THE OPTIMIZATION OF A REVENUE FUNCTION IN A FUZZY LINEAR PROGRAMMING MODEL USED FOR INDUSTRIAL PRODUCTION PLANNING

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Abstract

In this paper, the S-curve membership function methodology is used in a real-life industrial problem in which there are various products, each of which requires a certain mix of raw materials selected from a set of available raw materials. This problem occurs in the chocolate manufacturing industry where decision makers and implementers play important roles that enable successful manufacturing of the products in an uncertain environment. The analysis in this paper tries to find a solution that helps a decision maker when deciding on what to implement. This problem is considered because it can be modeled with the help of fuzzy parameters (for example, the availability of raw materials is not always certain, and so can be treated as a fuzzy parameter). With 29 constraints and 8 variables the problem here is sufficiently large for the S-curve methodology employed because this methodology is applicable to problems with as few as 1 constraint and 1 variable. A decision maker can specify which vagueness parameter α is suitable for achieving a revenue which through the analysis results in an initial solution that can be implemented. From the results of this implementation the decision maker can then suggest some possible and practicable changes in fuzzy intervals for improving the revenue. Within the framework of the analysis this interactive process has to go on between the decision maker and the implementer until an optimum solution is achieved and implemented.

Key words: Production planning, Vague environment, Membership function, Fuzzy interval, Satisfactory solutions

1 Introduction

A non linear membership function, referred to as the "S–Curve Membership Function" has been used in problems involving interactive fuzzy systems. A modified S-curve membership function [16, 9, 23] can be applied and tested for its suitability for a given problem. For the problem in this paper, a modified S-curve membership function is applied to reach a decision when factors, such as the objective function, technical coefficients and resources related to the product-mix selection are fuzzy. The solution obtained is suitable to be given to a decision maker and implementer for final implementation. The problem in this paper will be referred to as a fuzzy product-mix selection (FPS) problem, and in fact it is only one of eight cases of such problems that occur in real-life applications. It is of interest to investigate the fuzzy solution patterns of our problem and to do so we consider the case of Chocolate Manufacturing. The data for this problem are taken from the data-bank of Chocoman Inc, USA [22]. Chocoman produces varieties of chocolate bars, candy and wafers using a number of raw materials and processes. The objective is to use the modified S-curve membership function for obtaining a revenue maximization procedure through the fuzzy linear programming (FLP) approach.

Many authors have studied fuzzy linear programming models and used different methodologies to solve problems related to fuzzy optimization [19, 3, 18, 24, 14, 13, 21]. Zimmermann has offered a general formulation for fuzzy linear programming [27]. Fuzzy linear programming models are robust and flexible [10, 11, 13]. Decision-makers consider the existing alternatives under given constraints, but also develop new alternatives by considering all possible situations [28].

Various types of membership functions expressing a vague aspiration level of a decision maker are proposed such as linear membership functions [12, 6], a tangent form membership function [14], an interval linear membership function [30], an exponential membership function and an inverse tangent membership function [20]. As tangent, exponential, and inverse tangent membership functions are a non-linear, a fuzzy mathematical programming problem defined with a non-linear membership function results in non-linear programming. Usually a linear membership function is employed in order to avoid non-linearity. Nevertheless, there are some difficulties in selecting the solution of a problem that uses a linear membership function. Therefore a logistic membership function is employed by Watada [23] to overcome the difficulties that arise with linear functions; this function is nonlinear but Watada's treatment of it avoids many of the difficulties usually encountered when dealing with nonlinear functions – see Reference [23] for more details. In this paper a flexible modified logistic membership function of S-curve membership type is employed to solve the product mix selection problem.

The paper is arranged in the following order. The construction of an S-curve membership function for the selection problem followed by the formulation of the corresponding fuzzy linear programming (FLP) problem. The mathematical model arising from the FLP problem is solved using the Linear Programming (LP) toolbox in MATLAB®. The paper ends with some conclusions, and suggestions for future research work.

2 The FLP methodology for the selection problem

The methodology for FLP is described in the literature in many references, see for example, References [2, 9, 23, 29] and [31]. The approach proposed here is based on an interaction between the decision maker and the implementer working with the analysis in order to find a satisfactory solution for a fuzzy linear programming problem (FLP). In a decision process using a FLP model, source resource variables may be fuzzy, instead of precisely given numbers as in a crisp linear programming (CLP) model. For example, machine hours, labor force, materials needed and so on in a manufacturing center, are always imprecise because of incomplete information and uncertainty in various potential suppliers and environments. Therefore, they should be considered as fuzzy resources, and the FLP problem should be solved by using the fuzzy set theory.

