Nonlinear Vibrations of the Partially Tensioned System Taking into Account the Asymmetry of the Stiffness of Supports

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Abstract

The nonlinear vibrations of the partially tensioned slender column are presented in this paper. The considered system is subjected to Euler’s load, which direction of action is consistent with the non-deformed axis of the column. The magnitude of the external load is variable and its application point is located at different heights between the upper and lower supports. In addition, the longitudinal displacement and rotation of both of the system ends are limited by the discrete elements in the form of translational and rotational springs. This nonlinear system is based on the screw drive used in the vertical lift platform for disabled people or cargo lift equipped with an engine room located in the lower part of the frame. The boundary problem of free vibrations of the mentioned system has been formulated on the basis of Bernoulli – Euler theory and due to nonlinear expressions the solution of the problem was conducted with small parameter method. The results of numerical simulations are concern on linear and nonlinear component of vibrations in relation to the location of external load application and influence of asymmetric value of supports stiffness on the free vibration frequency.

Keywords: slender nonlinear systems, free vibrations, Euler's load

1. Introduction

Columns and beams are widely used as load-bearing or driving elements in mechanical engineering. Slender elements are increasingly used in construction to reduce production costs or perform specific functions. Therefore, the issue of free vibrations and stability is very important for scientific considerations. Particular attention should be paid to geometrically nonlinear systems in which the nonlinear theory and the theory of Bernoulli–Euler are used to formulate the boundary problem [1, 2]. Slender columns loaded with compressive forces usually form responsible support structures that can be exposed to dangerous resonance vibrations causing damage to the system. External loads can be classified into non-conservative [3] and conservative loads, which include Euler's force [4]. The load direction is consistent with the non-deformed axis of the system when tilting the system from the static equilibrium position. Interesting results of numerical simulations showing the impact of changes in cross-section and structure geometry on free vibrations of a non-prismatic column subjected to the active and passive follower force directed towards the positive pole are presented in [5]. In addition to geometrical parameters and the type of system load, the use of discrete elements at supporting points has a significant impact on the system's behaviour during vibrations [4, 6, 8, 9] and critical force [7]. Partially tensioned columns including rotational and translational springs [8, 9] model the screw drive used in vertical platform lifts equipped with a machine room located in the lower part of the frame. Studies on slender partial tension systems have shown a significant effect of amplitude and susceptible elements to natural vibration frequency that can be controlled. This work concerns the numerical research on the mentioned systems regarding free vibration (linear and nonlinear component) at different locations of external load application, taking into account the asymmetry of the value of support stiffness.

2. Boundary value problem concerning free vibrations

The considered system is shown in Fig. 1. It is a column resiliently mounted at both ends (longitudinal (K_0, K_1) and rotational (C_0, C_1) elasticity limiting the rotation of the column ends were taken into account) and loaded with a force whose direction coincides with the non-deformed axis of the system. The system loading force occurs between the ends of the column. The position of the loading force was determined by the parameter ζ ($\zeta = l_1/l$). The part of the column situated above the load application point is tensioned while the part below this point is compressed. To develop the mathematical model, the column was divided into two parts.

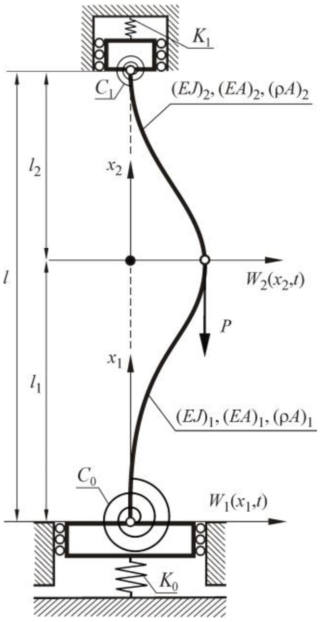

Figure 1. Physical model of considered column

The division point coincides with the point determining the position of the loading force. The lower (compressed) part is marked by index 1. The upper (tensioned) part is marked by index 2. Bending and longitudinal stiffness and mass per unit length of rods marked with indexes 1 and 2 are the same ($(EJ)_1 = (EJ)_2 = (EJ)$, $(EA)_1 = (EA)_2 = (EA)$, $(\rho A)_1 = (\rho A)_2 = (\rho A)$ (where: E_i – Young's modulus of column material, ρ_i – density of column material, A_i – cross-section area, J_i – geometrical axial moment of inertia of the cross-section of i-th element of the structure).

