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Bayesian Regression Based Approach for Beam Deflection Estimation

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ABSTRACT
Deflection of a beam is the movement of the beam from its initial position to another position depending on the applied load. Beam deflection estimation gives an indication about the possible deformation of the beam. A parametric Bayesian linear based model is introduced to mimic the experimentally collected data to estimate the stochastic deflection of a simply supported beam. A Gaussian noise is assumed to understand the stochastic behavior of the beam deflection as well as a Gaussian prior. The model mapping function used in this work is known as radial basis function, which can be linear or nonlinear. Three basis functions are compared, namely are linear, Gaussian and modified Gaussian function proposed in this work. The modified Gaussian function is a simple function introduced in this work. The performance of the functions is analyzed for three central concentrated loads. The best model can describe the observed data is found to be the modified Gaussian model with regularization factor of 0.9 for three loading cases. The prediction based linear basis function is better than the use of the Gaussian basis function prediction according to error of estimation. The maximum RMS error obtained for modified Gaussian radial basis function corresponding to central load of 4 kg is smaller than that of a theoretical based model for the same loading conditions.

Keywords: beam defelection, Bayesian inference, radial basis functions.

INTRODUCTION

Beam deflection estimation is a significant key factor for structure health monitoring and performance evaluation. The deflection of a beam can be estimated either from deterministic model or from experimental analysis. The deterministic model estimation is usually made based on simple assumptions by using bending or shear theory. A stochastic model can be generated based on a set of experimentally collected data. The prediction of a target variable *y(x)* corresponding to a D-dimensional input vector *x(t)* can be achieved by using regression. A simple regression model is a linear model that involves a linear combination of the input variables known as radial basis functions. Such functions can be linear or non-linear with respect to the input variables. The radial basis functions can handle the behaviour of the system based on the selected function [1,2]. Based on a prior knowledge the posterior of deflection can be estimated by using Bayesian linear parameter model taking into account an uncertainty of estimation. The deflection of a cantilever beam is analysed numerically and experimentally under concentrated load at the free end [3]. A simple nonlinear differential equation is derived to prove the nonlinearity of the system. The experimental results are compared to the Ansys software results. The dynamic deflection of a beam is derived from strain mode shapes, that estimated from experimental data based on stochastic subspace identification algorithm.

The performance of proposed method is evaluated by numerical simulations of two beams cantilever and simply supported beam, where the obtained results showed a good performance with small estimation error[4]. Finite element model updating for mode shapes and frequencies estimations is introduced based on a Bayesian model. The uncertainty estimation results show that the uncertainty decrease with the increase of the number of sensors used to collect the experimental data [5]. A meshless based on Gaussian basis function is developed for one dimensional convection-diffusion problems. Time dependent variable in this work considered as a normal space variable. Root mean square error (RMS) and average absolute error (AAE) are used to compare the obtained results with the literature [6]. Uncertainty quantification based Bayessian approach is used to update system parameters such as stiffness and mass of the structure. Two numerical examples for damage detection is implemented in this work and the obtained results showed a better performance based the proposed approach than the traditional ones [7]. The stochastic perspective of linear finite element method is studied for physical systems of discreate points to establish an uncertainty quantification approach for domain boundary with nodal coordinates.

The proposed work is validated by demonstrating a numerical example [8]. A numerical model based radial basis functions is developed for cantilever laminated beam composed of two materials aluminum and silicon dioxide. The estimated deflection values show a good correlation with experimental results and the suggested optimal design based on the developed numerical model is found to be light in weight and low in rigidity [9]. A stochastic non-parametric model by using MonteCarlo simulation is introduced to find the deflection of simply supported beam. The performance of the proposed model is evaluated by merging the model with the experimental data by using Kalman filter. The obtained results showed that the uncertainty associated with load is very small, whereas the dimension uncertainty has a significant effect on the uncertainty of estimation of beam deflection [10]. The effect of the uncertain system parameters on the deflection of a simply supported beam are studied by using stochastic finite volume method considering stochastic modulus of elasticity and stochastic load [11]. A bayessian finite element approach is adapted to update system parameters [12]. The method is examined by using the experimental data of Kraaij [13] of a cantilever steel beam. The system parameters in this study were the Young modulus and the beam geometry parameters. The stochastic sense is developed by using MonteCarlo method. Measurement uncertainty can contribute to understand the beam deflection uncertainty estimation. Digital image techniques are used to estimate the uncertainty of measuring of static in plane and dynamic out of plane displacements for a cantilever beam [14]. The strain of a simple composite beam is related to the temperature gradient under static load to study the effect of temperature on the structure damage of the beam [15].

