5.2.3. STRESS STRAIN ANALYSIS OF THE BALKAN PENINSULA

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5.2.3.1. Introduction

The investigations of the recent geodynamic processes are a purpose of many sciences which deal with the exploring of the Earth and with the applying of the proper measuring methods. The geodesy is one of these sciences which give us quantitative characteristics for these processes (Milev, 1978). But there is a feet-back link. Having the knowledge of these processes also has important practical meaning. During the geodetic measurements and re-measurements frequently preliminary information is necessary about where and what kind of movements of the earth crust should be expected. That is connected with the important task of processing of the results of these measurements which has been done during different epochs or during a more extensive period. And then appears the question whether the existing changes in the position of the points in the particular examined area can be accepted like negligible, so that the common processing of the measurements could be done correctly.

In the former practice one of the most frequently used methods for defining of the horizontal deformations are the methods of the designing geodetic networks. For defining of the vertical movements of the earth crust the method used is leveling. Although they are not so often used in the practice there are also other methods doing the same tasks: laser, satellite, interferometric, astronomic, etc.

5.2.3.2. Mathematical basis

During the processing of the results of the repeated measurements to find the most probable values of the coordinates of the points is necessary to know if the real movement of the same points exists. But the movements don't give a correct picture of the deformations every time because they are connected with the datum. Since the relatively deformations in the different directions have different values, the problem is to find the so called main deformations and the main directions in which they occur.

The assessment of the relative deformations can be done in different ways. The method of the finite elements is the most widespread. The required condition for using this method is the examined points to be apexes of not recovering triangles. And each one of them is considered like a finite element to which borders the surrounding elements are accepted isotropic. The main deformations are referred to the one medium point which belongs to the space of the triangle. The defining of the deformations is done using the displacements of the points. The extension of the displacements of each point is fixed by its differences to the coordinates, which are calculated for each epoch.

If the coordinates of the apexes of a triangle for two epochs are considered as x, y **and** x' y' **and we assume that the displacements are changing by a lineal way from the**

coordinates of the points, then the two components of the vector of the displacement in the two-dimensional space can be expressed by the following way:

(1)
$$dx_i = x^| - x = e_x + e_{11}x + e_{12}y$$
$$dy_i = y^| - y = e_y + e_{21}x + e_{22}y$$

In these expressions

(2) e_x, e_y **are the translations on *x* and *y*, while**

(3) $e_{11.}, e_{12}, e_{21}, e_{22}$ **are the coefficients which express the velocity of the displacement (the deformation) to the direction of the coordinate axes.**

The coefficients e_{ij} **are calculated by resolving two systems with three equations for the coordinates of the three points. A symmetric tensor is worked out:**

(4)
$$\mathbf{E} = \begin{vmatrix} e_{11} & e'_{12} \\ e'_{12} & e_{22} \end{vmatrix}$$

where

(5)
$$e^{'}_{12} = \frac{(e_{12} + e_{21})}{2}$$

In the literature different ways for deriving of the elements and the orientation of the main deformations are offered. The most popular solution of the problem is:

1.The major deformations are defined by solving a characteristic equation

(6)
$$\begin{vmatrix} e_{11} - \lambda & e©_{12} \\ e©_{12} & e_{22} - \lambda \end{vmatrix} = \lambda^2 - (e_{11} + e_{22})\lambda + (e_{11}.e_{22} - e^©_{12}e^©_{12}) = 0$$

which is composed with the help of the elements from the tensor (4).

For the main deformations is obtained:

(7)
$$a = \lambda_1 \qquad b = \lambda_2$$

where λ_1 **and** λ_2 **are the eigenvalues and** $\lambda_1 > \lambda_2$**.**

According to this method the values of *a* and *b* are the extreme relative deformations which appear in two perpendicular to each other directions.