The paper does not present in detail the boundary problem, which was formulated on the basis of the Hamilton principle and due to the occurrence of nonlinearity the small parameter method was taken into account (comp. [10]). Differential equations of motion in the transverse and longitudinal directions are as follows:

$$\frac{\partial^4 w_i(\xi_i,\tau)}{\partial \xi_i^4} + k_i^2(\tau)\frac{\partial^2 w_i(\xi_i,\tau)}{\partial \xi_i^2} + \Omega_i^2 \frac{\partial^2 w_i(\xi_i,\tau)}{\partial \tau^2} = 0 \tag{1}$$

$$u_i(\xi_i,\tau) - u_i(0,\tau) = -\frac{k_i^2(\tau)}{\Theta_i}\xi_i - \frac{1}{2}\int_0^{\xi_i}\left(\frac{\partial w_i(\xi_i,\tau)}{\partial \xi_i}\right)^2 d\xi_i \tag{2}$$

All nonlinear quantities describing the behavior of the system during vibrations were expanded into power series of a small parameter of amplitude. The dynamic properties of the column under consideration depend on the vibration amplitude. The following extensions of nonlinear components of equations into power series of the small parameter of vibration amplitude ε were used:

$$w_i(\xi,\tau) = \sum_{j=1}^{N} \varepsilon^{2j-1} w_{i2j-1}(\xi,\tau) + O\left(\varepsilon^{2(N+1)}\right) \tag{3}$$

$$u_i(\xi,\tau) = u_{i0}(\xi) + \sum_{j=1}^{N} \varepsilon^{2j} u_{i2j}(\xi,\tau) + O\left(\varepsilon^{2(N+1)}\right) \tag{4}$$

$$k_i^2(\tau) = k_{i0}^2 + \sum_{j=1}^{N} \varepsilon^{2j} k_{i2j}^2(\tau) + O\left(\varepsilon^{2(N+1)}\right) \tag{5}$$

$$\Omega_i^2 = \Omega_{i0}^2 + \sum_{j=1}^{N} \varepsilon^{2j} \Omega_{i2j}^2 + O\left(\varepsilon^{2(N+1)}\right) \tag{6}$$

$$\omega^2 = \omega_0^2 + \sum_{j=1}^{N} \varepsilon^{2j} \omega_{2j}^2 + O\left(\varepsilon^{2(N+1)}\right) \tag{7}$$

where:

$$w_{i1}(\xi,\tau) = \overset{(1)}{w}_{i1}(\xi)\cos\tau\,;\ w_{i3}(\xi,\tau) = \overset{(1)}{w}_{i3}(\xi)\cos\tau + \overset{(3)}{w}_{i3}(\xi)\cos 3\tau\,;\ \ldots \tag{8, 9}$$

$$u_{i2}(\xi,\tau) = \overset{(0)}{u}_{i2}(\xi) + \overset{(2)}{u}_{i2}(\xi)\cos 2\tau\,;\ \ldots \tag{10}$$

$$k_{i2}^2(\tau)=\overset{(0)}{k}{}_{i2}^2+\overset{(2)}{k}{}_{i2}^2\cos 2\tau\;;\;\ldots \tag{11}$$