From above it can be concluded that the mesh based methods required a computational cost higher than that needed for mesh less based methods. It is also important to note that, the selection of the radial basis function (mesh less function) play an important rule in building the best model to regress the collected data.

In the present work, a parametric stochastic beam deflection estimator is proposed based on Bayessian linear model with Gaussian noise, prior and posterior. The model relating the deflection of the beam to the position by using three radial basis functions, the functions used in this work are (linear, Gaussian, modified Gaussian functions). The contribution of the proposed model is included the use of a Bayesian model resulted in a distribution estimation of deflection instead of a value estimation corresponding to deterministic model. The model also comprised the modification of the Gaussian function to adapt the model with observed data.

BAYESSIAN LINEAR PARAMETER MODEL

Regression linear model is consisted of linear mapping function to predict the deflection of beam, by considering linear combinations of a set of nonlinear functions, known as basis functions. Such models are mesh less models that used in wide range of engineering applications as in [2, 9]. Building the model by taking into account a Gaussian noise and apply the Bays rule. This can expresses our uncertainty about the value of deflection for each value of location x. Consider the model of the form:

$$y = f(x; w) + б \tag{1}$$

$$f(x, w) = \sum_{i=1}^{B} w_i \varphi_i(x) = w^T \varphi(x)$$

where: w_i – are called 'weights, $\varphi_i(x)$ – the basis functions, the number of basis functions is dim $(w) = B$, and $б$ – is a Gaussian noise $б \sim N\ (б,\ 0,\ \sigma^2)$, we can use $\beta = 1/\sigma^2$ instead of σ.

Suppose a Gaussian prior distribution with a precision of is [16]:

$$p(w|\alpha) = N(w|0, \alpha^{-1}I) = \left(\frac{\alpha}{2\pi}\right)^{\frac{B}{2}} e^{\left(-\frac{\alpha}{2}w^T w\right)} \quad (2)$$

The likelihood based on model generate the data is:

$$p(D|w) = \prod_{n=1}^{N} p(y^n|w, x^n)p(x^n) \quad (3)$$

According to Bayes rule, the Gaussian weight posterior can be expressed as [17,18]:

$$p(w|\,\Gamma, D) = N(w|m, S) \quad (4)$$

where: $\Gamma = \{\alpha; \beta\}$ is a hyperparameter set.

The covariance and mean are given by:

$$S = \left(\alpha I + \beta \sum_{n=1}^{N} \varphi(x^n)\varphi^T(x^n)\right)^{-1},$$

$$m = \beta S \sum_{n=1}^{N} y^n \varphi(x^n)$$

The model network representation is depicted in Figure 1, where the prior controlled by parameter and the observations noise is managed by parameter.

Model hyperparameters optimization

The marginal likelihood of D given model specific parameters is can be written for parameter as [16]:

$$p(D|\Gamma) = p(\mathrm{w}|\alpha) \quad (5)$$

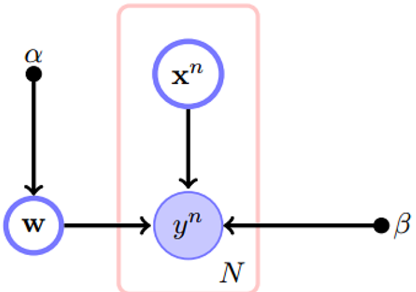

Fig. 1. Network of the model [16]