2. The orientation of the directions of the major deformation is defined by the formula

(8)
$$tg2\alpha = \frac{2e^©_{12}}{e_{11} - e_{22}}$$

3. In (Valev, 1995) the derivatives of the major deformations are connected with the main scales *a* and *b (*extreme values of the scale) with the expressions which are given by of the equations:

(9)
$$a^2\cos^2 U_1 + b^2\sin^2 U_1 = m_1^2$$
$$a^2\cos^2 U_2 + b^2\sin^2 U_2 = m_2^2$$
$$a^2\cos^2 U_3 + b^2\sin^2 U_3 = m_3^2$$

where m_1, m_2, m_3 are the scales on each one of the three sides of the triangle. The scales are calculated by the distances S and S' of the corresponding sides, measured (or calculated) in the two epochs by the formulas:

(10) $$m_1 = \frac{S'_{12}}{S_{12}}, \quad m_2 = \frac{S©_{13}}{S_{13}}, \quad m_3 = \frac{S©_{23}}{S_{23}},$$

The angles U_1, U_2 and U_3 are the angles between the sides of the finite element and the direction of the maximum scale *a* .They can be calculated with the help of the azimuths of the three sides and the direction of the main scale *a*

(11) $$\begin{aligned} U_1 &= \alpha_{12} - \alpha_a \\ U_2 &= \alpha_{13} - \alpha_a \\ U_3 &= \alpha_{23} - \alpha_a \end{aligned}$$

By eliminating the unknown quantities a^2 and b^2 in (11) we can reach to the equation

(12) $$\frac{\sin^2 U_2 - \sin^2 U_1}{\sin^2 U_3 - \sin^2 U_1} = \frac{m_2^2 - m_1^2}{m_3^2 - m_1^2}$$

Transforming the left side of (12) with functions of double angels by using (11) we get

(13) $$tg2\alpha_a = \frac{m_2^2 - m_1^2}{m_3^2 - m_1^2} - \frac{\cos 2\alpha_{12} - \cos 2\alpha_{13}}{\cos 2\alpha_{12} - \cos 2\alpha_{23}}$$

Using the calculated value of α_a we can derive the values of U_1, U_2 and U_3 by using (11). After that we use the offered formulas by (Valev, 1995) and the values of the semi-axes *a* and *b* can be calculated:

(14) $$\begin{aligned} &(b^2 - a^2) = (m_2^2 - m_1^2)/(\sin^2 U_2 - \sin^2 U_1), \\ &a = \sqrt{m_1^2 - (b^2 - a^2) * \sin^2 U_1}, \\ &b = \sqrt{a^2 + (b^2 - a^2)}. \end{aligned}$$

Since the deformations are shown with the quantity $1 - m = \frac{S'_{ik} - S_{ik}}{S_{ik}}$ **the main deformations would be as it has shown:**

(15) $$\begin{aligned} a &= 1 - b \\ b &= 1 - a \\ \alpha &= \alpha_a + 90 \end{aligned}$$

In conclusion it must be mentioned that the ellipse of the deformations gives the precision about the relative displacements. We must consider this circumstance during a common processing of the information from measurements, which are made in different times. If there is no deformation, connected with the earth surface, it shows that during the adjustment the differences, which we have as the results are only due to the measurements errors in the net. That means that it is possible to made common adjustment of the measurements of two epochs.

Using the offered method for each case and area of examination, depending on the received values of deformations and the accuracy of the observations, it could be decided

whether the common processing of the measurements or the results from them, made in the different epochs is possible and properly assessed.

5.2.3.3. Space case

The finite element method can be used in the case of space coordinates. It is relevant the space triangle to be considered like a plane triangle. When we investigate the geodynamics of a large region the finite elements (the triangles) are practically considered on the ellipsoid.

We apply the following method, which we find to be very suitable for deformation investigations with GPS on large territories (with big finite elements). The points measured with GPS are projected on the ellipsoid and later on we work with the triangles, formed by the ellipsoid hordes. Instead of bearings azimuths are used previously. The working stages and the formulas are the following:

1. Transformation of the spatial Cartesian coordinates X, Y, Z of the points into ellipsoid geographical coordinates φ, λ and heights h.

