMSE estimation method was proposed in [19] by assuming that the number of pilots is an exact integer multiple of channel length. This method depends on prior information regarding the channel's impulse response and on the appropriate placement of pilots across the OFDM subcarriers. Although the length of CIR can be obtained from the knowledge of the channel autocorrelation function by using the adaptive guard interval (GI) as given in [20], the length of the channel may not be guaranteed to be equal to exact integer multiples of the number of pilot subcarriers.

A compressed sensing (CS) algorithm based MMSE channel estimation is proposed in [21]. This method offers similar performance to that of LMMSE, with much lower computational complexity. However, this method assumes that the channel coefficients are sparse while estimating the channel. Recently, an LMMSE algorithm based on the vector quantization approach was proposed in [22] for OFDM systems. In this method, LMMSE filtering matrices corresponding to wireless channel parameters are calculated offline in the first phase. Subsequently, an appropriate LMMSE filtering matrix is selected according to the MSE criterion while estimating the channel in the online mode. Therefore, this method does not require the PDP to be known at the receiver and, hence, is characterized by negligible performance degradation with much lower computational complexity.

In this paper, a simple but efficient LMMSE channel estimation technique is proposed by exploiting discrete Fourier transformation (DFT) and circulant properties of the channel frequency autocorrelation matrix (R_{HH}) for OFDM systems in a frequency selective fading channel.

The symbols associated with the matrices and vectors are denoted with the use of boldface and underlined characters, respectively. Notations $(.)^H$, $(.)^{-1}$ denote the Hermitian and inverse operation, respectively. Similarly, $(.)_p$ denotes the position of the pilot signal and $E[.]$ symbolizes the expectation operator.

2. System Model

Let us consider an OFDM system with N number of subcarriers. After some signal manipulation consisting in addition of a cyclic prefix (CP), removal of CP, inverse fast Fourier transformation (IFFT) and FFT operation, the received signal vector in the frequency domain is given by:

$$\underline{Y} = \mathbf{X}\underline{H} + \underline{W} , \tag{1}$$

where $\underline{Y} = \left[Y(0), Y(1), \ldots, Y(N-1)\right]^T$.

The transmitted signal $\mathbf{X} = \mathrm{diag}\left[X(0), X(1), \ldots, X(N{-}1)\right]$ is an $N \times N$ diagonal matrix. Symbols $\underline{H} = \left[H(0), H(1), \ldots, H(N-1)\right]^T$ and $\underline{W} = \left[W(0), W(1), \ldots, W(N-1)\right]^T$ are the $N \times 1$ channel frequency response (CFR) and additive white Gaussian noise (AWGN) vector, respectively. In this paper, the comb-type pilot pattern is adopted for channel estimation purposes. After extraction of the pilot symbol at the receiver side, the received signal vector at the pilot position can be written as:

$$\underline{Y}_p = \mathbf{X}_p \underline{H}_p + \underline{W}_p . \tag{2}$$

Parameters $\underline{Y}_p$, $\mathbf{X}_p$ and $\underline{H}_p$ are the frequency domain received signal, transmitted signal and CFR at the pilot position, respectively. The CFR vector at the pilot subcarrier can be represented as:

$$\underline{H}_p = \boldsymbol{F}_p \underline{h} , \tag{3}$$

where $\boldsymbol{F}_p$ is an $N \times L$ unitary FFT matrix with $\boldsymbol{F}_p(k,l) = \mathrm{e}^{-\mathrm{j}2\pi\frac{kl}{N}}$, $k = 0 : p_s : (N_p{-}1)p_s$, $l = 0 : L-1$.

