IMPLEMENTATION OF ALTERNATIVE ALGORITHMS FOR DEFINING THE TRANSFORMATION PARAMETERS OF USK-2000 AND COORDINATE SYSTEMS OF GENERAL LAYOUT DURING THE MARKING OPERATIONS [1]

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The purpose of the research is designing and analyzing the algorithm of coordinates transformation from the general layout coordinate system in USK-2000 and conversely,minimizing the influence of random errors of RTN corrections and opportunity to eliminate crude errors in the coordinates of geodesic base points, by providing the optimal correlation of point coordinates in two systems with the help of iteration method and its experimental proof. To define the accuracy of the algorithm, both model and experimental works were accomplished on the reference polygon in the conditions of construction site. The suggested algorithm of calculation, that is based on the method of iteration and objective function, that minimizes the lengths of vector, that do not comply with the coordinates received from the measurements allows to transform the point coordinates of the construction area with the mm accuracy. The work focuses on using the iteration method for transformation of the points and implementation of the objective, that minimizes the vector lengths, that are not consistent with the coordinates, resulting from the measurements and calculations. Such approach solves the problem of the errors of the horizontal location of geodesic network points and errors in defining the coordinates of turning points of red lines (projected system of lines that regulates the construction process; any building should not cross the red lines) and existing constructions.

Keywords: GNSS, coordinates, RTN-measurement, planning works, total station.

1. INTRODUCTION

Providing the necessary accuracy of survey and marking engineering-geodesic works for construction is a important and time-consuming process. As the cities are developing at a fast pace, it is necessary to improve the existing and create new ways of geodesic support of constructions, that will comply with the regulatory requirements. Total stations, that operate without reflector [8] and

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GNSS receivers (Global Navigation Satellite Systems) with RTN [9, 10, 11] corrections may become the alternative to the current methods, that have a variety of drawbacks.

The coordinates of construction elements are known in the project system of general layout (construction network). Geodesic marking base, in its turn, is used for example in USK-2000. USK 2000 is the national geodetic reference system of coordinates for topographical and cartographical works on the territory of Ukraine. USK-2000 was created by fixation of ITRS system according to the scale, fixed deviation of the coordinates system beginning and system appliance to the period of the 2005 year. The reference ellipsoid of Krasovskyi was accepted as the surface of the coordinates system USK-2000. Accordingly, it is necessary to set up the parameters for conversion between the systems with account of both measurement errors and elimination of possible gross errors. Coordinates system of the construction netting (hereafter general layout) is even more convenient to use when planning the industrial objects, due to the simplified calculations for establishing the axes of both constructions and underground services.There are some cases, when construction network is used when planning the residential constructions in the modern cities.

2. CORRELATION OF THE WORK WITH SCIENTIFIC PROGRAMS

Nowadays, geodesic service of construction with the help of satellite observations, including RTN method, is to some extent standardised [1, 12, 14, 15]. However, this issue still remains unsettled during performing engineering geodesic works [10, 13]. In previous works [4], we have designed the recommendations on how to minimize the systematic errors. Implementation of RTN methods when marking the main axes [5] gives the chance to refuse from using and building the classic construction network on the new objects and allows to control the measurements without additional measurement series.

3. PURPOSE OF RESEARCH

Designing and analyzing the algorithm of transforming the coordinates from the general layout coordinate systems in USK-2000, and vice versa, minimizing the influence of random errors of RTN corrections and opportunity to eliminate gross errors in the coordinates of geodesic base points, by providing the optimal correlation of point coordinates in two systems with the help of iteration method and its experimental proof.

4. METHODOLOGY

According to the working regulatory documents [6] of construction line (main construction axis), it is recommended to stake and fix the points of geodesic network and, or red construction lines (see comments in abstract), fixed on the area, with the error of 50 – 80 mm depending on the construction type. The project of red lines may have the scale of 1:2000 [7]. At the same time, the mutual location of points, that fix the main axis (skylines), should have the error of 3 – 5 mm [6 – 7]. This leads to further reduction of the positioning of points, that fix the main axis. That's why, in the work [5], it is recommended to measure two pillars bases by RTN method, taking into account the coordinates of the red lines turning points, and/or points of the pillar geodesic network, in the way its points align with the main axis of the construction site. After this, all the construction elements have to be marked according to these base lines with the help of total station.

