NONLINEAR EQUATIONS OF ACOUSTICS - NUMERICAL SOLUTIONS AND THEIR VISUALIZATION

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The novel effective numeric solver of the nonlinear scalar wave equation describing the acoustic wave propagation in the attenuating media was derived. The solver was developed for the PC environment. The standard computation data include all stationary and dynamic characteristics of the radiated ultrasonic pressure field, especially its 4D (space/time) visualization. The results obtained with the solver can be used as the supporting tools (tool) in designing and developments of the multielement linear and phase array transducers applied in ultrasonography.

Theoretical analysis of the nonlinear scalar wave equation [1, 2], describing the ultrasonic wave propagation, makes it possible to create a new efficient numerical code for solutions of this wave equation by means of the PC technique for unilateral boundary pulse problems. The new method applied for axially symmetrical (2D) problems decreases the computation time at least many times – for weak nonlinearities. In the case of boundary problems without the axially symmetrical symmetry, also 3-dimentional, numerical costs, demand of memory and computation time decrease by two orders of magnitude when compared with the methods used up to now, enabling solutions of such problems by means of PC computers.

In the ultrasonography multielement transmitting probes became every year more popular. Fig. 1 presents, as an example, the principle of the arrangement of active elements (antennas) in such probes. They represent also the geometrical conditions of boundary problems characteristic for the 3-D problems. Electronic steering and control of the phase and amplitude of the active elements in the probe make it possible to scan and to process in time and in space the ultrasonic beams.

The developed numerical code supplies us with data which enable to obtain all stationary and dynamical field characteristics of such a probe, especially its time-space (4D) imaging.

Thus the created solver can be used for pure scientific applications by solving the nonlinear equation of acoustics, as well as a basic tool supporting and improving the process of developing the probes as a source of acoustic field of finite amplitudes. It makes possible 1. To select the conditions for the optimization of the beam shape. 2. Determination of the influence of technological defects and faults on the field distribution. 3. Identification of transmitting probe properties by comparison of numerical and measurement results. 4. Calibration of hydrophones – an important problem of all companies producing them for nonlinear measurements [3, 4]. 5. Determination of secondary beneficial effects like hyperthermia and destructive thermal effects and also mechanical effects – in general determination of harmless radiation doses.

Fig. 1 shows field distributions generated by a linear array probe composed of 64 elements, where $d = 0.2$ mm, $a = 0.14$ mm, $b = 6$ mm. Every element is exited by a pulse with the carrier frequency of fn = 7.5 MHz (see Fig. 2) with separate phase shift assuring the beam deflection by $\Theta = \Theta_{def} = 20^{\circ}$ (see Figs. 3, 4, 5, 6, 11).

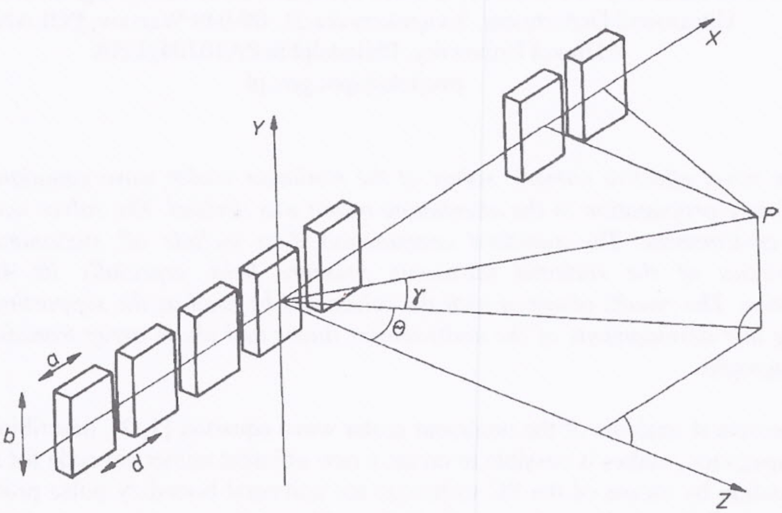


Fig. 1. Arrangement of active elements of a probe presented in a coordinate system.