The IFFT matrix is defined as $F_p^H = \frac{1}{N_p}(F_p)^H$. The channel impulse response (CIR) vector is defined as $\underline{h} = \left[h(0), h(1), \ldots, h(L-1)\right]$, where L is the total number of multipath channels. It is assumed that each multipath channel $h(l)$ is independent and identically distributed (i.i.d.) with a zero mean complex Gaussian random variable. The corresponding autocorrelation of the CIR $\underline{h}$ is given by $E[\underline{hh}^H] = \Delta$ with $\Delta = \mathrm{diag}\left[\Lambda(0), \Lambda(1), \ldots, \Lambda(L-1)\right]$ is the diagonal PDP matrix and $\Lambda(l)$ denotes the average power of the l-th delay path. The total power of the PDP $\mathrm{tr}(\Delta) = 1$ where $\mathrm{tr}(.)$ denotes the trace operation. The power delay profile (PDP) of the multipath is known to the receiver *a priori* or may be calculated using the method given in [23] with a computational complexity of $O(L^2)$. Channel estimation using the LS criterion is given by:

$$\underline{\tilde{H}}_{p,ls} = (X_p)^{-1} \underline{Y}_p . \tag{4}$$

In order to obtain the channel at all data subcarriers, the interpolation methods are to be deployed, such as linear interpolation, low pass interpolation, discrete Fourier transform (DFT)-based interpolation and so on [24]. In this paper, the DFT-based interpolation is adopted to obtain the CFR at all data subcarriers. After estimating the channel at all data subcarriers, one tap zero forcing equalization is performed to obtain the desired transmitted data signal at the receiver side. The LS method suffers from high MSE and

thus the LMMSE channel estimation method is typically adopted, as it is optimal in terms of MSE.
From [5], the LMMSE channel estimation method at pilot positions can be written as:

$$\underline{\tilde{H}}_{p,lmmse} = \boldsymbol{R}_{\underline{H}_p\underline{H}_p}\left(\boldsymbol{R}_{\underline{H}_p\underline{H}_p} + \frac{\beta}{SNR}\boldsymbol{I}_{Np}\right)^{-1}\underline{\tilde{H}}_{p,ls} \ . \quad (5)$$

The signal to noise ratio (SNR) is defined as:

$$SNR = \frac{E\left[|\underline{X}_p|^2\right]}{\sigma_W{}^2} \ .$$

Symbol $\beta = \frac{E\left[|\underline{X}_p|^2\right]}{E\left[|1/|\underline{X}_p|^2\right]}$ is a constant depending on the constellation. For QPSK and 16QAM modulation, the values of β are 1 and 17/9, respectively. The LMMSE method experiences high computational complexity of the order $O(N_p^3)$ due to the matrix inversion operations, as given in Eq. (5).

3. Proposed Low Complexity LMMSE Method

In this section, a low complexity LMMSE method is proposed that exploits the DFT technique. The channel frequency autocorrelation matrix is the Fourier transformation of the PDP and is given as:

$$\boldsymbol{R}_{\underline{H}_p\underline{H}_p} = E\left[\underline{H}_p\underline{H}_p^H\right] = N_p\boldsymbol{F}_p\Delta\boldsymbol{F}_p^H \ . \quad (6)$$

As $\boldsymbol{R}_{\underline{H}_p\underline{H}_p}$ and $\boldsymbol{R}_{\underline{H}_p\underline{H}_p} + \frac{\beta}{SNR}\boldsymbol{I}_{Np}$ are circulant matrices, hence they are commutative [25]. Thus, the LMMSE method can be rewritten as:

$$\underline{\tilde{H}}_{p,lmmse} = \left(\boldsymbol{R}_{\underline{H}_p\underline{H}_p} + \frac{\beta}{SNR}\boldsymbol{I}_{Np}\right)^{-1}\boldsymbol{R}_{\underline{H}_p\underline{H}_p}\underline{\tilde{H}}_{p,ls} \ . \quad (7)$$

Multiplying both sides of Eq. (7) with $(\boldsymbol{R}_{\underline{H}_p\underline{H}_p} + \frac{\beta}{SNR}\boldsymbol{I}_{Np})$ matrix, we have:

$$\left(\boldsymbol{R}_{\underline{H}_p\underline{H}_p} + \frac{\beta}{SNR}\boldsymbol{I}_{Np}\right)\underline{\tilde{H}}_{p,lmmse} = \boldsymbol{R}_{\underline{H}_p\underline{H}_p}\underline{\tilde{H}}_{p,ls} \ . \quad (8)$$

