# THE RELIABILITY MODEL FOR FAILURE CAUSE ANALYSIS OF PRESSURE VESSEL PROTECTIVE FITTINGS WITH TAKING INTO ACCOUNT LOAD-SHARING EFFECT BETWEEN VALVES

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Summary

In the paper reliability model for pressure vessel protective fittings is developed. The model is intended for the quantitative analysis of failure causes of such system. Reliability of the system is formalized by the dynamic fault tree in which load-sharing phenomena are mathematically described. Using the dynamic fault tree the split homogeneous Markov model is obtained. Reliability characteristics are calculated based on the Markov model. Life of protective fittings components is distributed by Weibull that provided by tensor splitting of Markov model. The result of the simulation is probability curve family obtained for different values of load-sharing coefficients. It is shown how the main cause of system failure changing with these coefficients changing.

Keywords: pressure vessel, safety valves, protective fittings, reliability model, dynamic fault tree, Markov model, failure cause.

MODEL NIEZAWODNOŚCI PRZYCZYN USZKODZEŃ ARMATURY OCHRONNEJ ZBIORNIKA CIŚNIENIOWEGO Z UWZGLĘDNIENIEM EFEKTU PODZIAŁU OBCIĄŻENIA POMIĘDZY ZAWORAMI

Streszczenie

W artykule przedstawiono model niezawodności armatury ochronnej zbiorników ciśnieniowych. Opracowany model przeznaczony jest do analizy ilościowej przyczyn awarii systemów takiego typu. Niezawodność systemu jest sformalizowana przez dynamiczne drzewa niesprawności, w których zjawiska podziału obciążenia zostały opisane matematycznie. Podział jednorodnego modelu Markowa otrzymywano za pomocą dynamicznego drzewa niesprawności. Otrzymane charakterystyki niezawodności obliczano na podstawie tak przyjętego modelu Markowa. Niezawodność elementów ochronnych armatury odpowiada rozkładowi Weibulla z uwzględnieniem podziału tensora tego modelu Markowa. Rezultatem wykonanych symulacji jest rodzina krzywych prawdopodobieństwa, uzyskana dla różnych wartości współczynnika podziału obciążenia. W artykule pokazano także jak zmienia się główna przyczyna awarii analizowanego systemu wraz z przebiegiem wartości tego współczynnika.

Słowa kluczowe: zbiorniki ciśnieniowe, zawory bezpieczeństwa, wyposażenie ochronne, model niezawodności, dynamiczne drzewa niesprawności, model Markowa, przyczyna awarii.

## 1. INTRODUCTION

Pressure vessels are hermetically sealed containers designed for physical and chemical processes, as well as for storage and transportation of substances under excessive pressure. These include autoclaves, compressors, steam and hot water boilers, gas containers, cylinders, pipelines for gas and hot water transport. Pressure vessels are taken to high-risk items. Pressure vessel destruction and, consequently, injury attendants and environment pollution can be caused by pressure increasing above the permissible level. Protective fittings are used for excess pressure preventing in the vessel. Protective fittings failure caused by boiler-scale, corrosion, sticking valves to saddles, leverage jamming can leads to the described above consequences. An important step in the design of protective fittings for pressure vessels is ensuring an acceptable level of reliability. It needs not only to determine the integral reliability index but also to analyze all failure causes for protective fittings and to develop recommendations for reliability improves.

## 2. THE PROBLEM FORMULATING

The purpose of the research is to develop mathematical description which takes into account load-sharing between the safety valves and load changing of the limiting pressure valve in the protective fittings reliability model as well as quantitative consideration of these phenomena in the system reliability characteristics.

## 3. LITERATURE REVIEW

For mathematical reliability model constructing of pressure vessels and their components and subsystems such approaches are distinguished. In the papers [1-3] mathematical models of physical processes such as crack corrosion propagation, wear, fatigue and more are used. The disadvantage of this approach is that even for simple systems derived model is sophisticated. In addition, the model parameters are known for researchers with some approximation that eliminates usage of precise models for physical processes. In the paper [4, 5] dynamic fault tree that combine logical and probabilistic approach and Bayesian network are used. The disadvantage of this approach is that whole range of phenomena connected with load-sharing processes cannot be adequately taken into account. In the papers [6, 7] reliability models based on Monte Carlo simulation are used. The results obtained by this method are distorted by fluctuations caused by random number generator using. This disadvantage is critical for high reliability systems, because investigated reliability characteristics are comparable with amplitude fluctuations. In the papers [8-10] Markov reliability models based on state space analysis of system are used. The main disadvantage of these models concerned with exponential distribution limit and high complexity of their construction, which increases in combinatorial order regarding component number. However, this approach in combination with dynamic fault tree is the most appropriate for solving the problem. Exponential distribution limit is avoided by state space splitting [11-13] that by fictitious state introducing provides arbitrary distribution using and component load-sharing history "remembering".

In the paper such goals are obtained:

- the reliability of protective fittings based on dynamic fault tree is mathematically described;
- the state and event model of system and split homogeneous Markov model are developed;
- the quantitative characteristics for all failure causes of protective fittings are determined.