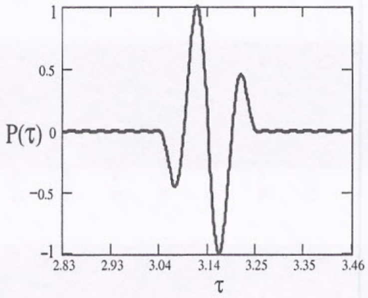
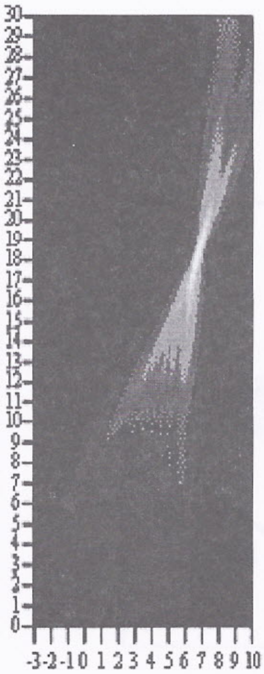


Fig. 2. Waveform of the exciting 2-cycle pulse at a frequency of 7.5 MHz



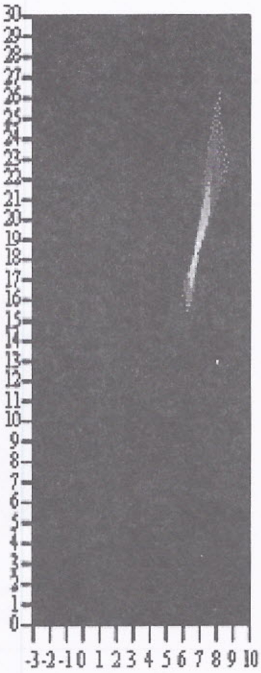
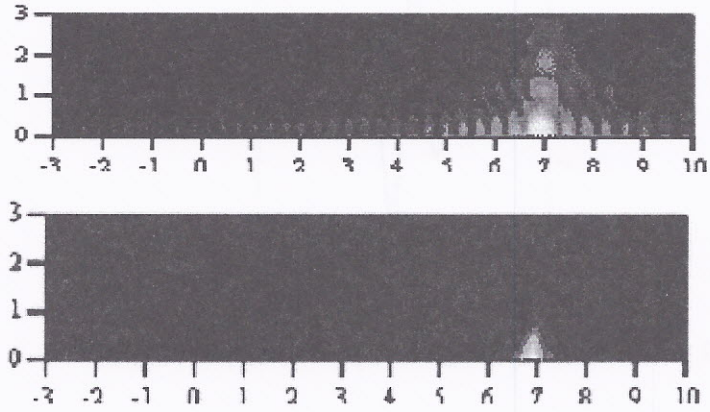


Fig.3. Distributions of the Fourier spectral components in the plane $y = 0$ for $f = \mathrm{fn} = 7.5$ MHz. Scale in mm.

Fig. 4. Distributions of the Fourier spectral components in the plane $y = 0$ for $f = 3 \cdot \mathrm{fn} = 22.5$ MHz. Scale in mm.



Figs. 5, 6 The Fourier spectral components distributions in the plane $z = 18$ mm for $f = \mathrm{fn}$ (top) and for $f = 3{\cdot}\mathrm{fn}$ (bottom)

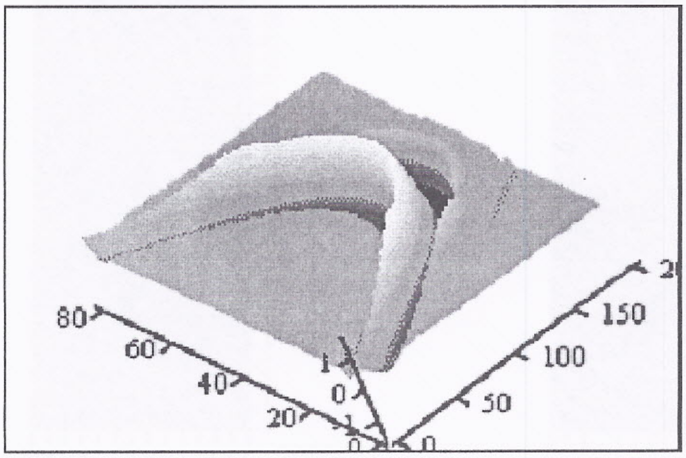


Fig. 7. The pressure wave in the plane $y = 0$, 10mm from the point $(x, y, z) = 0$ (before the focus).