In the expansions of formulas (3-7) $w_i(\xi_i,\tau)$, $u_i(\xi_i,\tau)$, $k_i(\tau)$ and Ω_i are dimensionless quantities, referred to the stiffness and length of each bars, related to transversal and longitudinal displacement, internal force and natural vibration frequency. All dependencies presented in the work were written using the following dimensionless quantities:

$$\xi_i=\frac{x_i}{l}\;;\zeta_i=\frac{l_i}{l}\;;\;w_i(\xi_i,\tau)=\frac{W_i(x_i,t)}{l}\;;\;u_i(\xi_i,\tau)=\frac{U_i(x_i,t)}{l}\;; \tag{12-15}$$

$$k_i^2(\tau)=\frac{S_i(\tau)l^2}{(EJ)_i}\;;\Omega_i^2=\frac{(\rho A)_i\,\omega^2 l^4}{(EJ)_i}\;;\;\tau=\omega t\;;\;\Theta_i=\frac{A_i l^2}{J_i}\;;\;i=1,\,2 \tag{16-19}$$

Where: $W_i(x_i,t)$, $U_i(x_i,t)$ and $S_i(t)$ – dimensional quantities respectively transversal and longitudinal displacements and internal force.

The next components of nonlinear quantities are marked with the indices 0, 1, 2, ... For example, ω_0 – linear component of natural vibration frequency, ω_2 – nonlinear component of natural vibration frequency. The natural frequency, which is calculated using both linear ω_0 and nonlinear ω_2 components is calculated on the basis of the presented expansion into power series of small parameter (7) from the formula:

$$\omega=\sqrt{\omega_0^2+\varepsilon^2\omega_2^2} \tag{20}$$

The method of calculating the individual components of the quantities that are necessary to determine the natural frequency ω is as follows (see [10]):

– the distribution of external force into the bars of the column (linear component of internal forces):

$$k_{10}^2=\frac{P\left(\frac{l^2}{(EJ)}\zeta_2\frac{1}{\Theta_2}+\frac{1}{lK_1}\right)}{\frac{(EJ)}{l^3}\left(\frac{1}{K_1}+\frac{1}{K_0}\right)+\frac{\zeta_1}{\Theta_1}+\frac{\zeta_2}{\Theta_2}}\;;k_{20}^2=k_{10}^2-P\frac{l^2}{(EJ)} \tag{21, 22}$$

– the linear component of the natural vibration frequency ω_0 is determined from the transcendental equation obtained after taking into account the solutions of differential equations associated with ω_0 in appropriate boundary conditions.

– nonlinear components of internal forces:

$$\overset{(0)}{k}{}_{22}^2=-\frac{\frac{1}{4}\sum_i\int_0^{\zeta_i}\left(\overset{(1)}{w}{}_{i1}^{I}(\xi_i)\right)^2 d\xi_i}{\frac{(EJ)_2}{l^3K_1}+\frac{(EJ)_2}{l^3K_0}+\frac{1}{\Theta_1}\frac{(EJ)_2\,\zeta_1}{(EJ)_1}+\frac{\zeta_2}{\Theta_2}}\;;\overset{(0)}{k}{}_{12}^2=\overset{(0)}{k}{}_{22}^2\frac{(EJ)_2}{(EJ)_1} \tag{23, 24}$$

$$\overset{(2)}{k}{}_{i2}^{2} = \overset{(0)}{k}{}_{i2}^{2} \tag{25}$$

– nonlinear components of free vibration frequency:

$$\omega_2^2 = \frac{\sum_i \left[\begin{array}{c} \int_0^{\zeta_i} \frac{3}{2} \frac{(EJ)_i}{l} \overset{(2)}{k}{}_{i2}^{2} \overset{(1)}{w}{}_{i1}^{II}(\xi_i) \overset{(1)}{w}_{i1}(\xi_i) d\xi_i + \\ + \frac{3}{2}\frac{(EJ)_1}{l} \overset{(0)}{k}{}_{12}^{2} \overset{(1)}{w}_{11}(0) \overset{(1)}{w}{}_{11}^{I}(0) - \frac{3}{2}\frac{(EJ)_2}{l} \overset{(0)}{k}{}_{22}^{2} \overset{(1)}{w}_{21}(\zeta_2) \overset{(1)}{w}{}_{21}^{I}(0) \end{array} \right]}{\sum_i \left[\int_0^{\zeta_i} (\rho A)_i l^3 \left(\overset{(1)}{w}_{i1}(\xi_i) \right)^2 d\xi_i \right]} \tag{26}$$

All quantities presented in this chapter (linear and nonlinear components defined by formulas (20-26)) are taken into account for presenting the results of numerical calculations.