## 4. DESCRIPTION OF THE APPROACH AND ACHIEVED RESULTS OF OWN RESEARCHES

### 4.1. Description of the system, dynamic fault tree

As required technology the working medium is given by a pipeline B to a pressure vessel A (fig. 1a). In the vessel the working medium is boiled by a heater C and is transported to a pipeline D under excess pressure. To avoid pressure increasing above acceptable level the protective fittings such as three-way valve 1, two safety valves 2 and 3 and limiting pressure valve 4 are installed. If pressure in the vessel exceeds the operating value, then the safety valves 2 and 3 are triggered, and the working medium is given to pipeline E, which is connected to the atmosphere.

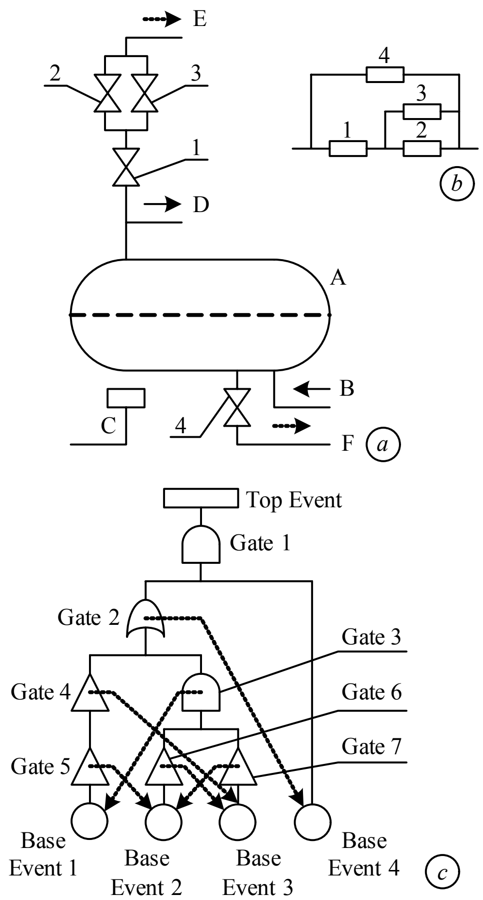



Fig. 1. Functional diagram (a), reliability block diagram (b) and dynamic fault tree (c) for protective fittings

If the pressure continues to rise further and exceeds the emergency level, the limiting pressure valve 4 is triggered, and the working medium is given by a pipeline F to a special tank. Duplicate safety valves 2 and 3 functions by loading redundancy algorithm, i.e. if both valves are operational, then the load are distributed between them in equal parts. If one of the safety valves is non-operational, then the load of other valve is doubled. The limiting pressure valve 4 functions by reduced load redundancy algorithm, i.e. if the three-way valve and at least one safety valve are operational, then this valve is under reduce load. If the three-way valve or both safety valves are non-operational, then the load of the limiting pressure valve s nominal. It is assumed that a diagnostics devices and switches are ideal as well as load-sharing processes are instantaneous. In reliability terms the logical block diagram of the protective fittings is formed the series-parallel combination of components, as shown in fig. 1*b*. Protective fittings reliability is formalized by dynamic fault tree (fig.1*c*). Dynamic fault tree is a mathematical model that describes the condition of non-operational state appearance of system as well as the conditions of load-sharing between its components based on logical and relation blocks. Non-operational state of protective fittings is given by "Top Event" block. It is assumed that such state is catastrophic, i.e. if system is non-operational, then any repair is disabled. But if system is operational, then repairing of any non-operational component can be done as many times as this is required. It is assumed that repaired component as good as new and other operational components have previous operating time. This event occurs when both pipelines F and E are blocked simultaneously that describes by "Gate 1" block. The type of this block is given by the logical operation AND. The pipeline E is blocked if the three-way valve 1 or the safety valves 2 and 3 group is non-operational. It is described by "Gate 2" block which type is given by the logical operation OR. The three-way valve non-operational state is described by "Base Event 1" block and its life is distributed by Weibull with $\alpha_1$ and $\beta_1$ parameters. Safety valves group non-operational state is occurred when both safety valves are non-operational. It is described by "Gate 3" block which type is given by the logical operation AND. The safety valves non-operational states are described by "Base Event 2" and "Base Event 3" blocks and their lives are distributed by Weibull with $\alpha_2$, $\beta_2$ and $\alpha_3$, $\beta_3$ parameters. The pipeline F is blocked if the limiting pressure valve 4 is non-operational. Such component non-operational state is described by "Base Event 4" block and its life is distributed by Weibull with $\alpha_4$ and $\beta_4$ parameters. Repair duration of all system components is distributed exponentially with $\mu$ parameter. In the protective fittings the following dynamic phenomena are occurred:

- load change of the limiting pressure valve 4 depending on the state of the pipeline E components,
- load change of the three-way valve 1 depending on the state of safety valves 2 and 3,
- load change of the safety valves 2 and 3 depending on the state of the three-way valve 1,
- mutual load change of safety valves 2 and 3 depending on their states.

