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SLIP REGIME MHD 2-LIQUID PLASMA HEAT TRANSFER FLOW WITH HALL CURRENTS BETWEEN PARALLEL PLATES

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The influence of the slip factor on the MHD 2-liquid heat transfer flow of ionized gases within a channel between two non-conducting plates with Hall currents is investigated theoretically. Slip conditions were used to obtain solutions for the velocity and temperature fields, as well as the heat transfer rates. The flow characteristics of the two liquids are studied for estimates of the leading parameters, for instance the magnetic parameter, Hall and slip factors, viscosity, density, height, electrical conductivity and the thermal conductivity ratios. It was observed that an upsurge in temperature in the two zones is caused by the thermal conductivity proportion. The rate of heat transfer coefficient diminishes up to a certain point, after that it starts to increase as the magnetic and Hall parameters increase.

Key words: immiscible slip flow, electric and magnetic fields, Hall effect, plasma, heat transfer.

1. Introduction

In a variety of applications, such as in solar technology, safety aspects of gas-cooled reactors, MHD generators, accelerators, pumps, flow meters, and crystal growth in liquids, etc., the study of MHD flows through channels under the influence of an applied transverse magnetic field with Hall effect is receiving a lot of attention in the literature [1-21]. Due to its many technological applications, the MHD slip flow of electrically conducting fluids (such as liquids and ionized gases) has recently attracted many researchers, scientists, and engineers. The particle adjacent to a solid surface no longer takes the surface's velocity in many practical applications. The particle slips over the surface due to its finite tangential velocity. Usages of fluid slippage at boundaries are diverse, such as in micro-channels. It is likely that in some be circumstances particles in a geothermal region may slide along a border. Magnetohydrodynamic flows with or without Hall effects in a slip-flow regime, have been discussed by several authors [22-37]. A vital challenge in cancer research is the delivery of medication to the affected part of the body through vessels that experience slippage. In this technique, the patient's drug is carefully administrated to the body using ferro-fluids. In view of these purposes, many researchers like [38-41], have studied in detail slippage in a single phase flow.

In fact, the two liquid flows frequently occur in a variety of scientific, engineering, and technological disciplines; in particular, they are used in the production of oil and gas, gas liquid flows in boilers, aerosol deposition in spray medications, etc. They can be seen, among other things, in inkjet printers, clouds, fog, ground water flow, ocean waves, and pest resistance. To improve conceptual stratagems for fusion reactors and liquid metal MHD power generators, one requires a detailed comprehension of 2-liquid flows of electrically conducting liquids stimulated by a transverse magnetic field. Applications of a two-liquid arrangement that intends to increase flow rates in an electromagnetic pump involve transportation and excavation of oil products, since there may be a reduction in the amount of power required to pump oil via a pipe due to an adequate water accumulation [42]. Numerous theoretic as well as investigational models on

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MHD two liquid flow issues have been developed and received copious devotion in the writings by various investigators [43-54].

Kalra *et al.* [55] examined the impact of Hall effect and resistivity on the stability of a gas and liquid system. Dun [56] discussed the single and phase MHD pipe flows. Lohrasbi and Sahai [57] studied the MHD thermal transfer effect on two-phase fluid flows with electrically conducting. Malashetty and Leela [58] studied the magneto-hydrodynamic heat transfer in a two-phase fluid flow assuming that the fluids of the zones are electrically conducting. Chamkha [59] investigated the magnetohydrodynamic of a two-phase fluid flow through a channel. Abbey and Bestman [60] studied the slip fluid flow in a two-layered radiative heat transfer plasma mode.

Umavathi *et al.* [61] studied an oscillatory Hartmann two-layered fluid flow and heat transfer within a horizontal channel and Tsuyoshi Inoue and Shu-Ichiro Inutsuka [62] investigated the magneto-hydrodynamic simulations on convergent two-phase fluid flows in an interstellar medium. L. Raju *et al.* [63] examined an MHD two-phase slip flow within the parallel walls. Abdul [64] studied a magneto hydrodynamic transient flow of two immiscible fluids in a horizontal channel. Zaheer *et al.* [65] studied the MHD two-layered heat transfer fluid flow with partial slip through an inclined channel. L. Raju [66] studied the effect of Hall currents on a two-ionized fluid flow between two parallel plates. Mallikarjuna *et al.* [67] studied the effect of slip on velocity and temperature of two-phase MHD convective radiative flows through an asymmetric non-uniform channel.

There has been much more theoretical research on single phase stream of electrically conducting liquids in channel systems with Hall effects employing slip conditions than there has been on two liquid flows of electrically conducting fluids like gases and liquids. So, the aim of this article is to examine the impact of the slip parameter on a two layered MHD flow of ionized gases in a straight channel.

2. Formulation of major equations

In this study, first order slip conditions are used to analyse the L. Raju [66] two-liquid MHD flow of ionised gases under control of a constant pressure gradient through a horizontal channel rather than no-slip conditions. At $y = h_1$ and $y = h_2$, the two parallel rigid plates surround the channel. The plates are assumed to be consisting of non-conducting materials. The plates are infinitely long in either direction when the origin is positioned halfway between the *X*-axis and *Z*-axis. The *X*-axis is oriented towards the hydrodynamic pressure gradient but not the plane perpendicular to the plates, the flow field is transverse to the *Y*-direction, where a constant magnetic field B_0 is applied. It is taken for granted that the thermal boundary conditions are valid all over on the infinite channel plates and ignored the thermal conduction in the flow direction. Because the plates are maintained at a consistent temperature (T_w), so the temperature on the upper plate (T_{w1}) is the same as that on the lower plate (T_{w2}). The regions $0 \le y \le h_1$ and $-h_2 \le 0 \le y$ in the channel are filled with two different liquids and are called zones I and II, respectively. These zones are comprised of two electrically conducting, immiscible, incompressible fluids with diverse viscosities (μ_1, μ_2), electrical conductivities (σ_{01}, σ_{02}) and thermal conductivities (K_1 and K_2). It is assumed that the two liquid flows are laminar and steady. Under the assumption that the magnetic field is extremely strong and the gases are fully ionised the Hall effect is studied. The magnetic Reynolds number is thought to be low in order to ignore the induced magnetic field. The ideal contact between the two immiscible fluids is undisturbed, stress-free, and flat.

The flow equations and conditions are also expressed as dimensionless using the non-dimensional quantities listed below.

$$y_i^* = \frac{y_i}{h_i}, \quad u_p = -\frac{\partial p}{\partial x}\frac{h_1^2}{\mu_1}, \quad u_1^* = \frac{u_1}{u_p}, \quad u_2^* = \frac{u_2}{u_p}, \quad w_1^* = \frac{w_1}{u_p}, \quad w_2^* = \frac{w_2}{u_p},$$
$$m_{ix} = \frac{E_{ix}}{B_0 u_p}, \quad m_{iz} = \frac{E_{iz}}{B_0 u_p}, \quad I_{ix} = \frac{J_{ix}}{\sigma_{0i} B_0 u_p}, \quad I_{iz} = \frac{J_{iz}}{\sigma_{0i} B_0 u_p}, \quad J_i^2 = J_{ix}^2 + J_{iz}^2 \quad (i = 1,2), \tag{2.1}$$

the Hartmann number: $H_a{}^2 = \sqrt{\dfrac{\sigma_{01} B_0^2 h_1^2}{\mu_1}}$, the viscosity ratio: $\alpha = \dfrac{\mu_1}{\mu_2}$,

the height ratio: $h = \dfrac{h_2}{h_1}$, the thermal conductivity ratio: $\beta = \dfrac{K_1}{K_2}$,

the electrical conductivity ratio: $\sigma_0 = \dfrac{\sigma_{01}}{\sigma_{02}}$,

$$\sigma_{01} = \frac{\sigma_{12}}{\sigma_{11}}, \quad \sigma_{02} = \frac{\sigma_{22}}{\sigma_{21}}, \quad \frac{1}{1+m^2} = \frac{\sigma_{11}}{\sigma_{01}}, \quad \frac{m}{1+m^2} = \frac{\sigma_{21}}{\sigma_{01}}, \tag{cont.2.1}$$

the Hall parameter: $m = \dfrac{\omega_e}{\left(\dfrac{1}{\tau} + \dfrac{1}{\tau_e}\right)}$, the Prandtl number: $P_{ri} = \dfrac{\mu_i C_{pi}}{K_i}$,

the temperature distribution $\theta_i = \dfrac{T_i - T_{wi}}{\left(u_p{}^2 \mu_1 / K_i\right)}$.

The following is a detailed description of the non-dimensional form of the equations of motion, energy and current, in two fluid zones as well as the interface and slip boundary conditions for a steady MHD stream of ionised gases under the consideration of Hall currents with slippage. For the sake of simplicity, the asterisks have been ignored.

Zone-I

$$\frac{d^2 u_1}{dy^2} - \frac{(m_{1z} + u_1) H_a^2}{1+m^2} + \frac{(m_{1x} - w_1) m H_a^2}{1+m^2} + p_1 = 0, \tag{2.2}$$

$$\frac{d^2 w_1}{dy^2} + \frac{(m_{1x} - w_1) H_a^2}{1+m^2} + \frac{(m_{1z} + u_1) m H_a^2}{1+m^2} + p_2 = 0, \tag{2.3}$$

$$\frac{d^2 \theta_1}{dy^2} + p_{r_1} \left[\left(\frac{du_1}{dy} \right)^2 + \left(\frac{dw_1}{dy} \right)^2 + H_a^2 I_1^2 \right] = 0, \tag{2.4}$$

$$I_{1x} = \frac{(m_{1x} - w_1)}{1+m^2} + \frac{(m_{1z} + u_1)\mathrm{m}}{1+m^2} - \frac{sm}{(1+m^2) H_a^2}, \tag{2.5}$$

$$I_{1z} = \frac{(m_{1z} + u_1)}{1+m^2} - \frac{(m_{1x} - w_1) m}{1+m^2} + \left(1 - \frac{m}{1+m^2} \right) \frac{s}{H_a^2} \tag{2.6}$$

and $I_1^2 = I_{1x}^2 + I_{1z}^2$.