2.1 Formulation of a logistic function

As mentioned in Reference [23], a trapezoidal membership function will have some difficulties such as degeneration while solving fuzzy linear programming problems. In order to deal with the issue of degeneration, we should employ a non-linear logistic function such as a tangent hyperbolic function which has asymptotes at 1 and 0 [4].

In this paper, we employ a logistic function for the non-linear membership function given by:

$$f(x) = \frac{B}{1 + Ce^{\alpha x}} \tag{1}$$

where B and C are scalar constants and α which has the range $0 < \alpha < \infty$, is a fuzzy parameter which measures the degree of vagueness, wherein α indicates a crisp value with no fuzziness. Fuzziness increases as $\alpha \to \infty$. Equation (1) will be of the form indicated by Figure 1 when $0 < \alpha < \infty$. The parameter α determines the shape of a membership function $f(x)$. The larger α becomes, the greater the fuzziness, meaning that the availability of fuzzy parameters such as $\tilde{a}_{ij}$ (technical coefficients), $\tilde{b}_i$ (resource variables) and $\tilde{c}_i$ (revenue values), becomes less certain. It is necessary that the value for α, which determines the figures of the membership functions, should be heuristically and empirically determined by experts.

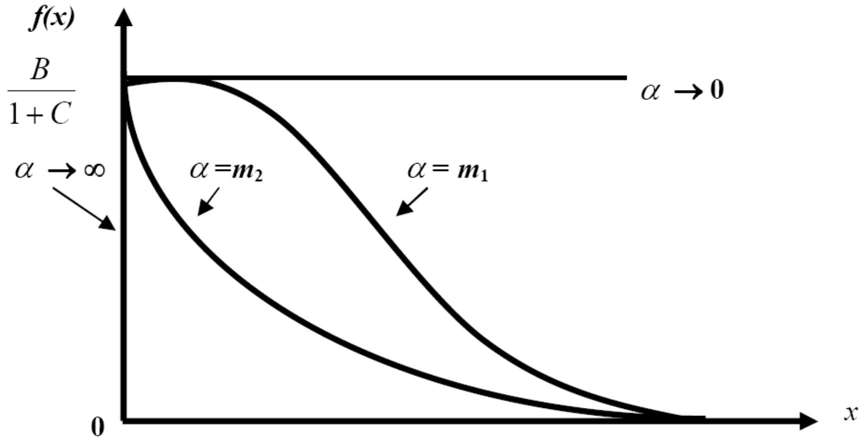

Figure 1. Variation of *f(x)* with respect to α ($m_2 > m_1$)

The reason why we use this logistic membership function is because it has a similar shape to that of the tangent hyperbolic function employed by Freeling [4], but it is more flexible [1]. It is also known that a trapezoidal membership function is an approximation to a logistic function. Therefore, the logistic function is a very appropriate function to represent the vague goal level. This function is found to be very useful in making decisions that lead to effective implementations [22].

The logistic function, equation (1), is a monotonically decreasing function [8], which will be employed as the fuzzy membership function. This is very important because, due to the uncertain environment, the availability of the variables is represented by the degree of vagueness.

3 S-curve membership function

There are many possible forms for a membership function: linear, exponential, hyperbolic, hyperbolic inverse , piece-wise linear, etc. [20]. Here we employ the modified S-curve form as it is not as restrictive as the linear form, being flexible enough to describe the vagueness in the fuzzy parameters [17]. The S-curve membership function is a particular case of a logistic function with specific values of B, C and α. These values are to be found out. This logistic function, as given by equation (2) and depicted in Figure 2, is an S-shaped membership function [5, 26].

If the obtained membership value of the solution is appropriate, that is, if it is included in the interval (0,1), then regardless of the shape of the membership function, linear and non-linear membership functions produce similar solutions. Nevertheless, it is possible that a non-linear membership function such as an S-curve membership function changes its shape according to the parameters values. Then a decision maker is able to apply his strategy to a fuzzy mix product selection problem using these parameters. Therefore, the non-linear membership function is much more convenient than a linear one (that this is true is consistent with the results of this paper and we intend to do more research in the future that will hopefully provide further justification for this).