The first phenomenon of load change is described by logic condition in the "Gate 2" block. If the logic signal FALSE is supplied to the block input, i.e. components 1–3 provide the pipeline E functioning, then operating intensity for the limiting pressure valve 4 is equal $k_4$ that regarding to reduce load mode.

The second phenomenon of load change is described by logic condition in the "Gate 3" block. If the logic signal TRUE is supplied to the block input, i.e. safety valves 2 and 3 are non-operational then operating intensity for the three-way valve 1 is equal 0.

For third phenomenon of load change "Gate 4" and "Gate 5" blocks are added to the dynamic fault tree structure. They are the logical signal repeaters and the logic condition of load change containers. If the logic signal TRUE is supplied to both block inputs, i.e. the three-way valve 1 is non-operational then operating intensities for both safety valves 2 and 3 are equal 0.

For fourth phenomenon of load change, which loading redundancy algorithm implements, "Gate 6" and "Gate 7" blocks are added to the dynamic fault tree structure. They are the logical signal repeaters and the logic condition of load change containers. If the logic signal TRUE is supplied to the block input, i.e. the safety valve 2 is non-operational then operating intensities for the safety valve 3 is equal $k_3$. Accordingly, if the logic signal TRUE is supplied to the "Gate 7" block input, i.e. the safety valve 3 is non-operational then operating intensities for the safety valve 2 is equal $k_2$.

### 4.2. The state and event model

Based on the above dynamic fault tree for the protective fittings according to the formalized rules [13] the state and event model is developed. This model is a mathematical description of states in which the system may be, and events that can occur in the system. The diagram of the model is shown in fig. 2 and its parameters are given in the table.

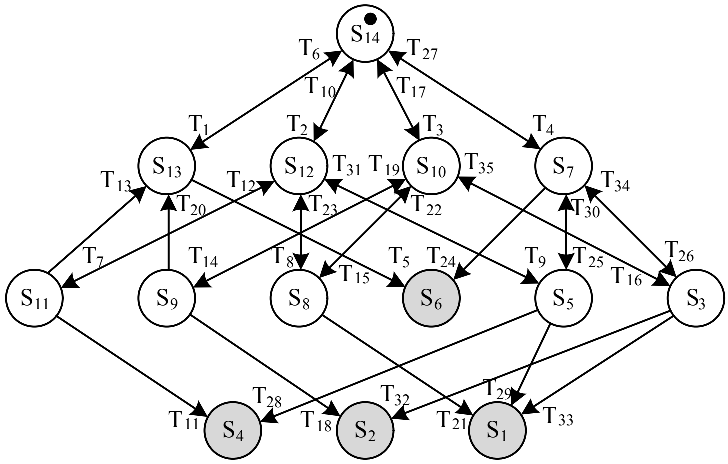
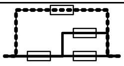
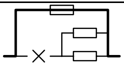
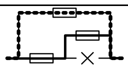
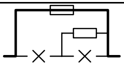
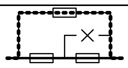
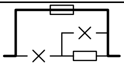
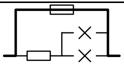
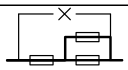
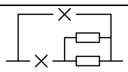
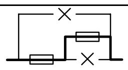
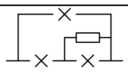
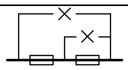
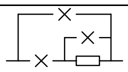
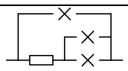



Fig. 2. State and transition diagram for state and event model of protective fittings