Zone-II

$$\frac{d^2u_2}{dy^2}-\frac{(m_{2z}+u_2)\alpha\sigma_1 h^2H_a^2}{1+m^2}+\frac{(m_{2x}-w_2)m\alpha\sigma_2 h^2H_a^2}{1+m^2}+p_3\alpha h^2=0, \tag{2.7}$$

$$\frac{d^2w_2}{dy^2}+\frac{(m_{2z}-w_2)\alpha\sigma_1 h^2H_a^2}{1+m^2}+\frac{(m_{2z}+u_2)m\alpha\sigma_2 h^2H_a^2}{1+m^2}+p_4\alpha h^2=0, \tag{2.8}$$

$$\frac{d^2\theta_2}{dy^2}+p_{r_2}\left[\frac{\beta}{\alpha}\left\{\left(\frac{du_2}{dy}\right)^2+\left(\frac{dw_2}{dy}\right)^2\right\}+\sigma\beta h^2H_a^2I_2^2\right]=0, \tag{2.9}$$

$$I_{2x}=\frac{(m_{2x}-w_2)\sigma_0\sigma_1}{1+m^2}+\frac{(m_{2x}+u_2)m\sigma_0\sigma_2}{1+m^2}-\frac{sm\sigma_0^2\sigma_2}{(1+m^2)H_a^2}, \tag{2.10}$$

$$I_{2z}=\frac{(m_{2z}+u_2)\sigma_0\sigma_1}{1+m^2}-\frac{(m_{2x}-w_2)m\sigma_0\sigma_2}{1+m^2}+\left(1-\frac{\sigma_0\sigma_1}{1+m^2}\right)\frac{s\sigma_0}{H_a^2}, \tag{2.11}$$

where, $p_1=1-\frac{sm^2}{1+m^2}$, $p_2=\frac{-sm}{1+m^2}$, $p_3=1-s\left(1-\frac{\sigma_0\sigma_1}{1+m^2}\right)$, $p_4=\frac{-sm\sigma_0\sigma_2}{1+m^2}$ and $I_2^2=I_{2x}^2+I_{2z}^2$.

The corresponding boundary conditions are

$$u_1(1)=-\Gamma\frac{du_1(1)}{dy},\quad w_1(1)=-\Gamma\frac{dw_1(1)}{dy}, \tag{2.12}$$

$$u_2(-1)=\Gamma\frac{du_2(-1)}{dy},\quad w_1(-1)=\Gamma\frac{dw_2(-1)}{dy}, \tag{2.13}$$

$$u_1(0)=u_2(0),\quad w_1(0)=w_2(0), \tag{2.14}$$

$$\frac{du_1}{dy}=\frac{1}{\alpha h}\frac{du_2}{dy}\ \text{ and }\ \frac{dw_1}{dy}=\frac{1}{\alpha h}\frac{dw_2}{dy}\ \text{ at } y=0, \tag{2.15}$$

$$\theta_1(1)=-\Gamma\frac{d\theta_1(1)}{dy},\ \theta_2(-1)=\Gamma\frac{d\theta_2(-1)}{dy},\ \theta_1(0)=\theta_2(0)\ \text{ and }\ \frac{d\theta_1}{dy}=\frac{1}{\beta h}\frac{d\theta_2}{dy}\ \text{ at } y=0. \tag{2.16}$$

where, the subscripts 1 and 2 in the above governing equations mention the quantities for zone-I and zone-II. The symbols u_1, u_2 and w_1, w_2 are the velocity components in the x and z axes of both the liquids, and are named as primary and secondary velocity distributions, respectively. The notations J_{ix} and J_{iz}, E_{ix} and E_{iz} are the elements of the current densities, electric field in the x and z axes. The quantity $s=\frac{p_e}{p}$ is the proportion of electron pressure to the total pressure. T_1 and T_2 are the temperatures of both fluid flows, and $C_{pi}\,(i=1,2)$ is the specific heat at constant pressure. The symbols, σ_{11}, σ_{12} and σ_{21}, σ_{22} are the modified conductivities perpendicular and parallel to the direction of the electric field. Also, ω_e is the gyration frequency of electrons, τ, τ_e are the mean collision time in between the electron and ion, electron and neutral particles.

3. The solution process

The system of Eqs (2.2-2.3, 2.7-2.8) and (2.4 and 2.9) are to be resolved subject to the boundary and interface conditions (2.12-2.16) for the velocity and temperature profiles in the two liquid zones. For easiness, the system of equations is represented in terms of the symbolized notations as:

Zone-I

$$\frac{d^2q_1}{dy^2}-a_1q_1=a_2, \tag{3.1}$$

$$\frac{d^2\theta_1}{dy^2}=b_1e^{b_{29}y}+b_2e^{b_{30}y}+b_3e^{b_{31}y}+b_4e^{b_{32}y}+b_5e^{\overline{a_9}y}+b_6e^{-\overline{a_9}y}+b_7e^{a_9y}+b_8e^{-a_9y}+b_9. \tag{3.2}$$

Zone-II

$$\frac{d^2q_2}{dy^2}-a_3q_2=a_4, \tag{3.3}$$

$$\begin{aligned}\frac{d^2\theta_2}{dy^2}&=b_{13}e^{b_{33}y}+b_{14}e^{b_{34}y}+b_{15}e^{b_{35}y}+b_{16}e^{b_{36}y}+\\&+b_{17}e^{\overline{a_9}y}+b_{18}e^{-\overline{a_9}y}+b_{19}e^{a_9y}+b_{20}e^{-a_9y}+b_{21}.\end{aligned} \tag{3.4}$$

The boundary-interface conditions are

$$q_1(1)=-\Gamma\frac{dq_1(1)}{dy},\ \ q_2(-1)=\Gamma\frac{dq_2(-1)}{dy},\ \ q_1(0)=q_2(0)\ \text{ and }\ \frac{dq_1}{dy}=\frac{1}{\alpha h}\frac{dq_2}{dy}\ \text{ at }\ y=0, \tag{3.5}$$

$$\theta_1(1)=-\Gamma\frac{d\theta_1(1)}{dy},\ \ \theta_2(-1)=\Gamma\frac{d\theta_2(-1)}{dy},\ \ \theta_1(0)=\theta_2(0)\ \text{ and }\ \frac{d\theta_1}{dy}=\frac{1}{\beta h}\frac{d\theta_2}{dy}\ \text{ at }\ y=0. \tag{3.6}$$

where $q_1=u_1+iw_1$, $q_2=u_2+iw_2$.

As the side plates are held at great distance in the X-axis and are made of insulating (non-conducting) material, the induced electrical current does not leave the horizontal channel but circulates through the fluid. As a result, the following conditions are obtained [2]:

$$\int_0^1 I_{1x}dy=0,\quad \text{and}\quad \int_0^1 I_{2x}dy=0. \tag{3.7}$$

In the same way, at a large distance along the Z-direction, other relations are obtained as;

$$\int_0^1 I_{1z}dy = 0, \quad \text{and} \quad \int_0^1 I_{2z}dy = 0. \tag{3.8}$$

The conditions (3.5) are used to find constants in the solutions, and then satisfy the expressions of m_{ix} and m_{iz} $(i = 1,2)$ using conditions (3.7) and (3.8) as a result; the solutions are obtained for both the velocities and also mean velocity distributions. As a result, the energy Eqs (3.2 and 3.4) are solved using conditions (3.6) for temperature fields in the two zones, and then the heat transfer coefficient rates at the two plates are obtained.

Zone-I

$$q_1(y) = A_1 e^{a_5 y} + A_2 e^{-a_5 y} - a_6\,, \tag{3.9}$$

$$I_1 = M_1 a_{16} + q_1 a_{17} - a_{18}\,, \quad M_1 = a_{31} + a_6 a_{32}\,,$$

$$\begin{aligned}\theta_1(y) = {} & \frac{b_1}{b_{29}^2}e^{b_{29}y} + \frac{b_2}{b_{30}^2}e^{b_{30}y} + \frac{b_3}{b_{31}^2}e^{b_{31}y} + \frac{b_4}{b_{32}^2}e^{b_{32}y} + \frac{b_5}{\bar{a}_5^2}e^{\bar{a}_5 y} + \\ & + \frac{b_6}{\bar{a}_5^2}e^{-\bar{a}_5 y} + \frac{b_7}{a_5^2}e^{a_5 y} + \frac{b_8}{a_5^2}e^{-a_5 y} + \frac{b_9}{2}y^2 + B_1 y + B_2.\end{aligned} \tag{3.10}$$

The coefficient of heat transfer rate at the upper plate is $Nu_1 = \left(-\frac{d\theta_1}{dy}\right)$ at $y = 1$:

$$\begin{aligned}Nu_1 = -\Bigg(& B_1 + \frac{b_1}{b_{29}}e^{b_{29}} + \frac{b_2}{b_{30}}e^{b_{30}} + \frac{b_3}{b_{31}}e^{b_{31}} + \frac{b_4}{b_{32}}e^{b_{32}} + \\ & + \frac{b_5}{\bar{a}_5}e^{\bar{a}_5 y} - \frac{b_6}{\bar{a}_5}e^{\bar{a}_5 y} + \frac{b_7}{a_5}e^{a_5} - \frac{b_8}{a_5}e^{a_5} + b_9\Bigg)\end{aligned} \tag{3.11}$$

where the mean velocity is given by:

$$\begin{aligned}& q_{1m} = u_{1m} + iw_{1m} = \int_0^1 q_1 dy, \quad u_{1m} = \frac{q_{1m} + \overline{q_{1m}}}{2}, \quad w_{1m} = \frac{q_{1m} - \overline{q_{1m}}}{2i}, \\ & q_{1m} = \frac{A_1}{a_5}\left(e^{a_5} - 1\right) - \frac{A_2}{a_5}\left(e^{-a_5} - 1\right) - a_6.\end{aligned} \tag{3.12}$$

