ARCHIVES of FOUNDRY ENGINEERING

Published quarterly as the organ of the Foundry Commission of the Polish Academy of Sciences

ISSN (1897-3310)
Volume 15
Issue 1/2015

51 – 68

11/1

# Prediction of Secondary Dendrite Arm Spacing in Squeeze Casting Using Fuzzy Logic Based Approaches

**M.G.C. Patel [a], P. Krishna [a], M.B. Parappagoudar [b, *]**
[a]Department of Mechanical Engineering, National Institute of Technology Karnataka-Surathkal-575025, India
[b]Department of Mechanical Engineering, Chhatrapati Shivaji Institute of Technology, Durg (C.G) 491001, India
[*]Corresponding author.: E-mail address: maheshpg@gmail.com

## Abstract

The quality of the squeeze castings is significantly affected by secondary dendrite arm spacing, which is influenced by squeeze cast input parameters. The relationships of secondary dendrite arm spacing with the input parameters, namely time delay, pressure duration, squeeze pressure, pouring and die temperatures are complex in nature. The present research work focuses on the development of input-output relationships using fuzzy logic approach. In fuzzy logic approach, squeeze cast process variables are expressed as a function of input parameters and secondary dendrite arm spacing is expressed as an output parameter. It is important to note that two fuzzy logic based approaches have been developed for the said problem. The first approach deals with the manually constructed mamdani based fuzzy system and the second approach deals with automatic evolution of the Takagi and Sugeno's fuzzy system. It is important to note that the performance of the developed models is tested for both linear and non-linear type membership functions. In addition the developed models were compared with the ten test cases which are different from those of training data. The developed fuzzy systems eliminates the need of a number of trials in selection of most influential squeeze cast process parameters. This will reduce time and cost of trial experimentations. The results showed that, all the developed models can be effectively used for making prediction. Further, the present research work will help foundrymen to select parameters in squeeze casting to obtain the desired quality casting without much of time and resource consuming.

**Keywords:** Squeeze casting process, Secondary dendrite arm spacing, Fuzzy logic, Adaptive network based fuzzy interface system (ANFIS)

**Nomenclature**

| | | | |
|---|---|---|---|
| FLC | Fuzzy logic controller | $a_1,........a_6$ | Half base widths |
| L | Low | $p_i$, $q_i$, $r_i$ & $u_i$ | Coefficient of consequent part |
| M | Medium | SDAS | Secondary dendrite arm spacing |
| H | High | GA-NN | Genetic algorithm neural network |
| A | Time delay | BPNN | Back propagation neural network |
| B | Pressure duration | MAPE | Mean absolute percent error |
| C | Squeeze pressure | $R^2$ | Co-efficient of correlation determination |
| D | Pouring temperature | ANFIS | Adaptive network based fuzzy interface system |
| E | Die temperature | Mctrimf | Manually constructed triangular membership function |

| | | | |
|---|---|---|---|
| μ | Membership function | Mcbellmf | Manually constructed bell shape membership function |
| ANNs | Artificial neural networks | Mcgaussmf | Manually constructed gaussian membership function |
| GA | Genetic algorithms | Angaussmf | Adaptive network gaussian membership function |
| FL | Fuzzy logic | Antrimf | Adaptive network triangular membership function |
| RMSE | Root mean square error | Anbellmf | Adaptive network bell shape membership function |

# 1. Introduction

The second lightest material next to magnesium is aluminium and it is also the third largest material abundantly available in the earth crust. Aluminium alloys, widely used as a casting material from the past few decades due to its inherent properties such as light weight, recycling potential, reduce fuel consumptions to save energy and provides better environmental protection for the future generation [1]. Silicon (Si) found to be the better alloying element in aluminium alloys, since it improves fluidity, abrasion resistance, reduces melting temperature, lowers density, cost effective and easily available [2]. Addition of Copper (Cu) and Magnesium (Mg) are necessary to enhance the strength of Al-Si alloys [2]. It is interesting to note that Al-Si-Cu-Mg alloys have shown better casting characteristics with improved cooling rate, minimum porosity, reliability, good dimensional accuracy, modified eutectic silicon particles, better mechanical, micro and macro-structure properties.[3]. However it is important to note that the LM20 alloy used for the present study constitutes these alloy elements. .

The squeeze casting process is based on the principle of pressurized solidification concept, suggested by D.K. Chernov in the early 1878. Porosity, shrinkage, segregations are the major limitations and have drawn much attention of the researchers in squeeze casting process development. Squeeze casting process combines the desirable features of gravity, pressure die casting and forging processes. It is important to note that mechanical and micro-structure properties are largely influenced by its cooling rate of cast alloys. Higher cooling rate reduces grain size, grain boundary, shrinkage porosity, segregation between dendrites, modifies eutectic silicon particles and decreases secondary dendrite arm spacing [4]. Higher solidification rate can be achieved with proper control of squeeze cast process parameters like time delay, pressure duration, squeeze pressure, die temperature and pouring temperature. Improper choice and levels of the aforementioned parameters may lead to possible casting defects such as oxide inclusions, over/under filling, extrusion, die sticking, segregations, cold laps, poor surface quality, dimensional inaccuracy and case debonding [5]. It is important to note that these defects finally affect the microstructure characteristics like secondary dendrite arm spacing (SDAS), which can be minimized by proper control of squeeze cast process variables. Hence it is of paramount importance to develop the squeeze cast process model and analyze the input (squeeze cast process variables) and output (secondary dendrite arm spacing) relationships of the process.

The potential applications of squeeze cast components are found in aerospace and automobile sectors. This made researchers/investigators to carry out a great deal of research work on micro-structural characteristics during 1990's and 2000's throughout the world. It is important to note that majority of the research work happened during those periods was based on theoretical and experimental work. Lee et al., (1998) made an attempt to investigate the effects of gap distance on cooling rate and secondary dendrite arm spacing of gravity and squeeze cast wrought aluminium alloy using numerical and experimental approach [6]. It is important to note that the applied squeeze pressure reduces the air gap between the melt and the die interface leads to higher heat transfer rate (cooling rate) results in lower secondary dendrite arm spacing. Yang (2007) studied the effect of solidification time on the mechanical properties of LM6 and ZA3 alloys utilizing two analytical models namely steady state heat flow model and gracia's virtual model [7]. In addition, performance of the developed models was compared with the practical castings and the average percent deviations were found to be equal to 27 for LM6 and 20 for ZA3 alloys respectively. The effect of pouring temperatures and squeeze pressures on cast structure and tensile properties of wrought aluminium 7010 alloy had been investigated by Yue (1997) [8]. It is important to note that experiments performed by keeping the die temperature and pressure duration as constant. Moreover in their work, it was observed that the time delay parameter acts as a crucial role wherein fine grain structure and better tensile properties were achieved when the alloy was pressurized between its liquidus and solidus temperature. Ming et al., (2007) investigated the effects of different squeeze pressures on secondary dendrite arm spacing, tensile strength and percentage elongation of squeeze cast Al-Cu based alloys [9]. It is also important to mention that experiments were performed by keeping pressure duration, die and pouring temperature at fixed values. In addition the results showed that, the applied pressure eliminates porosity and the alloy grain structure was clearly characterized by reduced secondary dendrite arm spacing (SDAS) with increase in applied pressure. Hajari and Divandari (2008) investigated the influence of different squeeze pressure on secondary dendrite arm spacing and the mechanical properties of 2024 wrought aluminium alloy [10]. However, it is important to note that pouring temperature, die temperature and pressure duration was kept at fixed values while performing experiments. Hong et al., (2000) [11], made an investigation to analyse the effects of pouring temperature, applied pressure, time delay, die temperature, degassing and inoculation treatments on formation of macro-defects in Al7%Si alloy. It is important to note that, the experiments were performed utilizing the classical engineering approach (that is, varying one parameter at a time and keeping the rest at the fixed values). Maleki et al., (2009) [12] used a classical engineering approach for performing experiments and analyse the effects of squeeze pressure, melt and die temperatures on secondary dendrite arm spacing and aspect ratio of LM13 alloy. It is to be noted that the effect of pressure duration and time delay parameters was left out in their analysis.

Following observations have been made from the above discussed literature.

– Most of the author's attempted conventional engineering approach for conducting and analysis, wherein large number of experiments are to be conducted with an increase in number of process variables and their levels.
– The results obtained from the said approach will not reveal the complete information about the impact of the interaction (combined) effect of the process variables on the response.
– The practical guidelines suggested by the authors to optimize squeeze casting process may not help the foundry men for selection of process parameters unless input-output relationships are expressed in mathematical form.

In recent years limited research efforts made by the authors to develop input-output relationship using statistical and taguchi parametric design. Vijian and Arunachalam (2006) [13] utilized taguchi method for experimentation and developed multi variable linear regression equation which includes output (hardness and tensile strength) as a function of input (squeeze pressure, pressure duration and die temperature) parameters. . However, the developed regression equation includes only linear terms and neglected the effects of square and interaction terms, moreover, pouring temperature variations was left out in their analysis. Bin SB et al., (2013) investigated the strength and ductility of squeeze cast AlSi9Cu3 alloys via taguchi tools [14]. The authors failed to develop the model, which could predict the response, in addition pressure duration contributions was left out in their analysis. The research efforts was made by Senthil and Amrithagadeswaran to study the effects of squeeze cast process variable on hardness, ultimate tensile strength and yield strengths of LM24 alloy [15 & 16]. In addition, the developed regression equations are not used for the prediction, percent contribution of square and interaction terms were not estimated and moreover the influence of time delay process parameter was left out in their analysis. In recent years, many researchers applied soft computing tools, such as artificial neural networks (ANNs), fuzzy logic (FL) and genetic algorithm (GA) approaches and their different combinations to model and analyse the manufacturing processes [17]. GA has been adopted to solve multi-objective optimization of various responses of squeeze casting process, Vijian and Arunachalam (2006) [13]. It is important to note that the objective function includes only main effect parameters and the paramount importance of square and interaction parameters in identifying the non-linear effects are neglected in their research work. Wang RJ et al., (2012) [18] used artificial neural networks to predict the temperature difference of the squeeze cast part. It was observed that, ANNs finds better prediction and reduces the need of costly simulation software, and need of experts to interpret the results. Research efforts were made by some authors to develop an auxiliary hybrid system (combining desirable features of GA and ANNs) to tackle the problems related to different moulding sand and pressure die casting process [19-22]. It is also important to make a note that some authors made efforts to model and analyze the important manufacturing processes with the help of embedded type hybrid systems (combining desirable features of GA and FL, ANNs and FL) [23-27]. It is interesting to note that many authors have successfully implemented embedded type hybrid systems for various manufacturing processes and proved it as a cost effective tool to model and analyze the complex manufacturing processes.