We define here a modified S-curve membership function as follows:

$$\mu(x) = \begin{cases} 1 & x < x^a \\ 0.999 & x = x^a \\ \dfrac{B}{1 + Ce^{\alpha x}} & x^a < x < x^b \\ 0.001 & x = x^b \\ 0 & x > x^b \end{cases} \tag{2}$$

where, following fuzzy set theory, μ is the degree of the membership function. Figure 2 shows the S-curve. In Equation (2) the membership function range is redefined as $0.001 \le \mu(x) \le 0.999$. This range is selected because in a manufacturing system the work force need not be always 100% of the requirement. At the same time, the work force will not be 0%. Therefore, there is a range between x^a and x^b with $0.001 \le \mu(x) \le 0.999$ (see Watada [23] for a further discussion why we use an upper limit of 0.999 rather than 1). It is this range that is applied to our real life problem of product-mix selection.

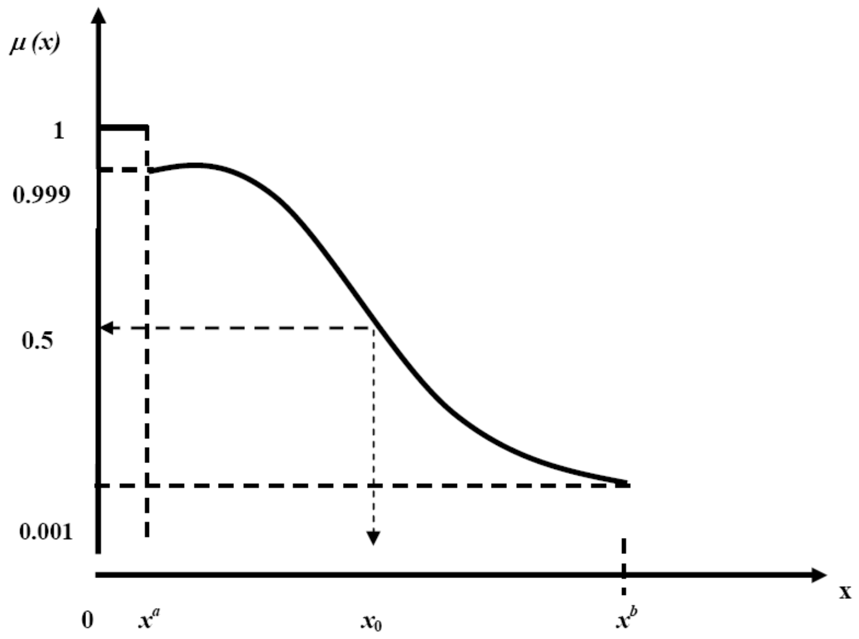

Figure 2. S-Curve Membership Function

We rescale the x axis as $x^a = 0$ and $x^b = 1$ in order to find the values of B, C and α. In Reference [15] such rescaling was done for analysis related to the social sciences. The values were calculated analytically as B=1, C=0.001001001 and $\alpha = 13.8135$.

Here we only consider one fuzzy product-mix selection problem in which the objective coefficients, technical coefficients and resource variables are all fuzzy. The FLP model for this problem is given in equation (3) below. The objective function is the revenue for the product-mix problem.

$$\begin{aligned} &\text{Maximize } z = \sum_{j=1}^{8} \tilde{c}_j x_j \\ &\text{subject to } \sum_{i=1}^{29} \sum_{j=1}^{8} \tilde{a}_{ij} x_j \le \tilde{b}_i \end{aligned} \tag{3}$$

Equation (3) is solved by using a parametric programming approach [2] and an S-curve membership function from the methodology in Reference [16] is employed. The input data are as follows. The c_j are the fuzzy revenue values, the a_{ij} are technical coefficients and the b_i are the resource variables for the FPS problem. There are 29 constraints and 8 products and hence in equation (8), i = 1, 2,...,29 and j = 1,2,...,8. The membership function and its c_j coefficients are constructed, with the values that the coefficients take being decided on by the decision maker. The FLP problem has been formulated and all the coefficients are parameterized. However, it will not be possible to use a linear parametric formulation to solve the FLP problem since the membership functions are non-linear [27]. Instead, it is necessary to carry out a series of numerical experiments using 21 membership values: $\mu_{aij} = \mu_{bi} = \mu_{cj} = \mu =$ 0.0010, 0.0509, 0.1008,..., 0.9990 with an interval of 0.0499. These experiments were carried out by using the Simplex Method in the Optimization Tool Box of MATLAB®.