Zone-II

$$q_2(y) = A_3 e^{a_9 y} + A_4 e^{-a_9 y} - a_{10}\,, \tag{3.13}$$

$$I_2 = M_2 a_{19} + q_2 a_{20} - a_{21}, M_2 = a_{33} + a_{10} a_{34},$$

$$\theta_2(y) = \frac{b_{13}}{b_{33}^2} e^{b_{33} y} + \frac{b_{14}}{b_{34}^2} e^{b_{34} y} + \frac{b_{15}}{b_{35}^2} e^{b_{35} y} + \frac{b_{16}}{b_{36}^2} e^{b_{36} y} + \frac{b_{17}}{\overline{a_9}^2} e^{\overline{a_9} y} + \tag{3.14}$$
$$+ \frac{b_{18}}{\overline{a_9}^2} e^{-\overline{a_9} y} + \frac{b_{19}}{a_9^2} e^{a_9 y} + \frac{b_{20}}{a_9^2} e^{-a_9 y} + \frac{b_{21}}{2} y^2 + B_3 y + B_4 .$$

The coefficient of heat transfer rate at the lower plate is $N Nu_2 = \left(\frac{1}{\beta h} \frac{d\theta_2}{dy} \right)$ at $y = -1$:

$$Nu_2 = \frac{1}{\beta h} \left(B_3 + \frac{b_{13}}{b_{33}} e^{-b_{33}} + \frac{b_{14}}{b_{34}} e^{-b_{34}} + \frac{b_{15}}{b_{35}} e^{-b_{35}} + \frac{b_{16}}{b_{36}} e^{-b_{36}} + \right.$$
$$\left. + \frac{b_{17}}{\overline{a_9}} e^{\overline{a_9} y} - \frac{b_{18}}{\overline{a_9}} e^{\overline{a_9} y} + \frac{b_{19}}{a_9} e^{-a_9} - \frac{b_{20}}{a_9} e^{a_9} - b_{21} \right) . \tag{3.15}$$

The mean velocity is given by:

$$q_{2m} = u_{2m} + i w_{2m} = \int_0^1 q_2 dy, \quad u_{2m} = \frac{q_{2m} + \overline{q_{2m}}}{2}, \quad w_{2m} = \frac{q_{2m} - \overline{q_{2m}}}{2i},$$
$$q_{2m} = \frac{A_3}{a_9} \left(e^{a_9} - 1 \right) - \frac{A_4}{a_9} \left(e^{-a_9} - 1 \right) - a_{10} . \tag{3.16}$$

4. Results and discussion

In order to achieve the physical characteristics of the issue and to discuss the results, numerical evaluations are made for the velocity and temperature profiles in the two liquid zones. In Figs 1-21, the estimations of the velocity and temperature fields for sets of values of the governing parameters are graphically depicted. For all evaluations, the specific parameters $\sigma_1 = 1.2, \sigma_2 = 1.5$ and $P_{r1} = 1 = P_{r2}$ are fixed, and the cause of supporting vital flow characteristics on the velocity and temperature is examined. As expected the solutions are not dependent on the ionisation parameter, which is the ratio of the electron pressure to the total pressure and this is in agreement with the investigation of L. Raju [66] for no-slip boundary conditions.

Figures 1, 2 and 3 illustrate the impact of the Hartmann number H_a on both the velocity profiles and temperature distribution in the two liquid zones. From Figs 1 and 2, it is seen that a rise in the Hartmann number H_a causes a drop in the velocity profiles in the two zones. This is due to the effect of slippage and a resistive force known as the Lorentz force which is produced when a transverse magnetic field is applied an electrically conducting fluid. However, it is identified that the greatest velocity profiles in the channel tend to move above the channel axis towards zone-I while H_a increases. From Fig.3 it is discovered that for small values of the Hartmann number H_a the temperature profile decreases in the two liquid zones but for large values of the Hartmann number H_a the temperature distribution increases in the two zones. The extreme temperature within the channel tends to move below the central axis of the channel towards zone-I as H_a rises to the point where the remaining characteristics are fixed.

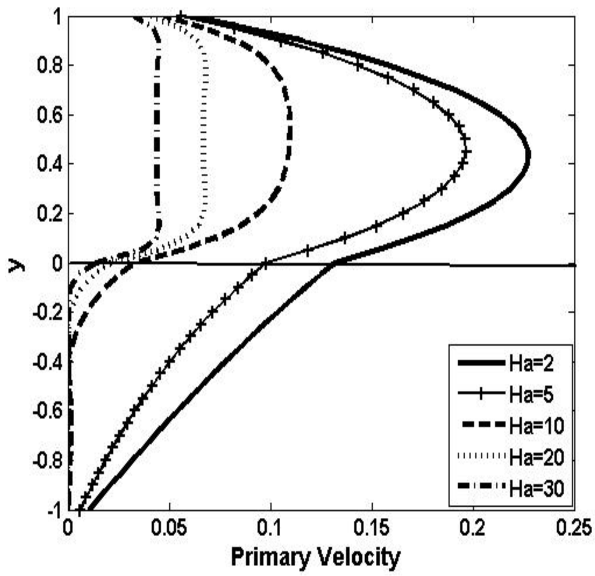

Fig.1. Primary velocity profiles u_1, u_2 for various H_a and $\alpha = 0.333$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $\Gamma = 0.1$, $h = 1$, $m = 2$

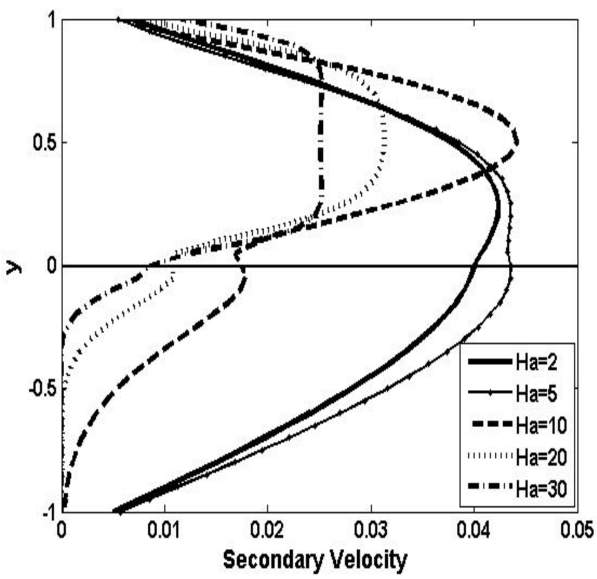

Fig.2. Secondary velocity profiles w_1, w_2 for various H_a and $\alpha = 0.333$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $\Gamma = 0.1$, $h = 1$, $m = 2$.

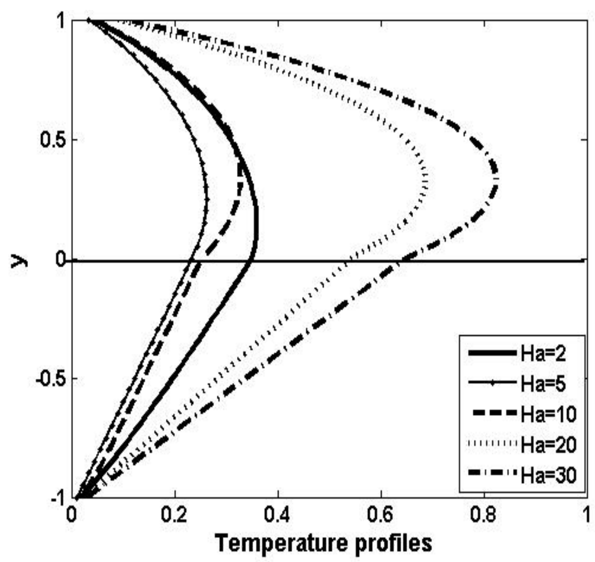

Fig.3. Temperature profiles θ_1, θ_2 for various H_a and $\alpha = 0.333$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $\beta = 0.5$, $\Gamma = 0.1$, $h = 1$, $m = 2$.

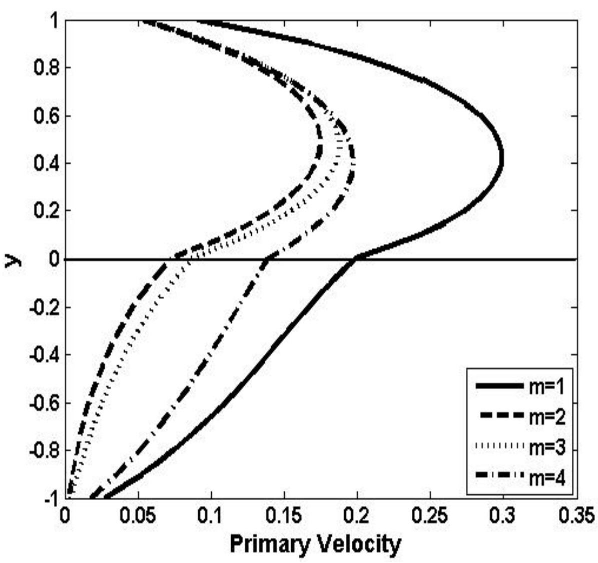

Fig.4. Primary velocity profiles u_1, u_2 for various m and $\alpha = 0.333$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $\Gamma = 0.1$, $h = 1$, $H_a = 4$.