To the best of author's knowledge, no much of the work has been reported in literature to carry out the forward mapping of squeeze casting process utilizing fuzzy logic based approaches. In the present work an attempt has been made to predict the secondary dendrite arm spacing (SDAS) utilizing Mamdani and Takagi and Sugeno based fuzzy logic approaches. Approach 1 deals with development of mamdani based fuzzy logic system where in consequent, rule base and antecedent parts are constructed with the help of human expertise. Approach 2 follows development of adaptive network based fuzzy interface system popularly known as Takagi and Sugeno's model. This model deals with the automatic evolution of consequent and antecedent parts. It is to be noted that linear and non-linear membership function distributions are used for both of the approaches.Finally, the performance all models are compared in making the prediction of SDAS in squeeze casting.

## 2. Experimental details

The casting quality depends mainly on the composition of the alloy, processing method and machine related parameters. In squeeze casting process the solidification occurs with applied pressures and is considered as one of the near net shape manufacturing process. It is important to note that, the microstructure of the squeeze castings depends on machine related process parameters. Figure 1, shows the schematic diagram of input (Time delay, pressure duration, squeeze pressure, pouring temperature and die temperature) and output (secondary dendrite arm spacing) of the squeeze casting process. The range of input squeeze cast process parameters considered in the present study is shown in Table 1. The process parameters and their levels have been finalized based on available literature and consulting experts

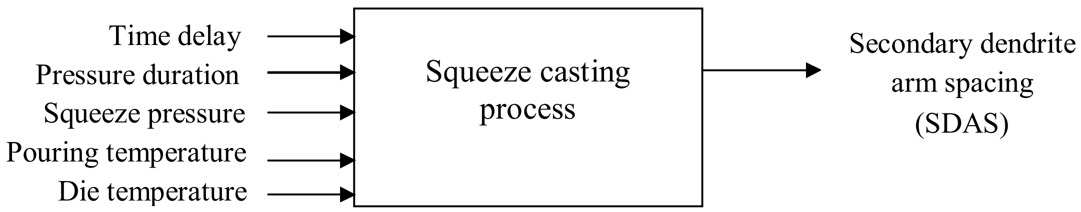


Fig. 1. Input and Output model of the squeeze casting process

Table 1.
Squeeze cast process parameters and their ranges

| Process parameters | Notation | Units | Level-1 | Level-2 | Level-3 | Level-4 | Level-5 |
|---|---|---|---|---|---|---|---|
| Time delay, ($T_d$) | A | S | 03 | 05 | 07 | 09 | 11 |
| Pressure duration, ($D_p$) | B | S | 10 | 20 | 30 | 40 | 50 |
| Squeeze pressure, ($S_p$) | C | MPa | 0.1 | 50 | 100 | 150 | 200 |
| Pouring temperature, ($P_t$) | D | ˚C | 630 | 660 | 690 | 720 | 750 |
| Die temperature, ($D_t$) | E | ˚C | 100 | 150 | 200 | 250 | 300 |

**Secondary dendrite arm spacing (SDAS)**

Micro-structure examination is carried out on the prepared squeeze cast specimen. The sectioned surface was initially grounded using belt grinder, followed by series of silicon carbide papers with increasing fineness. Continuous circulation of water was maintained during grinding. Disc polisher is used with 400 mesh $Al_2O_3$ powder, 1000 mesh SiC powder with water and diamond paste with hyfin liquid to get scratch free surface of test specimen. The prepared samples are cleaned with soap solution followed by alcohol and dried. The samples are etched with kellers reagent (2.5% $HNO_3$ + 1.5% HCl + 1%HF + 95%$H_2O$) solution to reveal the micro-structure. The prepared samples have been examined using optical microscope and images of micro-structure is recorded. Biovis image analysis software is used to determine the SDAS values. The linear intercept method has been adopted to measure the secondary dendrite arm spacing. The quantification of secondary dendrite arm spacing is done by drawing the lines measuring the distance between the adjacent sides on the longitudinal part of a primary dendrite as a function of the distance from the dendrite tip (Zeren (2005) [38]). The SDAS is measured by using the dendrites which possess more than 5 dendrite arms. The average value of the SDAS in each casting sample is determined at three different locations by taking at least to a minimum of 15 different primary dendrites containing more than 5 secondary dendrite arms. SDAS is measured by using Eq. i and Eq. ii,

$$\overline{X_{SDAS\,i}} = \frac{X_i}{M_i} \qquad (i)$$

$$Average\ SDAS = \frac{1}{n}\sum_{i=1}^{n}\frac{X_i}{M_i} \qquad (ii)$$

Where $X_i$ is the length of the $i^{th}$ dendrite, n is the number of measurements, $m_i$ is the number of dendrite arms and 'i' is the index term of the measured dendrites.

Table 2.
Summary results of input-outputs of the test cases

| Exp. No | Squeeze casting process parameters | | | | | Response |
|---|---|---|---|---|---|---|
| | Td | $D_P$ | Sp | Pt | Dt | SDAS, μm |
| 1 | 11 | 30 | 101 | 671 | 263 | 48.43 |
| 2 | 7 | 14 | 110 | 635 | 192 | 49.74 |
| 3 | 6 | 37 | 63 | 674 | 236 | 47.64 |
| 4 | 5 | 40 | 142 | 731 | 254 | 33.78 |
| 5 | 5 | 10 | 71 | 723 | 142 | 46.86 |
| 6 | 9 | 33 | 110 | 738 | 261 | 48.33 |
| 7 | 9 | 48 | 96 | 637 | 174 | 50.63 |
| 8 | 11 | 32 | 172 | 712 | 189 | 44.86 |
| 9 | 4 | 21 | 196 | 646 | 213 | 35.66 |
| 10 | 4 | 23 | 89 | 742 | 284 | 41.34 |

The micro-structures obtained for few test samples are shown in the Fig. 2. It is interesting note that the micro-structure of the squeeze cast samples shown in Fig. 2 (c) (Test case 9) yields lower secondary dendrite arm spacing compared to the Fig. 2 (b) (Test case 10) and Fig. 2. (a) (Test case 7) (refer table 2).

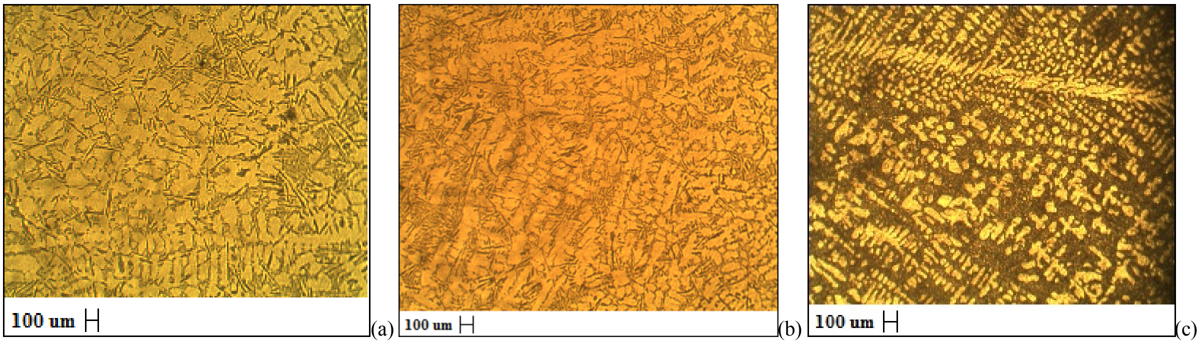


Fig. 2. Micro-structure of squeeze cast samples of different test cases shown in table 6, (a) Test case 7, (b) Test case 10 and (c) Test case 9

This is because the dendrites are broken into small pieces due to the better squeeze casting conditions. This might be due to low time delay value (that is, molten metal with high fluidity) and the higher applied pressure. The liquid metal with high fluidity and pressure will result in eliminating gas entrapment, improved heat transfer co-efficient and higher solidification rate. These results will yield micro-structure with smaller dendrite arm spacing values.

## 2.1. Data collection

The performance of the developed model predictions in artificial intelligence techniques depends on the quality and quantity of the training data used. In soft computing applications training has to be done with huge (say 500) data base and should cover all possible combinations of the input variable ranges. The collection of such data through real experiments is tedious and not feasible, since, it leads to large amount of material waste, labour waste and time consuming. It is to be noted that huge amount of training data has been generated at random using the response equation. This response equation (input-output relation) is obtained earlier by same authors [Refer 28]. Further, the data used to test the models has been collected through experiments and not used in training the FLC. The non linear regression equation for secondary dendrite arm spacing (SDAS) expressed in terms of squeeze cast technical parameters is shown in Eq. [1].

$$SDA = 313.45 + 3.71889T_d - 0.318161D_p - 0.120364S_p - 0.632729P_t - 0.22336D_t - 0.144949T_d^2 + 0.00238712D_p^2 + 0.000198798S_p^2 + 0.000402338P_t^2 + 0.000513454D_t^2 \quad (1)$$

## 2.2. Fuzzy Modelling

The method of identifying, analysing and establishing the input-output relationship of the physical system is referred as modelling. The fuzzy concept is adopted to develop the relationship between squeeze cast process variables and the secondary dendrite arm spacing (SDAS). In the present work, fuzzy modelling aims at predicting the output for the known set of inputs. SDAS is expressed as a function of squeeze cast process variables. Takagi & Sugeno's and Mamdani approach of FLC have been developed and used to predict the SDAS. The performance of the developed models is tested for both linear (Triangular) and non-linear (Generalized bell shape and Gaussian) membership function distributions with the help of 10 test cases. The input-output model of the squeeze casting process using fuzzy logic is shown in Fig.3.

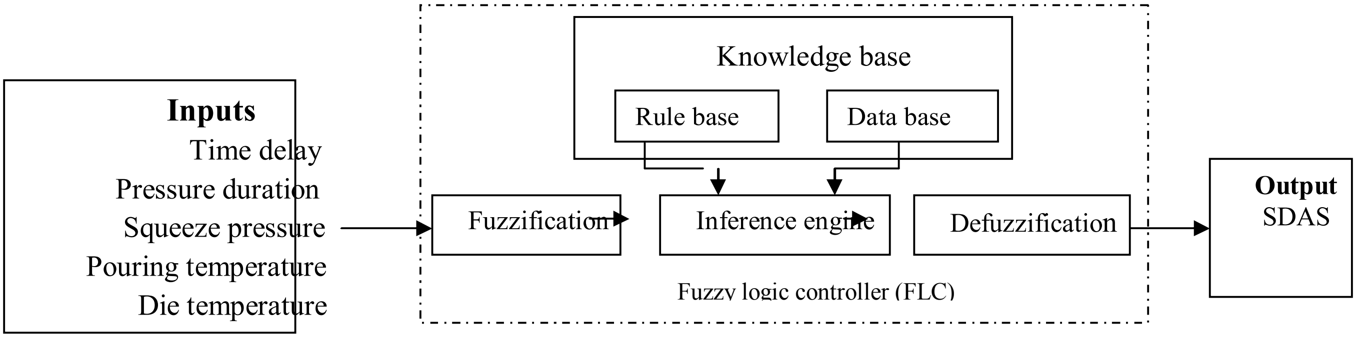


Fig. 3. Model representation of squeeze casting process using fuzzy interface system

# 3. Fuzzy logic controller

Due to rapid development in the application of fuzzy logic to solve complex real world problems, researcher/investigators are more interested to develop the fuzzy input-output relationships. Fuzzy concept works based on the thinking and reasoning capabilities of our human behaviour and the same is used to establish the input-output relationships of the system. Easy to understand and implement, capable to handle uncertainty and exact mathematical formulation is not required are the potential advantages of the fuzzy logic systems (Pratihar DK (2008)) [29]. Takagi Sugeno and Mamdani based models of fuzzy logic system (refer table 3) have been developed in the present study to model the squeeze casting process.