3.1 Fuzzy Coefficient for Objective Function $\tilde{c}_j$.

The membership function for $\tilde{c}_j$ is given by:

$$\mu_{c_j} = \frac{B}{1 + Ce^{\alpha\left(\frac{c_j - c_j^a}{c_j^b - c_j^a}\right)}} \tag{4}$$

$$e^{\alpha\left(\frac{c_j - c_j^a}{c_j^b - c_j^a}\right)} = \frac{1}{C}\left(\frac{B}{\mu_{c_j}} - 1\right)$$

$$\alpha\left(\frac{c_j - c_j^a}{c_j^b - c_j^a}\right) = ln\frac{1}{C}\left(\frac{B}{\mu_{c_j}} - 1\right)$$

$$c_j = c_j^a + \left(\frac{c_j^b - c_j^a}{\alpha}\right) ln\frac{1}{C}\left(\frac{B}{\mu_{c_j}} - 1\right)$$

Since c_j is a fuzzy coefficient for the objective function as in equation (4), it is denoted as $\tilde{c}_j$. Therefore

$$\tilde{c}_j = c_j^a + \left(\frac{c_j^b - c_j^a}{\alpha}\right) ln\frac{1}{C}\left(\frac{B}{\mu_{c_j}} - 1\right) \tag{5}$$

The membership function for μ_{c_j} and the fuzzy interval , c_j^a *to* c_j^b , for $\tilde{c}_j$ is given in Figure 3 .

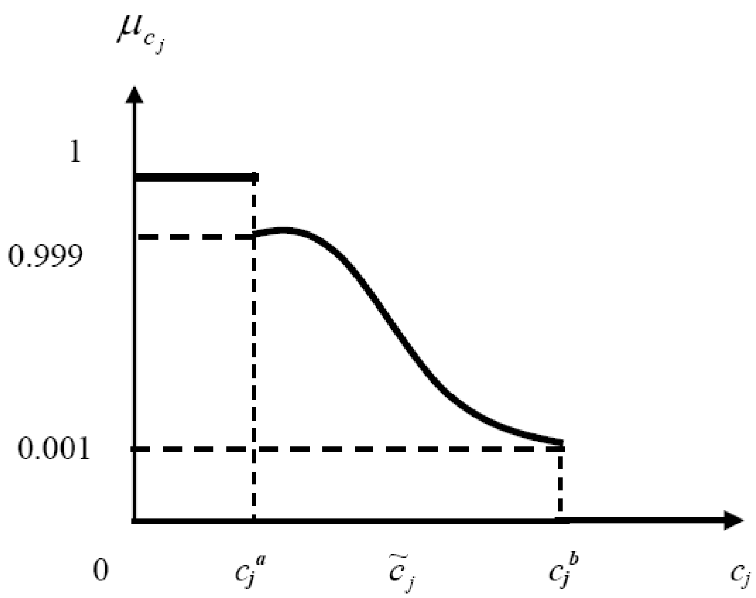

Figure 3. Membership Function μ_{c_j} and Fuzzy Interval for $\tilde{c}_j$

4 Case study analysis and fuzzy modeling

Due to limitations in resources for manufacturing a product and the need to satisfy certain conditions of manufacturing and demand, a degree of fuzziness occurs in production planning systems. This also occurs in chocolate manufacturing when deciding on the various mixes of raw materials required to produce a variety of products. This is referred to here as Product- mix Selection [22].

Our FPS problems is stated as follows:

There are *n* products to be manufactured by mixing *m* raw materials in different proportions and by using *k* varieties of processing. There are limitations on the availability of the raw materials. There are also some constraints imposed by the marketing department such as product–mix requirements, main product line requirements and the lower and upper limit of demand for each product. All the above requirements and conditions are fuzzy. It is necessary to obtain maximum revenue with a certain degree of satisfaction by using a fuzzy linear programming approach.

Chocoman Inc manufactures 8 chocolate products. There are 8 raw materials to be mixed in different proportions and 9 processes (facilities) to be utilized. The product demand, material and facility available are as illustrated in Table 1 and Table 2, respectively. Table 3 and Table 4 give the mixing proportions and facility usage required for manufacturing each product.