The influence of the Hall parameter m on the velocity and temperature profiles is illustrated in Figs. 4, 5 and 6. It is observed that when the Hall parameter value increases up to 2, then both the velocity distributions decrease and when the Hall parameter value is more than 2 then both the velocity profiles increase in the two zones. Likewise, it can be noticed that the greatest distribution of both the velocities in the horizontal channel tends to move along the axis of the channel towards zone-I as m increases. From Fig.6, it is noticed that the temperature profile enhanced up to a specific estimate of the Hall factor and thereafter is reduced with an increase in the Hall parameter *m*. The outermost temperature distribution in the channel tends to shift over the axis of the channel towards zone-I as the Hall factor increases.

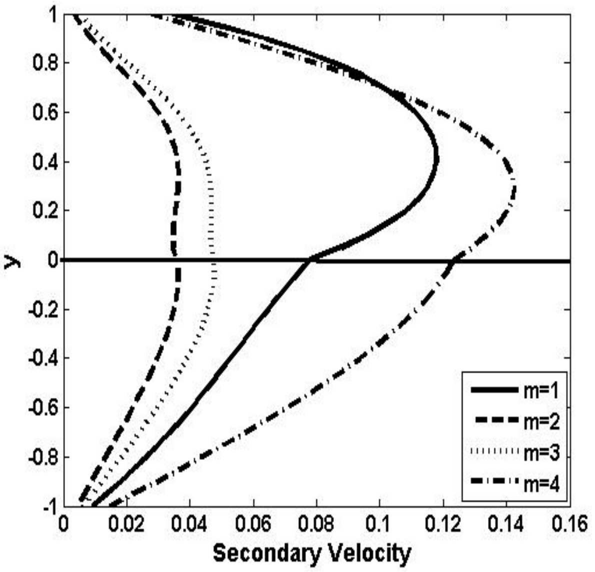

Fig.5. Secondary velocity profiles w_1, w_2 for various m and $\alpha = 0.333$, $\sigma_0 = 2, \sigma_1 = 1.2$, $\sigma_2 = 1.5$, $\Gamma = 0.1, h = 1$, $H_a = 4$.

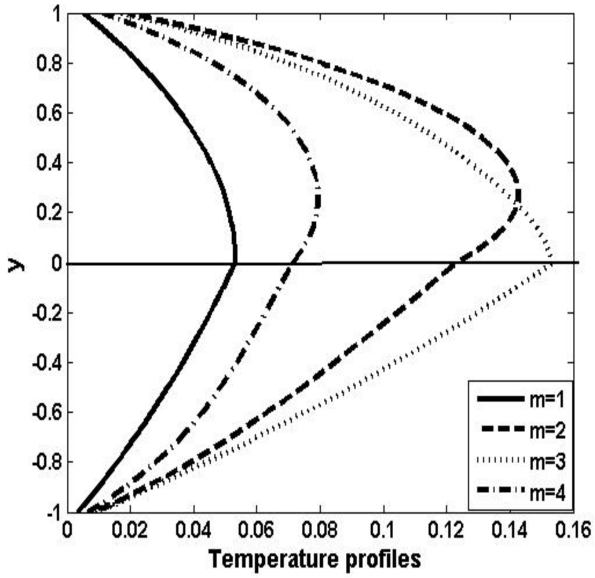

Fig.6. Temperature profiles θ_1, θ_2 for various m and $\alpha = 0.333$, $\sigma_0 = 2$, $\sigma_1 = 1.2, \sigma_2 = 1.5$, $\beta = 0.5$, $\Gamma = 0.1, h = 1$, $H_a = 4$.

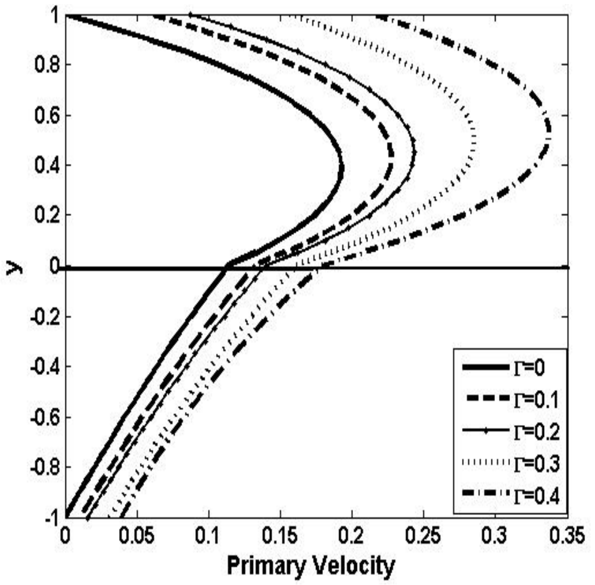

Fig.7. Primary velocity profiles u_1, u_2 for various Γ and $\alpha = 0.333$, $\sigma_0 = 2, \sigma_1 = 1.2, \sigma_2 = 1.5$, $h = 1$, $m = 2$, $H_a = 2$.

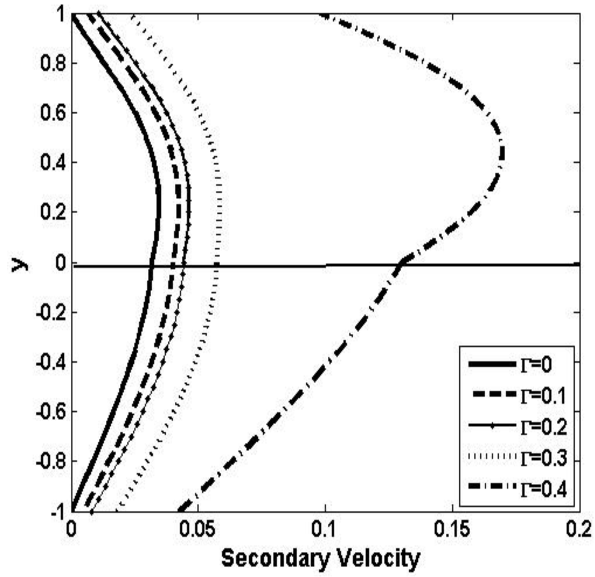

Fig.8. Secondary velocity profiles w_1, w_2 for various Γ and $\alpha = 0.333$, $\sigma_0 = 2, \sigma_1 = 1.2$, $\sigma_2 = 1.5$, $h = 1$, $m = 2$, $H_a = 2$.

The impact of the slippage parameter Γ on both the velocity profiles is shown in Figs. 7 and 8. In Fig.7, we can see that a rise in the slip parameter increases the primary velocity in both zones. Figure 8 shows clearly that the growing in Γ up to the estimation of $\Gamma = 0.5$ increases the secondary velocity profiles in both the zones. Likewise, it can be noticed that the extreme velocity distributions within the channel tend to move along the axis of the channel towards zone-I as Γ rises. The effect of varying the slippage parameter Γ on temperature distribution is displayed in Fig.9. With a increase in the slip parameter Γ, the temperature profile within the fluids was found to oscillate in both zones. The greater temperature of the channel tends to move beneath the channel focus line towards zone-I as the slip parameter Γ increases when the remaining governing parameters are fixed.

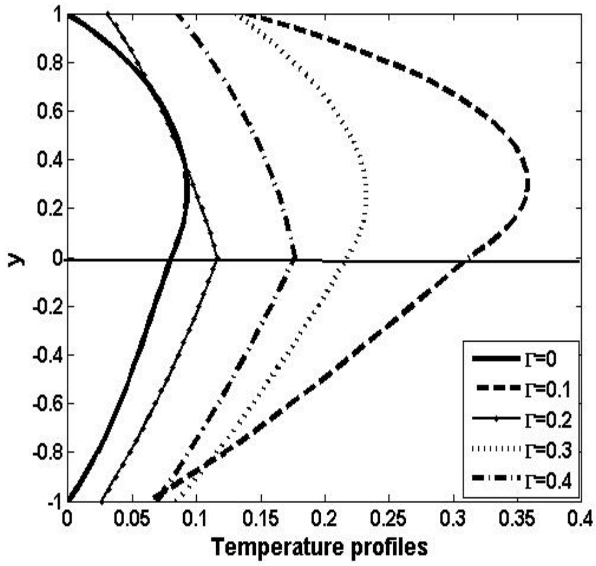

Fig.9. Temperature profiles θ_1, θ_2 for various Γ and $\alpha = 0.333$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $h = 1$, $m = 2$, $H_a = 2$.

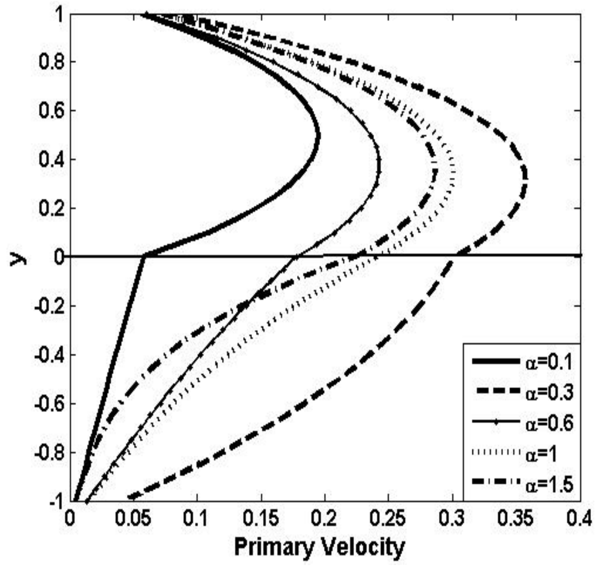

Fig.10. Primary velocity profiles u_1, u_2 for various α and $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $h = 1$, $\Gamma = 0.1$, $m = 2$, $H_a = 2$.