The calculation of *λ* is carried out with the formula

(16) $$\lambda = \text{arctg}\left(\frac{Y}{X}\right).$$

The ellipsoid height and the ellipsoid width are calculated

(17) $$h = \frac{A}{N}(A - a)$$

(18) $$\text{tg}\varphi = k.\text{tg}\varphi',$$

where:

$$\text{tg}\varphi' = \frac{Z(1+e'^2)}{\sqrt{X^2+Y^2}}$$

(19) $$k = \left(1 + \frac{H}{N}e'^2\right)^{-1}$$

$$A = \sqrt{X^2+Y^2+Z^2(1+e'^2)}$$

$$N = \sqrt{X^2+Y^2+Z^2(1+e'^2)^2}$$

<u>**For *h* < 10 000 m the accuracy is the following:**</u> $\delta h < 0.1\text{mm}, \delta\varphi < 0.000001''$

2. Calculation of the spatial Cartesian coordinates *Xo, Yo, Zo* of the projections of the points.

The transformation of the coordinates *φ, λ* и *h* into spatial ellipsoid orthogonal coordinates *Xo*, *Yo* и *Zo* is carried out with the formulas:

(20) $$X_o = No.\cos\varphi.\cos\lambda,$$
$$Y_o = No.\cos\varphi.\sin\lambda,$$
$$Z_o = No.(1-e^2).\sin\varphi\ .$$

where *e* is the first offset of the ellipsoid, and *No* is the cross radius of a curve, which is defined with the formula:

(21) $$N_o = \frac{a}{\sqrt{(1-e^2\sin^2\varphi)}}$$

3. Calculation of the azimuths of the sides of the ellipsoid horde triangle.

(22) $$tg\alpha_{ik} = \frac{(\vec{b}.\vec{r})}{(\vec{t}.\vec{r})},$$

Where $\vec{b}$ (- sinλ, cosλ, 0) and $\vec{t}$ (-sinφ.cosλ, -sinφ.sinλ, cosφ) are vectors from the natural trihedron on the ellipsoid, and $\vec{r}$ ($X_k - X_i$, $Y_k - Y_i$, $Z_k - Z_i$) is the horde vector.

4. Calculation of the lengths of the sides of the ellipsoid horde triangle

(23) $$S_{ik} = \sqrt{(X_k^o - X_i^o)^2 + (Y^o{}_k - Y_i^o)^2 + (Z^o{}_k - Z^o{}_i)^2}$$

5. Calculations of the vectors of the movements of the apexes of the triangle.

(24)
$$dx_i = X_i - X_i$$
$$dy_i = Y_i - Y_i,$$
$$dz_i = Z_i - Z_i$$

6. Calculations of the deformations of the sides of the triangle. We use formulas (10)

7. Calculation of the main deformations and their directions. We use formulas (13). (14) и (15).

With big triangles the relative deformations are small and the respective tensions which the cause, are also small, and that’s why the latter are rarely calculated.

5.2.3.4. Application to the Balkan Peninsula

Below some results from geodynamic researches from the special GPS campaigns on the territory of Balkan Peninsula are given. The stations are as follows (Fig. 5.2.3.1.):