In general fuzzy logic model performances depend on the knowledge base, which consists of data base and rule base. In data base, the membership function is decided, based on the distributed data of variability in the process. Triangular and trapezoidal membership functions are used for the linear type data distribution. Whereas, generalized bell shape, sigmoid, gaussian membership functions can be used if the data distributions are assumed to be non-linear. In fuzzy logic systems the variables need to be expressed in the form of linguistic terms such as low, medium, high, small, medium etc., and the input-output relationships are expressed as a function of linguistic terms in the form of rules. It is important to note that the number of rules vary with linguistic terms and process variables.

Table 3. Fuzzy logic system modelling approaches (Azar and Taher (2010)) [30].

| Type | Linguistic fuzzy modelling | Precise fuzzy modelling |
|---|---|---|
| Approach | Mamdani approach | Takagi sugeno's approach |
| Advantage | Better interpretability | High accuracy |
| Limitation | Low accuracy | Low interpretability |
| Number of parameters | Few parameters | More parameters |
| Number of rules | Few rules | More rules |

## 3.1. Approach 1:

*Development of manually constructed Mamdani based FLC*

In this approach, Mamdani based fuzzy logic controller (FLC) has been developed to carry out forward mapping of squeeze casting process. Squeeze cast process variables such as time delay, pressure duration, squeeze pressure, pouring temperature and die temperature are considered as the inputs and secondary dendrite arm spacing (SDAS) is treated as an output.

In fuzzy systems the input and output parameters need to expressed in linguistic terms. In the present case, three linguistic terms such as low (L), medium (M) and high (H) were used to represent the input-output variables of the present system. For simplicity, triangular membership function distributions representing the input-output variables of the squeeze casting process with fuzzy logic system is shown in Fig. 3. It is important to mention that squeeze casting process is complex in nature, since it consists of large number of parameters and the output may behaves linear or non-linear with respect to change in the output. Hence, the fuzzy logic models were developed with both linear and non-linear type membership function distributions.

The 'a' values shown in Fig. 4 indicates the half base widths of isosceles triangles and the base widths of the right angled triangles.

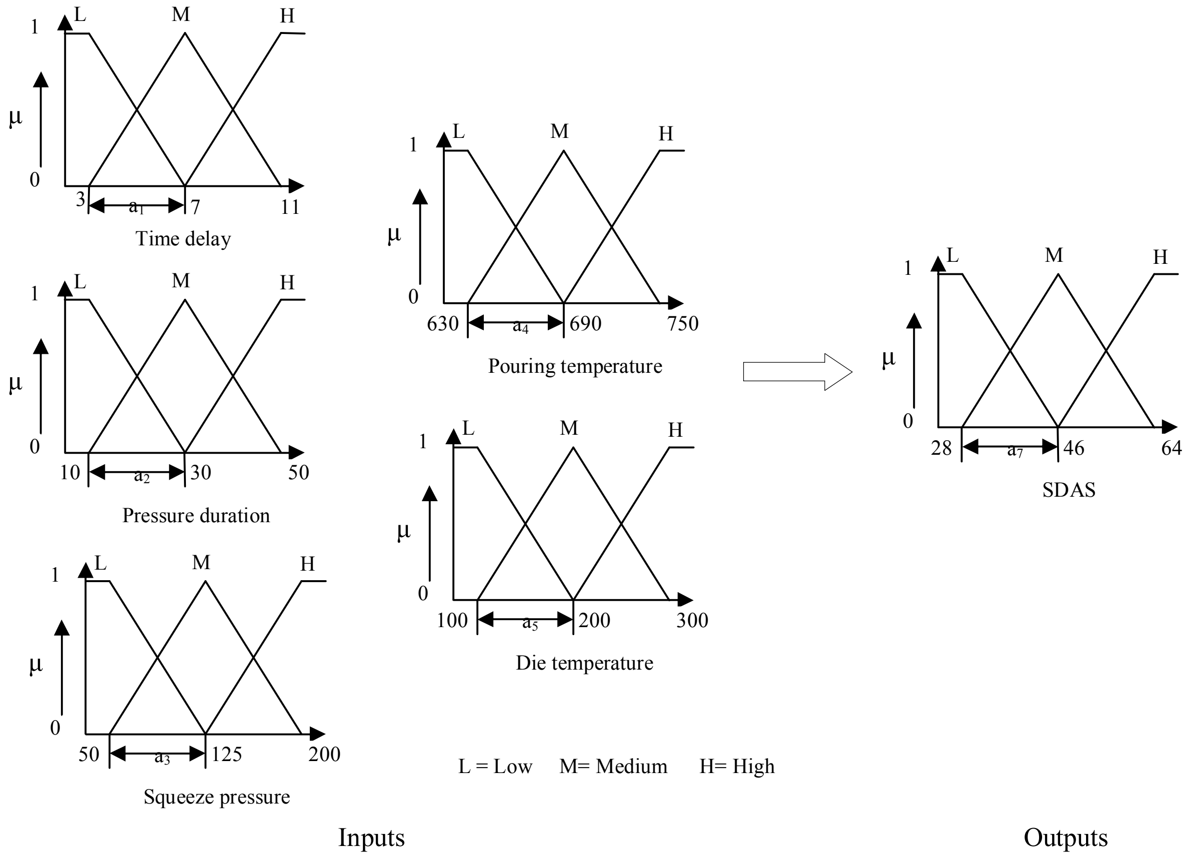


Fig. 4. Manually constructed membership function distribution for input-output variables

The base width of the right angle triangular membership function distributions namely $a_1$, $a_2$, $a_3$, $a_4$, $a_5$ and $a_6$ values are kept equal to 4, 20, 75, 60, 100 and 18 respectively. As there are five input variables and each input variable is expressed using three linguistic terms, the number of rules to be defined for the present system is found to be equal to 243 ($3 \times 3 \times 3 \times 3 \times 3$). The manually constructed rule base of the fuzzy logic system is shown in Table 3. The typical rule base of the fuzzy logic system will looks as shown below,

IF A is M AND B is H AND C is L AND D is H AND E is M THEN SDAS is M

It is important to mention that the knowledge base of the fuzzy system consists of rule base as well as data base. The manually constructed rule base depends completely on the human expertise in their relative field and is not considered to be optimal always. Hence attempts required to automatically emulate the rule and data base utilizing better learning capabilities of artificial neural networks (ANNs).

## 3.2. Approach 2:

Development of adaptive network based fuzzy interface system to automatically retrieve the data and derive the rule base

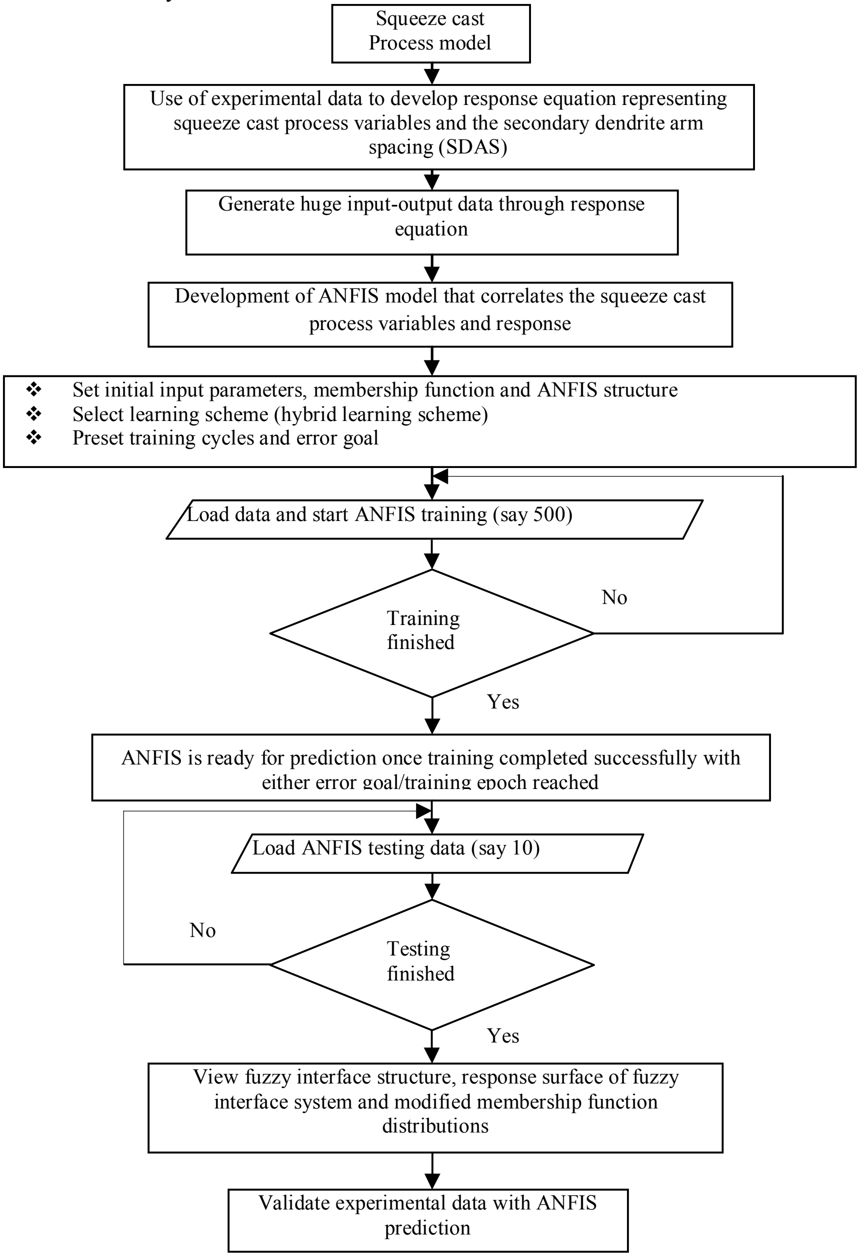


Fig. 5. Flow chart representing methodology followed for predicting density and SDAS using ANFIS

Artificial neural networks are excellent, cost effective modelling tool to map complex manufacturing processes, Parappagoudar and Vundavilli (2012) [17]. The reason might be due to better learning capabilities and generalize (forecast reasonable output for the inputs which are not used during the learning phase) (Hykin S. (2006)) [31]. Some of the ANNs limitations are precision of output is limited, problem with solutions getting trapped in local optimum , huge data that cover entire process variable range is required to train the network and large number of training epochs, Rajasekaran and Pai (2003) [32].