Figures 10, 11 and 12 show the impact of the viscosity ratio α of the two fluids. It can be observed from Fig.10 that an increase of α up to the estimate $\alpha = 1.5$ enhances the primary velocity profiles in both zones. It is noticed from Fig.11 that with an increase in α, the secondary velocity profile is increases in both the zones. The highest primary velocity profile in the channel tends to move over the channel centre-line towards zone-I as α rises and the most extreme secondary velocities in the channel tend to shift under the channel centre-line towards zone-II as α increases. Figure 12 illustrates that with an increase in α, the temperature distribution decreases in both the zones except at $\alpha = 1$. Moreover, the temperature profile within the channel tends to shift along the channel centre-line towards zone-I as α increases.

Figures 13, 14, and 15 show the impact of the variance of the height ratio *h* on temperature and velocity profiles. In region-I, it is noticed from Figs. 13 and 14 that rising *h* decreases both the velocity profiles, while in zone-II, it increases them. As *h* increases, the most extreme velocity distributions in the channel have a tendency to cross the channel focus line and travel towards zone-II. The temperature field declines in zone I and II with an increase in *h* as can be seen in Fig.15. As *h* grows, the zone-I with the highest temperature in the horizontal channel tends to move over the central axis of the channel.

Figures 16, 17, and 18 illustrate how the ratio of electrical conductivity σ_0 influences the velocity and temperature fields. The temperature distributions are found to increase as σ_0 increases, but there is a minimal change in both velocity distributions. As σ_0 rises, the channel's maximum primary, secondary, and temperature distributions have a tendency to cross the channel central axis and move towards zone-I.

Figure 19 shows the effect of the thermal conductivity ratio β on the temperature profiles. It has been noticed that a rise in β causes the temperature profiles in the two zones to increase. As the temperature rises, it has a tendency to shift from the channel centre axis towards zone-I.

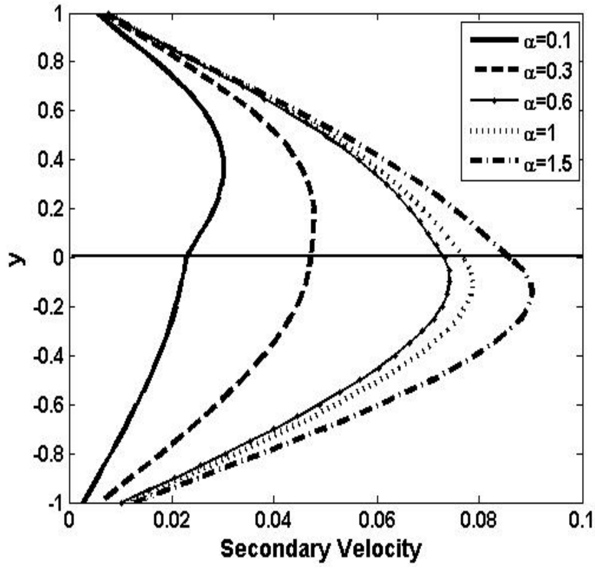

Fig.11. Secondary velocity profiles w_1, w_2 for various α and $\sigma_0 = 2, \sigma_1 = 1.2, \sigma_2 = 1.5$, $h = 1$, $\Gamma = 0.1$, $m = 2$, $H_a = 2$.

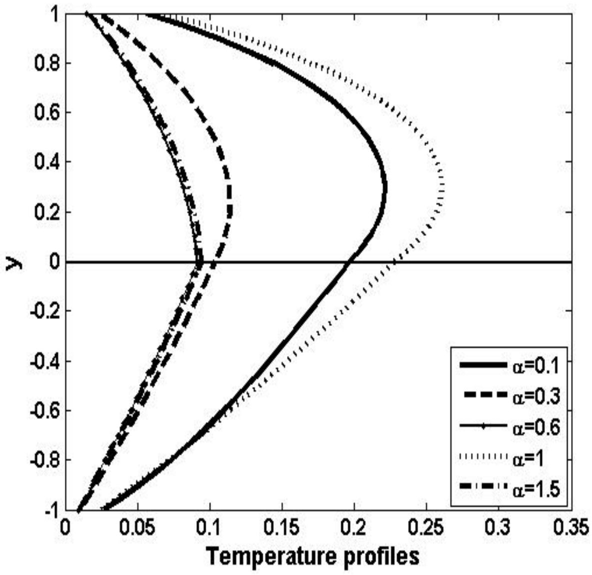

Fig.12. Temperature profiles θ_1, θ_2 for various α and $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $h = 1$, $\beta = 0.5$, $\Gamma = 0.1$, $m = 2$, $H_a = 2$.

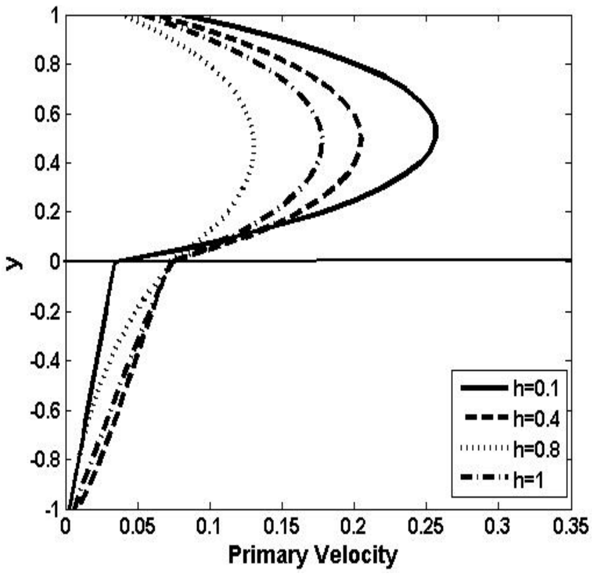

Fig.13. Primary velocity profiles u_1, u_2 for various h and $\alpha = 0.333$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $\Gamma = 0.1$, $H_a = 4$, $m = 2$.

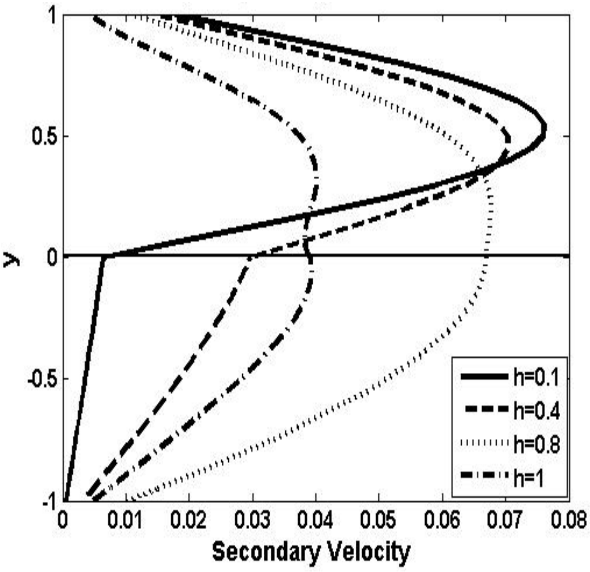

Fig.14. Secondary velocity profiles w_1, w_2 for various h and $\alpha = 0.333$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $\Gamma = 0.1$, $H_a = 4$, $m = 2$.

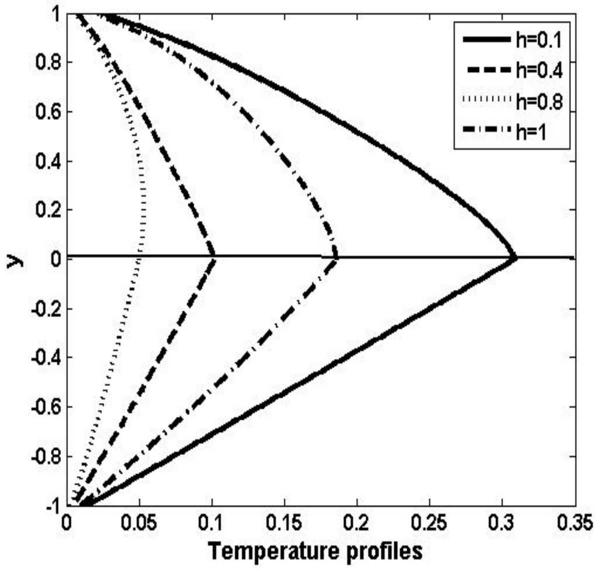

Fig.15. Temperature profiles θ_1, θ_2 for various h and $\alpha = 0.333$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $\beta = 0.5$, $\Gamma = 0.1$, $H_a = 4$, $m = 2$.

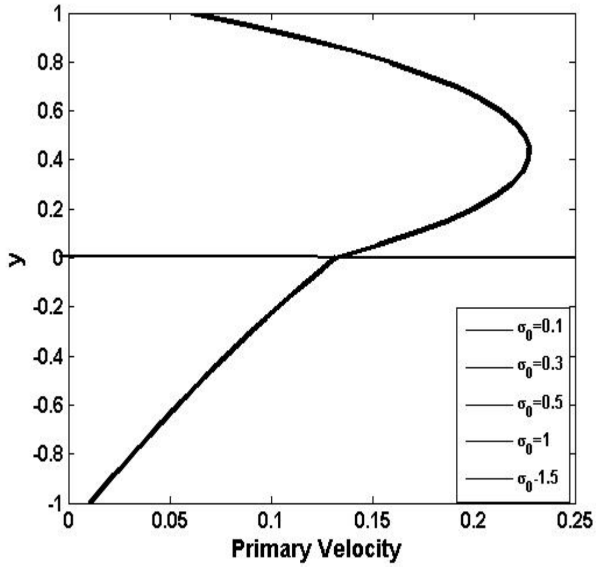

Fig.16. Primary velocity profiles u_1, u_2 for various σ_0 and $\alpha = 0.333, \sigma_1 = 1.2, \sigma_2 = 1.5$, $\Gamma = 0.1$, $h = 1$, $H_a = 2$, $m = 2$.