As $\frac{\beta}{SNR}$ is a scalar quantity, then $\frac{\beta}{SNR}\boldsymbol{I}_{Np}$ can be written as $N_pF_p(\frac{1}{N_p}\frac{\beta}{SNR}I_{\mathbf{L}})F_p^H$ and Eq. (8) can be simplified to:

$$\left[N_pF_p\Delta F_p^H + N_pF_p\left(\frac{1}{N_p}\frac{\beta}{SNR}\boldsymbol{I}_L\right)F_p^H\right]\underline{\tilde{H}}_{p,lmmse} =$$
$$N_pF_p\Delta F_p^H\underline{\tilde{H}}_{p,ls}$$
$$\Rightarrow N_pF_p\left(\Delta + \frac{1}{N_p}\frac{\beta}{SNR}\boldsymbol{I}_L\right)F_p^H\underline{\tilde{H}}_{p,lmmse} =$$
$$N_pF_p\Delta F_p^H\underline{\tilde{H}}_{p,ls}$$
$$\Rightarrow \left(\Delta + \frac{1}{N_p}\frac{\beta}{SNR}\boldsymbol{I}_L\right)\underline{\tilde{h}}_{lmmse} = \Delta\underline{\tilde{h}}_{ls}$$
$$\Rightarrow \underline{\tilde{h}}_{lmmse} = \delta\underline{\tilde{h}}_{ls} \ , \quad (9)$$

where $\delta = \text{diag}\left[\frac{\Lambda(0)}{\Lambda(0)+\frac{1}{N_p}\frac{\beta}{SNR}}, \ldots, \frac{\Lambda(L-1)}{\Lambda(L-1)+\frac{1}{N_p}\frac{\beta}{SNR}}\right]$. The parameters $\underline{\tilde{h}}_{ls}$ and $\underline{\tilde{h}}_{lmmse}$ are the estimated channel in the time domain using LS and MMSE criterion, respectively. The estimated channel frequency response is given as $\underline{\tilde{H}}_{lmmse} = \boldsymbol{F}_l\underline{\tilde{h}}_{lmmse}$, where $\boldsymbol{F}_l$ is an $N \times L$ FFT unitary matrix.

4. Performance Analysis

In this section, the mean square error of the proposed DFT-based LMMSE channel estimation method is derived. The time domain LS method is given by $\underline{\tilde{h}}_{ls} = F_p^H\underline{\tilde{H}}_{p,ls} = \underline{h} + F_p^H(X_p)^{-1}W_p$. The mean square error of the proposed channel estimation method can be written as:

$$\text{mse} = \frac{1}{L}\text{tr}E\left[|\underline{h} - \underline{\tilde{h}}_{lmmse}|^2\right] =$$
$$\frac{1}{L}\text{tr}E\left[|\underline{h} - \delta(\underline{h} + F_p^H(X_p)^{-1}W_p)|^2\right] =$$
$$\frac{1}{L}\text{tr}E\left[\underline{hh}^H - \underline{hh}^H\delta^H - \delta\underline{hh}^H + \right.$$
$$\left.\delta\underline{hh}^H\delta^H + \delta F_p^H\frac{\beta}{SNR}I_{N_p}F_p\delta^H\right] =$$
$$\frac{1}{L}\left(\sum_{l=0}^{L-1}\delta(l)[1-\delta(l)]^2 + \delta(l)^2\frac{1}{N_p}\frac{\beta}{SNR}\right) \ , \quad (10)$$

where $\delta(l) = \frac{\Lambda(l)}{\Lambda(l)+\frac{1}{N_p}\frac{\beta}{SNR}}$ and the parameter $\Lambda(l)$ is power of the l-th multipath channel.