The fuzzy concepts are successfully implemented to address the problems related to the production and operation management field, Wong and Lai (2011) [33]. Greater flexibility in formulating system and capability to handle imprecise input-output data are the major advantages of fuzzy logic approach. It is important to note that, no systematic procedure available to define the membership function distributions is the major limitation of the fuzzy logic system, Yurdusev and Firat (2009) [34]. In recent years research efforts were made to develop hybrid systems to solve complex real world problems by combining the desirable features of artificial neural networks and fuzzy logic tools. The embedded type hybrid system developed to limit the weakness of one of the soft computing tool with the strengths of the other. Adaptive neuro-fuzzy interface system (ANFIS) is one such hybrid system found to have some attractive features such as easy to implement, better generalization capabilities, fast and precise learning, excellent descriptions via fuzzy rules, easy to include both numerical and linguistic knowledge for solving complex problems (Azar and Taher (2010)) [30]. It is important to note that some of the authors successfully implemented ANFIS model and proved it as the cost-effective modelling tool [35-37].

In ANFIS, an artificial neural network with the fuzzy system is used to automatically evolve the rule base. It is interesting to note that the ANFIS works with the use of hybrid learning algorithm (Gradient descent and least square estimator) to map the input-output relationships. The fuzzy rule is composed of both antecedent (includes membership function parameters and its shape) parameter and consequent (functional parameter of input signals which describes network output) parameters. The training of hybrid learning algorithm includes both forward and backward pass calculations. In forward computation the antecedent parameters are fixed initially and consequent parameters were identified by means of least square principle. The summation of outputs of consequent layer determines the network output. It is to be noted that the major objective of any training algorithm is to reduce the error (between the actual and network predicted), so the network parameters needs to updated and this can be accomplished with backward pass calculation. Later in backward pass calculation the consequent parameters are fixed and the premise parameters were updated by means of gradient descent method, Yurdusev and Firat (2009) [34]. The steps followed for the ANFIS model in the present work is shown in Fig. 5.

The structure of the adaptive neuro-fuzzy interface systems for the squeeze casting process looks is shown in the Fig. 6. The rectangle and circle symbols used in the network architecture indicate adaptive and fixed nodes respectively. Similar to artificial neural network architecture the network includes input, output and hidden layers. Squeeze cast process variables are expressed as a function of input nodes in the input layer, whereas secondary dendrite arm spacing function as an output node. The nodes functioning in the hidden layer includes membership function and the rules. In the present work there exists five inputs and one output parameter. Each input parameter is expressed in the form of three linguistic terms and the 243 possible combinations of rules exist. For the first-order Takagi and Sugeno's model a typical output with three fuzzy rules can be expressed as shown in Eq. [3], [4] and [5].

Rule 1: if ($T_d$ is $A_1$) and ($D_p$ is $B_1$) and ($S_p$ is $C_1$) and ($P_t$ is $D_1$) and ($D_t$ is $E_1$) then

$$f_1 = p_1T_d + q_1D_p + r_1S_p + s_1P_t + t_1D_t + u_1 \tag{3}$$

Rule 2: if ($T_d$ is $A_2$) and ($D_p$ is $B_2$) and ($S_p$ is $C_2$) and ($P_t$ is $D_2$) and ($D_t$ is $E_2$) then

$$f_2 = p_2T_d + q_2D_p + r_2S_p + s_2P_t + t_2D_t + u_2 \tag{4}$$

Rule 3: ($T_d$ is $A_i$) and ($D_p$ is $B_i$) and ($S_p$ is $C_i$) and ($P_t$ is $D_i$) and ($D_t$ is $E_i$) then

$$f_i = p_iT_d + q_iD_p + r_iS_p + s_iP_t + t_iD_t + u_i \tag{5}$$

Where, i = 1, 2, 3, ..........243, $p_i$, $q_i$, $r_i$, $s_i$, $t_i$ and $u_i$ are the consequent parameters, f is the output parameter, $A_i$, $B_i$, $C_i$, $D_i$ and $E_i$ are the linguistic labels used to define the membership function.

The adaptive network based fuzzy interface system architecture consists of six layers namely input layer, fuzzification layer, product layer, normalization layer, de-fuzzification layer and output layer. The systematic procedure in developing the input-output relationship and the proper functioning of each layer is described as follows,

***Layer 1:*** In layer 1, squeeze cast process variables are expressed as a function of input nodes of the input layer. The layer 1 transmits the same input values to the next corresponding layer using linear transformation function.

***Layer 2:*** The layer 2 is the fuzzification layer, in which membership function values are determined corresponding to the assigned linguistic labels shown in Eq. [6], Eq. [7], Eq. [8], Eq. [9] and Eq. [10]. $T_d$, $D_p$, $S_p$, $P_t$ and $D_t$ are the input nodes expressed as membership functions in terms of $A_i$, $B_i$, $C_i$, $D_i$ and $E_i$ of layer 2. Where $O_{2,i}$ is the output of $i^{th}$ node of layer 2.

$$O_{2,i} = \mu A_i\,(T_d) \qquad \text{for } i = 1, 2, 3 \tag{6}$$

$$O_{2,1} = \mu B_{i-3}(D_p) \qquad \text{for } i = 4, 5, 6 \tag{7}$$

$$O_{2,i} = \mu C_{i-6}(S_p) \qquad \text{for } i = 7, 8, 9 \tag{8}$$

$$O_{2,i} = \mu D_{i-9}(P_t) \qquad \text{for } i = 10, 11, 12 \tag{9}$$

$$O_{2,i} = \mu E_{i-12}(D_t) \qquad \text{for } i = 13, 14, 15 \tag{10}$$

Triangular, generalized bell shape and gaussian are the most commonly used membership functions and the values always lies between zero and one corresponding to the input conditions.

***Layer 3:*** The layer 3 referred as product layer, which determines the number of all possible rules ($3^5$=243), 243 nodes present in the layer 3 and is usually labelled using the term Π. A maximum of 32 nodes will be activated for the particular set of input conditions and each node represents the possible combination of input variables. The information from layer 2 is received and generates the output by multiplying all the input signals as shown in Eq. [11].

$$O_{3,i} = w_i = \mu A_i(T_d) \times \mu B_{i-3}(D_p) \times \mu C_{i-6}(S_p) \times \mu D_{i-9}(P_t) \times \mu E_{i-12}(D_t) \quad (11)$$

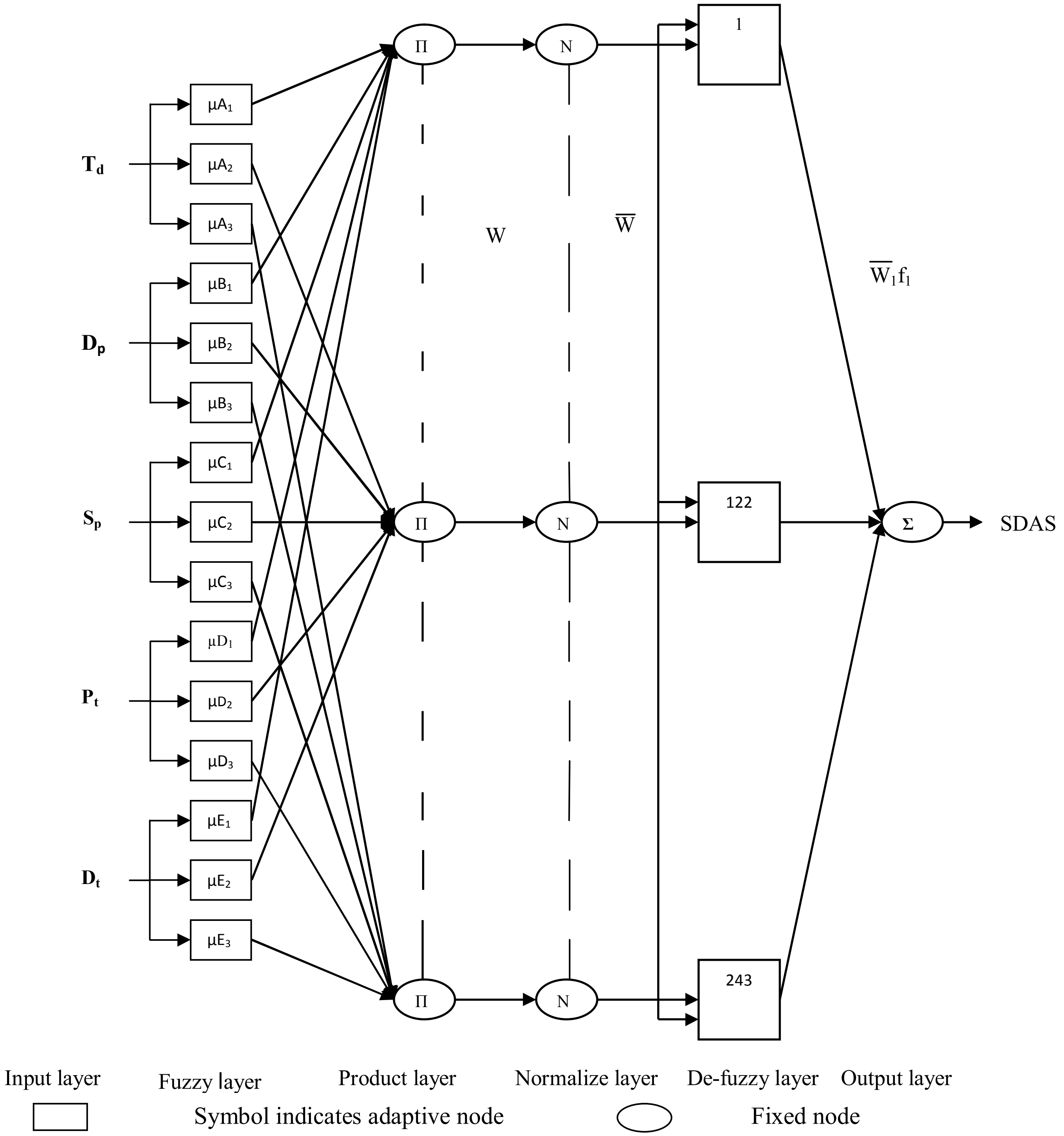


Fig. 6. ANFIS network architecture for predicting the response-SDAS

***Layer 4:*** The layer 4 is also referred as normalization layer and nodes presented in that layer is usually labelled as N. The major function of this layer is to normalize the weight functions using Eq. [12]. The output calculation of $i^{th}$ node is the ratio of $i^{th}$ rule firing strength to the sum of all the fired rules.

$$O_{4,i} = \bar{w}_i = w_i/(w_1 + w_2 + w_3 + \cdots + w_{243}) \qquad (12)$$

***Layer 5:*** The fifth layer is termed as the de-fuzzification (centre of area method) layer and each node is calculated using the product of all the normalized firing strengths and the output of the corresponding fired rule is calculated using Eq. [13]. The layer 5 consists of 243 nodes and maximum of 32 nodes will be activated for the particular input variables combination.

$$O_{5,i} = \bar{w}_i f_i \, \bar{w}_i \left(p_i T_d + q_i D_p + r_i S_p + s_i P_t + t_i D_t + u_i\right) \quad (13)$$

***Layer 6:*** The last layer is the output layer, since only one output variable is used for the present study only single node is present. The output calculation is performed using the summation of all the received input signals from the $5^{th}$ layer shown in Eq. [14].

$$O_{6,i} = \sum_i \bar{w}_i f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \qquad (14)$$

# 4. Results and discussion

The performance of the developed models to predict secondary dendrite arm spacing in squeeze casting has been tested with the help of 10 test cases generated at random. The following section presents the information about the results obtained and comparison of developed model performances with experimental values.