In each Table, the entries are given as non fuzzy data as well as fuzzy data with two limits; the lower limit is crisp data whereas the upper limit is fuzzy data, and hence the range is fuzzy. For example, in Table 2, MC 250 (Milk Chocolate 250 gm) a demand of 500,000 units is known exactly with no fuzziness. But the range 625,000 – 500,000 = 125,000 is fuzzy. This fuzziness is due to various reasons such as availability and usage of raw material, availability and usage of process facilities, etc. Of course, fuzziness is inevitable in any large manufacturing center such as Chocoman Inc.

Using equation (5) and similar equations for $\tilde{a}_{ij}$ and $\tilde{b}_j$ the formulation in equation (5) becomes

$$\text{Max} \sum_{j=1}^{8} \left(c_j^a + \left[\frac{c_j^b - c_j^a}{\alpha} \right] \ln \frac{1}{C} \left[\frac{B}{\mu_{c_j}} - 1 \right] \right) x_j$$

Subject to

$$\sum_{i=1}^{29} \left(a_{ij}^a + \left[\frac{a_{ij}^b - a_{ij}^a}{\alpha} \right] \ln \frac{1}{C} \left[\frac{B}{\mu_{a_{ij}}} - 1 \right] \right) x_j \le b_i^a + \left[\frac{b_i^b - b_i^a}{\alpha} \right] \ln \frac{1}{C} \left[\frac{B}{\mu_{b_i}} - 1 \right] \quad (6)$$

where

$$x_j \ge 0, \quad j = 1,2,3...,29 \ , \quad 0 < \mu_{c_j}, \mu_{a_{ij}}, \mu_{b_i} < 1 \ , \quad 0 < \alpha < \infty \, .$$

In equation (6), the best value for the objective function at a fixed level of μ is reached when [4]

$$\mu = \mu_{c_j} = \mu_{a_{ij}} = \mu_{b_i} \ for \ i = 1,2,\dots,29; j = 1,2,\dots,8 \quad (7)$$

Using equation (5) with the above values of α, B and C, values of $\tilde{c}_j$ are generated and computed for the range $\mu_{c_j} = 0.001$ to $\mu_{c_j} = 0.999$. The interval between two adjacent μ_{c_j} values can be arbitrary but has to be so small so that it would be possible to reach a good level of precision when obtaining an optimal solution. Here an interval for μ_{c_j} of 0.0499 is used. Kuzmin [9] has indicated that the membership function μ_{c_j} can be obtained in several ways. One of the ways is by using a functional rule for determining μ_{c_j}. This observation is adopted in forming a function for μ_{c_j} as given in equation (2). Carlsson and Korhonen [2] have also used their own functional rule for μ_{c_j}.

The interval for μ_{c_j} for computation of c_j in their work of 0.1 is significantly larger than the one we are using.

Table 1. Product Demand

Synonym		Product	Fuzzy Interval ($\times 10^3$ units)
x_1	MC 250	Milk chocolate, 250 g	[500,625)
x_2	MC 100	Milk chocolate, 100 g	[800,1000)
x_3	CC 250	Crunchy chocolate, 250 g	[400,500)
x_4	CC 100	Crunchy chocolate, 100 g	[600,750)
x_5	CN 250	Chocolate with nuts, 250g	[300,375)
x_6	CN 100	Chocolate with nuts, 100 g	[500,625)
x_7	CANDY	Chocolate candy	[200,250)
x_8	WAFER	Wafer	[400,500)

Table 2. Raw Material and Facility Availability

Raw Material/Facility (units)	Fuzzy Interval for Availability
Coco (kg)	[75000,125000)
Milk (kg)	[90000,150000)
Nuts (kg)	[45000,75000)
Confectionery sugar (kg)	[150000,250000)
Flour (kg)	[15000,25000)
Aluminum foil (ft2)	[375000,625000)
Paper (ft2)	[375000,625000)
Plastic (ft2)	[375000,625000)
Cooking (ton-hours)	[750,1250)
Mixing (ton-hours)	[150,250)
Forming (ton-hours)	[1125,1875)
Grinding (ton-hours)	[150,250)
Wafer making (ton-hours)	[75,125)
Cutting (hours)	[300,500)
Packaging 1 (hours)	[300,500)
Packaging 2 (hours)	[900,1500)
Labor (hours)	[750,1250)

Table 3. Mixing Proportions (Fuzzy)