3. Results of numerical simulations

Results of numerical simulations of free vibrations frequency of the considered partially tensioned slender system were presented in non-dimensional form, defined as:

$$\lambda = \frac{Pl^2}{(EJ)};\ c_j = \frac{C_j l}{(EJ)};\ k_j = \frac{K_j l^3}{(EJ)};\ \zeta = \frac{l_1}{l};\ \Omega^2 = \frac{\omega^2 (\rho A) l^4}{(EJ)};\ j = 0, 1 \tag{27-31}$$

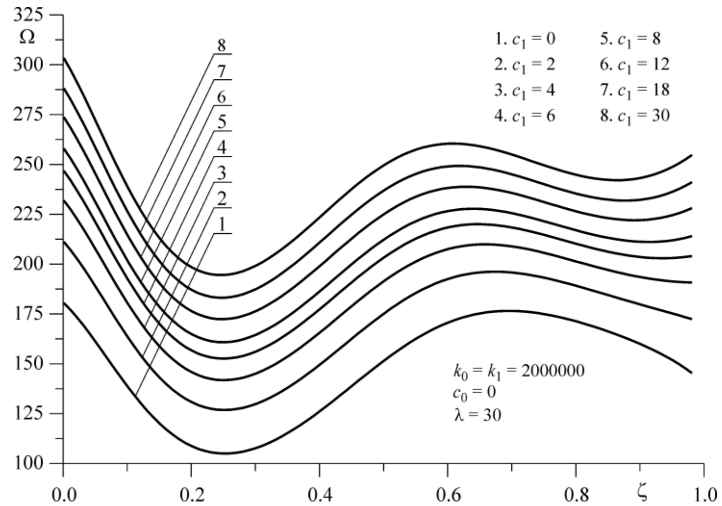

Figure 2. The relationship between natural vibration frequency Ω and the rotational springs stiffness c_0 and c_1, at different points of external load application ζ taking into account the parameters: $k_0 = k_1 = k$; $c_0 = 0$; $\lambda = 30$

The amplitude of vibrations in numerical calculations is assumed as a double radius of gyration. Figures 2, 3, 4 show the relationship between the parameter of the first natural vibration frequency Ω and the point of external load application for different values of rotational spring stiffness limiting the rotation of the column ends. The assumed values of rotational spring stiffness are for the lower fastening c_0 = 0, 15, 30 and the upper c_1 = 0, 2, 4, 6, 8, 12, 18, 30. The value of the load parameter is constant $\lambda = 30$, while the point of its application moves along the length of the system $\zeta \in (0,1)$. The same stiffness of the translation springs at both attachment points $k_0 = k_1 = 2 \cdot 10^6$) was assumed. Based on the numerical simulations performed, it has been observed that as the stiffness of both the lower and upper fastening increases, the natural vibration frequency increases (frequency containing both linear and nonlinear components – formula (20)).

The natural vibration frequency is strongly dependent on the position of the loading force. The difference between the highest and the lowest value of the natural vibration frequency from the factor ζ variation range depends on the stiffness of the rotational springs. With the rotational rigidity of the lower fastening $c_0 = 0$ (Figure 2) at $c_1 = 30$, the difference between the largest value and the smallest is about 35% of the value of the highest frequency from the considered factor ζ.