According to the current regulations, all construction elements should have the coordinates in the civil geodesic coordinate system of Ukraine. However, the construction used to be managed by the earlier adopted local coordinate systems or the coordinate system SK-63, which proves the necessity to provide the connection between them and USK-2000. That's why one of the conditions of creating the geodesic network on the construction [7] was the opportunity to turn the elements into different coordinate systems, as well as define the parameters of transformation between these coordinates.

It is essential to take into account that using of two coordinate systems on large industrial objects during planning works is more convenient. One of such systems is the system of general layout, in which the coordinate axes are parallel to the construction lines. Another one is the civil geodesic coordinate system USK-2000, in which the coordinates of geodesic base points are known. Red lines for the residential construction or the points of construction network on the industrial area are taken out from these points.

Fig. 1 is adopted from a famous monograph of Baran [2] and it demonstrates the benefits of using the general layout coordinate system during the marking operations. One of the benefits is that the point coordinates (in case they are parallel to the construction lines of the coordinates axis) are calculated by summing up the technological sizes of the construction and projected distances between them, that almost eliminates the calculation errors. One more important factor is that the mutual positioning and size of the buildings defined in this coordinate system theoretical (free of measurement errors). Instead, the point coordinates (especially of red lines and even the points of geodesic base) may contain gross errors. All these facts should be considered when designing an algorithm of defining the transformation elements between these systems. This is exactly what we have accomplished in this work.

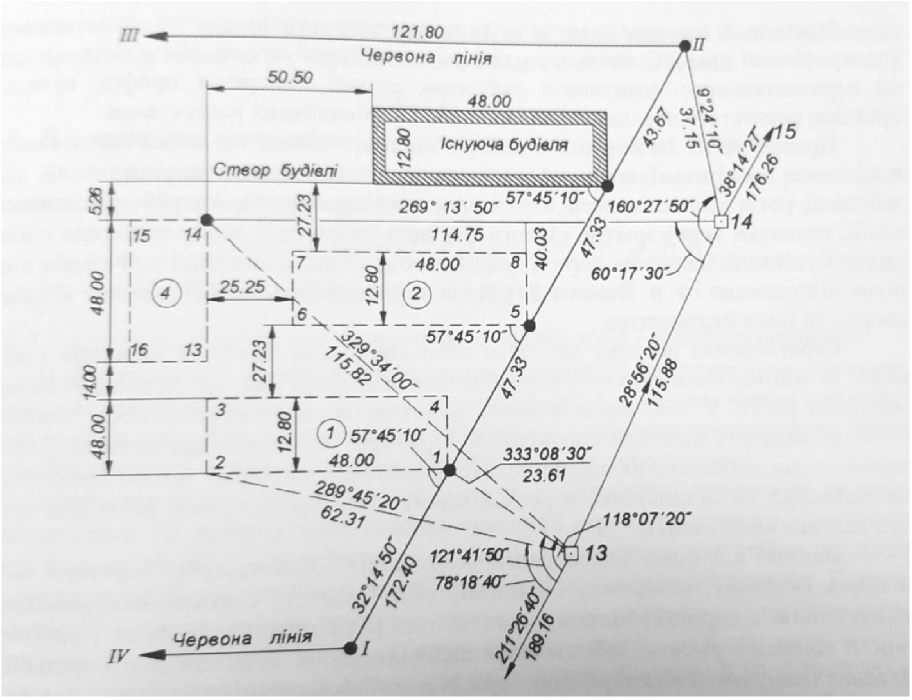

Fig 1. Layout drawing of residential area buildings: points on

To obtain the coordinates of the points not only in the system GNSS receiver works with (USK-2000), but in the project system of coordinates as well, it is necessary to take RTN measuring on the points of building contours and existing buildings that have coordinates in the general layout system and then find the parameters of conversion between the systems mentioned above.