Table 1 Parameters of state and event model for protective fittings

| No. | State description | | | | | | | | | | | Event description | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Source state | State load flow diagram | Operational intensity multiplier | | | | | | | | $y$ | Event name | Finished process | Destination state |
| | | | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | | | | |
| 1. | $S_{14}$ |  | 1 | 1 | 1 | $k_4$ | 0 | 0 | 0 | 0 | 1 | $T_1$ | $P_1$ | $S_{13}$ |
| 2. | | | | | | | | | | | | $T_2$ | $P_2$ | $S_{12}$ |
| 3. | | | | | | | | | | | | $T_3$ | $P_3$ | $S_{10}$ |
| 4. | | | | | | | | | | | | $T_4$ | $P_4$ | $S_7$ |
| 5. | $S_{13}$ |  | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 1 | $T_5$ | $P_4$ | $S_6$ |
| 6. | | | | | | | | | | | | $T_6$ | $P_5$ | $S_{14}$ |
| 7. | $S_{12}$ |  | 1 | 0 | $k_3$ | $k_4$ | 0 | 1 | 0 | 0 | 1 | $T_7$ | $P_1$ | $S_{11}$ |
| 8. | | | | | | | | | | | | $T_8$ | $P_3$ | $S_8$ |
| 9. | | | | | | | | | | | | $T_9$ | $P_4$ | $S_5$ |
| 10. | | | | | | | | | | | | $T_{10}$ | $P_6$ | $S_{14}$ |
| 11. | $S_{11}$ |  | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | $T_{11}$ | $P_4$ | $S_4$ |
| 12. | | | | | | | | | | | | $T_{12}$ | $P_5$ | $S_{12}$ |
| 13. | | | | | | | | | | | | $T_{13}$ | $P_6$ | $S_{13}$ |
| 14. | $S_{10}$ |  | 1 | $k_2$ | 0 | $k_4$ | 0 | 0 | 1 | 0 | 1 | $T_{14}$ | $P_1$ | $S_9$ |
| 15. | | | | | | | | | | | | $T_{15}$ | $P_2$ | $S_8$ |
| 16. | | | | | | | | | | | | $T_{16}$ | $P_4$ | $S_3$ |
| 17. | | | | | | | | | | | | $T_{17}$ | $P_7$ | $S_{14}$ |
| 18. | $S_9$ |  | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 1 | $T_{18}$ | $P_4$ | $S_2$ |
| 19. | | | | | | | | | | | | $T_{19}$ | $P_5$ | $S_{10}$ |
| 20. | | | | | | | | | | | | $T_{20}$ | $P_7$ | $S_{13}$ |
| 21. | $S_8$ |  | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 1 | $T_{21}$ | $P_4$ | $S_1$ |
| 22. | | | | | | | | | | | | $T_{22}$ | $P_6$ | $S_{10}$ |
| 23. | | | | | | | | | | | | $T_{23}$ | $P_7$ | $S_{12}$ |
| 24. | $S_7$ |  | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | $T_{24}$ | $P_1$ | $S_6$ |
| 25. | | | | | | | | | | | | $T_{25}$ | $P_2$ | $S_5$ |
| 26. | | | | | | | | | | | | $T_{26}$ | $P_3$ | $S_3$ |
| 27. | | | | | | | | | | | | $T_{27}$ | $P_8$ | $S_{14}$ |
| 28. | $S_6$ |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | — | — | — |
| 29. | $S_5$ |  | 1 | 0 | $k_3$ | 0 | 0 | 1 | 0 | 1 | 1 | $T_{28}$ | $P_1$ | $S_4$ |
| 30. | | | | | | | | | | | | $T_{29}$ | $P_3$ | $S_1$ |
| 31. | | | | | | | | | | | | $T_{30}$ | $P_6$ | $S_7$ |
| 32. | | | | | | | | | | | | $T_{31}$ | $P_8$ | $S_{12}$ |
| 33. | $S_4$ |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | — | — | — |
| 34. | $S_3$ |  | 1 | $k_2$ | 0 | 0 | 0 | 0 | 1 | 1 | 1 | $T_{32}$ | $P_1$ | $S_2$ |
| 35. | | | | | | | | | | | | $T_{33}$ | $P_2$ | $S_1$ |
| 36. | | | | | | | | | | | | $T_{34}$ | $P_7$ | $S_7$ |
| 37. | | | | | | | | | | | | $T_{35}$ | $P_8$ | $S_{10}$ |
| 38. | $S_2$ |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | — | — | — |
| 39. | $S_1$ |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | — | — | — |

In the state and event model functioning and repairing for the three-way valve 1 is marked as $P_1$ and $P_5$, for the safety valve 2 – $P_2$ and $P_6$, for the safety valve 3 – $P_3$ and $P_7$, and for the limiting pressure valve 4 – $P_4$ and $P_8$. The system can be in fourteen states, four of which correspond to non-operational states $S_1$, $S_2$, $S_4$ and $S_6$. The thirty five events can be occurred in the system, nine of which cause catastrophic failure $T_5$, $T_{11}$, $T_{18}$, $T_{21}$, $T_{24}$, $T_{28}$, $T_{29}$, $T_{32}$ and $T_{33}$. The state parameters are operational intensity multiplier value for $P_1$–$P_8$ processes and logical function $y$, which takes the value “1” if the system is operational and value “0” otherwise. The event parameters are source state name, finished process name, and destination state name.

### 4.3. Markov model

Based on the state and event model for the protective fittings according to formalized rules [13] split homogeneous Markov model is developed. This model is given by a system of Kolmogorov-Chapman differential equations:

$$\frac{d}{dt}\mathbf{p}(t) = \mathbf{A}\,\mathbf{p}(t), \qquad \mathbf{y}(t) = \mathbf{C}\,\mathbf{p}(t). \tag{1}$$

where $t$ – time; $\mathbf{p}(t)$ – vector which contains phase probability functions; $\mathbf{y}(t)$ – vector which contains system probability characteristics functions.