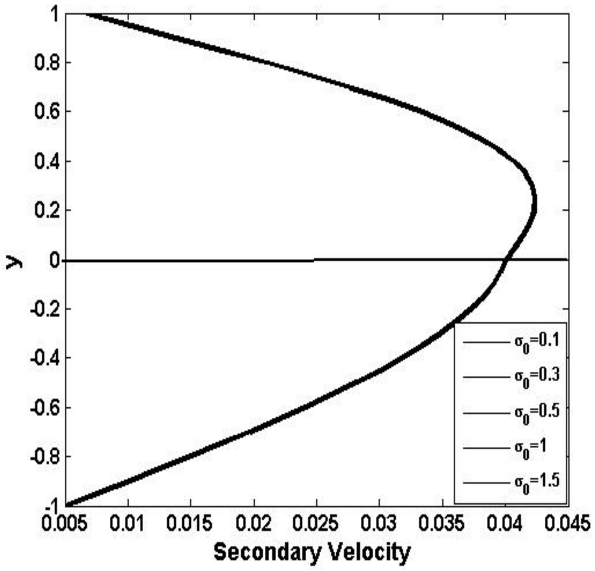

Fig.17. Secondary velocity profiles w_1, w_2 for various σ_0 and $\alpha = 0.333$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $h = 1$, $\Gamma = 0.1$, $H_a = 2$, $m = 2$.

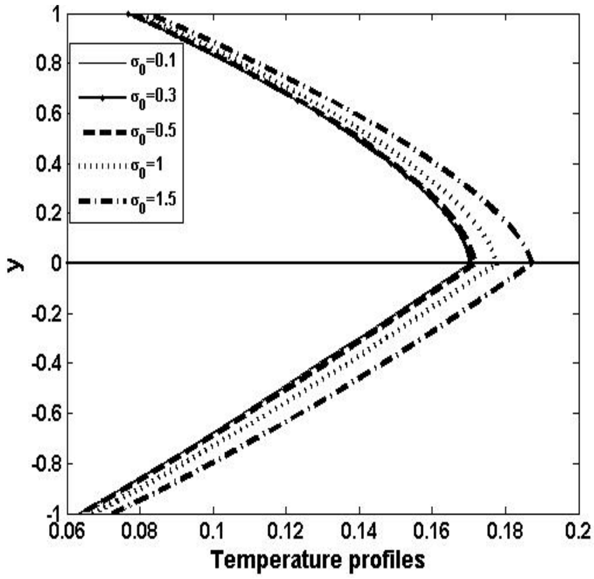

Fig.18 Temperature profiles θ_1, θ_2 for various σ_0 and $\alpha = 0.333$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $h = 1$, $\Gamma = 0.1$, $\beta = 0.5$, $H_a = 2$, $m = 2$.

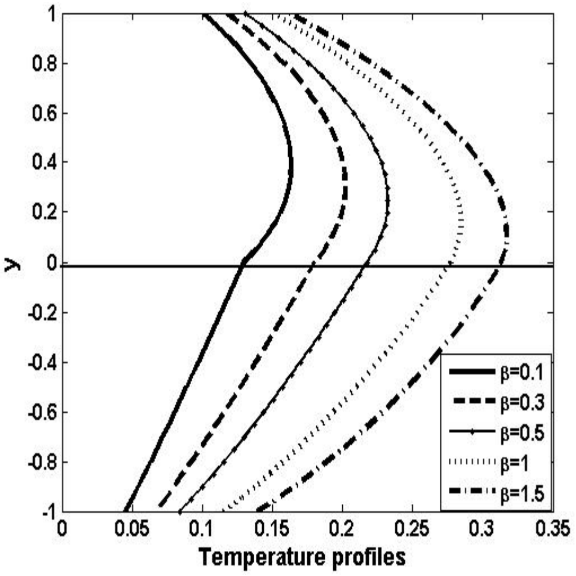

Fig.19. Temperature profiles θ_1, θ_2 for various β and $\alpha = 0.333$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $h = 1$, $\Gamma = 0.1$, $H_a = 3$, $m = 2$.

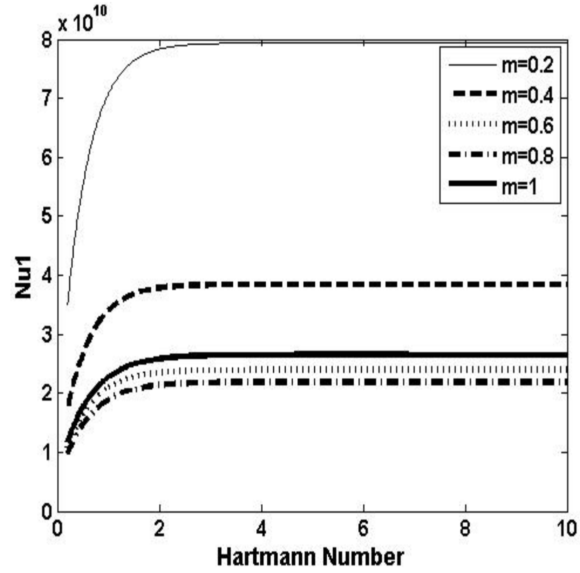

Fig.20. Nusselt number Nu_1 for various m and $\alpha = 0.333, \sigma_0 = 2, \sigma_1 = 1.2, \sigma_2 = 1.5$, $\Gamma = 0.1$, $\beta = 0.5$, $h = 1$.

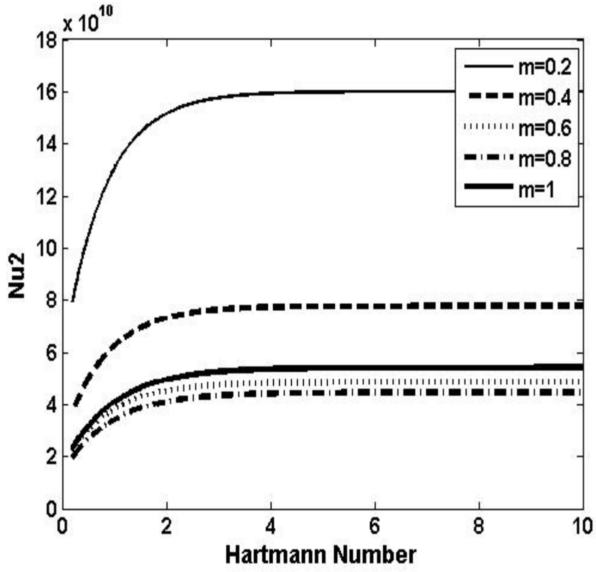

Fig.21. Nusselt number Nu_2 for various m and $\alpha = 0.333$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $\Gamma = 0.1$, $\beta = 0.5$, $h = 1$.

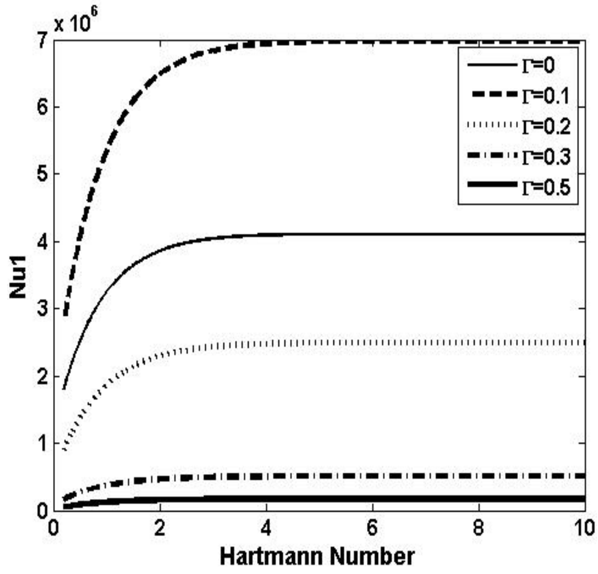

Fig.22. Nusselt number Nu_1 for various Γ and $\alpha = 0.333$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5$, $m = 2$, $\beta = 0.5$, $h = 1$.

The rate of heat transfer coefficients and with the Hartmann number are compared in Figs. 20 and 21. It is observed that the rate of heat transfer coefficient enhanced as H_a increases at the point where other parameters are fixed. It can also be seen that increasing the Hall parameter to a certain value decreases the rate of heat transfer coefficients and past this value it increases the coefficient of heat transfer rate at the two plates.

Figures 22 and 23 show that as the Hartmann number or slip parameter is increased, the heat transfer coefficients at the bounding plates increase while all other factors are constant.

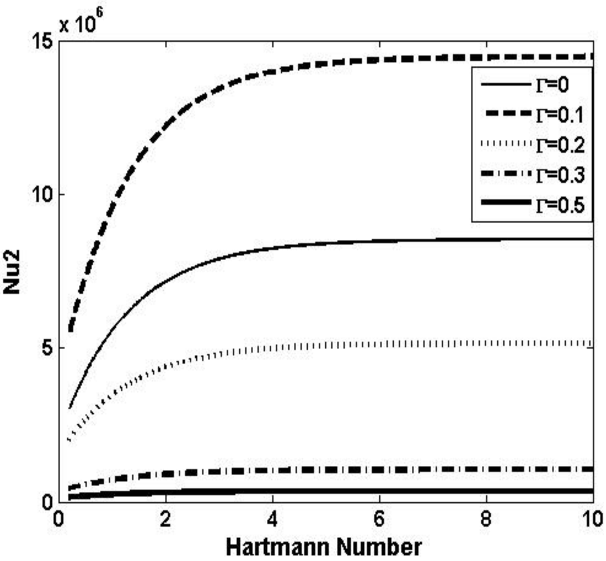

Fig.23. Nusselt number Nu_2 for various Γ and $\alpha = 0.333$, $\sigma_0 = 2$, $\sigma_1 = 1.2$, $\sigma_2 = 1.5, m = 2$, $\beta = 0.5$, $h = 1$.

