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## VEHICLE DRIVING CYCLE OPTIMISATION ON THE HIGHWAY

**Summary.** This paper is devoted to the problem of reducing vehicle energy consumption. The authors consider the optimisation of highway driving cycle a way to use the kinetic energy of a car more effectively at various road conditions. The model of a vehicle driving control at the highway which consists of elementary cycles, such as accelerating, free rolling and deceleration under forces of external resistance, was designed. Braking, as an energy dissipation regime, was not included. The influence of the various longitudinal profiles of the road was taken into consideration and included in the model. Ways to use the results of monitoring road and traffic conditions are presented. The method of non-linear programming is used to design the optimal vehicle control function and phase trajectory. The results are presented by improved typical driving cycles that present energy saving as a subject of choice at a specified schedule.

## OPTYMALIZACJA RUCHU POJAZDU NA AUTOSTRADZIE

**Streszczenie.** Artykuł poświęcony jest problemowi redukcji zużycia paliwa pojazdu. Autorzy uważają optymalizację cyklu jazdy autostradą, jako sposób na bardziej efektywne wykorzystanie energii kinetycznej samochodu w różnych warunkach drogowych. Został zaprojektowany model kontroli jazdy pojazdu na autostradzie. Składa się on z cykli elementarnych, takich jak przyspieszenie, wolne toczenie, hamowanie siłą oporu zewnętrznego. Nie został wpisany tutaj reżim hamowania, jako proces rozpraszania energii. Wpływ różnych profili podłużnych drogi został wzięty pod uwagę i uwzględniony w modelu. Jest możliwość korzystania z wyników monitorowania drogi i warunków ruchu. Aby zaprojektować funkcję kontroli pojazdu, używany był optymalny sposób programowania nieliniowego. Wyniki są przedstawiane przez ulepszone typowe cykle ruchu, które zapewniają oszczędność energii jazdy, jako wybór w określonym harmonogramie.

### 1. INTRODUCTION

One of the promising ways of the automotive industry is the creation and use of autonomous vehicles designed to consume energy efficiently and be safe in the transportation of goods and

passengers on highways. To do this, one must use the available information on road and traffic conditions, taking into account the required performance schedule tasks and transport energy potential of the car. These conditions are mostly contradictory. Many attempts were made for their coordination to build appropriate optimisation models [1, 2]. However, most of them were not implemented in practice. Others do not reflect the progressive tendencies of road transport development.

## 2. LITERATURE SOURCES REVIEW

The amount of publications in which researchers deal with information provision on road transport is getting bigger, particularly on highways. This is accomplished by using the devices of collecting and transmitting signals from distance sensors (radar), speed and acceleration. The quality of operational control, namely, accuracy and other functionality of these devices are increasing. However, there is no comprehensive data on the efficiency of using different media devices to optimise driving cycle.

An automobile autonomy is based on information obtained from highway conditions and special algorithms of data transmission. It was introduced for consideration a long time ago, and the first studies in this direction were made in Japan in the 1970s of the XX century [1]. This area is still under development today around the fast-paced world. Autonomous vehicles have already driven hundreds of kilometers, and the founders of the technology argue that it will help to reduce the amount of traffic congestion, to increase road capacity and, more importantly, to make the movement much more secure without the driver. Grounds for such a claim are that, according to statistics, about 90% of all traffic accidents occur with the participation of a man [2]. But, simultaneously, researchers omitted to pay attention to the energy efficiency of self-driven cars.

Fuel consumption rates with normal driving, Japanese eco-driving and German eco-driving on a straight open road were compared quantitatively in some researches, which enable the performance of experiments repeatedly under the same conditions [1]. For a sample, the result of 700-m open road experiment indicated that German eco-driving reduced the fuel consumption rate by approximately 4% compared to Japanese eco-driving. There were no statistically significant differences between normal driving and Japanese eco-driving or German eco-driving. This result suggests that Japanese eco-driving would not always be helpful to reduce fuel consumption. Furthermore, the relationship between driving manner and headway distance was shown, focusing on the difference in the driving behavior for each style of eco-driving. The result suggested that Japanese eco-driving creates headway distance about twice as long as that by normal driving. It should be noted that this result is not exactly common, as these approximations were obtained under the condition that the vehicles should stop two times at traffic signals during their 700-m drive.