Zero gradient is considered to maximise the marginal likelihood:

$$\frac{\partial}{\partial\Gamma} p(D|\Gamma) = \frac{\partial}{\partial\alpha} p(\mathrm{w}|\alpha)$$

$$\frac{\partial}{\partial\Gamma} logp(D|\Gamma) = \frac{\partial}{\partial\alpha} logp(\mathrm{w}|\alpha)$$

and

$$logp(\mathrm{w}|\alpha) = -\frac{\alpha}{2} w^T w + \frac{B}{2} loga + constant$$

$$\frac{\partial}{\partial\alpha} p(\mathrm{w}|\alpha) = -\frac{1}{2} w^T w + \frac{B}{\alpha}$$

$$-\frac{1}{2} w^T w + \frac{B}{2\alpha} = 0$$

As a result:

$$\alpha = \frac{B}{w^T w} \quad (6)$$

Similarly, hyperparameter β can be updated from:

$$\beta = \frac{1 - \beta\ trace(S\hat{S})}{\frac{1}{N}\sum_{n=1}^{N}[y^n - m^T \boldsymbol{\varphi}(x^n)]^2} \quad (7)$$

where:

$$\hat{S} = \frac{1}{N} \sum_{n=1}^{N} \varphi(x^n)\varphi^T(x^n)$$

The complete solution based on the proposed model is depending on parameters β, α and on the

Algorithm 1. Regression based beam deflection estimation algorithm

select a proper basis function $\boldsymbol{\varphi(x^n)}$
intialise $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$
while not converged do
$\boldsymbol{i \leftarrow i + 1}$

$$\boldsymbol{S^i \leftarrow \left(\alpha^{i-1} I + \beta^{-1} \sum_{n=1}^{N} \varphi(x^n)\varphi^T(x^n)\right)^{-1}}$$

$$\boldsymbol{m^i \leftarrow \beta^{i-1} S^{i-1} \sum_{n=1}^{N} y^n \varphi(x^n)}$$

update parameters

$$\boldsymbol{\beta^i = \frac{1 - \beta^{i-1}\ trace(S\hat{S})^{i-1}}{\frac{1}{N}\sum_{n=1}^{N}\left[y^n - m^{T^{i-1}}\varphi(x^n)\right]^2}}$$

$$\boldsymbol{\alpha^i = \frac{B}{w^T w}}$$

end while

$$\boldsymbol{RMS = \sqrt{\frac{\left[y \quad - m^T \quad \varphi\right]^2}{N}}}$$

end

selection of the basis function $\varphi(x^n)$, which can be either linear or non linear function. The whole algorithm for updating the model parameters is depicted in Algorithm 1.

RESULTS AND DISCUSSION

The deflection of a simply supported beam is measured to represent the target (y) of estimation and the location of point of estimation represent the (x) vector. The beam has the following dimensions: 75 cm length, width 24 mm and thickness 4 mm, E = 2×10^{11} N/m^2. The deflection is measured by using a dial gauge in the applied mechanics lab in the technical college of Basra. The experimental setup is shown in Figure 2, where a central concentrated load is applied and the deflection is measured in 7 positions from left side to the middle of the beam as depicted in Figure 2. Regression based model is created with three different basis functions. The basis functions used in this work are linear function, Gaussian function and modified Gaussian function. For comparison of the obtained results RMS value is estimated to test the correlation of the results. For linear function the factors β and α are assumed constant, whereas the Algorithm 1 is used to estimate both of them in case of using the Gaussian and modified Gaussian functions. The Gaussian function used in this work of the form:

$$\varphi_i(x) = e^{\left(-\frac{\mu(x-c_i)^2}{\lambda}\right)} \quad (8)$$

where: c_i – are centers spread out from -2 to 2, λ – is the width, μ – represent a regularization factor.

The RMS error is used for the selection of optimal λ and μ factors. A new function adapted has the form of:

$$\varphi_i(x) = e^{\left(-\frac{\mu\sqrt{(x-c_i)}}{\lambda}\right)} \quad (9)$$

Which known in this work as the modified Gaussian function.