### 4.1. Approach 1

The approach 1 deals with the manual construction of the rule and data base of the fuzzy logic system with the help of human expertise. For simplicity, linear type triangular shape membership function distribution is used for the present system and shown in Fig. 3. The base width of 'a' values shown in the triangular shape membership function distributions is same as explained in the earlier section. The performance of both approaches has been compared. Further, for each of the approach comparison of performance has been made for three different membership function distribution. The performance comparison results are summarized and presented in Table 5. The prediction accuracy of the developed models relies majorly on accurate construction of the rule base defined with the knowledge of human expertise. The developed manually constructed rule base to predict the secondary dendrite arm spacing is presented in Table 4.

Table 4.
Manually constructed rule base of the fuzzy logic system

| Rule No. | A | B | C | D | E | SDAS, μm | Rule No. | A | B | C | D | E | SDAS, μm | Rule No. | A | B | C | D | E | SDAS, μm |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | L | L | L | L | L | H | 82 | M | L | L | L | L | H | 163 | H | L | L | L | L | H |
| 2 | L | L | L | L | M | M | 83 | M | L | L | L | M | H | 164 | H | L | L | L | M | H |
| 3 | L | L | L | L | H | M | 84 | M | L | L | L | H | H | 165 | H | L | L | L | H | H |
| 4 | L | L | L | M | L | M | 85 | M | L | L | M | L | H | 166 | H | L | L | M | L | H |
| 5 | L | L | L | M | M | M | 86 | M | L | L | M | M | M | 167 | H | L | L | M | M | H |
| 6 | L | L | L | M | H | M | 87 | M | L | L | M | H | M | 168 | H | L | L | M | H | H |
| 7 | L | L | L | H | L | M | 88 | M | L | L | H | L | M | 169 | H | L | L | H | L | H |
| 8 | L | L | L | H | M | M | 89 | M | L | L | H | M | M | 170 | H | L | L | H | M | M |
| 9 | L | L | L | H | H | M | 90 | M | L | L | H | H | M | 171 | H | L | L | H | H | H |
| 10 | L | L | M | L | L | M | 91 | M | L | M | L | L | H | 172 | H | L | M | L | L | H |
| 11 | L | L | M | L | M | M | 92 | M | L | M | L | M | M | 173 | H | L | M | L | M | H |
| 12 | L | L | M | L | H | M | 93 | M | L | M | L | H | M | 174 | H | L | M | L | H | H |
| 13 | L | L | M | M | L | M | 94 | M | L | M | M | L | M | 175 | H | L | M | M | L | H |
| 14 | L | L | M | M | M | M | 95 | M | L | M | M | M | M | 176 | H | L | M | M | M | M |
| 15 | L | L | M | M | H | M | 96 | M | L | M | M | H | M | 177 | H | L | M | M | H | M |
| 16 | L | L | M | H | L | M | 97 | M | L | M | H | L | M | 178 | H | L | M | H | L | M |
| 17 | L | L | M | H | M | M | 98 | M | L | M | H | M | M | 179 | H | L | M | H | M | M |
| 18 | L | L | M | H | H | M | 99 | M | L | M | H | H | M | 180 | H | L | M | H | H | M |
| 19 | L | L | H | L | L | M | 100 | M | L | H | L | L | M | 181 | H | L | H | L | L | H |
| 20 | L | L | H | L | M | M | 101 | M | L | H | L | M | M | 182 | H | L | H | L | M | M |
| 21 | L | L | H | L | H | M | 102 | M | L | H | L | H | M | 183 | H | L | H | L | H | M |
| 22 | L | L | H | M | L | L | 103 | M | L | H | M | L | M | 184 | H | L | H | M | L | M |
| 23 | L | L | H | M | M | M | 104 | M | L | H | M | M | M | 185 | H | L | H | M | M | M |
| 24 | L | L | H | M | H | M | 105 | M | L | H | M | H | M | 186 | H | L | H | M | H | M |
| 25 | L | L | H | H | L | L | 106 | M | L | H | H | L | M | 187 | H | L | H | H | L | M |

| | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 26 | L | L | H | H | M | L | 107 | M | L | H | H | M | M | 188 | H | L | H | H | M | M |
| 27 | L | L | H | H | H | M | 108 | M | L | H | H | H | M | 189 | H | L | H | H | H | M |
| 28 | L | M | L | L | L | M | 109 | M | M | L | L | L | H | 190 | H | M | L | L | L | H |
| 29 | L | M | L | L | M | M | 110 | M | M | L | L | M | M | 191 | H | M | L | L | M | H |
| 30 | L | M | L | L | H | M | 111 | M | M | L | L | H | H | 192 | H | M | L | L | H | H |
| 31 | L | M | L | M | L | M | 112 | M | M | L | M | L | M | 193 | H | M | L | M | L | H |
| 32 | L | M | L | M | M | M | 113 | M | M | L | M | M | M | 194 | H | M | L | M | M | M |
| 33 | L | M | L | M | H | M | 114 | M | M | L | M | H | M | 195 | H | M | L | M | H | H |
| 34 | L | M | L | H | L | M | 115 | M | M | L | H | L | M | 196 | H | M | L | H | L | H |
| 35 | L | M | L | H | M | M | 116 | M | M | L | H | M | M | 197 | H | M | L | H | M | M |
| 36 | L | M | L | H | H | M | 117 | M | M | L | H | H | M | 198 | H | M | L | H | H | M |
| 37 | L | M | M | L | L | M | 118 | M | M | M | L | L | M | 199 | H | M | M | L | L | H |
| 38 | L | M | M | L | M | M | 119 | M | M | M | L | M | M | 200 | H | M | M | L | M | M |
| 39 | L | M | M | L | H | M | 120 | M | M | M | L | H | M | 201 | H | M | M | L | H | H |
| 40 | L | M | M | M | L | M | 121 | M | M | M | M | L | M | 202 | H | M | M | M | L | M |
| 41 | L | M | M | M | M | L | 122 | M | M | M | M | M | M | 203 | H | M | M | M | M | M |
| 42 | L | M | M | M | H | M | 123 | M | M | M | M | H | M | 204 | H | M | M | M | H | M |
| 43 | L | M | M | H | L | M | 124 | M | M | M | H | L | M | 205 | H | M | M | H | L | M |
| 44 | L | M | M | H | M | L | 125 | M | M | M | H | M | M | 206 | H | M | M | H | M | M |
| 45 | L | M | M | H | H | L | 126 | M | M | M | H | H | M | 207 | H | M | M | H | H | M |
| 46 | L | M | H | L | L | M | 127 | M | M | H | L | L | M | 208 | H | M | H | L | L | M |
| 47 | L | M | H | L | M | M | 128 | M | M | H | L | M | M | 209 | H | M | H | L | M | M |
| 48 | L | M | H | L | H | M | 129 | M | M | H | L | H | M | 210 | H | M | H | L | H | M |
| 49 | L | M | H | M | L | M | 130 | M | M | H | M | L | M | 211 | H | M | H | M | L | M |
| 50 | L | M | H | M | M | L | 131 | M | M | H | M | M | M | 212 | H | M | H | M | M | M |
| 51 | L | M | H | M | H | L | 132 | M | M | H | M | H | M | 213 | H | M | H | M | H | M |
| 52 | L | M | H | H | L | L | 133 | M | M | H | H | L | M | 214 | H | M | H | H | L | M |
| 53 | L | M | H | H | M | L | 134 | M | M | H | H | M | L | 215 | H | M | H | H | M | M |
| 54 | L | M | H | H | H | L | 135 | M | M | H | H | H | M | 216 | H | M | H | H | H | M |
| 55 | L | H | L | L | L | M | 136 | M | H | L | L | L | H | 217 | H | H | L | L | L | H |
| 56 | L | H | L | L | M | M | 137 | M | H | L | L | M | M | 218 | H | H | L | L | M | H |
| 57 | L | H | L | L | H | M | 138 | M | H | L | L | H | M | 219 | H | H | L | L | H | H |
| 58 | L | H | L | M | L | M | 139 | M | H | L | M | L | M | 220 | H | H | L | M | L | H |
| 59 | L | H | L | M | M | M | 140 | M | H | L | M | M | M | 221 | H | H | L | M | M | M |
| 60 | L | H | L | M | H | M | 141 | M | H | L | M | H | M | 222 | H | H | L | M | H | M |
| 61 | L | H | L | H | L | M | 142 | M | H | L | H | L | M | 223 | H | H | L | H | L | M |
| 62 | L | H | L | H | M | L | 143 | M | H | L | H | M | M | 224 | H | H | L | H | M | M |
| 63 | L | H | L | H | H | M | 144 | M | H | L | H | H | M | 225 | H | H | L | H | H | M |
| 64 | L | H | M | L | L | M | 145 | M | H | M | L | L | M | 226 | H | H | M | L | L | M |
| 65 | L | H | M | L | M | M | 146 | M | H | M | L | M | M | 227 | H | H | M | L | M | M |
| 66 | L | H | M | L | H | M | 147 | M | H | M | L | H | M | 228 | H | H | M | L | H | M |
| 67 | L | H | M | M | L | M | 148 | M | H | M | M | L | M | 229 | H | H | M | M | L | M |
| 68 | L | H | M | M | M | L | 149 | M | H | M | M | M | M | 230 | H | H | M | M | M | M |
| 69 | L | H | M | M | H | L | 150 | M | H | M | M | H | M | 231 | H | H | M | M | H | M |
| 70 | L | H | M | H | L | L | 151 | M | H | M | H | L | M | 232 | H | H | M | H | L | M |
| 71 | L | H | M | H | M | L | 152 | M | H | M | H | M | M | 233 | H | H | M | H | M | M |
| 72 | L | H | M | H | H | L | 153 | M | H | M | H | H | M | 234 | H | H | M | H | H | M |
| 73 | L | H | H | L | L | M | 154 | M | H | H | L | L | M | 235 | H | H | H | L | L | M |
| 74 | L | H | H | L | M | L | 155 | M | H | H | L | M | M | 236 | H | H | H | L | M | M |
| 75 | L | H | H | L | H | L | 156 | M | H | H | L | H | M | 237 | H | H | H | L | H | M |
| 76 | L | H | H | M | L | L | 157 | M | H | H | M | L | M | 238 | H | H | H | M | L | M |
| 77 | L | H | H | M | M | L | 158 | M | H | H | M | M | M | 239 | H | H | H | M | M | M |
| 78 | L | H | H | M | H | L | 159 | M | H | H | M | H | M | 240 | H | H | H | M | H | M |
| 79 | L | H | H | H | L | L | 160 | M | H | H | H | L | M | 241 | H | H | H | H | L | M |
| 80 | L | H | H | H | M | L | 161 | M | H | H | H | M | L | 242 | H | H | H | H | M | M |
| 81 | L | H | H | H | H | L | 162 | M | H | H | H | H | L | 243 | H | H | H | H | H | M |

## 4.2. Approach 2

The steps followed to predict the secondary dendrite arm spacing (SDAS) using ANFIS model is shown in the Fig. 4. It is to be noted that huge amount of training data (say 500) is generated artificially at random by utilizingthe response equation, obtained earlier by the same authors. Five input parameter (time delay, squeeze pressure, pressure duration, pouring temperature and die temperature) and single output (SDAS) parameter have been considered to develop input-output relationship in squeeze casting using ANFIS model and is shown schematically in Fig. 5. As explained in the previous sections both linear (triangular) and non-linear (generalized bell shape and gaussian) type membership function distributions have been used and the prediction performance of the developed models are compared with ten different test cases. The test results are summarized in Table 5. The input-output values of ten different squeeze casting conditions (that is test cases, collected through experiments) are presented in Table 6.