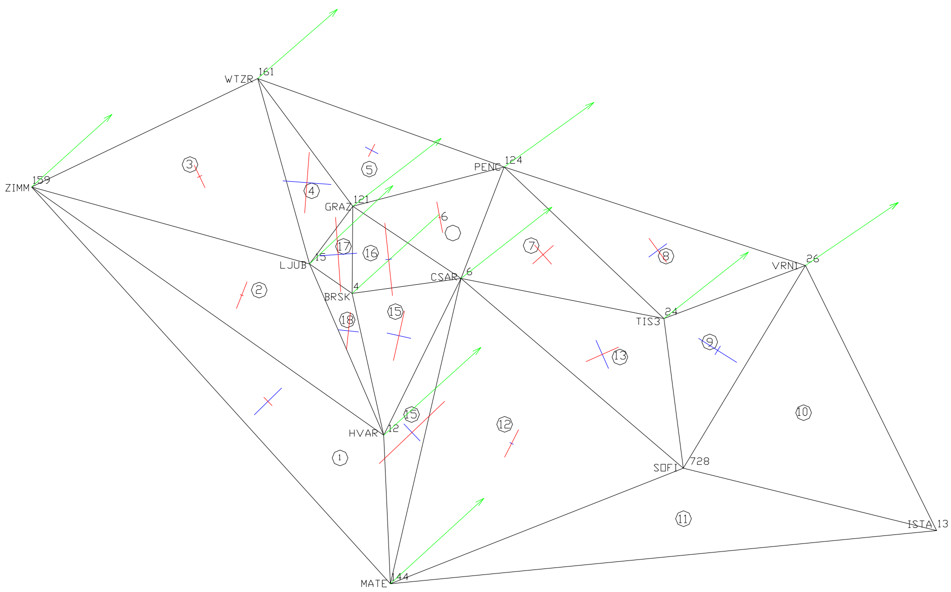

Fig. 5.2.3.1. Vectors of displacements and deformations

Epochs 1997 - 2003

Displacements

No	dX	dY	dZ	dS	
	dN	dE	ds	Az	dH
121	-0.1014	0.1071	0.0549	0.1574	
	0.0880	0.1303	0.1572	55.9681	-0.0069
124	-0.1007	0.1075	0.0505	0.1557	
	0.0780	0.1347	0.1557	59.9180	-0.0026
144	-0.1031	0.1167	0.0855	0.1776	
	0.1073	0.1414	0.1775	52.7973	0.0062
159	-0.0950	0.1173	0.0463	0.1579	
	0.0893	0.1286	0.1566	55.2409	-0.0202
161	-0.0976	0.1024	0.0476	0.1493	
	0.0858	0.1216	0.1488	54.7766	-0.0113
728	-0.1017	0.1108	0.0497	0.1584	
	0.0700	0.1421	0.1584	63.7760	-0.0027
4	-0.7230	-0.0460	-0.5863	0.9320	
	0.0959	0.1498	0.1778	57.3724	-0.9149
6	-0.1092	0.1097	0.0553	0.1644	
	0.0883	0.1383	0.1641	57.4380	-0.0086
12	-0.1477	0.1164	0.0461	0.1936	
	0.1080	0.1535	0.1876	54.8648	-0.0477
15	-0.0833	0.1001	0.0848	0.1554	
	0.0989	0.1178	0.1538	49.9818	0.0225
24	-0.0518	0.1101	0.1183	0.1697	
	0.0866	0.1216	0.1493	54.5480	0.0807
26	-0.1191	0.1014	0.0373	0.1608	
	0.0697	0.1440	0.1600	64.1710	-0.0157

Deformations

Points			Baseline length deformation			Main deformations		
A	B	C	AB	AC	BC	a	b	azimuth
159	144	12	-0.21	-0.06	-0.03	-0.68	0.28	54.98
159	12	15	-0.06	-0.41	0.10	-0.14	0.93	142.87
159	161	15	-0.18	-0.41	-0.45	0.15	0.54	42.29
161	121	15	-0.04	-0.45	-0.10	-0.79	0.58	87.95
161	124	121	0.10	-0.04	-0.02	-0.10	0.38	105.92
121	124	6	-0.02	0.06	-0.42	-0.12	0.43	102.87
124	24	6	-0.54	-0.42	-0.58	0.42	0.58	12.61
124	26	24	0.01	-0.54	0.53	-0.54	0.78	68.69
728	26	24	0.09	0.62	0.53	-1.29	-0.08	127.51
144	728	6	-0.25	-0.29	0.10	-0.10	0.38	126.78
728	24	6	0.62	0.10	-0.58	-0.63	0.80	171.06
144	6	12	-0.29	-0.03	-0.67	-0.02	1.70	166.41
12	6	4	-0.67	-0.46	-0.70	0.42	0.77	146.46
121	6	4	0.06	-0.44	-0.70	-0.15	1.15	135.64
121	15	4	-0.10	-0.44	2.71	-3.15	0.66	101.80