In general, the performance of three proposed models are compared in Table 1, where it can be seen clearly that the minimum RMS value is obtained when the Modified Gaussuian function based model is used and the Gaussian function performance is very week with maximum RMS for all loading cases. Modified Gaussian function correlate closely to the experimental data and the best value for regularization factor is 0.9 with different values of function width λ for all loads as shown in Table 1. The RMS values for the linear model is less than that of the Gaussian model, but it is greater than the RMS of the Modified model. It can be conclude that the modified model is the best model of the proposed model that can predict the deflection with minimum RMS error.

Linear function-based estimation is displayed in Figure 3 with maximum RMS error of 0.904 when the load is 4 kg. Figure 4 shows the estimation of deflection based on Modified Gaussian function for set of values of λ for load of 4 kg.

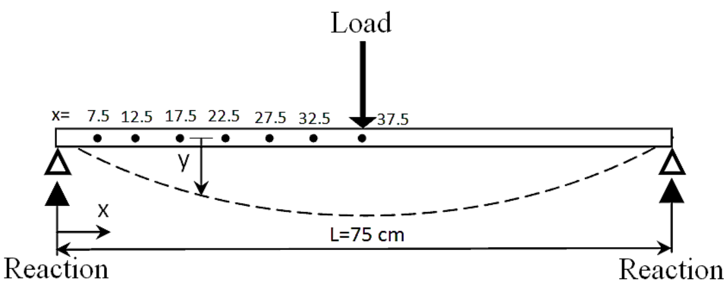

Fig. 2. Schematic diagram of the experimental setup

Table 1. RMS value corresponding for estimation of bending by three models for three loads

Load (kg)	Model									
	Linear	Gaussian					Modified Gaussian			
	RMS	RMS	μ	λ	α	β	RMS	μ	λ	α	β
4 kg	0.904	8.465	0.7	1.8	1	0.014	0.608	0.9	-1.6	0.598	0.017
6 kg	0.722	3.48	0.5	1	1	0.006	0.539	0.9	-1.4	1	0.006
8 kg	0.34	16.11	0.5	1.2	1	0.0039	0.165	0.9	-1.55	1.033	0.2045