One of the solutions to the above task is the dynamic braking system, which means the conversion of kinetic energy into electricity or some kind of it, in contrast to friction braking. This kind of braking does not generate wear and tear, heat or sound. Today, the widespread way of using this system is rheostatic braking, which the regenerated electricity dissipates in banks of variable resistors. The second option, called regenerative braking, is reusing regenerated electricity within the transport network itself [3]. The recovered energy from the braking of vehicle may be returned into the power supply for use at acceleration [4]. To increase the efficiency of this model, the synchronisation of acceleration and deceleration of vehicles by optimising scheduled timetables is needed. Some attempts of different methods of utilising regenerative braking in the chapter about power grid management of train driving were made in the studies [3]. It can be concluded that energy saving of about 25%, especially in metro lines, can be achieved by utilising ESSs. To experience significant benefits from smart rail infrastructure, smart systems should go beyond national boundaries and be implemented on a larger scale. But regenerating braking, while converting energy into another form, leads to inevitable power losses. To avoid braking, one must get and use sufficient information.

The pilot promising project of Volvo Company is a combination of automatic transmission and the GPS system with an intelligent cruise control called I-See [5]. It is a unique system that learns the topography of the road and stores it in a central database. When a truck is being driven, it

automatically uses its knowledge to save fuel—up to 5%—during a driving cycle. A typical cycle includes some techniques. 1) Before going up the hill: I-See lets the speed increase, approaching the upper speed limit, to gain momentum. The truck stays longer in a higher gear. 2) On the uphill: keeping from downshifting. When climbing, I-See uses its stored knowledge to avoid unnecessary downshifts towards the top. It approaches the crest smoothly without wasting fuel in a lower gear at the top of the climb. 3) When approaching a downhill, I-See keeps the truck from accelerating unnecessarily. 4) Before the slope, the driveline temporarily disengages, allowing the truck to roll free. This saves energy and minimises the need for braking. 5) When gaining speed downhill, I-See can apply the engine brake smoothly in the process—rather than abruptly at the end—to prepare for the upcoming topography. 6) When a downhill is followed by an immediate uphill, I-See really comes on its own. It lets the truck roll, gaining speed and momentum to drive uphill with less effort [5]. This technology does not deal with appropriate scheduling of truck driving. Transport cycle made by I-See is not optimal after criteria of minimum process delay.

Some researchers determined optimal composition program mode motion by the criterion of minimum energy [7, 8]. It was believed that the transport cycle, in which the length of constant speed motion mode of the vehicle and its intensity of braking at the end of the cycle are maximal, is the most fuel efficient. The structure of the optimal cycle depends on the average speed and the length of the road passed. So, a car driving was seen as a material point at the given road conditions, without taking into account the parameters of traffic flow. The problem of choosing the optimal control procedure of two variables: the driving force to the driving wheels and braking is still unsolved. Thus, beforehand known inefficient regimes of forced braking was taken into account.

Road resistance movement was studied by many scientists. Everyone puts a different objective: to simulate the dynamics of the car, to estimate the quality of the tires or undercarriage car properties and so on. In general, all the studies have shown that rolling resistance is one of the components of the resistance movement, the value of which depends on many factors. So it should be defined empirically. Analytical researches are not entirely adequate in this case. There were attempts to predict the folding road profile, as its numerous characteristics—time series—were based on a posteriori evaluation of vehicle suspension. Predictions were designed in the form of truncated matrix parameters, which is a so-called "sliding window" [13]. However, applying the proposed algorithm is a narrow range for the formulation of the problem mentioned above—concerns about suspension control cannot relate to the assessment of the rolling resistance of a car.

Thus, developing the schedule of a car or trailer driving, which would ensure minimum energy consumption for the whole route along long-distance routes, with the principle of implementation of the transport problem "just in time" due to the evaluating of road and traffic conditions is a task with little attention paid to it.

The purpose of the research is to build a model and to get conceptual solutions to optimal control problem of a vehicle if the road conditions are characterised by great variability after a criterion of minimum energy consumption and limitations—namely the timely completion of the transport task.