5. Computational Complexity

The efficacy of LS, LMMSE and the proposed method can be compared by evaluating the computational complexity related to obtaining CFR at all subcarriers. The LS estimation technique at the pilot positions given in Eq. (4) requires N_p number of complex multiplications. In order to obtain the CFR at for all subcarriers, the DFT interpolation is used. This requires N_p numbers of IFFT and N number of FFT operations. Therefore, the overall computational complexity of the LS estimation approach requires $N_p + N_p\log_2 N_p + N\log_2 N$. The conventional LMMSE channel estimation at the pilot positions, as given in Eq. (5), requires multiplication and inversion of the CFR autocorrelation matrix $\boldsymbol{R}_{\underline{H}_p\underline{H}_p}$ of size $N_p \times N_p$. In order to determine the $\boldsymbol{R}_{\underline{H}_p\underline{H}_p}$ matrix in Eq. (6), N_p point FFT/IFFT operations of the diagonal channel autocorrelation matrix $E[hh^H]$ are required. As a result, the computation complexity of $\boldsymbol{R}_{\underline{H}_p\underline{H}_p}$ is $N_p\log_2 N_p$. Thus, the LMMSE channel estimation at the pilot positions requires computational complexity of $N_p^2 + N_p^3 + N_p\log N_p$.
Similarly, CFR at all subcarriers, as discussed in the previous paragraph, requires $N_p\log N_p + N\log N$ operations. The overall computational complexity of obtaining CFR using the LMMSE technique requires $N_p^2 + N_p^3 + 2N_p\log_2 N_p + N\log_2 N$. The proposed DFT-based low complexity LMMSE method is given in Eq. (9). The δ in Eq. (9) requires a diagonal matrix inversion of size $L \times L$. Therefore, the calculation of the δ parameter needs an L number of complex multiplications. CFR at all subcarriers

can be obtained by N point FFT operations. Therefore, the overall computational complexity of the proposed low complexity LMMSE method requires $L + N\log_2 N$ operations. The overall computational complexities of various channel estimation methods are compared and is listed in Table 1.

Table 1
Computational complexity

Methods	Computational complexity
LS	$N_p + N_p \log_2 N_p + N \log_2 N$
Classical LMMSE	$N_p^2 + N_p^3 + 2N_p \log_2 N_p + N \log_2 N$
Proposed LMMSE	$L + N \log_2 N$

6. Simulation Results

In this section, the performance of the proposed LMMSE method is compared with classical LS and LMMSE channel estimation methods in terms of MSE and BER. The following parameters are considered for an OFDM system model: the number of subcarriers $N = 128$, length of cyclic prefix $N_{CP} = 16$, system bandwidth $B = 1$ MHz and QPSK/16QAM modulation. If no modulation scheme is specified in the simulation, it is considered to be the of the 16QAM variety. The channel is assumed to be of the exponential decaying power delay profile (PDP) [26] with power of the l-th path given as $\Lambda(l) = \Lambda(0)\mathrm{e}^{-\frac{l}{d}}$ where:

$$\Lambda(0) = \frac{1 - \mathrm{e}^{-\frac{1}{d}}}{1 - \mathrm{e}^{-\frac{L}{d}}}$$

is the power of the first multipath channel. Parameter $d = \frac{-\tau_{rms}}{T_s}$ is the normalized delay spread, where T_s is the sampling period and τ_{rms} is the root mean squared (rms) delay of the channel. The number of multipaths is given by $L = \frac{\tau_{\max}}{T_s}$ where τ_{max} is the maximum excess delay and is defined as $\tau_{\max} = \tau_{rms} \ln A$ with A being the ratio of non-negligible path power to first path power. In this paper, the value of A is taken as $A = -40$ dB and normalized delay spread d =1.5, giving the total number of multipaths of

Table 2
Simulation parameters

Parameters	Value
Number of subcarriers	128
Number of FFT	128
Number of CP	16
Modulation type	QPSK/16QAM
System bandwidth	1 MHz
Subcarrier spacing	7.8125 kHz
Channel type	Exponential decaying PDP
Pilot spacing	8
Normalized delay spread	1.5
Number of multipath	14

$L = 14$. The total number of pilots is considered as $N_p = 16$ with pilot spacing of $p_s = 1{:}8$. All simulation parameters are listed in the Table 2. Perfect time and frequency synchronizations are assumed at the receiver side.