The Markov model is a set of matrices which define the transition intensity between phases A, the initial phase probability $\mathbf{p}(0)$, and relation of phase probabilities with system reliability characteristics C. For the system the Markov model is

$$\mathbf{A} = \left[\begin{array}{cccccccccccccc}
 & & \mathbf{A}_{T_{33}} & & \mathbf{A}_{T_{29}} & & & \mathbf{A}_{T_{21}} & & & & & & \\
 & & \mathbf{A}_{T_{32}} & & & & & & \mathbf{A}_{T_{18}} & & & & & \\
 & & \mathbf{A}_{S_{3}} & & & & \mathbf{A}_{T_{26}} & & & \mathbf{A}_{T_{16}} & & & & \\
 & & & & \mathbf{A}_{T_{28}} & & & & & & \mathbf{A}_{T_{11}} & & & \\
 & & & & \mathbf{A}_{S_{5}} & & \mathbf{A}_{T_{25}} & & & & & \mathbf{A}_{T_{9}} & & \\
 & & & & & & \mathbf{A}_{T_{24}} & & & & & & \mathbf{A}_{T_{5}} & \\
 & & \mathbf{A}_{T_{34}} & & \mathbf{A}_{T_{30}} & & \mathbf{A}_{S_{7}} & & & & & & & \mathbf{A}_{T_{4}} \\
 & & & & & & & \mathbf{A}_{S_{8}} & & \mathbf{A}_{T_{15}} & & \mathbf{A}_{T_{8}} & & \\
 & & & & & & & & \mathbf{A}_{S_{9}} & \mathbf{A}_{T_{14}} & & & & \\
 & & \mathbf{A}_{T_{35}} & & & & & \mathbf{A}_{T_{22}} & \mathbf{A}_{T_{19}} & \mathbf{A}_{S_{10}} & & & & \mathbf{A}_{T_{3}} \\
 & & & & & & & & & & \mathbf{A}_{S_{11}} & \mathbf{A}_{T_{7}} & & \\
 & & & & \mathbf{A}_{T_{31}} & & & \mathbf{A}_{T_{23}} & & & \mathbf{A}_{T_{12}} & \mathbf{A}_{S_{12}} & & \mathbf{A}_{T_{2}} \\
 & & & & & & & & \mathbf{A}_{T_{20}} & & \mathbf{A}_{T_{13}} & & \mathbf{A}_{S_{13}} & \mathbf{A}_{T_{1}} \\
 & & & & & & \mathbf{A}_{T_{27}} & & & \mathbf{A}_{T_{17}} & & \mathbf{A}_{T_{10}} & \mathbf{A}_{T_{6}} & \mathbf{A}_{S_{14}}
\end{array}\right], \mathbf{p}(0) = \left[\begin{array}{c}
\mathbf{p}_{S_{14}} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\
\end{array}\right] \tag{1}$$

$$\mathbf{C} = \left[\begin{array}{cccccccccccccc}
\mathbf{I} & & & & & & & & & & & & & \\
 & \mathbf{I} & & \mathbf{I} & & \mathbf{I} & & & & & & & &
\end{array}\right].$$

Markov model components are formed based on auxiliary Markov models for $P_1$–$P_8$ processes. Parameters for Markov model processes are determined accordingly to the criterion of equality the first and the second centered moments of the actual distribution process and its auxiliary Markov model. Is assumed that for the process $P_1\{\alpha_1, \beta_1\}$ auxiliary Markov model parameters equal $\{\mathbf{A}_1, \mathbf{p}_1(0), \mathbf{C}_1\}$, for $P_2\{\alpha_2, \beta_2\}$ — $\{\mathbf{A}_2, \mathbf{p}_2(0), \mathbf{C}_2\}$, for $P_3\{\alpha_3, \beta_3\}$ — $\{\mathbf{A}_3, \mathbf{p}_3(0), \mathbf{C}_3\}$, for $P_4\{\alpha_4, \beta_4\}$ — $\{\mathbf{A}_4, \mathbf{p}_4(0), \mathbf{C}_4\}$, for $P_5\{\mu\}$ — $\{\mathbf{A}_5, \mathbf{p}_5(0), \mathbf{C}_5\}$, for $P_6\{\mu\}$ — $\{\mathbf{A}_6, \mathbf{p}_6(0), \mathbf{C}_6\}$, for $P_7\{\mu\}$ — $\{\mathbf{A}_7, \mathbf{p}_7(0), \mathbf{C}_7\}$ and for $P_8\{\mu\}$ — $\{\mathbf{A}_8, \mathbf{p}_8(0), \mathbf{C}_8\}$. According to these parameters the Markov model components of the system are calculated by using the following formulas, in particular, for operating state $S_{14}$:

$$\begin{aligned}
&\mathbf{A}_{S_{14}} = \\
&= \mathbf{A}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\
&+ \mathbf{E}_1 \otimes \mathbf{A}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\
&+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{A}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\
&+ k_4\, \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{A}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8, \\
&\mathbf{p}_{S_{11}}(0) = \mathbf{p}_1(0) \otimes \mathbf{p}_2(0) \otimes \mathbf{p}_3(0) \otimes \mathbf{p}_4(0) \otimes \\
&\otimes \mathbf{p}_5(0) \otimes \mathbf{p}_6(0) \otimes \mathbf{p}_7(0) \otimes \mathbf{p}_8(0),
\end{aligned} \tag{2}$$

where $\otimes$ – tensor multiplication operator; $E_1$–$E_8$ – the identity matrix which dimension is equal to $A_1$–$A_8$ matrices dimension.