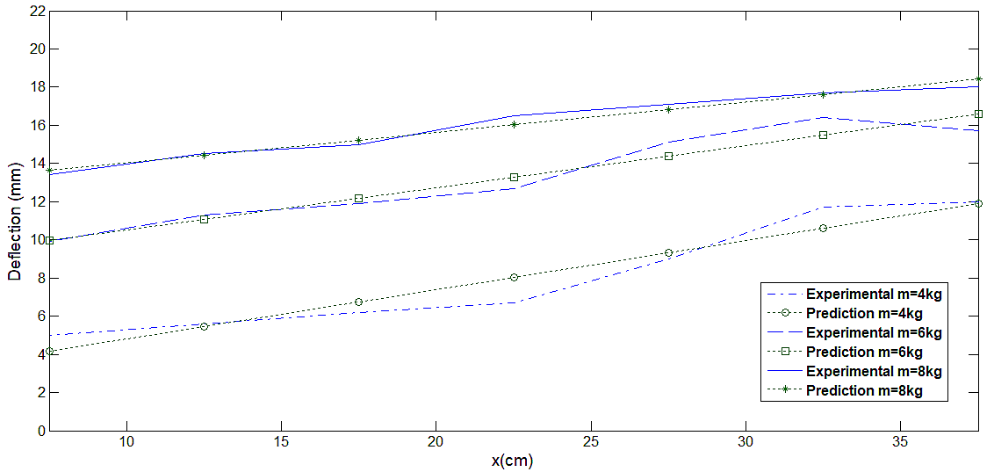

Fig. 3. Deflection estimation with linear function

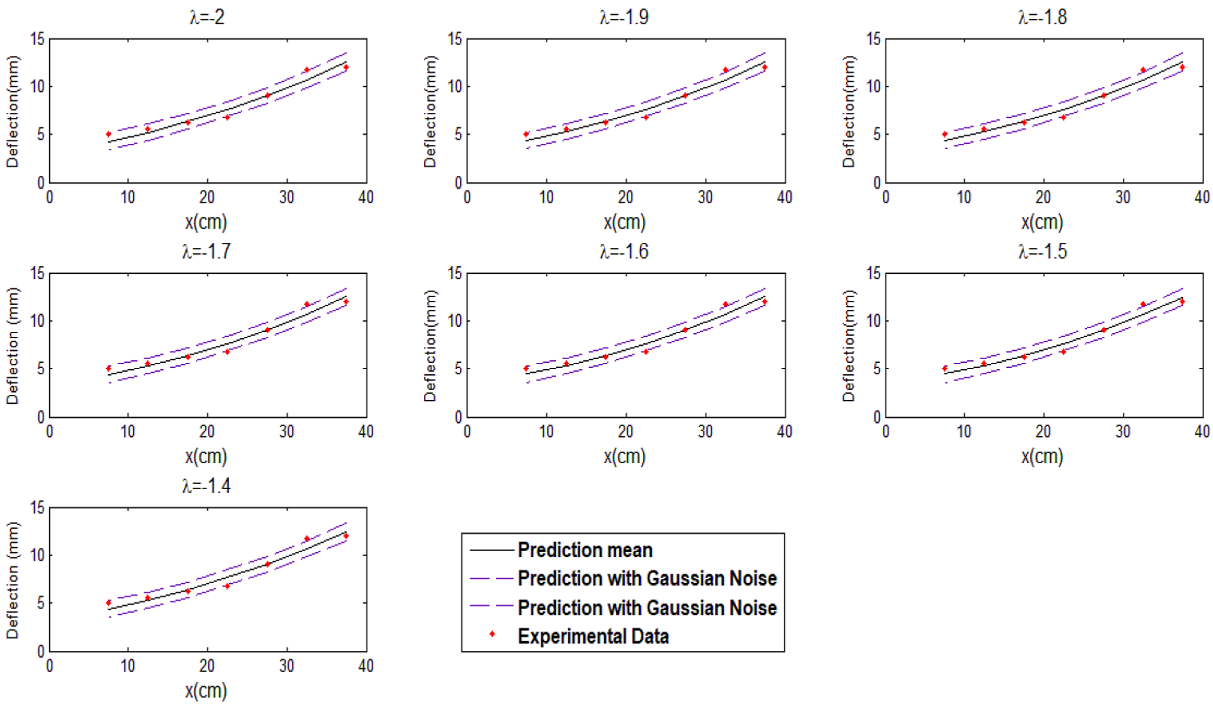

Fig. 4. Comparison of the modified Gaussian based model with experimental data for m = 4 kg

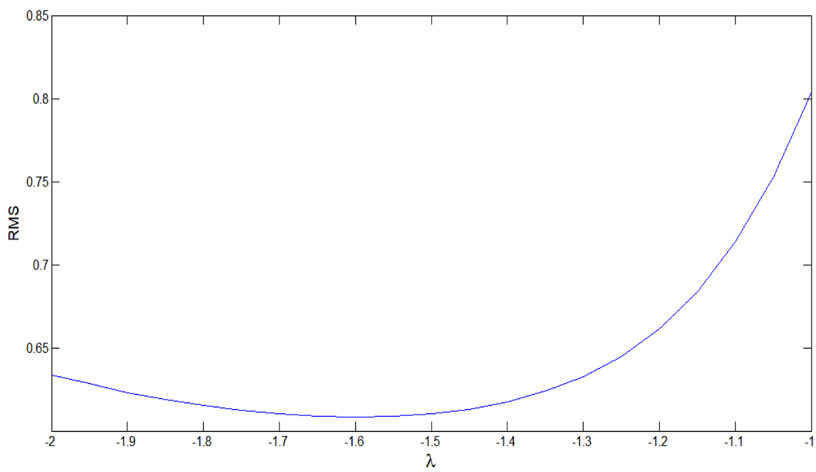

Fig. 5. RMS value for different values of λ of the modified Gaussian based model for m = 4 kg