### 3. FORMULATION OF THE PROBLEM

The vehicle may be considered as a material point with sufficient accuracy at intercity highways, so its basic equation of motion, taking into consideration recent results of vehicle rolling resistance, is [14, 16]:

$$m\ddot{x} = P_k(t) - P_o(x) - P_i(x, \dot{x}) - P_w(x), \quad N, \quad (1)$$

where  $m$  – mass of the vehicle which is concentrated in the centre of mass, kg;  $x$  – current location (distance) of the car, m;  $P_k(t)$  – driving force applied to the drive wheels, N;  $P_o(x)$  – rolling resistance associated with the deformation of the roadway, N;  $P_i(x)$  – rolling resistance associated with the longitudinal profile of the road, N;  $P_w(x)$  – the resistance of air flow, N.

Assuming that the road conditions can be defined as some distance  $S$  forward by the onboard control system of the car or truck with sufficient accuracy, the equation 1 can be written as follows:

$$x = u(t) - f_o \pm f_i x - f_v x - f_w x^2, \text{ m/s}^2, \quad (2)$$

where  $u(t)$  – the driving force per unit of weight of the complete vehicle  $G_a$ , which is  $(P_k \cdot g) / (\delta \cdot G_a)$ ,  $\text{m/s}^2$ ;  $f_o$  – coefficient of rolling resistance which takes into account the deformation of the tire and the road and is defined as constant  $f_o = P_o / G_a$ ;  $f_i$  – coefficient of rolling resistance caused by longitudinal profile of road and is defined as  $1/R$ , where  $R$  – radius of the curve of it,  $1/\text{m}$ ;  $f_v$  – additional component of wheels rolling resistance caused by bumpy road and is defined as a variable which depends on vehicle velocity and from index of road undulation;  $f_w$  – values that reflect the relative resistance movement, depending on the air flow,  $N \cdot \text{s}^2 / \text{m}^2$  [16].

Vehicle traffic control is optimised after the criterion of minimal energy costs. It was assumed that energy is not dispersed over the forced reduction of car speed during the transport cycle and is not expended on braking, unlike the existing models [8, 9]. This decelerating occurs due to the movement resistant losses. Also, the positive work can be performed not only by the driving force  $P_k(t)$ , but also the horizontal component of the gravitational forces  $P_i(x)$  on the slopes of the road was taken into account. Therefore, we write the expression for the criterion of problem solving as follows:

$$E = \int_{t_o}^T (P_k(t) - P_i(x)) x dt \rightarrow \min, \quad (3)$$

where  $t_o$ ,  $T$  – the start and end of the cycle.

Distance  $S$  is much smaller than the length of the route of the car  $S_m$  on the intercity roads with known conditions, so the final time  $T$  cycle is unknown and the integral in expression (3) has a movable right border. Considering the Bellman principle of optimality, the route can be divided into sections so that the total traffic on highways program  $u(x)$ ,  $x = 0 \dots S_m$ , consisting of partial optimal programs  $u(x_j)$ ,  $j = x_{j,o} \dots S_j$ , will be optimal too. One needs to provide such a schedule of an automobile in the main issue of the paper, that at  $x_o(t) = 0$ ,  $x_o(T) = S$ , where  $x = \overline{x_o, S_m}$  – distance;  $T$  – limiting time of the route performance of length  $S_m$ , to arrive at destination point with a minimum energy consumption. Let us introduce new variables  $x_1(t) = x(t)$ ,  $x_2(t) = x(t)$ . Let us define  $f_x = f_o + f_i x$ . Then the function objective becomes:

$$E = \int_{t_o}^T (u(t) \pm f_x \cdot x_1) x_2 dt \rightarrow \min, \quad (4)$$

and the system of conjugate equations:

$$\begin{cases} \dot{x}_2(t) = u(t) \pm f_x x_1 - f_v x_2 - f_w x_2^2 \\ x_2(t) = x_1(t) \end{cases}, \quad (5)$$

where the sign of  $f_x$  is taken depending on the profile of the road: on the rise – "-", on the descent – "+".

It is necessary to find a phase trajectory  $x_2 = F(x_1(t))$  and program of object control  $u(x)$  with such restrictions:  $x_1(t_o) = 0$ ;  $x_2(t_o) = V_o$  – left end phase trajectory is fixed;

$x_1(T) \geq S$  – follows from the terms of the schedule that if the limit of time of any part of the passed distance  $S$  it is not maintained then a whole schedule is disrupted;

$x_2(t) \leq V_{\max}$  – limit of the maximum speed of traffic safety conditions;

$u(x) \leq u_{\max}$  – limiting the power of the vehicle.