There is a big number of methods of calculating the transformation parameters. They may be divided into theoretical and empirical. Empirical methods solve this task both by the classic least square method and method of linear programming [3], that allows to solve this task not only by minimizing the sum of error squares, but also by minimizing their maximum deviation.

However, these works don't dwell upon the problem of gross errors elimination. At the same time, transformation parameters are significantly garbled by the fact that the errors of red lines may exceed acceptable 0.05 m, when the mutual positioning of points should have the errors of 2 – 3 mm [6, 7]. For this reason we have designed and experimentally verified an algorithm, that solves this problem by providing an optimal correlation of point coordinates with the opportunity to eliminate the gross errors.

Here is the main point of the algorithm. The correlation between the general layout coordinates and USK-2000 is calculated by using the famous formulas (1). For this, it is necessary to define three parameters of transformation – angle

Q of the axes rotation of two systems and displacement of the coordinate axes of one system relatively to another X_0, Y_0.

$$\begin{cases} X_i' = X_0 + X_i \cos Q - Y_i \sin Q \\ Y_i' = Y_0 + X_i \sin Q + Y_i \cos Q \end{cases} \quad (1)$$

where $X_i X_i'$ i $Y_i Y_i'$ – coordinates of the point i both in USK-2000 and the system of general outlay coordinates.

To simplify the further calculations and use the standard MathCad procedures, let's make the system (1) linear. For this, let us take $cos\,Q$ from the first and second equation as the coordinate function and update the right sides of the obtained expressions. After the modifications, we obtain the following equation for every measured point

$$\sin Q = \frac{X_i(Y_i' - Y_0) - Y_i(X_i' - X_0)}{X_i^2 + Y_i^2} \quad (2)$$

After taking out the unknown transformation parameters from the equation (2), we receive

$$\sin Q - X_0 \frac{Y_i}{X_i^2 + Y_i^2} + X_0 \frac{X_i}{X_i^2 + Y_i^2} = \frac{X_i Y_i' - Y_i X_i'}{X_i^2 + Y_i^2} \quad (3)$$

To make the further calculation more comprehensive, let's replace the values in the formula (3) by the coefficients k, a, b, c and get

$$aX_0 + bY_0 + k = c \quad (4)$$

$$k = \sin Q\,, a = -\frac{Y_i}{X_i^2 + Y_i^2}, b = \frac{X_i}{X_i^2 + Y_i^2}, \quad c = \frac{X_i Y_i' - Y_i X_i'}{X_i^2 + Y_i^2} \quad (5)$$

The equation (4) has 3 unknown transition parameters, that's why to get a single-valued solution we have to take the measurements at least at three points and compose the following system of equations:

$$\begin{cases} a_1 X_0 + b_1 Y_0 + k_1 = c_1 \\ a_2 X_0 + b_2 Y_0 + k_2 = c_2 \\ a_3 X_0 + b_3 Y_0 + k_3 = c_3 \end{cases} \quad (6)$$

A number of points will vary on practice depending on the complexity of the object. The primary task is to define the location of turning points of red lines and existing buildings (with known coordinates in the general layout system) by GNSS receiver and then find the transition parameters between these systems and the following coordinates calculation in USK-2000. Block scheme of the designed algorithm and its implementation in MathCad environment is shown on the Fig. 2