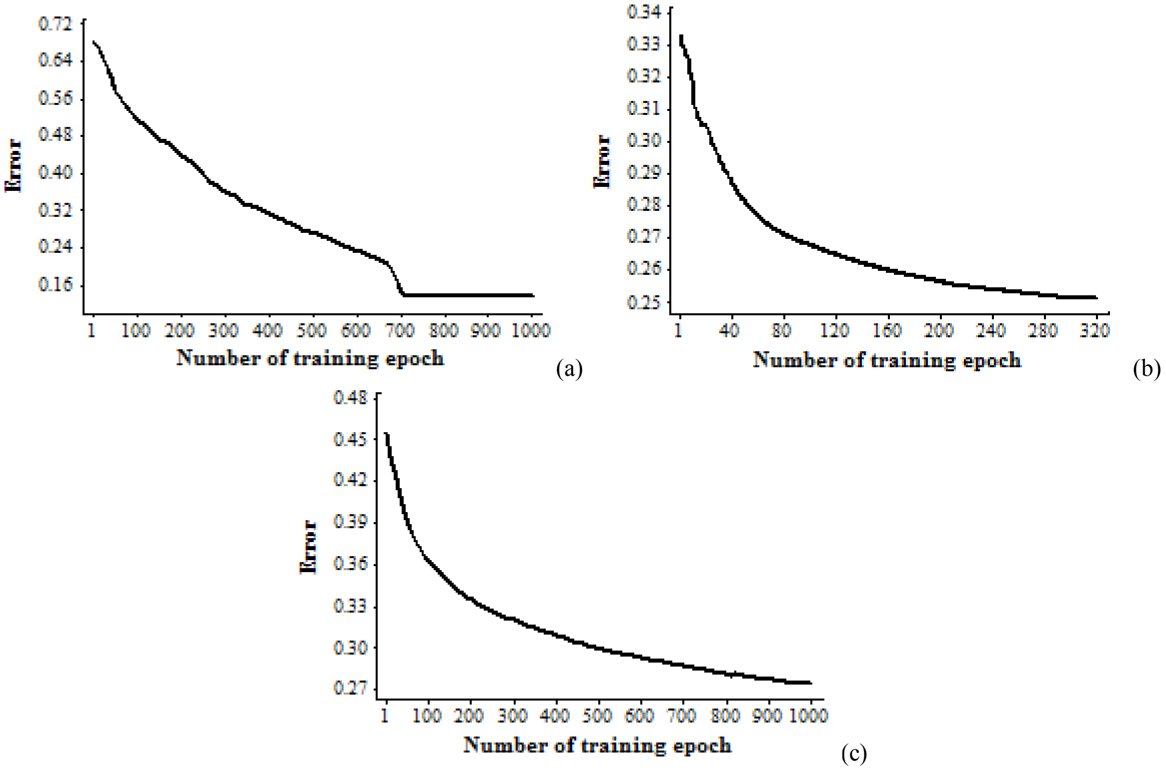


Fig. 7. Convergence of ANFIS training (RMSE v/s Number of training epoch) for response-SDAS; (a) Triangular membership function, (b) Generalized bell shape membership function and (c) Gaussian membership function distribution

It is of paramount importance to note that the prediction accuracy of the developed models rely on the closeness of the predicted and the actual values during training and is usually measured using root mean squared error (RMSE). The root mean squared error obtained at the end of the training for different membership function distributions for the response secondary dendrite arm spacing is shown in Fig. 7.

## 4.3. Comparison of the developed models

The performance of developed models (that is Approach1 and Approach 2 with different membership distribution functions) has been compared with the help of test cases. The results are summarized in Table 5.

### 4.3.1. Approach 1

It is interesting to note that the prediction of triangular membership function distribution shown in Fig. 8 (a) are close to the ideal line as compared with Fig. 8 (b) of generalized bell shape and Fig. 8 (c) of gaussian membership function distributions respectively. The summary results of the test cases in predicting the SDAS is presented in Table. 2. However, the accuracy and prediction capability of the developed model performances are evaluated based on mean absolute percentage error (Refer Eq. [15]). 8.911, 8.888 and 9.408 are the values of mean absolute percentage deviation obtained in predicting the SDAS for triangular, generalized bell shape and gaussian membership function distribution respectively and the same is presented in Table 5.

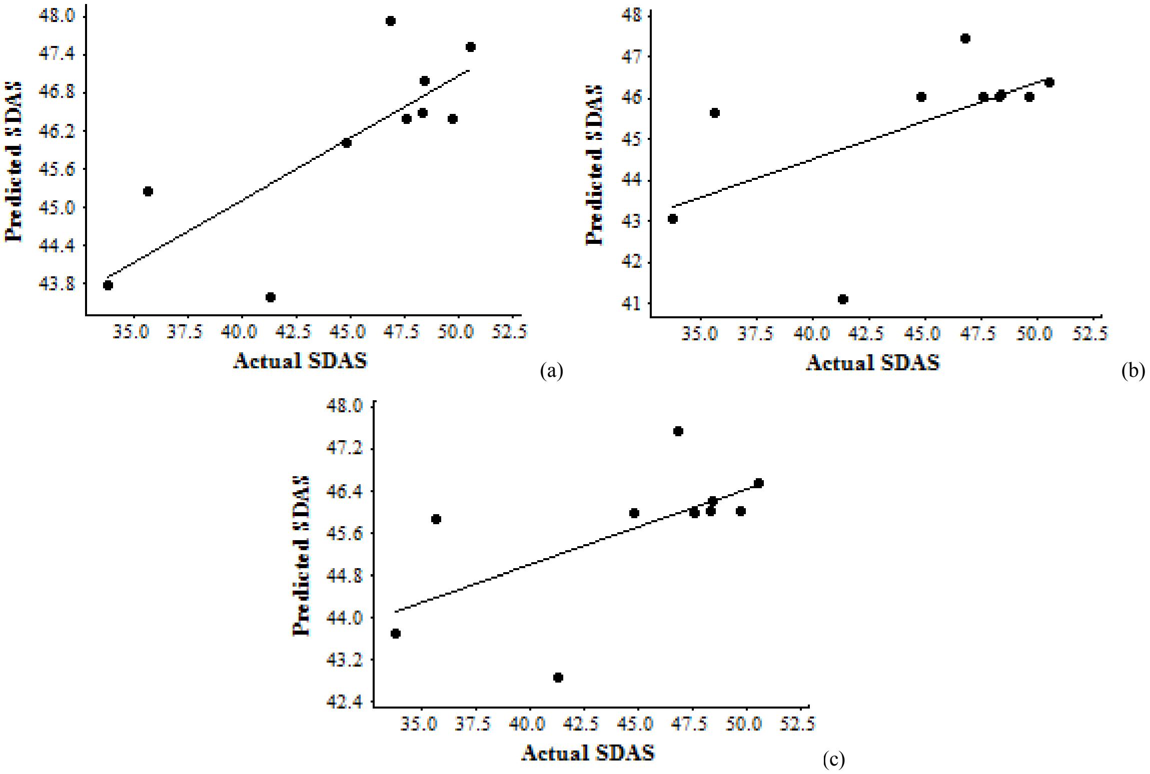


Fig. 8. Comparison of predicted and actual SDAS via approach 1, (a) Triangular membership function distributions, (b) Bell shape membership function distributions and (c) Gaussian membership function distributions

The mean absolute percent error is obtained by using the following equation:

$$MAPE = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{O_i - P_i}{O_i}\right| \tag{15}$$

Where $P$ is the predicted, n is the number of data sets and O is the observed values.

### 4.3.2. Approach 2

In approach 2, ANNs receives fuzzy inputs, processes it and extracts fuzzy outputs. The function of artificial neural networks in this approach is to automatically define the structure, tune the fuzzy parameters, rule base, data base and membership function distributions. The performance of the developed models can be further enhanced by utilizing different membership function distributions [26 and 27]. It is also important to note that performance of the developed models also rely on the quantity and quality of the training data, degree of closeness with actual and network predicted values and is usually determined using the root mean squared error at the end of the training. Five hundred set of input-output data is used for training and the network training was terminated with the error reaching steady state. The RMSE values obtained at the end of the training for triangular, generalized bell shape and gaussian membership function distributions are found to be equal to 0.1328,0.2512 and 0.2734 respectively (Refer Fig. 6). The adjusted 'a' values of six variables for different membership function distributions are presented in Table 5

Table 5.
The optimized or adjusted 'a' values of fuzzy parameters

| Membership function distributions | Half base width of right angled triangle | | | | | |
|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ |
| Triangular | 3.7786 | 19.3613 | 74.8241 | 59.3171 | 98.9814 | 18.2993 |
| Generalized bell | 3.4442 | 19.6671 | 74.8939 | 60.0567 | 100.1465 | 18.2993 |
| Gaussian | 3.8413 | 19.4823 | 74.8434 | 59.8642 | 99.1276 | 18.2993 |

The developed models performance is compared with the help of few experimental test cases and shown in Fig. 8. The best fit line is used to compare the model predicted and the actual values of SDAS. It is interesting to note that the best fit line of all the models looks similar. However, the triangular membership function shown in Fig. 9 (a) performs slightly better in comparison with Fig. 9 (b) of generalized bell shape and Fig. 9(c) of gaussian membership function distributions. It is also important to observe that majority of the data points of triangular shape membership function of the fuzzy logic system falls close to the ideal line compared to rest. The summary result of the test cases for secondary dendrite arm spacing prediction is presented in the Table 6. Moreover the developed model performances are evaluated by means of MAPE values for 10 different test cases. The mean absolute percent error values for triangular, generalized and Gaussian membership function distribution are found to be equal to 4.571, 5.298 and 5.422 respectively (Refer Table 5).