Figure 1 shows MSE performance with respect to SNR for the proposed LMMSE, classical LS and LMMSE methods with a normalized delay spread of $d = 1.5$ and $L = 14$. The simulation results show that the LS method offers poorer MSE performance than the LMMSE method, due to the noise enhancement problem. It is observed that the MSE performance of the proposed LMMSE estimation method exactly matches that of the classical LMMSE method. This is due to the fact, the proposed method is directly derived from the classical LMMSE method, without any approximations.

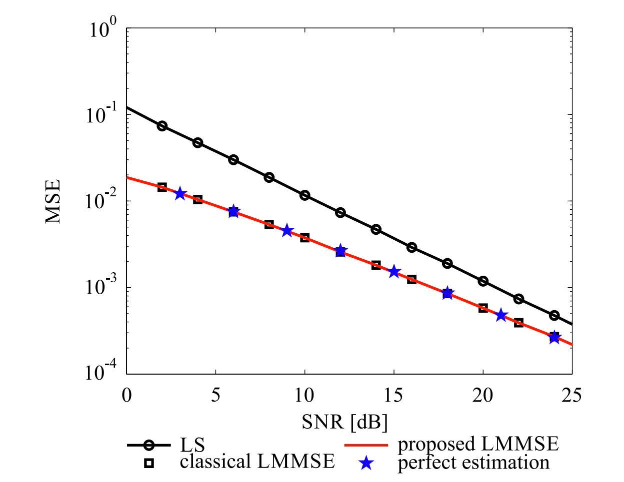

Fig. 1. Comparison of MSE vs SNR performance between the proposed LMMSE, classical LS and LMMSE channel estimation methods.

In order to analyze the effect of normalized delay spread d on the performance of various channel estimation methods, multiple values of d are taken into considerations, e.g. $d = [0.3\ 0.6\ 0.9\ 1.2\ 1.5\ 1.8]$. This leads to the total number of multipath channels equaling $L = [3\ 6\ 8\ 12\ 14\ 16]$. The MSE vs. normalized delay spread (d) for various channel estimation methods is shown in Fig. 2. The result is obtained for pilot spacing of $p_s = 8$ at 25 dB SNR. The simulation result shows that the performance of the proposed LMMSE approach exactly matches that of LMMSE, irrespective of the value of normalized delay spread d. It is also noticed that the performance of LS is close to that of the LMMSE method for low values of d. However, the performance gap increases with an increase in normalized delay spread value.

BER performance comparison of the proposed LMMSE, classical LS, LMMSE and perfect channel estimation methods for pilot spacing of $p_s = 8$ is shown in the Fig. 3. The simulation results show that BER performance of the LS estimation method is poorer when compared with the LMMSE estimation method. Figure 3 that performance of the proposed LMMSE method is very close to that of

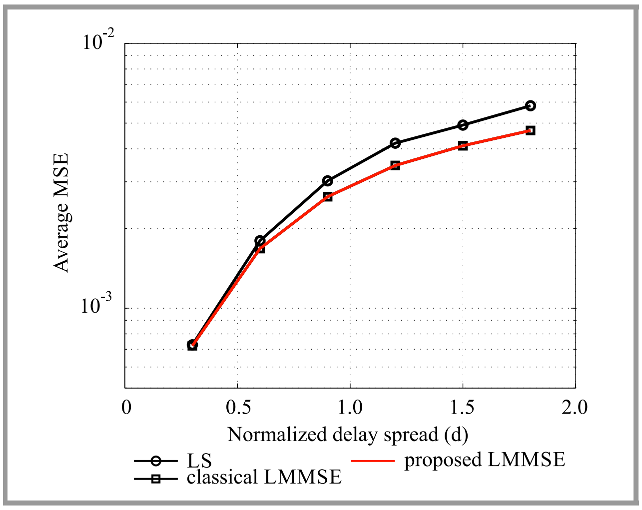

Fig. 2. MSE vs. normalized delay spread of the proposed LMMSE, classical LS and LMMSE channel estimation methods.