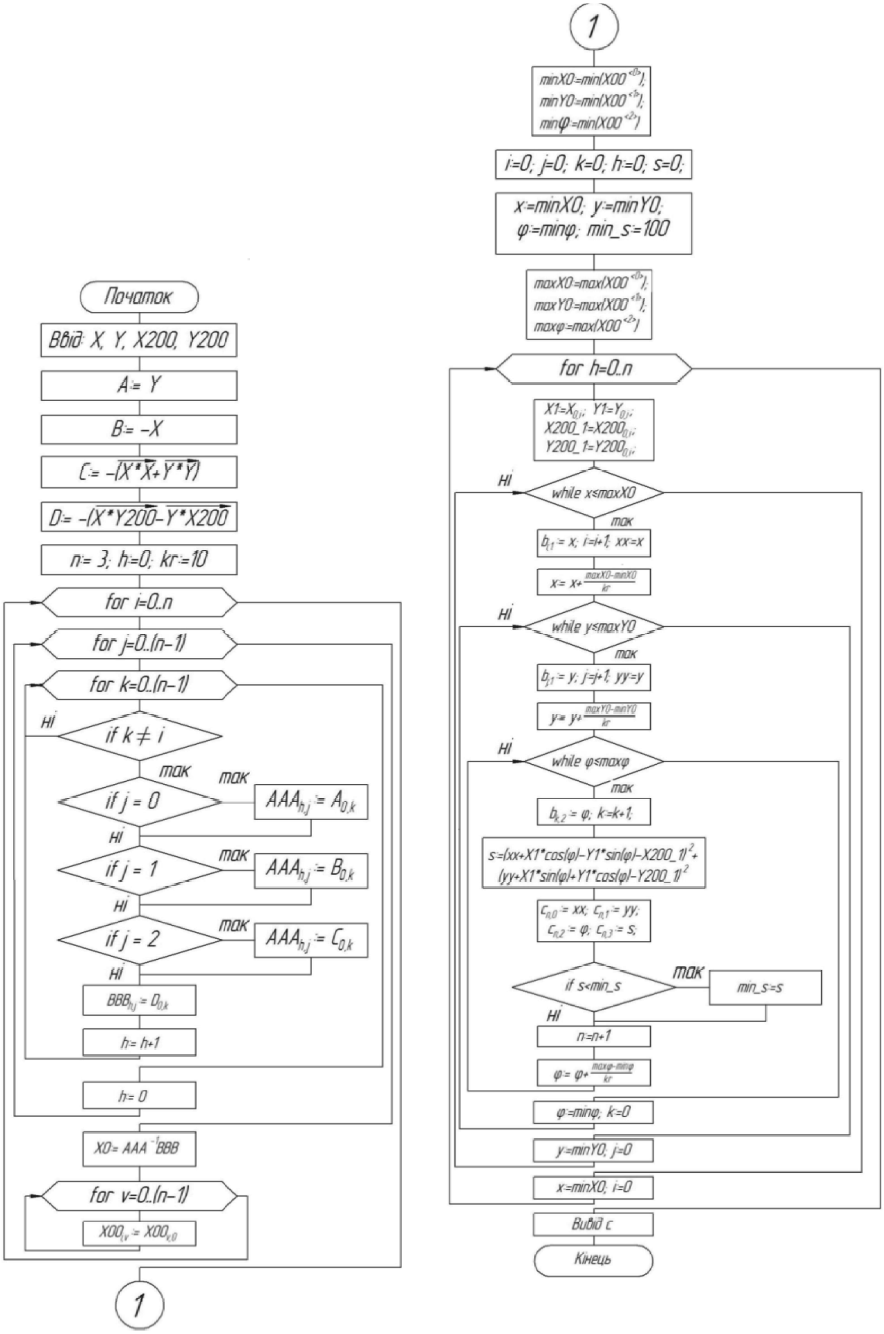

Fig 2. Block scheme of the suggested algorithm of calculating the transformation parameters

The algorithm solves the task by providing the optimal correlation of points coordinates in two systems. It consists of two subprograms: first part calculates the approximate value of transformation parameters, from which the exact calculation of values is determined in the second block with the help of objective function (7).

$$S_i^2 = (X_0 + X_i \cos Q - Y_i \sin Q - X_i^*)^2 + (Y_0 + X_i \sin Q + Y_i \cos Q - Y_i^*)^2 \rightarrow min \quad (7)$$

The used objective function, described in the work [3] has to minimize the vector lengths that are not correlated to the coordinates Si, obtained from the GNSS measurements and calculated from the transition parameters on each point.

Let's review how this algorithm functions on the real example of transformation of coordinates obtained during their verification.

The reference polygon consists of 4 points. Points with forced centering (Fig. 3) are located on the guarded territory. The territory of the construction area consists of 1 – instructive industrial center with technical equipment, 2 – derrick (52 m), 3 – electric station, 4 – boiler house (Fig. 3).