#### 4. METHODS OF SOLUTION

The solution to such a global problem can be found by methods of variations, including using the Pontryagin maximum principle, providing that the system of equations 5 is differentiated by variables

$x_1, x_2$ , and that the corresponding Hamiltonian should reach a maximum at the range  $t_o \dots T$  [11]. This problem, for the route with a constant resistance of driving, has been solved before [14]. However, the maximum principle is sufficient if the construction phase trajectory of both its beginning and end is fixed. The right end of the phase trajectory is movable in these studies. The system of equations 5 is nonlinear, and functions  $f_o - f_x x_1 - f_{vx2}$  may not have sustainable solutions, in particular, the form of membership. In this regard, the initial reduction of (4) and (5) to some finite-dimensional problems of mathematical programming was applied. If one considers the assumption that vehicle traffic control is carried out not continuously, but sometimes  $t_o < t_1 < \dots < t_{N-1} < t_N = T$ , then the initial problem may be replaced by finite-difference analogue:

$$\sum_{i=0}^{N-1} (u^i \pm f_x^i) (t_{i+1} - t_i) x_2^i \rightarrow \min, \quad (6)$$

where  $i$ —number of areas in which the distance  $S$  was divided. Conjugated system of equations is:

$$\left\{ \begin{array}{l} \frac{x_2^{i+1} - x_2^i}{t_{i+1} - t_i} = u^i \pm f_x^i x_1^i - f_v x_2^i - f_w (x_2^i)^2 \\ \frac{x_2^{i+1} - x_2^i}{t_{i+1} - t_i} = x_2^{i+1} \end{array} \right. ,$$

and restrictions:

$$x_2^0 = V_o; x_1^0 = 0; x_2^i \leq V_{\max}, i = 0, 1, \dots, N; u^i \leq u_{\max}; x_1^N \geq S, \quad (7)$$

where we can get:

$$x_2^{i+1} = \left( u^i \pm f_x^i x_1^i - f_v x_2^i - f_w (x_2^i)^2 \right) (t_{i+1} - t_i) + x_2^i, \quad (8)$$

$$\text{and } x_1^{i+1} = x_2^{i+1} (t_{i+1} - t_i) + x_1^i. \quad (9)$$

Variables of new mathematical programming problem are  $x_1^i, i = 1, \dots, N$  and  $u^i, i = 0, \dots, N-1$ . Their number and the effectiveness of the solution depend essentially on the length of the distance  $S$  which has a wide longitudinal profile. A lot of options were analysed, but it was found that any type of profile can contain nine typical elements, which are interdependent (Fig. 1).

There are fractures in the design of the longitudinal profile of intersections of roads in neighbouring areas with different slopes as a rule. The close distance between the longitudinal profile of fractures, especially frequent alternation of uphill and downhill makes worse the comfort, dynamics and efficiency of movement as required to change the mode of driving, gear shifting, and even braking. Therefore, Ukraine's building codes contain new requirements of increasing the distance between fractures and design conjugations of such fractures for European roads specified category. Raised fractures make worse smooth driving and reduce visibility in the car. Otherwise, a dynamic force, namely, centrifugal force, occurs on the concave fractures. The radius of the curves in the longitudinal profile should not be less than: convex – 70000 m, concave – 8000 m. Assuming there are only highway roads of the first and second categories, the following parameters were adopted for this task.

The option of the longitudinal profile of the road, shown in Fig. 1, is the most typical of the cyclical driving on the highway. There are other options such as downhill, lasting descent, ascent, and climbing the step. However, a forced braking is present in these cases or the potential energy of the vehicle is not used properly.