Table 6.
Summary results of test cases of fuzzy models for the response SDAS

| Test case no | Actual SDAS, μm | Approach 1 | | | | | | Approach 2 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Triangular | | G bell shape | | Gaussian | | Triangular | | G bell shape | | Gaussian | |
| | | Predicted | Abs.% deviation | Predicted | Abs.% deviation | Predicted | Abs. deviation | Predicted | Abs.% deviation | Predicted | Abs. % deviation | Predicted | Abs. % deviation |
| 1 | 48.43 | 46.96 | 3.035 | 46.08 | 4.852 | 46.20 | 4.605 | 53.11 | 9.663 | 52.64 | 8.693 | 52.71 | 8.837 |
| 2 | 49.74 | 46.38 | 6.755 | 46.00 | 7.519 | 46.01 | 7.499 | 53.53 | 7.620 | 54.07 | 8.705 | 53.36 | 7.278 |
| 3 | 47.64 | 46.37 | 2.666 | 46.02 | 3.401 | 45.99 | 3.463 | 47.85 | 0.441 | 47.74 | 0.210 | 47.62 | 0.042 |
| 4 | 33.78 | 43.77 | 29.574 | 43.05 | 27.442 | 43.69 | 29.34 | 33.89 | 0.326 | 35.69 | 5.654 | 32.43 | 3.996 |
| 5 | 46.86 | 47.91 | 2.241 | 47.44 | 1.238 | 47.55 | 1.472 | 49.44 | 5.506 | 49.24 | 5.079 | 49.42 | 5.463 |
| 6 | 48.33 | 46.48 | 3.836 | 46.00 | 4.829 | 46.01 | 4.808 | 45.79 | 5.263 | 45.76 | 5.325 | 45.56 | 5.739 |
| 7 | 50.63 | 47.50 | 6.182 | 46.37 | 8.414 | 46.56 | 8.039 | 52.62 | 3.930 | 52.58 | 3.851 | 52.51 | 3.713 |
| 8 | 44.86 | 46.00 | 2.541 | 46.00 | 2.541 | 46.00 | 2.541 | 42.75 | 4.704 | 43.55 | 2.920 | 42.02 | 6.331 |
| 9 | 35.66 | 45.24 | 26.865 | 45.64 | 27.987 | 45.88 | 28.660 | 38.32 | 7.459 | 39.59 | 11.021 | 39.49 | 10.740 |
| 10 | 41.34 | 43.58 | 5.418 | 41.07 | 0.653 | 42.85 | 3.653 | 41.01 | 0.798 | 40.71 | 1.524 | 40.48 | 2.080 |
| MAPE | | | 8.911 | | 8.888 | | 9.408 | | 4.571 | | 5.298 | | 5.422 |

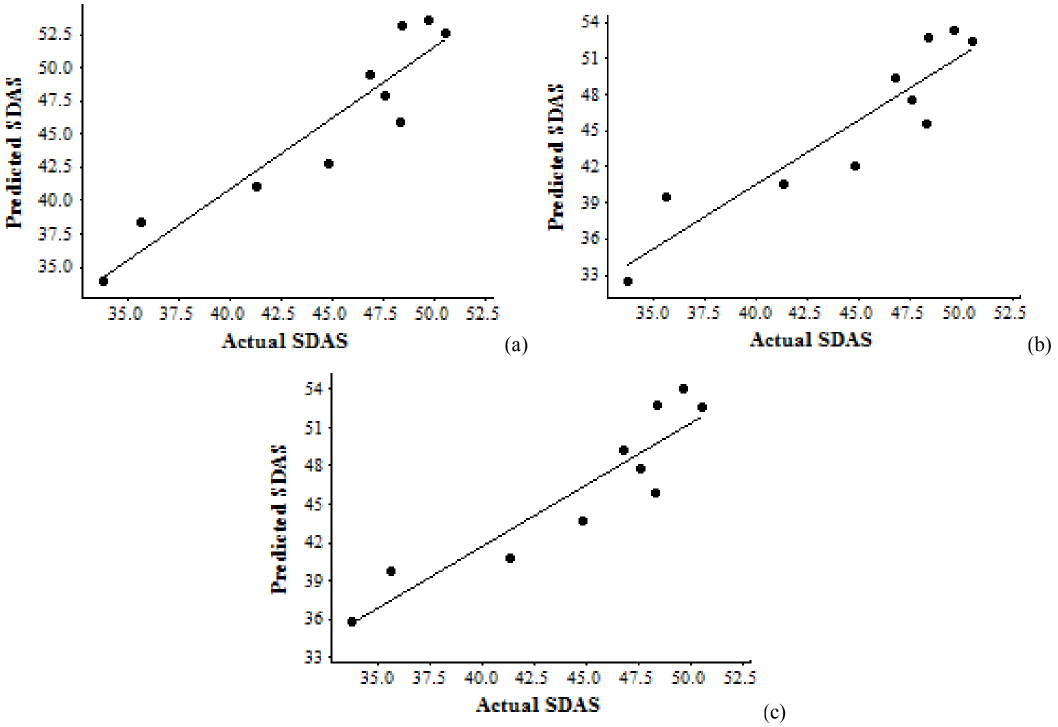


Fig. 9. Comparison of predicted and actual SDAS values using approach 2; (a) Triangular membership function, (b) Bell shape membership function and (c) Gaussian membership function distribution

## 4.4. Comparison of the developed approaches using percent deviations

The variation in predicting the SDAS for approach 1 and approach 2 is shown in Fig. 10 (a) and Fig. 10 (b) respectively. The percent deviation in predicting SDAS values for both the approaches using triangular membership function distribution is found to lie in the range of (-29.574%, +6.755%) for approach 1 and (-9.664%, +5.263%) for approach 2. Similarly, for generalized bell shape and gaussian membership function distribution the percent deviation in prediction values are found to vary in the range of (-27.987%, +8.414%), (-29.337%, +8.0387%) for approach 1 and (-11.208%, +5.325%), (-10.740%, +5.739%) for approach 2 (Refer Figs. 10). It is interesting to note that the percent deviation pattern is found to be similar for all three models (that is, approach 1 with different membership function distributions, Refer Fig. 10 (a)).

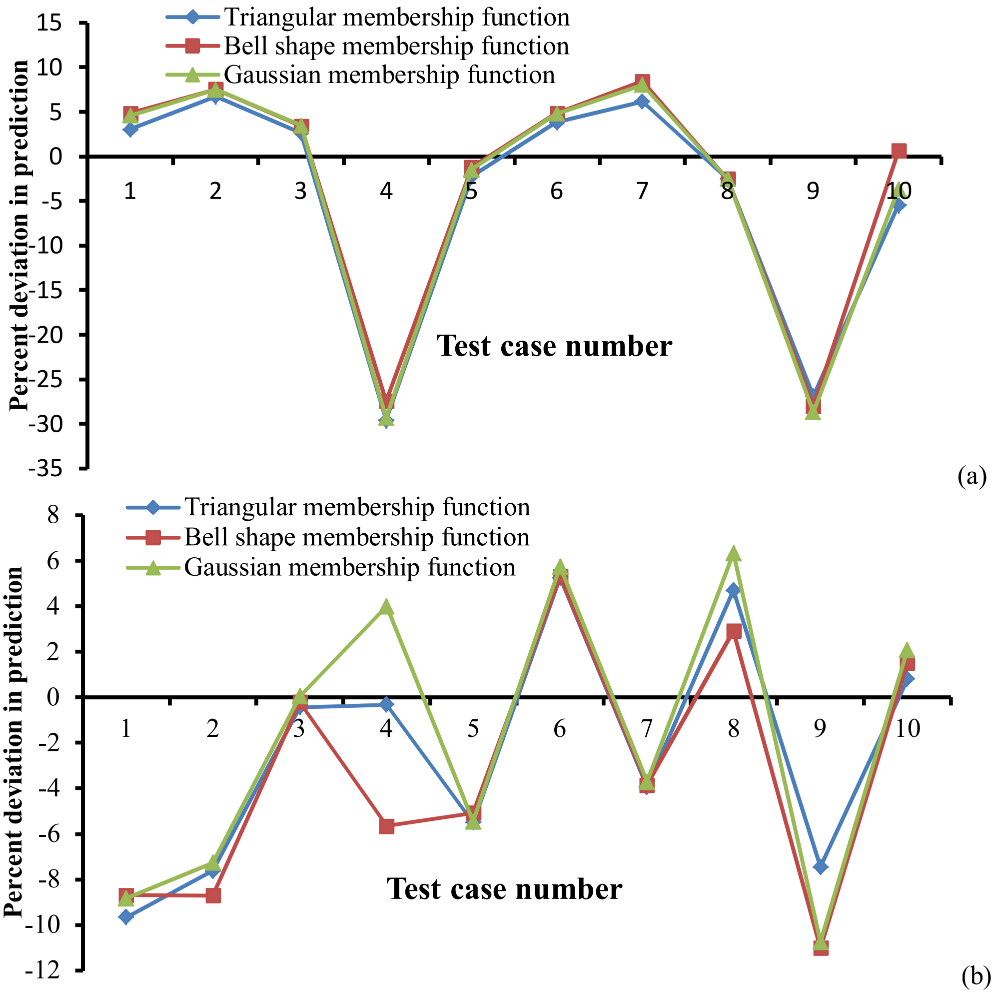


Fig. 10. Comparison of different approaches of the developed models with different membership function distribution in terms of percent deviation in prediction for the response-SDAS, (a) Manually constructed fuzzy logic system and (b) Adaptive network based fuzzy logic system

## 4.5. Comparison of the developed approaches using average absolute percent deviation

The prediction ability of the developed approaches, with three different membership function distributions have been evaluated using mean absolute percent error and shown in Fig. 11. It is also important to note that prediction of approach 2 would show slightly better performance as compared with approach 1. However, it is has been observed that the performance of the approach 2 also varies with linear and non-linear type membership function distributions, the reason might be due to the nature of error surface during training. The improved performance of approach 2 might be due to the automatic evolution of antecedent and consequent parts of fuzzy logic system utilizing huge training data through better learning capabilities of artificial neural networks. On the other hand, the antecedent and the consequent parts of the fuzzy logic system developed in approach

1 is with the help of human expertise and are not considered to be optimal always. It is important to note that the generalized bell shape of approach 1 and triangular shape membership function of approach 2 perform better compared to other membership function distributions. Moreover, the triangular shape membership function distributions of approach 2 outperforms all other models in terms of mean absolute percent error as shown in Fig. 11

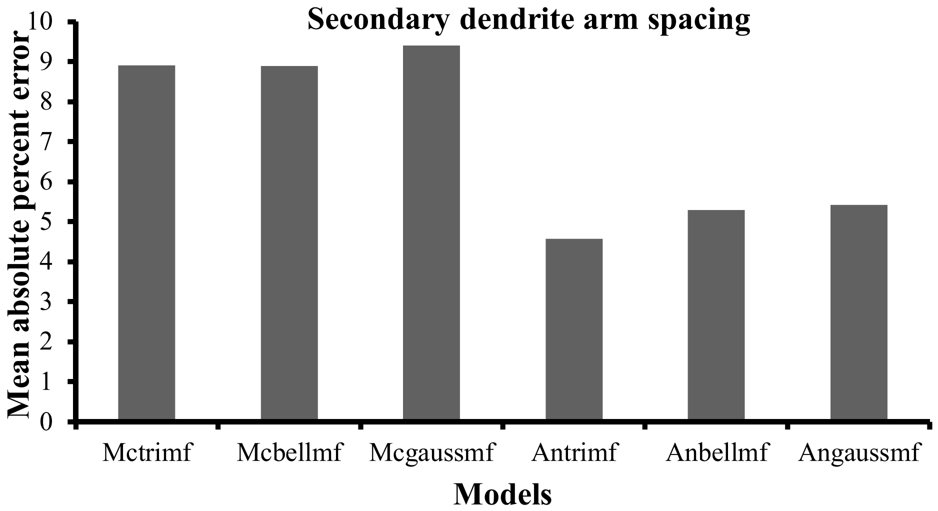


Fig. 11. Comparison of different model performances in terms of average absolute percent deviation in prediction for the responses-Secondary dendrite arm spacing