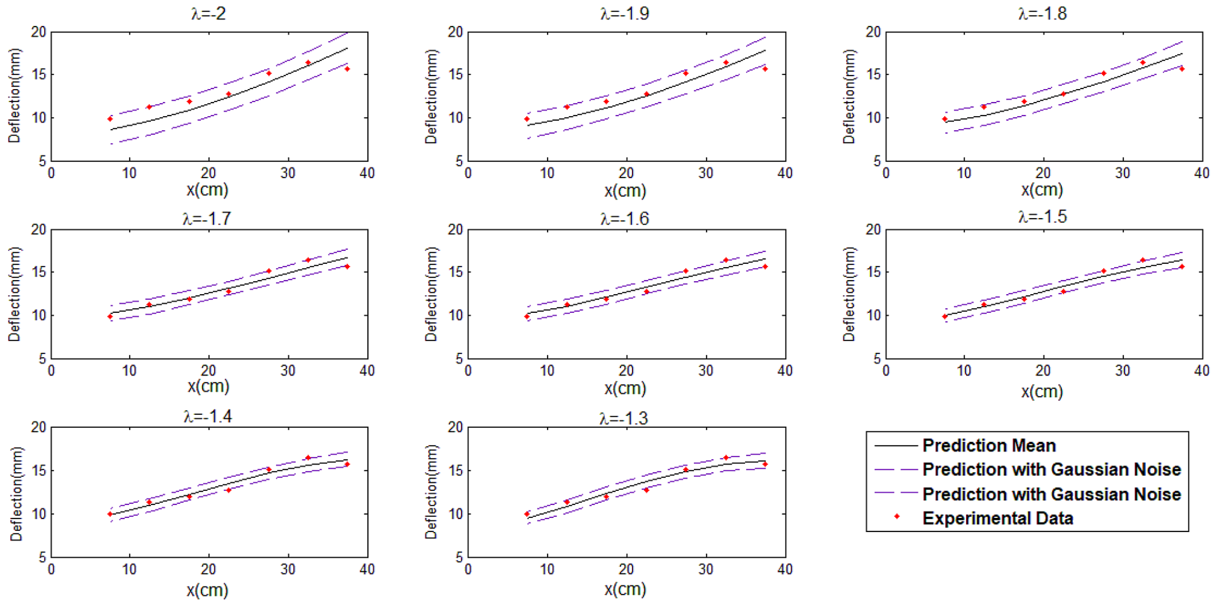

Fig. 6. Comparison of the modified Gaussian based model with experimental data for m = 6 kg

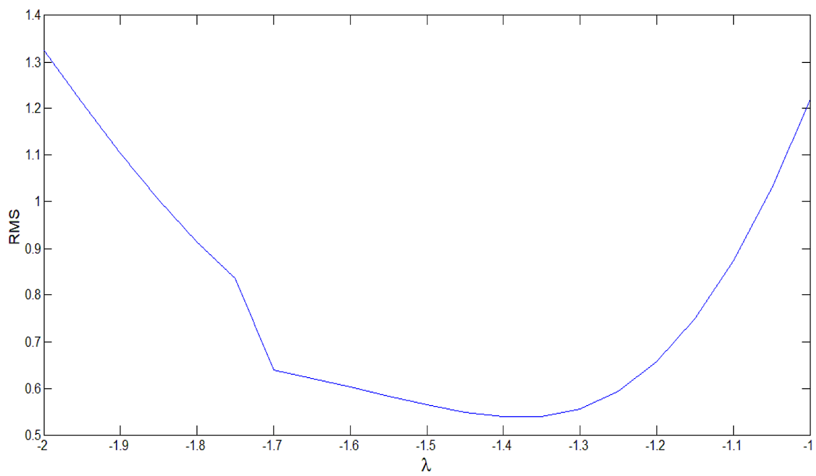

Fig. 7. RMS value for different values of λ of the modified Gaussian based model for m = 6 kg