	Product Types – Fuzzy Interval							
Materials required (per 1000 units)	MC 250	MC 100	CC 250	CC 100	CN 250	CN 100	CANDY	WAFER
Cocoa (kg)	[66,11)	[26,44)	[56,94)	[22,37)	[37,62)	[15,25)	[45,75)	[9,21)
Milk (kg)	[47,78)	[19,31)	[37,62)	[15,25)	[37,62)	[15,25)	[22,37)	[9,21)
Nuts (kg)	0	0	[28,47)	[11,19)	[56,94)	[22,37)	0	0
Cons. sugar (kg)	[75,13)	[30,50)	[66,11)	[26,44)	[56,94)	[22,37)	[157,26)	[18,30)
Flour (kg)	0	0	0	0	0	0	0	[54,90)
Alum. foil (ft2)	[375,63)	0	[375,63)	0	0	0	0	[187,31)
Paper (ft2)	[337,56)	0	[337,56)	0	[337,56)	0	0	0
Plastic (ft2)	[45,75)	[95,15)	[45,75)	[90,15)	[45,75)	[90,15)	[1200,2)	[187,31)

Table 4. Facility Usage (Fuzzy)

	Product Types – Fuzzy Interval							
Facility usage required (per 1000 units)	MC 250	MC 100	CC 250	CC 100	CN 250	CN 100	CANDY	WAFER
Cooking (ton-hours)	[0.4,0.6)	[0.1,0.2)	[0.3,0.5)	[0.1,0.2)	[0.3,0.4)	[0.1,0.2)	[0.4,0.7)	[0.1,0.12)
Mixing (ton-hours)	0	0	[0.1,0.2)	[0.04,0.07)	[0.2,0.3)	[0.07,0.12)	0	0
Forming (ton-hours)	[0.6,0.9)	[0.2,0.4)	[0.6,0.9)	[0.2,0.4)	[0.6,0.9)	[0.2,0.4)	[0.7,1.1)	[0.3,0.4)
Grinding (ton-hours)	0	0	[0.2,0.3)	[0.07,0.12)	0	0	0	0

Wafer making (ton-hours)	0	0	0	0	0	0	0	[0.2,0.4)
Cutting (hours)	[0.07,0.12)	[0.07,0.12)	[0.07,0.12)	[0.07,0.12)	[0.07,0.12)	[0.07,0.12)	[0.15,0.25)	0
Packag-ing 1 (hours)	[0.2,0.3)	0	[0.2,0.3)	0	[0.2,0.3)	0	0	0
Packag-ing 2 (hours)	[0.04,0.06)	[0.2,0.4)	[0.04,0.06)	[0.2,0.4)	[0.04,0.06)	[0.2,0.4)	[1.9,3.1)	[0.1,0.2)
Labor (hours)	[0.2,0.4)	[0.2,0.4)	[0.2,0.4)	[0.2,0.4)	[0.2,0.4)	[0.2,0.4)	[1.9,3.1)	[1.9,3.1)

The following constraints were established by the sales department of Chocoman:

1. Product mix requirements. Large – sized products (250g) of each type should not exceed 60% (a crisp value) of the small-sized product (100g), so that:

$x_1 \leq$ [45%,75%) x2 (8)

$x_3 \leq$ [45%,75%) x4 (9)

$x_5 \leq$ [45%,75%) x6 (10)

2. Main product line requirement. The total sales from candy and wafer products should not exceed 15% (non fuzzy value) of the total revenues from the chocolate bar products, so that :

[300,500) x_7 + [112.5,187.5) $x_8 \leq$ [42.19, 70.31) x_1 + [16.87, 28.12) x_2 + [45, 77) x_3 + [18, 30) x_4 + [47.25, 78.75)x_5 + [19.69, 32.81) x_6 (11)

Table 5. Objective Coefficients

Raw Material/Facility (units)	**Fuzzy Interval for Availability**
Milk chocolate, 250 g	[135,225)
Milk chocolate, 100g	[62,104)
Crunchy chocolate, 250g	[115,191)
Crunchy chocolate, 100g	[54,90)
Chocolate with nuts, 250g	[97,162)
Chocolate with nuts, 100g	[52,87)
Chocolate candy	[156,261)
Chocolate wafer	[62,104)

By using a linear programming technique we are able to solve the fuzzy mix product selection problem and a fuzzy frontier solution [23] for the revenue function to be obtained. The obtained results are summarized in the following section.

5 Result of fuzzy frontier solution

The FPS problem is solved by using MATLAB® and its Linear Programming (LP) tool box. The vagueness is given by α, and μ is the degree of satisfaction. The LP tool box has two inputs namely α and μ in addition to the fuzzy parameters. There is one output z^*, the optimal revenue.