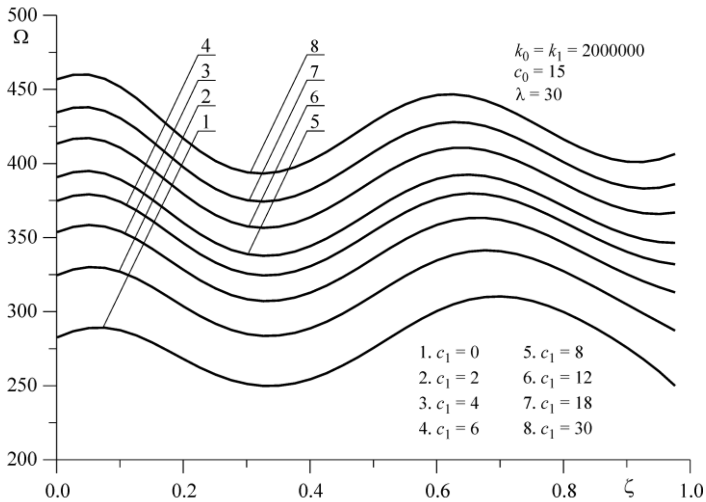

Figure 3. The relationship between natural vibration frequency **Ω** and the rotational springs stiffness c_0 and c_1, at different points of external load application ζ taking into account the parameters: $k_0 = k_1 = k$; $c_0 = 15$; $\lambda = 30$

When the rotational rigidity of the lower attachment increases $c_0 = 30$, the difference considered is only about 15%. It was also observed that the highest value of the natural frequency (for bigger value of parameter c_1) occurs when the force loading the column is near the lower fastening. Depending on the rotational stiffness values, two or one local minimum and maximum of the considered curves are obtained.

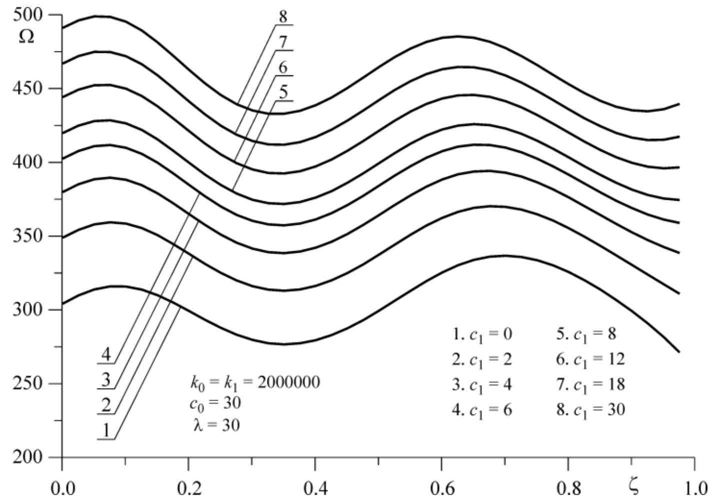

Figure 4. The relationship between natural vibration frequency **Ω** and the rotational springs stiffness c_0 and c_1, at different points of external load application ζ taking into account the parameters: $k_0 = k_1 = k$; $c_0 = 30$; $\lambda = 30$

4. Conclusions

In this paper, the boundary problem of free vibrations of partially tensioned columns subjected to Euler's load was considered. The discussion includes both the linear component of the first vibration frequency and its nonlinear component, whose impact on the frequency depends on the vibration amplitude. The effect of asymmetry of the system mounting rigidity on its dynamic properties was analyzed. The considerations took into account the asymmetry in rotational stiffness limiting the rotation of both ends of the column. Based on the numerical tests carried out, it was shown that the considered natural vibration frequency is strongly dependent on the parameters of rotational stiffness. In addition, the frequency of free vibrations depends on other parameters such as location of external force, load magnitude, translational springs stiffness and physical and geometrical properties of the system. In the future, it is planned to study the effect of circular-shaped heads on free vibrations to prevent resonance in screw-driven platform lifts.

Acknowledgments

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