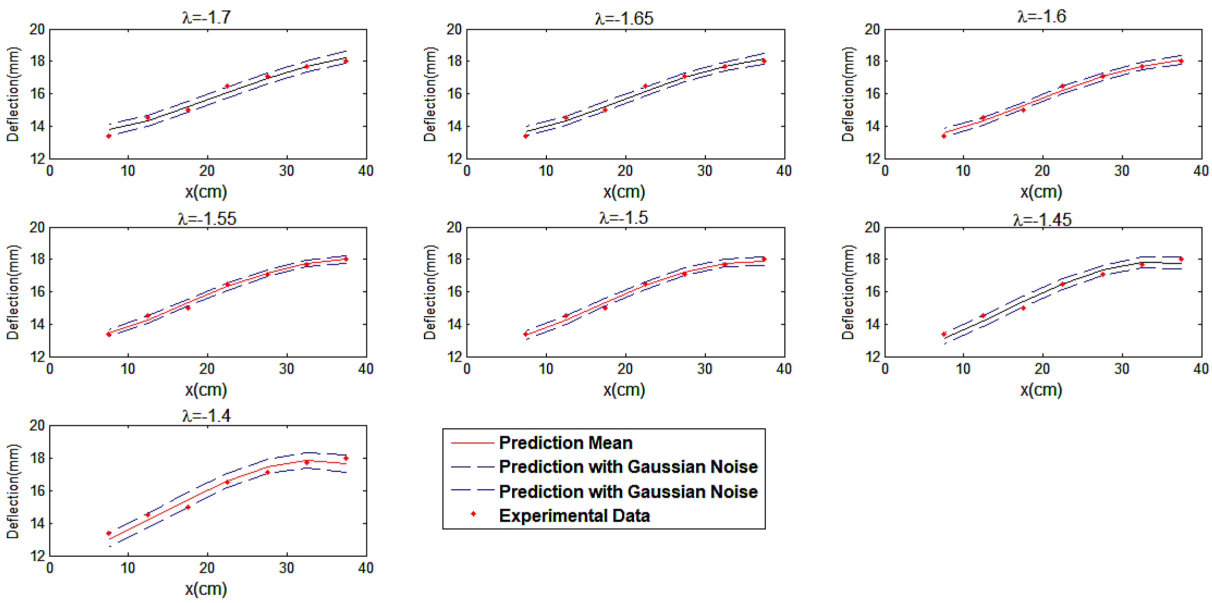

Fig. 8. Comparison of the modified Gaussian based model with experimental data for m = 8 kg

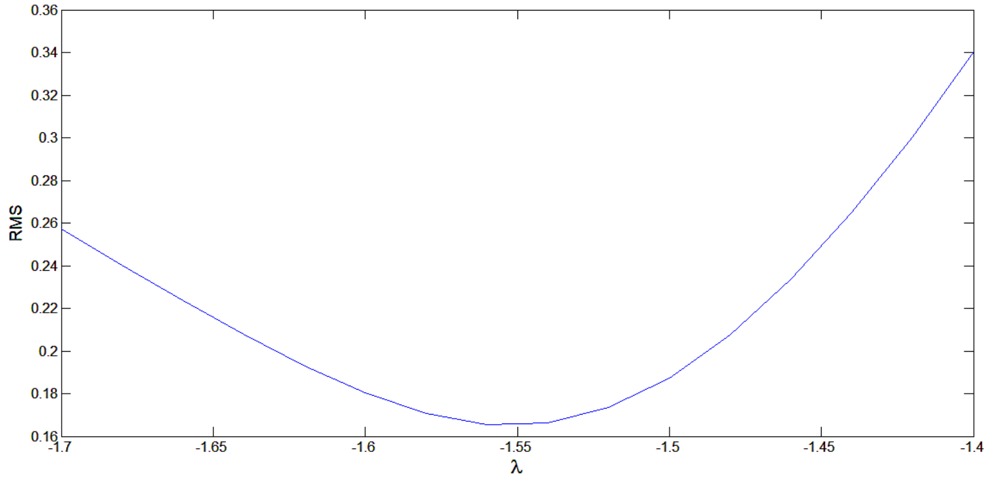

Fig. 9. RMS value for different values of λ of the modified Gaussian based model for m = 8 kg