The given values of various parameters of Chocolate Manufacturing are fed to the tool box. The solution can be tabulated and presented as 2 and 3 dimensional graphs.

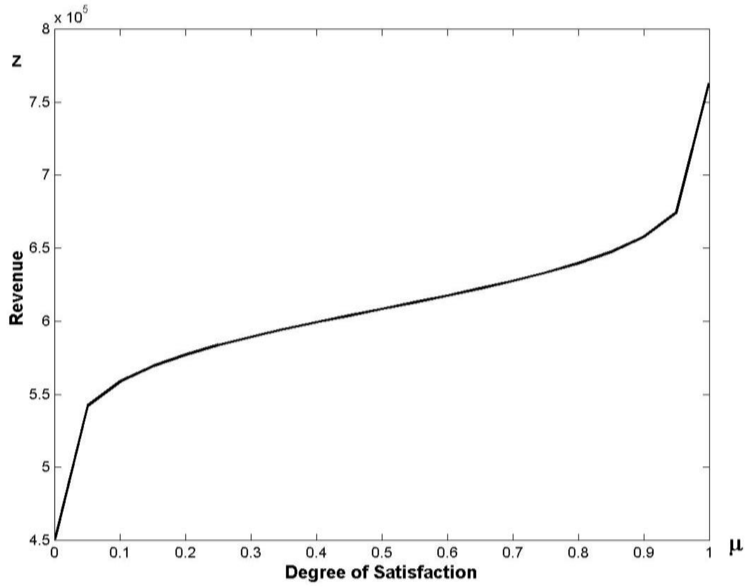

Figure 4. Revenue and Degree of Satisfaction at α = 13.8135

From Table 6 and Figure 4, we can conclude that a higher degree of satisfaction gives a higher value of revenue for a particular value of vagueness. When the vagueness increases then the revenue value decreases. But a realistic solution for the above problem exists for a of degree of satisfaction of 50% [2], that is 608440. From Fig. 4, we can see that the fuzzy outcome of the revenue function, z^* is an increasing function

Table 6. Fuzzy Optimal Revenue at $\alpha = 13.8135$

No	**D. S (μ) %**	**Revenue z^***
1	00.10	451190
2	05.09	542170
3	10.08	558910
4	15.07	569330
5	20.06	577180
6	25.05	583670
7	30.04	589340
8	35.03	594490
9	40.02	599310
10	45.01	603920
11	50.00	608440
12	54.99	612960
13	59.98	617570
14	64.97	622370
15	69.96	627500
16	74.95	633140
17	79.94	639590
18	84.93	647380
19	89.92	657690
20	94.91	674230
21	99.90	762940

D. S: Degree of Satisfaction

5.1 Revenue z^* For Various Values Of Vagueness, α

The membership value μ in Figure 5 represents the degree of satisfaction and z^* is the revenue function for the FPS problem. We can observe that when the vagueness increases, the revenue value at a particular μ value decreases. This phenomenon actually occurs in real life problems in a fuzzy environment.

The ideal solution in a fuzzy environment occurs at $\mu = 0.5$ [2, 23]. Hence the result of 50% for the degree of satisfaction with $2 \leq \alpha \leq 40$; the corresponding values for z^* are presented in Table 7

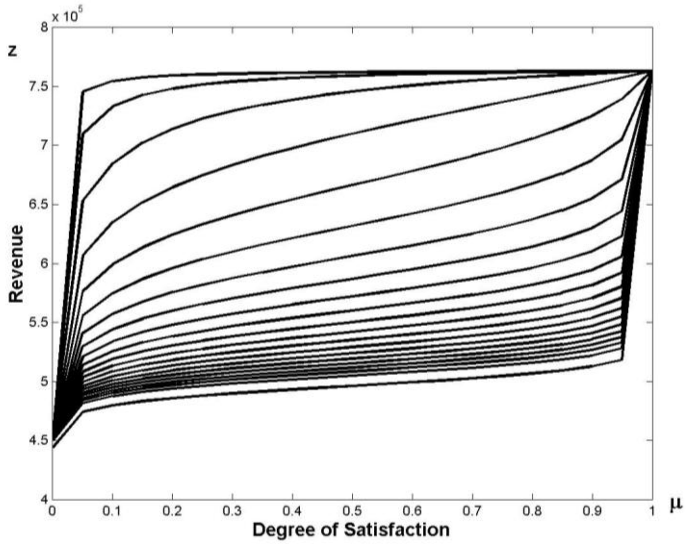

Figure 5. Revenue and Degree of Satisfaction for $2 \leq \alpha \leq 40$

Table 7. Vagueness α and Revenue z^* at 50% Degree of Satisfaction

Vagueness α	Revenue z^*
2	761960
4	758940
6	745660
8	709900
10	666350
12	631840
14	606370
16	587130
18	572130
20	560110
22	550260
24	542050
26	535090
28	529120
30	523950
32	519420
34	515420
36	511850
38	508560
40	504920