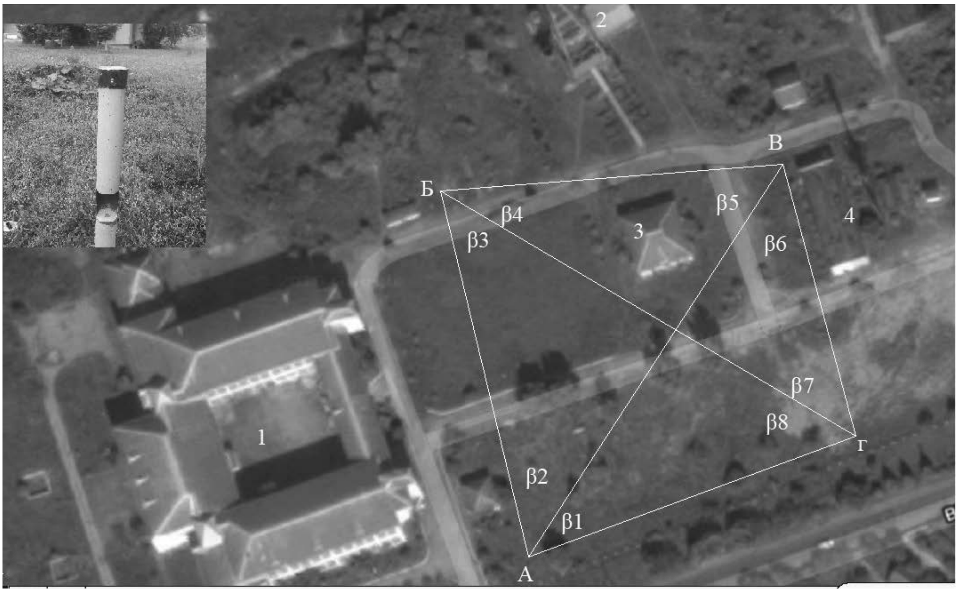

Fig. 3. Scheme of the reference polygon location

The coordinates of points of the reference zone were determined in two coordinate systems, local one and national one (USK-2000).

Coordinates of the same points in the local system were obtained with the help of three series (sets) of measurements taken by total station South NTS-350 (X_i,Y_i). Eight angles and six distances (see Fig. 3) were sequentially measured in two faces of telescope. Each of the distances was measured 12 times. Standard deviation of the reference polygonwere calculated by the Bessel formula , the errors measurement of angles was equal 0.1” – 2.1”, and of sides – 0.18 mm – 0.3 mm.

The coordinates of points of the reference zone in USK-2000 system were obtained with GNSS receiver QStar 8+. Real-time measurements (X_RTN, Y_RTN) were carried out at each of these points within a few seconds (30 averaging mode), the obtained values were used during the calculations of transformation parameters. To enable control over the obtained results, a series of static measurements (X_(st.),〖 Y〗_(st.)) was executed (4 hours at each point). The obtained data are specified in Table 1.

The coordinates from the static measurements are different from RTN measurements in the range of 3 – 9 mm.

Tab 1. Coordinates of the base stations in the local coordinate system and USK 2000

	X_i	Y_i	X'_i	Y'_i	$X_{ст.}$	$Y_{ст.}$
A	1000	1000	5424202,8624	5331275,0148	5424202,8649	5331275,0127
Б	1204.855	1000	5424305,2853	5331452,4295	5424305,2912	5331452,4226
B	1176.551	847.461	5424423,2368	5331351,6435	5424423,2410	5331351,6410
Г	1016.511	855.397	5424336,3466	5331217,0131	5424336,3496	5331217,0101

By using the compared values of measurement for finding the parameters of transformation between the systems of local coordinates of the reference polygon and USK-2000 (from RTN measurements), 4 equations with 3 unknown quantities were composed. To solve the system of equations, the number of equations has to correspond to the number of unknown quantities, that’s why we have to receive 4 different systems (C_4^3=4). By solving the system of equations (6), we receive the results, each of which comply with only 3 points, included in the system. As a result, we get 4 variants of transformation parameters, values of which slightly differ from each other. This error occurs because of the errors of RTN measurements, that were inherently propagated to the determined transformation parameters.