Mean prediction is plotted in solid line bounded by standard deviation doted lines. The best model corresponding to minimum RMS error value is observed when λ = -1.6 and regularization factor μ of 0.9. The relation between RMS value and λ is depicted in Figure 5 to show the best performance of the selected model.

The prediction of deflection under 6 kg load is illustrated in Figure 6 for different values of λ, for regularization factor of 0.9 as mentioned before, where a set of values of μ is used and the factor corresponding to minimum RMS value is considered. Figure 7 indicates the RMS error of prediction against model width (λ), where it can be observed that the minimum error is reached at λ = -1.4. For 8 kg loading case the minimum obtained error is 0.165 corresponding to model with λ = -1.55 as seen in Figures 8 and 9. The Gaussian model prediction for all loading cases is not acceptable as it has a very high RMS error compared to linear model and modified Gaussian function-based model. For all three loading cases it can be seen that the optimal regularization factor is 0.9 and the model width is in the range (-1.6 to -1.4) for modified Gaussian function

In order to check the validity of the proposed model a comparison of the loading case study that has the maximum RMS error of 0.608, with a deterministic model based bending theory is made. It can be seen from Figure 10 that the modified Gaussian based estimation is close to experimental data better than the deterministic model estimation, where the RMS error of deterministic bending based model is 2.337 which greater than 0.608.

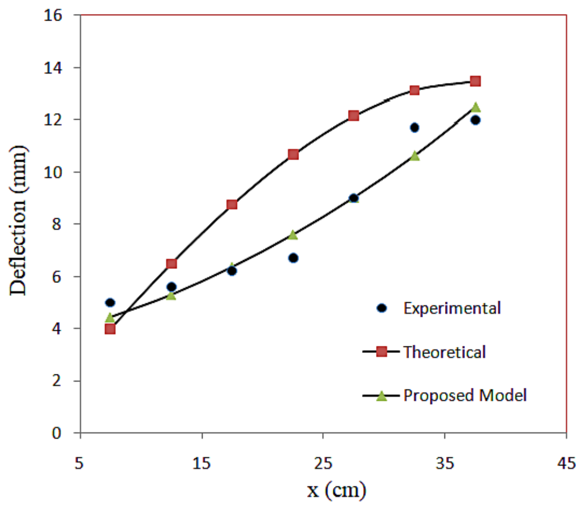

Fig. 10. Deflection estimation comparison

CONCLUSIONS

A Bayesian linear model for deflection estimation is introduced for simply supported beam, the model based on a basis mapping function, which can be linear or nonlinear. In this work, three types of functions are employed: linear, Gaussian and modified Gaussian. Two factors characterize the performance of the Gaussian and modified Gaussian functions are regularization factor μ and the width λ. The ability of three functions to predict the deflection of three loading weights is compared. From the obtained results, it is clear that the prediction of Gaussian is unable to catch the behavior of the system. On the other hand the modified Gaussian function show a high ability to mimic the collected data. The prediction based linear basis function has an accepted RMS values, but it still higher than the modified function introduced in this work. Three cases are studied in this work with central concentrated load of 4 kg, 6 kg and 8 kg applied on simply supported beam. The obtained results showed that the optimal regularization factor is 0.9 and the model width is in the range (-1.6 to -1.4) for modified Gaussian function. The comparison of modified Gaussian function results with a classic deterministic model based bending showed that the Gaussian function performance is better than that of the deterministic model. The proposed work introduced a new simple function (modified Gaussian function) with low computational cost and based on stochastic approach (Bayesian inference).

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