For operational state $S_{13}$:

$$\begin{aligned}
&\mathbf{A}_{S_{13}} = \\
&= \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{A}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\
&+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{A}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8.
\end{aligned} \tag{3}$$

For operational state $S_{12}$:

$$\begin{aligned}
&\mathbf{A}_{S_{12}} = \\
&= \mathbf{A}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\
&+ k_3\, \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{A}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\
&+ k_4\, \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{A}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\
&+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{A}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8.
\end{aligned} \tag{4}$$

For operational state $S_{11}$:

$$\begin{aligned}
&\mathbf{A}_{S_{11}} = \\
&= \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{A}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\
&+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{A}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\
&+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{A}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8.
\end{aligned} \tag{5}$$

For operational state $S_{10}$:

$$\begin{aligned}
&\mathbf{A}_{S_{10}} = \\
&= \mathbf{A}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\
&+ k_2\, \mathbf{E}_1 \otimes \mathbf{A}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\
&+ k_4\, \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{A}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\
&+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{A}_7 \otimes \mathbf{E}_8.
\end{aligned} \tag{6}$$

For operational state $S_9$:

$$\begin{aligned}&\mathbf{A}_{S_9} = \\ &= \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{A}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\ &+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{A}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\ &+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{A}_7 \otimes \mathbf{E}_8.\end{aligned} \quad (7)$$

For operational state $S_8$:

$$\begin{aligned}&\mathbf{A}_{S_8} = \\ &= \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{A}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\ &+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{A}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\ &+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{A}_7 \otimes \mathbf{E}_8.\end{aligned} \quad (8)$$

For operational state $S_7$:

$$\begin{aligned}&\mathbf{A}_{S_7} = \\ &= \mathbf{A}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\ &+ \mathbf{E}_1 \otimes \mathbf{A}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\ &+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{A}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\ &+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{A}_8.\end{aligned} \quad (9)$$

For operational state $S_5$:

$$\begin{aligned}&\mathbf{A}_{S_5} = \\ &= \mathbf{A}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\ &+ k_3\, \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{A}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\ &+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{A}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\ &+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{A}_8.\end{aligned} \quad (10)$$

For operational state $S_3$:

$$\begin{aligned}&\mathbf{A}_{S_3} = \\ &= \mathbf{A}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\ &+ k_2\, \mathbf{E}_1 \otimes \mathbf{A}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8 + \\ &+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{A}_7 \otimes \mathbf{E}_8 + \\ &+ \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{A}_8.\end{aligned} \quad (11)$$

For event $T_1$, $T_7$, $T_{14}$, $T_{24}$, $T_{28}$ та $T_{32}$ caused by $P_1$ process completion:

$$\begin{aligned}&\mathbf{A}_{T_1} = \mathbf{A}_{T_7} = \mathbf{A}_{T_{14}} = \mathbf{A}_{T_{24}} = \mathbf{A}_{T_{28}} = \mathbf{A}_{T_{32}} = \\ &= \mathbf{p}_1\mathbf{C}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8.\end{aligned} \quad (12)$$

For event $T_2$, $T_{15}$, $T_{25}$ та $T_{33}$ caused by $P_2$ process completion:

$$\begin{aligned}&\mathbf{A}_{T_2} = \mathbf{A}_{T_{25}} = \\ &= \mathbf{E}_1 \otimes \mathbf{p}_2\mathbf{C}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8, \quad (13) \\ &\mathbf{A}_{T_{15}} = \mathbf{A}_{T_{33}} = k_2\, \mathbf{A}_{T_2}.\end{aligned}$$

For event $T_3$, $T_8$, $T_{26}$ та $T_{29}$ caused by $P_3$ process completion:

$$\begin{aligned}&\mathbf{A}_{T_3} = \mathbf{A}_{T_{26}} = \\ &= \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{p}_3\mathbf{C}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8, \quad (14) \\ &\mathbf{A}_{T_8} = \mathbf{A}_{T_{29}} = k_3\, \mathbf{A}_{T_3}.\end{aligned}$$

For event $T_4$, $T_5$, $T_9$, $T_{11}$, $T_{16}$, $T_{18}$ та $T_{21}$ caused by $P_4$ process completion:

$$\begin{aligned}&\mathbf{A}_{T_5} = \mathbf{A}_{T_{11}} = \mathbf{A}_{T_{18}} = \mathbf{A}_{T_{21}} = \\ &= \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{p}_4\mathbf{C}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8, \\ &\mathbf{A}_{T_4} = \mathbf{A}_{T_9} = \mathbf{A}_{T_{16}} = k_4\, \mathbf{A}_{T_5}.\end{aligned} \quad (15)$$

For event $T_6$, $T_{12}$ та $T_{19}$ caused by $P_5$ process completion:

$$\begin{aligned}&\mathbf{A}_{T_6} = \mathbf{A}_{T_{11}} = \mathbf{A}_{T_{18}} = \\ &= \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{p}_5\mathbf{C}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8.\end{aligned} \quad (16)$$

For event $T_{10}$, $T_{13}$, $T_{22}$ та $T_{30}$ caused by $P_6$ process completion:

$$\begin{aligned}&\mathbf{A}_{T_{10}} = \mathbf{A}_{T_{13}} = \mathbf{A}_{T_{22}} = \mathbf{A}_{T_{30}} = \\ &= \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{p}_6\mathbf{C}_6 \otimes \mathbf{E}_7 \otimes \mathbf{E}_8.\end{aligned} \quad (17)$$