5. Conclusion

The MHD steady flow and the related heat transfer of two ionised fluids with Hall effect via a channel enclosed by two horizontal non-conducting plates in a slip flow regime is investigated theoretically. The effect of flow characteristics on both the velocities and temperature profile is examined in two liquid zones. These variables include the Hall and slip parameters, Hartmann number, and height, ratios of viscosity, electrical, and also thermal conductivities.

The major observations of this study are as follows:

1. Both the velocities diminish for higher values of the Hartman number in both the regions.
2. The temperature profile drop until they reach a certain specific Hartmann number level, after which they increases as the Hartmann number grows.
3. With a increase in the Hall parameter up to a specific value, the fluid velocity profiles decrease and the temperature profiles increases in both the zones. On the other hand, beyond that specific value the fluid velocity increase and the temperature profiles decrease in both the zones.
4. As the slip parameter rises, the velocity component rises in both zones. However, temperature variations were seen in both areas with an increasing slip parameter.
5. The velocity profiles and the temperature profiles are higher in the lower fluid zone than in the upper zone as the viscosity ratio rises.
6. The primary velocity field decreases for certain specific values of height ratio and it drops beyond this value in the upper zone while it increases in the lower fluid zone. The secondary velocity field enhanced in both zones with an increase in the height ratio.
7. Increasing the thermal conductivity proportion boosts the temperature field.
8. The coefficient of heat transfer rate decreases up to a specific value and then increases with increasing values of the magnetic and Hall parameters.
9. The rate of heat transfer coefficient diminishes up to a certain point, after that it starts to increase as the magnetic and Hall parameters increase.

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Appendix

$$a_1 = -\left(\frac{mi-1}{1+m^2}\right)H_a^2, \qquad a_2 = -K_{11} - \left(\frac{i+m}{1+m^2}\right)H_a^2, \qquad a_3 = \frac{\alpha h^2 H_a^2}{1+m^2}(\sigma_1 - mi\sigma_2),$$

$$a_4 = -K_{12}\alpha h^2 - \frac{(i\sigma_1 + m\sigma_2)\alpha h^2 H_a^2 M_2}{1+m^2}, \qquad M_1 = m_{1x} + im_{1z}, \qquad M_2 = m_{2x} + i\, m_{2z};$$

$$q_1 = u_1 + iw_1; \quad q_2 = u_2 + iw_2; \quad K_{11} = P_1 + iP_2; \quad K_{12} = P_3 + iP_4; \quad P_1 = 1 - 4\frac{sm^2}{1+m^2};$$

$$P_2 = \frac{-sm}{1+m^2}; \quad -s\left(1 - \frac{\sigma_0\sigma_1}{1+m^2}\right); \quad P_4 = \frac{-sm\sigma_0\sigma_2}{1+m^2}; \quad a_5 = \sqrt{a_1}, \quad a_6 = \frac{a_2}{a_1}, \quad a_7 = e^{a_5}(1+\Gamma a_5),$$

$$a_8 = e^{-a_5}(1-\Gamma a_5), \quad a_9 = \sqrt{a_3}, \quad a_{10} = \frac{a_4}{a_3}, \quad a_{11} = e^{-a_9(1-\Gamma a_9)}; \quad a_{12} = e^{a_9}(1+\Gamma a_9),$$

$$a_{13} = \left(1 - \frac{a_7}{a_8}\right)\left(a_5 + \frac{a_9}{\alpha h}\right) - 2a_5, \quad a_{14} = \left(1 - \frac{a_7}{a_8}\right)\left(-a_5 + \frac{a_9}{\alpha h}\right) + 2a_5; \quad a_{16} = \frac{1-mi}{1+m^2}, \quad a_{17} = \frac{i+m}{1+m^2},$$

$$a_{15} = 2a_5(a_6 - a_{10}) - \frac{2a_5a_6}{a_8} - a_5\left(1 - \frac{a_7}{a_8}\right)(a_6 - a_{10}), \qquad a_{18} = \left(\frac{m(1+i)}{1+m^2} - i\right)\frac{s}{H_a^2},$$

$$a_{19} = \frac{\sigma_0\sigma_1 - im\sigma_0\sigma_2}{1+m^2}, \qquad a_{20} = \frac{i\sigma_0\sigma_1 + m\sigma_0\sigma_2}{1+m^2}, \qquad a_{22} = a_{12}a_{13} + a_{11}a_{14},$$

$$a_{21} = \frac{sm\sigma_0^2}{(1+m^2)H_a^2} - \frac{is\sigma_0}{H_a^2} + \frac{is\sigma_0^2\sigma_1}{(1+m^2)H_a^2}, \qquad a_{23} = \frac{1}{a_{22}}\left[a_{13} + 2a_{11}a_5 - a_{11}a_5\left(1 - \frac{a_5}{a_8}\right)\right],$$

$$a_{24} = \frac{1}{a_{22}}\left[2a_{11}a_5 - \frac{2a_{11}a_5}{a_8} + a_{11}a_5\left(1 - \frac{a_7}{a_8}\right)\right], \qquad a_{28} = \frac{1}{\left(1 - \frac{a_7}{a_8}\right)}[a_{26} - a_{23} + 1],$$

$$a_{25} = \frac{1}{a_{13}}\left[2a_5 - \frac{2a_5}{a_8} - a_5\left(1 - \frac{a_7}{a_8}\right) - a_{14}a_{24}\right], \qquad a_{30} = \frac{a_7a_{28}}{a_8}, \qquad a_{31} = \frac{a_{18}}{a_{16}}, \qquad a_{33} = \frac{a_{21}}{a_{19}},$$

$$a_{26} = \frac{1}{a_{13}}\left[2a_5 - a_5\left(1 - \frac{a_7}{a_8}\right) - a_{14}a_{23}\right], \qquad a_{27} = \frac{1}{\left(1 - \frac{a_7}{a_8}\right)}\left[a_{25} - a_{24} + 1 - \frac{1}{a_8}\right], \qquad a_{29} = \frac{1}{a_8}[1 - a_7a_{27}],$$

$$a_{34}=\frac{1}{a_{19}a_9}\left\{a_{20}a_{26}\left(e^{a_9}-1\right)-a_{20}a_{23}\left(1-e^{-a_9}\right)-a_{20}a_9-\left[\frac{a_7a_{28}-a_8a_{30}}{a_7a_{27}+a_8a_{29}-1}\right]\left(a_{20}a_{25}\left(e^{a_9}-1\right)-a_{20}a_{24}\left(1-e^{-a_9}\right)\right)\right\},$$

$$b_1=-p_{r_1}\left[A_1a_5\overline{A_1a_5}+H_a^2a_{17}A_1\overline{a_{17}A_1}\right],\quad b_2=p_{r_1}\left[A_1a_5\overline{A_2a_5}-H_a^2a_{17}A_1\overline{a_{17}A_2}\right],\quad a_{35}=a_{16}M_1-a_{18},$$

$$a_{36}=a_{19}M_2-a_{21},\quad b_3=p_{r_1}\left[A_2a_5\overline{A_1a_5}-H_a^2a_{17}A_2\overline{a_{17}A_1}\right],\quad b_4=-p_{r_1}\left[A_2a_5\overline{A_2a_5}+H_a^2a_{17}A_2\overline{a_{17}A_2}\right],$$

$$b_5=p_{r_1}H_a^2\left[a_6a_{17}\overline{A_1a_{17}}-a_{35}\overline{a_{17}A_1}\right],\quad b_6=p_{r_1}H_a^2\left[a_{17}a_6\overline{A_2a_{17}}-a_{35}\overline{a_{17}A_2}\right],$$

$$b_7=p_{r_1}H_a^2\left[a_6a_{17}A_1\overline{a_{17}}-a_{17}\overline{a_{35}}A_1\right],\quad b_8=p_{r_1}H_a^2\left[a_{17}a_6A_2\overline{a_{17}}-a_{17}\overline{a_{35}}A_2\right],$$

$$b_9=p_{r_1}H_a^2\left[a_{35}\overline{a_6a_{17}}-a_{35}\overline{a_{35}}+a_{17}\overline{a_{35}}a_{16}-a_{17}a_6\overline{a_6a_{17}}\right],\quad b_{13}=-p_{r_2}\beta\left[\frac{A_3}{\alpha}a_9\overline{A_3a_9}+\sigma h^2H_a^2a_{20}A_3\overline{a_{20}}A_4\right],$$

$$b_{10}=\frac{b_1}{b_{29}^2}e^{b_{29}}+\frac{b_2}{b_{30}^2}e^{b_{30}}+\frac{b_3}{b_{31}^2}e^{b_{31}}+\frac{b_4}{b_{32}^2}e^{b_{32}}+\frac{b_5}{\overline{a_5}^2}e^{\overline{a_5}}+\frac{b_6}{\overline{a_5}^2}e^{-\overline{a_5}}+\frac{b_7}{a_5^2}e^{a_5}+\frac{b_8}{a_5^2}e^{-a_5}+\frac{b_9}{2};b_{12}=-b_{10}-\Gamma b_{11};$$

$$b_{11}=\left(\frac{b_1}{b_{29}}e^{b_{29}}+\frac{b_2}{b_{30}}e^{b_{30}}+\frac{b_3}{b_{31}}e^{b_{31}}+\frac{b_4}{b_{32}}e^{b_{32}}\right)+\frac{b_5}{\overline{a_5}}e^{\overline{a_5}}-\frac{b_6}{\overline{a_5}}e^{-\overline{a_5}}+\frac{b_7}{a_5}e^{a_5}-\frac{b_8}{a_5}e^{-a_5}+b_9,$$