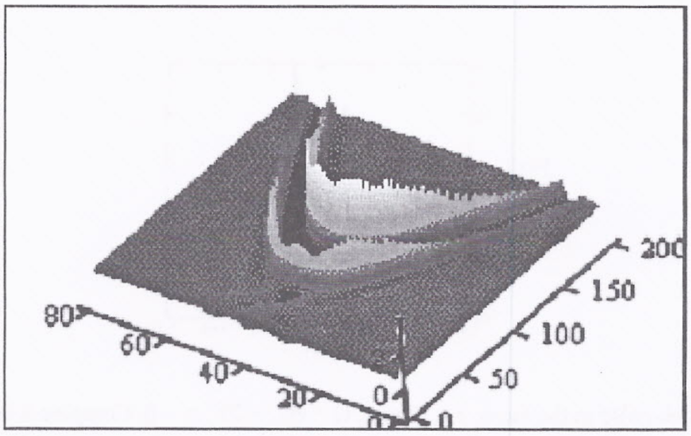


Fig. 8. The pressure wave in the plane $y = 0$; 20mm from the point $(x, y, z) = 0$ (in the focus).

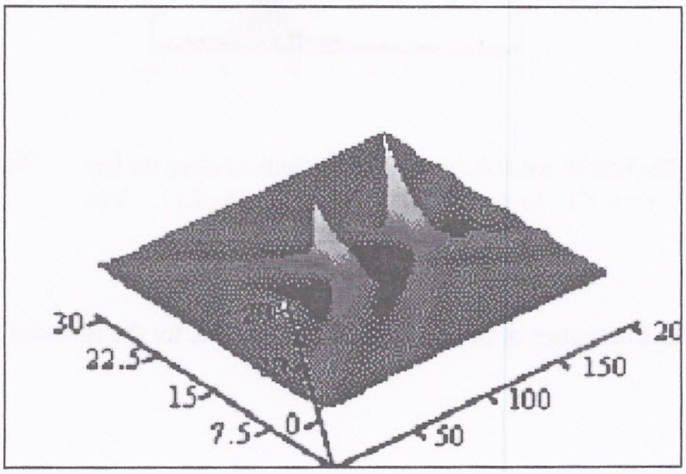


Fig. 9. The pressure wave in the plane $y = 0$; 27mm from the point $(x, y, z) = 0$ (behind the focus).

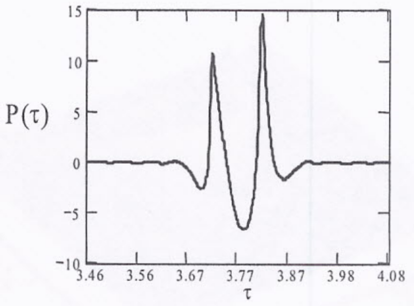


Fig.10. Pressure pulse in the focus, $z = 18$ mm, $\Theta = \Theta_{def} = 20^{\circ}$, $y = 0$. Compare with Fig. 2

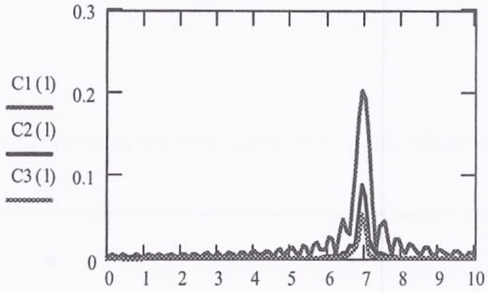


Fig. 11. The Fourier spectral components distributions along the line $z = 18$mm, $y = 0$, C1 - for f = fn; C2 - for f = 2·fn, C3 - for f = 3·fn.

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