15	12	4	0.10	2.71	-0.46	-4.55	0.59	84.79

Epochs 1997 - 2005

Displacements

No	dX	dY	dZ	dS	
	dN	dE	ds	Az	dH
121	-0.1281	0.1514	0.0743	0.2118	
	0.1114	0.1801	0.2118	58.2687	-0.0021
124	-0.1362	0.1455	0.0660	0.2099	
	0.1040	0.1823	0.2099	60.3027	-0.0052
144	-0.1464	0.1563	0.1097	0.2406	
	0.1453	0.1918	0.2406	52.8500	-0.0008
159	-0.1230	0.1501	0.0708	0.2066	
	0.1232	0.1648	0.2058	53.2256	-0.0184
161	-0.1328	0.1370	0.0640	0.2012	
	0.1167	0.1632	0.2006	54.4276	-0.0163
728	-0.1422	0.1418	0.0583	0.2091	
	0.0931	0.1866	0.2086	63.4770	-0.0152
4	-0.1513	0.1430	0.0819	0.2237	
	0.1340	0.1784	0.2231	53.0825	-0.0167
6	-0.1451	0.1475	0.0738	0.2197	
	0.1172	0.1855	0.2194	57.7064	-0.0109
12	-0.1878	0.1525	0.0759	0.2535	
	0.1490	0.1994	0.2490	53.2278	-0.0479
15	-0.1413	0.1404	0.0873	0.2175	
	0.1338	0.1713	0.2173	52.0115	-0.0077
24	-0.1426	0.1265	0.0736	0.2043	
	0.1097	0.1723	0.2043	57.5335	-0.0053
26	-0.1808	0.1199	0.0324	0.2193	
	0.0999	0.1883	0.2131	62.0406	-0.0518

Deformations

Points			Baseline length deformation			Main deformations		
A	B	C	AB	AC	BC	a	b	azimuth
159	144	12	-0.19	-0.08	0.09	-0.73	0.23	50.46
159	12	15	-0.08	-0.17	-0.19	0.07	0.55	124.03
159	161	15	-0.12	-0.17	-0.48	0.10	0.48	71.62
161	121	15	0.27	-0.48	-0.93	-0.92	1.16	104.79
161	124	121	0.18	0.27	-0.09	-0.28	0.27	130.36
121	124	6	-0.09	-0.21	-0.52	0.05	0.60	88.07
124	24	6	-0.50	-0.52	-0.52	0.49	0.53	153.73
124	26	24	-0.14	-0.50	0.36	-0.43	0.57	59.59
728	26	24	0.23	0.59	0.36	-0.86	-0.20	135.81
144	728	6	-0.45	-0.44	0.07	-0.08	0.59	129.88
728	24	6	0.59	0.07	-0.52	-0.59	0.68	173.33
144	6	12	-0.44	0.09	-0.97	-0.45	1.72	151.58
12	6	4	-0.97	-0.42	0.06	-0.47	0.97	113.90
121	6	4	-0.21	-1.38	0.06	-0.12	1.39	93.29
121	15	4	-0.93	-1.38	0.32	-0.72	1.42	95.66
15	12	4	-0.19	0.32	-0.42	-0.39	0.71	106.60