After obtaining 4 variants of transmission parameters (Table 2), that complied with the base stations, let’s solve the task of optimization that should eliminate the system errors.

Tab 2. Transformation parameters obtained from solving the 4 systems of view (4)

	bcd	acd	abd	abc
X_0	5424568.8606	5424568.8717	5424568.9127	5424568.908
Y_0	5329908.9185	5329908.9741	5329908.9658	5329908.9152
Q	1.0472462293553	1.04719293362607	1.04724220364913	1.04728818876528

With the help of MathCad program, using the equation of vector length minimization as the objective function (7), the optimization parameters were used – X_0,Y_0 Q, that correspond with the acceptable function values.

In such a way, the calculation of the points (A, B, C, D) with known coordinates in two systems was done. Maximum and minimum value of optimization parameters was chosen from the data, obtained after solving 4 systems of equations (6), that comply with the all base stations (Table 3). The number of iterations was 10, which gave 1000 results for each point. After analyzing the results of these samples, the following conclusion was made: the objective function had the minimal values at same transformation parameters on all points $X_0 = 5424568{,}8838$, $Y_0 = 5329908{,}9741$, $Q = 1{,}0471929$.

We have calculated the coordinates of reference polygon points with the help of found parameters and compared the received data with GNSS measurements during the static and RTN modes of work (Table 3).

$$\Delta_{\text{ст.}-\text{RTN}} = \sqrt{(X_{\text{st.}} - X_{\text{RTN}})^2 + (Y_{\text{st.}} - Y_{\text{RTN}})^2}, \quad (8)$$

$$\Delta_{\text{ст.}-\text{п.т.}} = \sqrt{(X_{\text{st.}} - X_{\text{T.P.}})^2 + (Y_{\text{st.}} - Y_{\text{T.P.}})^2}, \quad (9)$$

where $X_{\text{ст.}}, Y_{\text{ст.}}, X_{RTN}, Y_{RTN}$ i $X_{\text{п.т.}}, Y_{\text{п.т.}}$ the coordinates of explored points were obtained with GNSS receiver in the static RTN mode with the help of transformation parameters (T.P.).

For comparison, this task was solved not only by the iteration method (IM), but also by the least square method (LSM)

$$\mathbf{X} = (\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1} \cdot (\mathbf{A}^{\mathrm{T}}\mathbf{L}) \quad (10)$$

where $\boldsymbol{X}$ – a vector of unknown quantities $\boldsymbol{X} = \begin{bmatrix} C \\ S \end{bmatrix}$;

A, **L** – matrices of coefficients:

$$\boldsymbol{A} = \begin{bmatrix} (X_i' - X_0') & (Y_i' - Y_0') \\ (Y_i' - Y_0') & (X_i' - X_0') \end{bmatrix}, \qquad \boldsymbol{L} = \begin{bmatrix} X_0 - X_i \\ Y_0 - Y_i \end{bmatrix};$$

$$X_0' = \frac{1}{n}\sum_{i=1}^{n} X_i', \ Y_0' = \frac{1}{n}\sum_{i=1}^{n} Y_i'; \quad X_0 = \frac{1}{n}\sum_{i=1}^{n} X_i, \ Y_0 = \frac{1}{n}\sum_{i=1}^{n} Y_i. \quad (11)$$

Tab 3. The comparison of calculated points accuracy

	$\Delta_{st.-RTN}$ мм	S^2_{min}, мм	$\Delta_{st.-T.P.}$ мм	
			IM	LSM
A	3,2650	0.0066	5,489353	5,0352
Б	9,0785	0.0240	4,699648	6,4059
В	4,8877	0.0066	4,688047	6,0359
Г	4,2426	0.0007	4,708482	4,0917