## 5. Concluding remarks

An attempt has been made to carry out the forward mapping to predict the secondary dendrite arm spacing using Mamdani model and Takagi Sugeno model (ANFIS) of the fuzzy logic based approaches. It is to be noted that antecedent and the consequent parts of the fuzzy logic system is designed based on the human expertise for Mamdani model, whereas, for Takagi and Sugenos model the antecedent and consequent parts are automatically evolved by utilizing huge training data with the better learning capabilities of artificial neural networks. Batch mode of training has been employed, with huge training data (say 500) base for better training and accurate prediction. Huge data base collection through real experiments is impractical to achieve and are generated artificially at random using response equation obtained through real experiments. Ten different test cases generated at random were used to compare the performance of the developed approaches with both linear and non-linear type membership function distributions. It is also important to note that the test data collected through real experiments are not used during the learning phase of the ANFIS system. It is interesting to note that generalized bell shape membership function of approach 1 and triangular membership of approach 2 made better predictions. Moreover the approach 2 prediction performed better compared to approach 1 in the present case. The improved prediction performance of approach 2 depends mainly on the quality and quantity of the data used for training, membership function distributions and the nature of the error surface. In addition the prediction performance of also depends on the linear or non-linear behaviour of the response. It is interesting to note that all models with different membership function distributions of the fuzzy logic based approaches are capable of making prediction for SDAS values in squeeze casting. It is also important to mention that the approach 1 predictions can be further enhanced with increasing the number of linguistic terms, which will increase the number of rules and computational complexity. The developed fuzzy logic models can be effectively used for making predictions of secondary dendrite arm spacing at different squeeze casting conditions and eliminate the need of extensive experimental work. The present work is of paramount importance for the foundry men for the selection of most influential parameters to achieve the desired casting quality in squeeze casting process.


## Acknowledgement

The authors greatly acknowledge Dept. of Applied Mechanics and Hydraulics of NIT Karnataka, India, for their kind co-operation in carrying out the real experiments.


## References


[1] Miller, W.S., Zhuang, L., Bottema, J. & Wittebrood, A. J. (2000). Recent development in aluminium alloys for the automotive industry. *Materials Science and Engineering, A.* 280(1). 37-49.

[2] Hegde, S. & Prabhu, K. N. (2008). Modification of eutectic silicon in Al–Si alloys. *Journal of materials science.* 43(9). 3009-3027.

[3] Maleki, A. Shafyei, A. & Niroumand, B. (2009). Effects of squeeze casting parameters on the microstructure of LM13

alloy. *Journal of Materials Processing Technology.* 209.8, 3790-3797.
[4] Hosseini, V. A. Shabestari, S. G. & Gholizadeh, R. (2013). Study on the effect of cooling rate on the solidification parameters, microstructure, and mechanical properties of LM13 alloy using cooling curve thermal analysis technique. *Materials & Design.* 50, 7-14.
[5] Britnell, D.J. & Neailey, K. (2003). Macrosegregation in thin walled castings produced via the direct squeeze casting process. *Journal of materials processing technology*. 138(1), 306-310.
[6] Lee, J. H., Kim, H.S., Won, C.W. & Cantor, B. (2002). Effect of the gap distance on the cooling behavior and the microstructure of indirect squeeze cast and gravity die cast 5083 wrought Al alloy. *Materials Science and Engineering: A*. 338(1), 182-190.
[7] Yang, L. J. (2007). The effect of solidification time in squeeze casting of aluminium and zinc alloys. *Journal of materials processing technology*. 192, 114-120.
[8] Yue, T. M. (1997). Squeeze casting of high-strength aluminium wrought alloy AA7010. *Journal of materials processing technology*. 66(1), 179-185.
[9] Zhang, M. Zhang, W. W., Zhao, H. D., Zhang, D. T. & Li, Y. (2007). Effect of pressure on microstructures and mechanical properties of Al-Cu-based alloy prepared by squeeze casting. *Transactions of Nonferrous Metals Society of China.* 17(3), 496-501.
[10] Hajjari, E. & Divandari, M. (2008). An investigation on the microstructure and tensile properties of direct squeeze cast and gravity die cast 2024 wrought Al alloy. *Materials & Design*. 29(9), 1685-1689.
[11] Hong, C.P., Lee, S.M. & Shen, H.F. (2000). Prevention of macrodefects in squeeze casting of an Al-7 wt pct Si alloy. *Metallurgical and Materials Transactions B*. *31*(2), 297-305.
[12] Maleki, A., Shafyei, A. & Niroumand, B. (2009). Effects of squeeze casting parameters on the microstructure of LM13 alloy. *Journal of materials processing technology*. 209(8), 3790-3797.
[13] Vijian, P. & Arunachalam, V. P. (2007). Modelling and multi objective optimization of LM24 aluminium alloy squeeze cast process parameters using genetic algorithm. *Journal of materials processing technology*. 186(1), 82-86.
[14] Bin, S. B., Xing, S. M., Zhao, N. & Li, L. (2013). Influence of technical parameters on strength and ductility of AlSi9Cu3 alloys in squeeze casting. *Transactions of Nonferrous Metals Society of China*. 23(4), 977-982.
[15] Senthil, P. & Amirthagadeswaran, K.S. (2012). Optimization of squeeze casting parameters for non symmetrical AC2A aluminium alloy castings through Taguchi method. *Journal of mechanical science and technology*. *26*(4), 1141-1147.
[16] Senthil, P. & Amirthagadeswaran, K .S. (2014). Experimental Study and Squeeze Casting Process Optimization for High Quality AC2A Aluminium Alloy Castings. *Arabian Journal for Science and Engineering*. 39(3), 2215-2225.
[17] Parappagoudar, M. B. & Vundavilli, P. R. (2012). Application of modeling tools in manufacturing to improve quality and productivity with case study. *Proceedings in Manufacturing Systems*. *7*(4), 193-198.
[18] Wang, R. J., Zeng, J. & Zhou, D. W. (2012). Determination of temperature difference in squeeze casting hot work tool steel. *International journal of material forming*. 5(4), 317-324.
[19] Parappagoudar M. B., Pratihar, D. K. & Datta, G. L. (2008). Forward and reverse mapping,s in green sand mould system using neural networks. *Applied Soft Computing*. 8(1), 239-260.
[20] Parappagoudar, M. B., Pratihar, D. K. & Datta, G. L. (2007). Modelling of input–output relationships in cement bonded moulding sand system using neural networks. *International Journal of Cast Metals Research*. 20(5), 265-274.
[21] Kittur, J. K. & Parappagoudar, M. B. (2012). Forward and reverse mappings in die casting process by neural network-based approaches. *J. Manuf. Sci. Prod.* 12(1), 65-80.
[22] Parappagoudar, M. B., Pratihar, D. K. & Datta, G. L. (2008). Neural network-based approaches for forward and reverse mappings of sodium silicate-bonded, carbon dioxide gas hardened moulding sand system. *Materials and Manufacturing Processes*. 24(1), 59-67.
[23] Benguluri, S., Vundavilli, P. R., Bhat, R. P. & Parappagoudar, M. B. (2011). Forward and reverse mappings in metal casting—A step towards quality casting and automation, (11-009). *AFS Transactions-American Foundry Society*. 119(19), 1-15,
[24] Surekha, B., Vundavilli, P. R., Parappagoudar, M. B. & Srinath, A. (2011). Design of genetic fuzzy system for forward and reverse mapping of green sand mould system. *International Journal of Cast Metals Research*. 24(1), 53-64.
[25] Surekha, B., Vundavilli, P.R. & Parappagoudar, M.B. (2012). Forward and reverse mappings of the cement-bonded sand mould system using fuzzy logic, *The International Journal of Advanced Manufacturing Technology*. 61(9-12), 843-854.
[26] Vundavilli, P. R., Parappagoudar, M. B., Kodali, S. P. & Benguluri, S. (2012). Fuzzy logic-based expert system for prediction of depth of cut in abrasive water jet machining process. *Knowledge-Based Systems*. 27, 456-464.
[27] Surender, Y. & Pratihar, D. K. (2013). Fuzzy Logic-Based Techniques for Modeling the Correlation between the Weld Bead Dimension and the Process Parameters in MIG Welding. *International Journal of Manufacturing Engineering*. http://dx.doi.org/10.1155/2013/230463.
[28] Patel, G. C. M., Mathew, R., Krishna, P. & Parappagoudar, M. B. (2014). Investigation of squeeze cast process parameters effects on secondary dendrite arm spacing using statistical regression and artificial neural network models, (Accepted for publication in Procedia Technology Elsevier Journal).
[29] Pratihar, D. K. (2008). *Soft computing*, Narosa publishing house pvt. Ltd, India.
[30] Azar, A. T. (2010). Adaptive neuro-fuzzy systems, *Fuzzy systems*, 85-110, Feburaury 2010, INTECH, Croatia, ISBN 978-953-7619-92-3.
[31] Haykin, S. (1994). *Neural networks: a comprehensive foundation*. Prentice Hall PTR.
[32] Rajasekaran, S. & Pai, G.V. (2003). *Neural networks, fuzzy logic and genetic algorithm: synthesis and applications,* PHI Learning Pvt. Ltd.

[33] Wong, B. K. & Lai, V. S. (2011). A survey of the application of fuzzy set theory in production and operations management: 1998–2009. *International Journal of Production Economics*. 129(1), 157-168.

[34] Yurdusev, M. A. & Firat, M. (2009). Adaptive neuro fuzzy inference system approach for municipal water consumption modeling: An application to Izmir, Turkey. *Journal of hydrology*. 365(3), 225-234.

[35] Daoming, G. & Jie, C. (2006). ANFIS for high-pressure waterjet cleaning prediction. *Surface and Coatings Technology*. 201(3), 1629-1634.

[36] Lo, S. P. (2003). An adaptive-network based fuzzy inference system for prediction of workpiece surface roughness in end milling. *Journal of Materials Processing Technology*. 142(3), 665-675.

[37] Maji, K. & Pratihar, D. K. (2010). Forward and reverse mappings of electrical discharge machining process using adaptive network-based fuzzy inference system. *Expert Systems with Applications*. 37(12), 8566-8574.

[38] Zeren, M. (2005). Effect of copper and silicon content on mechanical properties in Al–Cu–Si–Mg alloys. *Journal of Materials Processing Technology.* 169(2), 292-298.