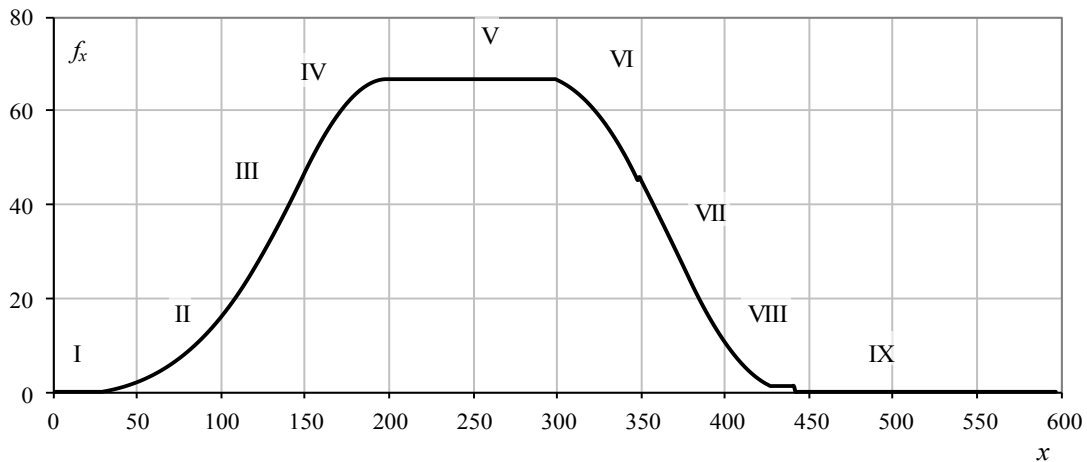


Fig. 1. Scheme longitudinal profile of a typical area highway: I-IX – typical elementary components plots  
 Fig. 1. Schemat podłużny typowych dróg: I-IX – elementarne składniki

The current requirements of highway design consists of allowable maximum values of the road slope, convex radius (IV, VI) and convex length (II, VIII) of the transition elements of profile parts and so on. Profiles are various and can be such that may consist of horizontal (I, V, IX), sloping (III, VII) or some transition areas, and their length can be arbitrary.

However, all elements of the profile are such that they could be accurately described by analytical dependencies:

$$f_x^j = f_j \pm \frac{x^{j-1} - x^j}{R^j}, \quad (10)$$

where  $f_j$  – is stable relative resistance movement;  $j = I, II, \dots IX$ ;  $R^j$  – transition radius area; sign of the right side of the formula depends on the type of curve: "+" if it is convex, "-" – concave;  $x^{j-1} - x^j = s^j$  – length of the horizontal component of the passed road. Thus, setting the parameters of  $s^j$ ,  $R^j$  and  $f_j$ , one can get a possibility to describe an analytical expression with a sufficient rate of accuracy of any profile of the highway as the function  $f_x(x)$ .

## 5. PROBLEM SOLVING AND ANALYSIS OF THE RESULTS

The problem of mathematical programming (6)-(9) is nonlinear if the variable profile of the road is taken into consideration. That is why it was necessary to apply the gradient method, the content of which was that the search procedure was directed towards increasing the length of the distance covered by  $t_N$  intervals. An example of one of the phase trajectories corresponding to the longitudinal profile is in Fig. 1, shown in Fig. 2.

In this example, path length, at which vehicle control drive was carried out is 500 m. The car would have to pass this distance in not more than 32 sec. The maximum speed of the vehicle is unrestricted, but it relates to the maximum value of the relative driving force. Optimal control of the car (changing the driving force of the driving wheels) is a discrete monotonic function (Fig. 3) with the first type of discontinuities.

As shown in Figs. 2 and 3, optimum driving is in pulsed mode (accelerating-free rolling) on the horizontal sections of the highway and at the entrance to uphill. The velocity of the vehicle is within the range of 15,9 ... 16 m/ ec. (see. Fig. 2).

The car drives at a pulsed cycle with restriction up the hill, which contains such regimes as acceleration, motion with constant velocity and free rolling. The rate of its speed does not undergo significant fluctuations, as the process of kinetic energy accumulation is going on. It will spend next

time on the downhill. Further, a car would drive along the horizontal section of the road in pulse mode without restrictions again.