After defining the values of unknown vector X, the coordinates of the points in the new coordinate system were calculated with the formulas:

$$\begin{cases} X_i = (X'_i - X'_0) * C + (Y'_i - Y'_0) * S + X_0 \\ Y_i = (Y'_i - Y'_0) * C + (X'_i - X'_0) * S + Y_0 \end{cases} \quad (12)$$

According to the results of minimizing the objective function S_min^2, we may assume that there are gross errors in the network, which allows us to find the weakest point and not take it into account during the calculations. As we can state from the calculation data in Table 3, in our case, it's the point B in which the error of RTN method was maximum and equal 9.0785 mm. It was influenced by the values of the objective function which equals 0.0240 mm. These results indicate that when there are gross errors in the network, the iteration method allows to find the coordinates of the points more accurately than the least square method described earlier. By using the suggested method, the errors of all points are displaced in one direction, when using another empirical methods their direction is stochastic, which leads to the accumulation of errors of values, calculated by the coordinates.

After excluding the B point (the weakest in the network), we have recalculated the point coordinates and compared them to the etalon data.

From the data, described in the Table 4, the following conclusion can be made. Depending on the objective, to provide the maximum accuracy of the points location, we recommend to repeat the calculations by the iteration method, and for obtaining the minimum declination from the initial coordinates in the national geodesic network – use the least square method.

Tab 4. Comparison of the accuracy of the calculated points

Пункти	$\Delta_{st.-T.P.}$		Віддалі	$S_{T.S.} - S_{T.P.}$	
	LSM	IM		LSM	IM
A	3,257752954	3,445058	AB	3,487295005	1,41045999
B	3,978863126	5,072006	AГ	1,408951629	0,113440571
Г	4,146254775	3,057367	BГ	1,475830059	0,495216358

5. SUMMARY

The suggested algorithm of calculation, based on the method of iteration and objective function minimizes the lengths of the vector, that don't comply with the coordinates. This algorithm allows transforming the point coordinates of the construction area with the mm accuracy. The distinction of this methodology is that the obtained transformation parameters do not contain measurement errors as they are correlated by way of minimization of the suggested target function.

This method allows to eliminate errors both in geodesic base network and on points, measured by GNSS receiver without additional measurements. The analysis of the research results described in Table 4 allows to recommend the optimization iteration methods for comparing special linear-angular networks, both by creating the template of electronic table and including the corresponding software unit in the system of automatic processing.

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IMPLEMENTACJA ALTERNATYWNYCH ALGORYTMÓW DO DEFINIOWANIA PARAMETRÓW TRANSFORMACJI SYSTEMU USK-2000 I UKŁADU WSPÓŁRZĘDNYCH OGÓLNEGO POŁOŻENIA PODCZAS STABILIZACJI

Streszczenie

Projektowanie i analiza algorytmu transformacji współrzędnych z ogólnego układu współrzędnych USK-2000 i odwrotnie, minimalizacja wpływu losowych błędów decyzji RTN i możliwość wyeliminowania błędów podstawowych we współrzędnych geodezyjnych punktów bazowych, poprzez zapewnienie optymalnej korelacji współrzędnych punktu w dwóch układach za pomocą metody iteracji i jej eksperymentalnego dowodu. Aby określić dokładność algorytmu, wykonano zarówno model, jak i prace eksperymentalne na wieloboku odniesienia w warunkach budowy. Proponowany algorytm obliczania, oparty na metodzie iteracji i funkcji celu, minimalizujący długości wektora, które nie są zgodne z współrzędnymi otrzymanymi z pomiarów, pozwala na przekształcenie współrzędnych punktu w obszarze konstrukcyjnym z dokładnością 1 mm. Praca koncentruje się na zastosowaniu metody iteracji do transformacji punktów i realizacji celu, która minimalizuje długości wektorów, które nie są zgodne ze współrzędnymi, wynikającymi z pomiarów i obliczeń. Takie podejście rozwiązuje problem błędów poziomej lokalizacji geodezyjnych punktów sieci i błędów w definiowaniu współrzędnych punktów zwrotnych czerwonych linii i istniejących konstrukcji.

Słowa kluczowe: GNSS, współrzędne, pomiar RTN, prace planistyczne, suma stacji.

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