$$b_{14}=p_{r_2}\beta\left[\frac{A_3}{\alpha}a_9\overline{A_4a_9}-\sigma h^2H_a^2a_{20}A_3\overline{a_{20}}A_4\right],\quad b_{15}=p_{r_2}\beta\left[\frac{A_4}{\alpha}a_9\overline{A_3a_9}-\sigma h^2H_a^2a_{20}\overline{A_3a_{20}}A_4\right],$$

$$b_{16}=-p_{r_2}\beta\left[\frac{A_4}{\alpha}a_9\overline{A_4a_9}+\sigma h^2H_a^2a_{20}\overline{A_4a_{20}}A_4\right],\quad b_{17}=p_{r_2}\alpha\beta h^2H_a^2\left[a_{20}a_{10}\overline{A_3a_{20}}-a_{36}\overline{A_3a_{20}}\right],$$

$$b_{18}=p_{r_2}\alpha\beta h^2H_a^2\left[a_{20}a_{10}\overline{A_4a_{20}}-a_{36}\overline{A_4a_{20}}\right],\quad b_{19}=p_{r_2}\alpha\beta h^2H_a^2\left[a_{20}A_3\overline{a_{10}a_{20}}-a_{20}A_3\overline{a_{36}}\right]$$

$$b_{20}=p_{r_2}\alpha\beta h^2H_a^2\left[a_{20}A_4\overline{a_{10}a_{20}}-a_{20}\overline{A_4a_{36}}\right],$$

$$b_{21}=-p_{r_2}\alpha\beta h^2H_a^2\left[a_{36}\overline{a_{36}}-a_{36}\overline{a_{10}a_{20}}-a_{20}a_{10}\overline{a_{36}}+a_{10}a_{20}\overline{a_{10}a_{20}}\right]$$

$$b_{22}=\frac{b_{13}}{b_{33}^2}e^{-b_{33}}+\frac{b_{14}}{b_{34}^2}e^{-b_{34}}+\frac{b_{15}}{b_{35}^2}e^{-b_{35}}+\frac{b_{16}}{b_{36}^2}e^{-b_{36}}+\frac{b_{17}}{\overline{a_9}^2}e^{-\overline{a_9}}+\frac{b_{18}}{\overline{a_9}^2}e^{\overline{a_9}}+\frac{b_{19}}{a_9^2}e^{-a_9}+\frac{b_{20}}{a_9^2}e^{a_9}+\frac{b_{21}}{2}$$

$$b_{23}=\left(b_{21}+\frac{b_{13}}{b_{33}}e^{-b_{33}}+\frac{b_{14}}{b_{34}}e^{-b_{34}}++\frac{b_{16}}{b_{36}}e^{-b_{36}}\right)+\frac{b_{17}}{\overline{a_9}}e^{-\overline{a_9}}-\frac{b_{18}}{\overline{a_9}}e^{\overline{a_9}}+\frac{b_{19}}{a_9}e^{-a_9}-\frac{b_{20}}{a_9}e^{a_9},\quad b_{24}=b_{22}-\Gamma b_{23},$$

$$b_{26}=\frac{1}{\beta h},\quad b_{28}=\frac{b_{25}-b_{24}-b_{12}+(1+\Gamma)b_{27}}{-(1+\Gamma)(1+b_{26})},\quad b_{29}=a_5+\overline{a_5},\quad b_{30}=a_5-\overline{a_5},\quad b_{31}=-a_5+\overline{a_5},$$

$$b_{25}=\left(\frac{b_{13}}{b_{33}^2}+\frac{b_{14}}{b_{34}^2}+\frac{b_{15}}{b_{35}^2}+\frac{b_{16}}{b_{36}^2}+\frac{b_{17}}{\overline{a_9}^2}+\frac{b_{18}}{\overline{a_9}^2}+\frac{b_{19}}{a_9^2}+\frac{b_{20}}{a_9^2}\right)-\left(\frac{b_1}{b_{29}^2}+\frac{b_2}{b_{30}^2}+\frac{b_3}{b_{31}^2}+\frac{b_4}{b_{32}^2}+\frac{b_5}{\overline{a_5}^2}+\frac{b_6}{\overline{a_5}^2}+\frac{b_7}{a_5^2}+\frac{b_8}{a_5^2}\right),$$

$$b_{27} = b_{26}\left[\frac{b_{13}}{b_{33}} + \frac{b_{14}}{b_{34}} + \frac{b_{15}}{b_{35}} + \frac{b_{16}}{b_{36}} + \frac{\overline{b_{17}}}{a_9} - \frac{\overline{b_{18}}}{a_9} + \frac{b_{19}}{a_9} - \frac{b_{20}}{a_9}\right] - \left[\frac{b_1}{b_{29}} + \frac{b_2}{b_{30}} + \frac{b_3}{b_{31}} + \frac{b_4}{b_{32}} + \frac{\overline{b_5}}{a_5} - \frac{\overline{b_6}}{a_5} + \frac{b_7}{a_5} - \frac{b_8}{a_5}\right],$$

$$b_{32} = -a_5 - \overline{a_5}, \quad b_{33} = a_9 + \overline{a_9}, \quad b_{34} = a_9 - \overline{a_9}, \quad b_{35} = -a_9 + \overline{a_9}, \quad b_{36} = -a_9 - \overline{a_9},$$

$$A_1 = a_6 a_{27} - a_{10} a_{23}, \quad A_2 = a_6 a_{29} + a_{10} a_{30}, \quad A_3 = a_6 a_{25} - a_{10} a_{26}, \quad A_4 = a_{10} a_{23} - a_6 a_{24}, .$$

$$B_1 = b_{27} + b_{26} b_{28}, \quad B_2 = b_{12} - (1+\Gamma)(b_{27} + b_{26} b_{28}), \quad B_3 = b_{28}, \quad B_4 = b_{12} - (1+\Gamma)(b_{27} + b_{26} b_{28}) - b_{25} .$$

Nomenclature

B_0 – applied magnetic field
$c_{p_i}\,(i=1,2)$ – specific heat at constant pressure in the two liquid zones
$E_{ix}, E_{iz}, (i=1,2)$ – applied electric fields in the x- and z- directions, where $\overline{E_i} = (E_{ix}, 0, E_{iz})$
h – ratio of the heights of the two regions
h_1 – height of the channel in the upper zone (zone-I)
h_2 – height of the channel in the lower zone (zone-II)
$I_{ix}, I_{iz}\,(i=1,2)$ – dimensionless current densities along x- and z- directions in zone-I and -II
I_1, I_2 – $I_1 = I_{1x} + iI_{1z}$ and $I_2 = I_{2x} + iI_{2z}$ symbols for currents in two liquids
J_{ix}, J_{iz} – current densities along x- and z- directions in two liquid zones
K_{11}, K_{12} – notations used for simplicity: $K_{11} = P_1 + iP_2$, $K_{12} = P_3 + iP_4$
H_a – Hartman number $H_a = \sqrt{\frac{\sigma_{01} B_0^2 h_1^2}{\mu_1}}$
m – Hall parameter where $m = \frac{\omega_e}{\left(\frac{1}{\tau} + \frac{1}{\tau_e}\right)}$
$m_{ix}, m_{iz}, (i=1,2)$ – dimensionless electric fields in zone-I and-II as $m_{1x}, m_{1z}, m_{2x}, m_{2z}$
M_1, M_2 – notations where $M_1 = m_{1x} + im_{1z}$, $M_2 = m_{2x} + im_{2z}$
P_r – Prandtl number
$q_1(y), q_2(y)$ – solutions of velocity distributions in complex for $q_1(y) = u_1(y) + iw_1(y)$, $q_2(y) = u_2(y) + iw_2(y)$
q_{1m}, q_{2m} – mean velocities as $q_{1m} = u_{1m} + iw_{1m}$ and $q_{2m} = u_{2m} + iw_{2m}$
$s = \frac{p_e}{p}$ – ionization parameter
T – temperature
$T_i\,(i=1,2): T_1, T_2$ – temperatures of the liquids in zone-I and zone-II
$u_i, (i=1,2): u_1$, u_2 – primary velocity distributions in zone-I and –II
u_{1m}, u_{2m} – primary mean velocity distributions in the two liquid zones
u_p – = () : the characteristic velocity
$w_i, (i=1,2): w_1, w_2$ – secondary velocity distributions in the two liquid zones
w_{1m}, w_{2m} – secondary mean velocity distributions in the two liquid zones
(x, y, z) – space co-ordinates in rectangular Cartesian co-ordinate system
$-\frac{\partial p}{\partial x}$ – common constant pressure gradient

Greek symbols

α – $= \frac{\mu_1}{\mu_2}$, ratio of the viscosities

β – $=\frac{K_1}{K_2}$, thermal conductivity ratio

Γ – slip parameter

$\mu_i(i=1,2):\mu_1,\mu_2$ – viscosities of the two liquids

σ_0 – ratio of electrical conductivities $\sigma_0=\frac{\sigma_{01}}{\sigma_{02}}$

$\sigma_{0i}\,(i=1,2),\,\sigma_{01},\,\sigma_{02}$ – electrical conductivities of the two liquids

$\sigma_{11},\sigma_{12},\sigma_{21},\sigma_{22}$ – modified conductivities

σ_1,σ_2 – symbols for the ratios $\sigma_1=\frac{\sigma_{12}}{\sigma_{11}},\,\sigma_2=\frac{\sigma_{22}}{\sigma_{21}}$

$\theta_1,\ \theta_2$ – dimensionless temperature distributions for two liquid zones

τ,τ_e – mean collision time between electron and ion, electron and particles

Subscripts

$1,\,2$ – denote the quantities in the upper and lower liquid zone

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