Epochs 2003 - 2005

Displacements

No	dX	dY	dZ	dS	
	dN	dE	ds	Az	dH
121	-0.0267	0.0443	0.0194	0.0552	
	0.0234	0.0498	0.0550	64.8533	0.0047
124	-0.0355	0.0380	0.0155	0.0543	
	0.0259	0.0476	0.0542	61.4078	-0.0026
144	-0.0433	0.0396	0.0242	0.0635	
	0.0380	0.0504	0.0631	52.9983	-0.0071
159	-0.0280	0.0328	0.0245	0.0496	
	0.0339	0.0362	0.0496	46.8470	0.0018
161	-0.0352	0.0346	0.0164	0.0520	
	0.0308	0.0416	0.0518	53.4244	-0.0050
728	-0.0405	0.0310	0.0086	0.0517	
	0.0231	0.0445	0.0502	62.5336	-0.0125
4	0.5717	0.1890	0.6682	0.8995	
	0.0381	0.0286	0.0477	36.8804	0.8982
6	-0.0359	0.0378	0.0185	0.0553	
	0.0289	0.0471	0.0553	58.5037	-0.0022
12	-0.0401	0.0361	0.0298	0.0616	
	0.0411	0.0460	0.0616	48.2387	-0.0002
15	-0.0580	0.0403	0.0025	0.0707	
	0.0349	0.0535	0.0639	56.9004	-0.0302
24	-0.0908	0.0164	-0.0447	0.1025	
	0.0231	0.0507	0.0557	65.5506	-0.0860
26	-0.0617	0.0185	-0.0049	0.0646	
	0.0302	0.0442	0.0535	55.6610	-0.0361
13	-0.0484	0.0319	0.0044	0.0581	
	0.0210	0.0514	0.0555	67.7987	-0.0173

Deformations

Points			Baseline length deformation			Main deformations		
A	B	C	AB	AC	BC	a	b	azimuth
159	144	12	0.01	-0.01	0.12	-0.12	0.03	7.62
159	12	15	-0.01	0.24	-0.29	-0.46	0.29	64.37
159	161	15	0.05	0.24	-0.03	-0.24	0.13	101.67
161	121	15	0.31	-0.03	-0.82	-0.38	0.82	124.33
161	124	121	0.08	0.31	-0.07	-0.36	0.07	159.15
121	124	6	-0.07	-0.28	-0.09	0.03	0.31	50.58
124	24	6	0.04	-0.09	0.07	-0.07	0.09	109.84
124	26	24	-0.14	0.04	-0.17	-0.28	0.20	179.05
728	26	24	0.14	-0.02	-0.17	-0.15	0.45	28.32
144	728	6	-0.19	-0.15	-0.02	0.02	0.22	137.64
728	24	6	-0.02	-0.02	0.07	-0.14	0.05	65.03
144	6	12	-0.15	0.12	-0.30	-0.75	0.33	125.76
12	6	4	-0.30	0.04	0.76	-1.01	0.32	107.04
121	6	4	-0.28	-0.94	0.76	-0.81	1.08	71.84
121	15	4	-0.82	-0.94	-2.40	0.70	2.48	19.24
15	12	4	-0.29	-2.40	0.04	-0.06	4.34	169.84
728	13	26	0.10	0.14	0.17	-0.18	-0.09	176.10
144	13	728	-0.03	-0.19	0.10	-0.10	0.51	111.02

Reference stations: 121(GRAZ), 124(PENC), 144(MATE), 159(ZIMM), 161(WTZR), 728(SOFI).

Additional stations: 4(BRSK), 6(CSAR), 12(HVAR), 13(ISTA), 15(LJUB), 24(TIS3) AND 26(VRN1). The comparison is made among 1997, 2003 and 2005.

The comparison shows that movements are not insignificant, and the deformations are very small.

5.2.3.5. Conclusions

1. **The calculated vectors of the displacements have equable values and directions (north-east) and are near to the publicized ones and given in the monograph.**
2. **The velocities are similar to these which are derived here.**
3. **The deformations of the sides of the finite elements are too small ($<10^{-7}$), because of very long distances between the points.**
4. **The main deformations are small too.**
5. **The directions of the main deformations are different.**
6. **There are deformation if compression and of a dilatation too.**
7. **The analyzing of the deformation, derived here has to be perform further together with tectonic and seismic situation.**

5.2.3.5. References

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