For event $T_{17}$, $T_{20}$, $T_{23}$ та $T_{34}$ caused by $P_7$ process completion:

$$\begin{aligned}&\mathbf{A}_{T_{17}} = \mathbf{A}_{T_{20}} = \mathbf{A}_{T_{23}} = \mathbf{A}_{T_{34}} = \\ &= \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{p}_7\mathbf{C}_7 \otimes \mathbf{E}_8.\end{aligned} \quad (18)$$

For event $T_{27}$, $T_{31}$ та $T_{35}$ caused by $P_8$ process completion:

$$\begin{aligned}&\mathbf{A}_{T_{27}} = \mathbf{A}_{T_{31}} = \mathbf{A}_{T_{35}} = \\ &= \mathbf{E}_1 \otimes \mathbf{E}_2 \otimes \mathbf{E}_3 \otimes \mathbf{E}_4 \otimes \mathbf{E}_5 \otimes \mathbf{E}_6 \otimes \mathbf{E}_7 \otimes \mathbf{p}_8\mathbf{C}_8.\end{aligned} \quad (19)$$

The identity vector **I** in **C** matrix has dimension which equal to the product of each $\mathbf{A}_1$–$\mathbf{A}_8$ matrix dimensions. Matrix **C** is constructed so that its lines corresponding to probability characteristics. The first line is set protective fittings failure probability due to non-operational states of both safety valves and the limiting pressure valve, which corresponds to $S_1$ non-operational state probability. The second line is set failure probability due to non-operational states of the three-way valve and the limiting pressure valve, which corresponds to the sum of $S_2$, $S_4$ and $S_6$ non-operational state probabilities. The model contains 224 differential equations.

### 4.4. The probability characteristics

The parameter values for protective fittings components are taken following $\alpha_1 = 3.0 \cdot 10^5$ h, $\beta_1 = 1.2$; $\alpha_2 = \alpha_3 = 1.5 \cdot 10^4$ h, $\beta_2 = \beta_3 = 1.3$; $\alpha_4 = 1.5 \cdot 10^5$ h, $\beta_4 = 1.1$, and repair intensity is $\mu = 0.01$ 1/h. The parameters of auxiliary Markov models according to [11] are taken the following values:

$$\mathbf{A}_1 = \begin{bmatrix} -\lambda_1 & \lambda_1 \\ 0 & -\lambda_1 \end{bmatrix}, \quad \mathbf{p}_1(0) = \begin{bmatrix} 0.36858 \\ 0.63142 \end{bmatrix}, \quad \mathbf{C}_1 = \begin{bmatrix} \lambda_1 & 0 \end{bmatrix}. \quad (20)$$

$$\mathbf{A}_2 = \mathbf{A}_3 = \begin{bmatrix} -\lambda_2 & \lambda_2 \\ 0 & -\lambda_2 \end{bmatrix}, \quad \mathbf{p}_2(0) = \mathbf{p}_3(0) = \begin{bmatrix} 0.19416 \\ 0.80584 \end{bmatrix}, \quad \mathbf{C}_2 = \mathbf{C}_3 = \begin{bmatrix} \lambda_2 & 0 \end{bmatrix}. \quad (21)$$

$$\mathbf{A}_4 = \begin{bmatrix} -\lambda_4 & \lambda_4 \\ 0 & -\lambda_4 \end{bmatrix}, \quad \mathbf{p}_4(0) = \begin{bmatrix} 0.58590 \\ 0.41410 \end{bmatrix}, \quad \mathbf{C}_4 = \begin{bmatrix} \lambda_4 & 0 \end{bmatrix}. \quad (22)$$

$\mathbf{A}_5 = \mathbf{A}_6 = \mathbf{A}_7 = \mathbf{A}_8 = [-\mu]$, $\mathbf{p}_5(0) = \mathbf{p}_6(0) = \mathbf{p}_7(0) = \mathbf{p}_8(0) = [1]$, $\mathbf{C}_5 = \mathbf{C}_6 = \mathbf{C}_7 = \mathbf{C}_8 = [\mu]$.

where $\lambda_1$ = 5.7811 $10^{-6}$ 1/h, $\lambda_2$ = 1,3035 $10^{-4}$ 1/h, $\lambda_4$ = –9,7702 $10^{-6}$ 1/h.

Using split homogeneous Markov model probability characteristics of the protective fittings are calculated. The calculation is performed by Rosenbrock method that caused Markov model stiffness. Such property is caused by parameters scattering for functioning and repairing, state space splitting algorithm features as well as coefficients $k_2$, $k_3$ and $k_4$ load-sharing impact. Fig. 3 presents a family of probability curves for the protective fittings regarding to coefficient $k_4$.