The data in Table 7 is the outcome of analyzing the FPS problem for the fuzzy mathematical model expressed by equation (6). This data is very useful for the decision maker when making a specific decision for implementation after consultation with the implementer.

The 3 dimensional plot for μ, α and z^* is shown in Figure 6. It is found that the S-curve membership function for various values of α offers a solution with an acceptable degree of satisfaction in a fuzzy environment. More vagueness results in less revenue.

The relationships between z^*, μ and α are given in Figure 6. These tables are very useful for the decision maker when finding the revenue at any given value of α with degree of satisfaction μ. From Figure 6, it can be seen that for the higher degree of satisfaction values, the revenue will not be higher. But at a 99% degree of satisfaction the revenue value will be the largest even with the higher value of α.

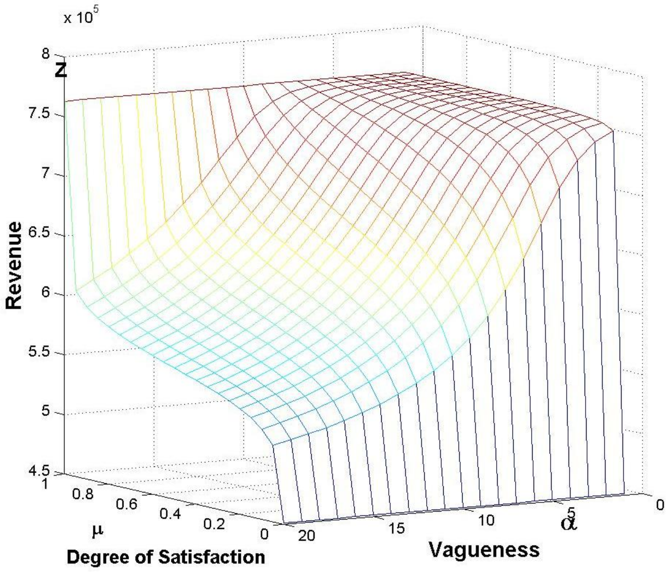

Figure 6. Variation of Revenue z^* in terms of μ and α

From the diagonal values in Figure 6, we can conclude that when the vagueness in the fuzzy parameters increases the revenue reduces. The result shows that the outcome hardly depends on the decisions made about the input level of the fuzzy parameters for the objective coefficients, technical coefficients and resource variables made in the early stages of the iteration. From the theory and numerical results, it can be seen that the method presented here for solving the fuzzy product-mix selection problem with a modified S-curve membership function is very promising and encouraging. Moreover in employing a modified S-curve membership function as a methodology for denoting fuzzy parameters, solution μ of the fuzzy mix product selection problem is within the range $0 < \mu < 1$ and therefore, exists on the efficient new frontier solution [23].

6 Summary and conclusion

An S-curve membership function has been used for generating fuzzy parameters in order to solve an industrial production planning problem. These parameters are defined in terms of a fuzzy linear programming problem and are called the fuzzy coefficients of the objective function, fuzzy technical coefficients and fuzzy resource variables. Membership values for these fuzzy parameters are created by using the S-curve membership function. This formulation is found to be suitable for applying the Simplex Method of Linear programming. This approach to solving the industrial production planning problem involves interactions between the decision maker and the implementer during the analysis. It is to be noted that higher revenue need not lead to a higher degree of satisfaction. The decision maker has to assess the acceptability of the revenue obtained through the FLP process with respect to the degree of satisfaction. Therefore there must be an interaction between the decision maker and implementer that continues until the decision maker is satisfied with the solution. The analysis in this paper has been formulated to work hand-in-hand with the decision maker and implementer to achieve the best outcome (highest degree of satisfaction) from this interactive process towards achieving a higher profit in a situation filled with uncertainty. Furthermore, for the problem considered, the optimal solutions obtained help us to show that incorporating fuzziness into a linear programming model results in a better level of satisfaction compared to non-fuzzy linear programming.

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