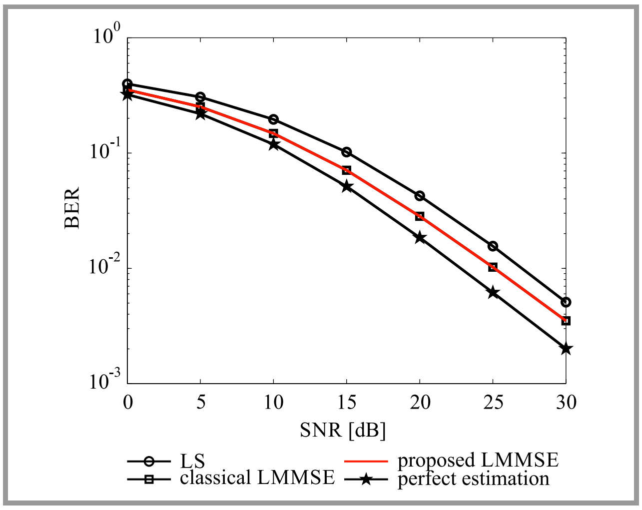

Fig. 3. BER vs. SNR performance comparison of the proposed LMMSE, classical LS, and LMMSE channel estimation methods for pilot spacing of $p_s = 8$.

the perfect estimation method, where it is assumed that complete channel state information (CSI) is known at the receiver side.

Comparison of BER vs. SNR performance of the various channel estimation methods with QPSK and 16QAM modulation for pilot spacing of $p_s = 8$ is shown in Fig. 4. The simulation results show that the performance of QPSK modulated channel estimation methods outperforms that of the 16QAM modulated channel estimation methods. It is also observed that the performance gap between LMMSE and perfect estimation with QPSK modulation is narrower than compared with 16QAM modulation.

7. Conclusion

In this paper, an optimal low complexity DFT-based LMMSE channel estimation method is proposed for an OFDM system in the frequency selective channel. The closed form MSE expression is also derived to validate the proposed method. The proposed LMMSE method is compared with conventional channel estimation methods in terms of performance and computational complexity. Simulation results show that the proposed LMMSE channel estimation approach exactly matches the theoretical assumptions and achieves the same performance as the classical LMMSE channel estimation method, with computational complexity of $L + N \log_2 N$ only. The limitation of the proposed LMMSE channel estimation technique is that it is not applicable to non-sample spaced channels, as it utilizes the DFT technique to estimate the channel.

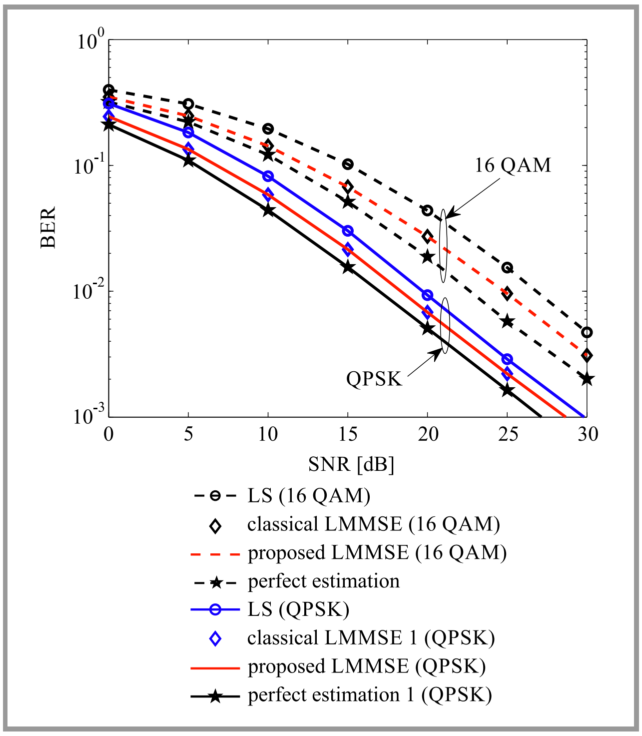

Fig. 4. BER vs. SNR performance comparison of the proposed LMMSE, LS, and LMMSE channel estimation methods with QPSK and 16QAM modulation for pilot spacing of $p_s = 8$.

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