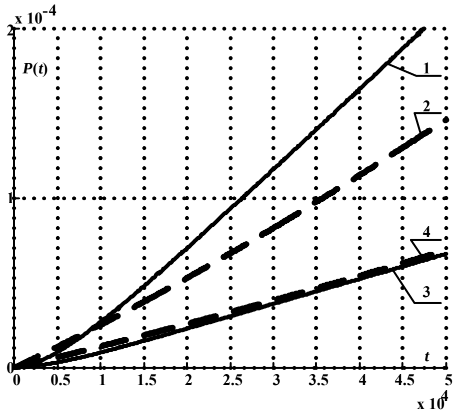



Fig. 3. Probability characteristic curves for protective fittings regarding coefficient $k_4$

The coefficient $k_4$ indicates how wearing intensity for the limiting pressure valve decrease in reduced load mode relative to the nominal mode. This coefficient can takes values in the range of 0 to 1. Curves 1 and 2 correspond to $k_4 = 1$, and curves 3 and 4 – $k_4 = 0$, provided that $k_2 = k_3 = 5$ for both cases. Solid curves 1 and 3 correspond to system failure probability caused by the safety valves and the limiting pressure valve non-operational states. Dashed curves 2 and 4 correspond to system failure probability caused by the three-way valve and the limiting pressure valve non-operational states.

Fig. 4 presents a family of probability curves for the protective fittings regarding to coefficients $k_2$ and $k_3$. Since the safety valves are the identical, it is assumed $k_2 = k_3$. The coefficients $k_2$ and $k_3$ indicate how wearing intensity for the safety valves increase in overload mode relative to the nominal mode. These coefficients can take values in the range of 1 to ∞. Solid curve family corresponds to system failure probability caused by the safety valves and the limiting pressure valve non-operational states, especially, curve 1 correspond to $k_2 = 1$, curve 2 – $k_2 = 2$, curve 3 – $k_2 = 5$, curve 4 – $k_2 = 10$, curve 5 – $k_2 = 20$ and curve 6 – $k_2 = 50$, provided that $k_4 = 0.2$ for all cases. Dashed curve 7 corresponds to system failure probability caused by the by the three-way valve and the limiting pressure valve non-operational states for all values of $k_2$ and $k_3$, provided that $k_4 = 0.2$.

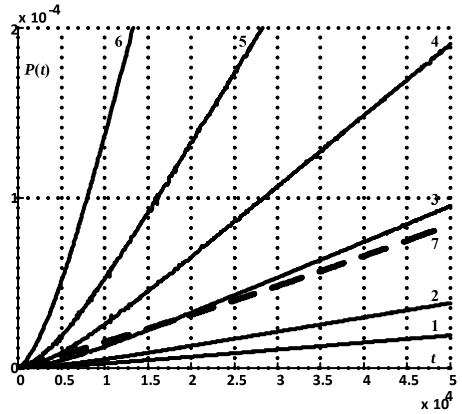



Fig. 4. Probability characteristic curves for protective fittings regarding coefficients $k_2$ and $k_3$

### 4.5. Discussion

Computational experiment results make it possible to investigate the influence of $k_2$, $k_3$ and $k_4$ coefficients on probabilistic characteristics of protective fittings. It is shown on fig. 3 that failure cause probabilistic characteristics increase linearly with $k_4$ increasing for $k_2$ and $k_3$ constant values. Step of increasing for probabilistic characteristics which corresponding failure of the safety valves and the limit pressure valve is greater than step of increasing probabilistic characteristics which meeting failure of the three-way valve and the limit pressure valve. It can be concluded that probabilistic characteristics, which corresponding the safety valves and the limiting pressure valve failure cause, increase with logarithmic step with $k_2$ and $k_3$ increasing for $k_4$ constant value by analyzing probabilistic characteristic family (fig. 4). But probabilistic characteristics which meet the three-way valve and the limiting pressure valve failure cause are insensitive to changes of these coefficients. It explains why for $k_4 = 0$ or $k_2 = k_3 = 1 \ldots 2$ the dominant protective fitting failure cause is the three-way valve and the limiting pressure valve failure and for other values the safety valves and the limit pressure valve failure cause is dominant. For curve family for $k_2 = k_3 > 10$ values the Markov model stiffness increases so that the numerical method results fluctuation on probabilistic characteristics. Also, the reliability model does not consider the three-way valve wearing in case of both safety valves failure. This phenomenon will be the basis for further research.

## 5. CONCLUSION

In this paper the mathematical model for pressure vessel protective fittings is developed. The model for failure cause quantitative analysis is intended. System reliability is mathematically described based on dynamic fault tree, which specified load-sharing logical conditions for the safety valves and the limiting pressure valve. System probabilistic characteristics are determined by the Markov model, which based on the tensor expressions of state space splitting. The Markov model takes into account load-sharing between protective fittings components, which life is distributed by Weibull. It is provided the prediction of the most probable cause of protective fittings failure depending on load-sharing parameters and pressure vessel exploitation duration by the model. The quantitative analysis of such system property cannot be adequately obtained either through classical fault tree using or by ordinary homogeneous Markov reliability model using. Further studies are aimed on developing of advanced reliability mathematical model for pressure vessel protective fittings which adequately taken into account three-way valve load-sharing effects.

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