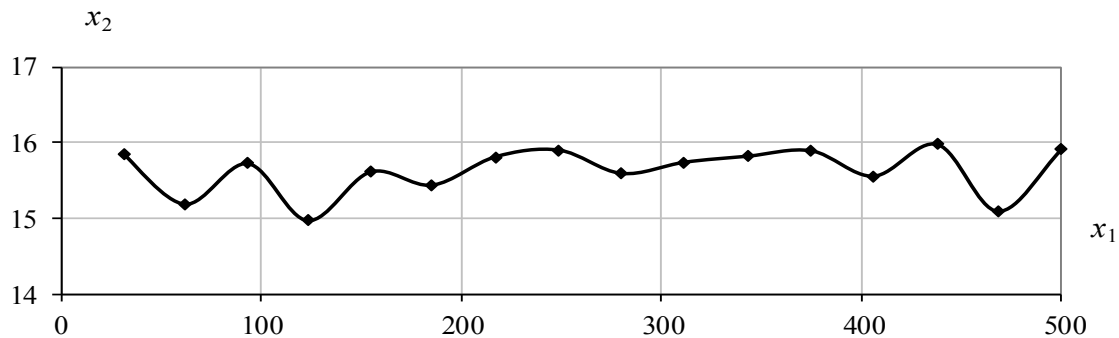


Fig. 2. The optimal phase trajectory  
Fig. 2. Optymalna faza trajektorii

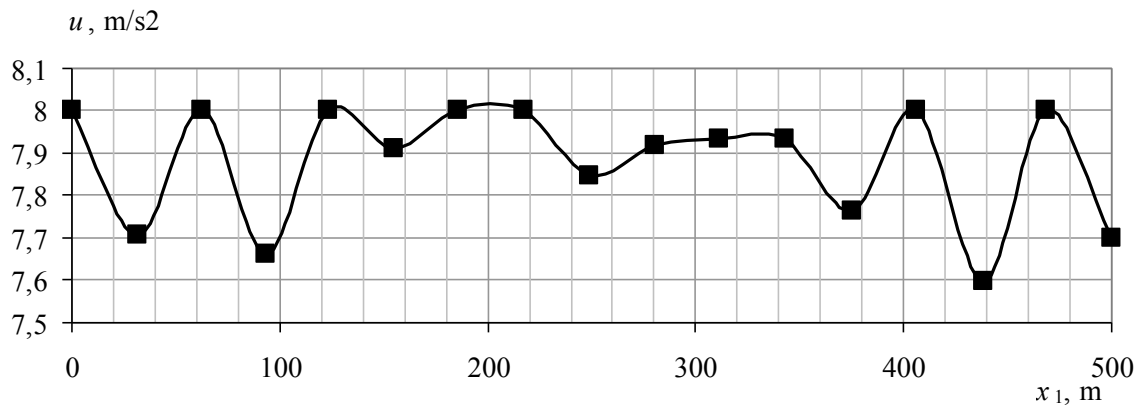


Fig. 3. The change of the relative value of the driving force at the optimum driving process  
Fig. 3. Zmiany względnej siły napędowej w optymalnym procesie jazdy

The average speed of this transport cycle is 15,6 m/s. Total energy consumption per unit of total mass of the car is 1,025 kW · h. / kg. If the parameter controls impose more restrictions (research carried out with alternating sense  $u_{max} = 6 \dots 10 \text{ m/s}^2$ ), this leads to a lack of impulse control modes, reducing the average speed of the same cycle and the growth of total energy consumption (6). By increasing the allowable values of the same control  $u(x)$ , its minimum value reaches zero, i.e. the free rolling with idling of vehicle's engine is appearing in the optimal transport cycle.

## 6. CONCLUSIONS AND PROSPECTS FOR FURTHER RESEARCH

The model of a motor vehicle control enables the optimisation of energy consumption when driving on the highway. This applies pulse transport cycle using available information about road conditions. The model differs from the complicated and the detailed and with the approximate solution of one which gives guaranteed optimal control of the problem, despite the considerable variability of the traffic conditions and the required amount of information. This eliminates the problem of finding stable solutions to nonlinear equations of driving.

The methodology of model building concerns highway optimisation of any line length and longitudinal profile. In addition, it does not depend on the type of vehicles and road categories. This

method of dynamic systems optimality principle can be used even if road and traffic conditions of vehicle driving along the route are not sufficiently known. Rather than the decision of the control mode, the choice should be less effective but still guarantee the optimal driving cycle.

The use of the proposed model of optimal control is possible by building a number of typical traffic cycles that differ by average speed, length and structure, but are characterised by minimal drive energy consumption without the use of unsustainable modes. Any complexity plan of the passed road with different conditions can be considered as the optimal set of elementary cycles that should be selected so that the overall traffic of this passed distance will be optimal in this case.

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