controllable selective assembling, incomplete production, density function, variation problem, parameters of control, one-step and multi-step algorithms

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CONTROL BY PARAMETERS OF THE SELECTIVE ASSEMBLING

Abstract. *Models for definition of the amount of incomplete production are offered. The influence of control by parameters of selective assembling on the amount of incomplete production is explored. One-step and multi-step control algorithms are surveyed. Outcomes of modelling are given.*

The amount of incomplete production (IP) is one of the most important parameters of selective assembling of products [1].

It is necessary to solve the definition task of the lower limit (minimum) of incomplete production with known dispersions of σ_1^2 and σ_2^2 mated part sizes for the control of selective assembling. The built model will be based on the control of selective assembling.

According to [1], the amount of incomplete production can be defined as:

$$\overline{Q} = Q \cdot P \, , \tag{1}$$

where $P = 0{,}5 \cdot \int_{-\beta}^{\beta} \left| p_1(x) - p_2(x) \right| dx$ - is the probability of non-assembling; Q - is the number of parts in a batch; $p_1(x)$ and $p_2(x)$ - are the densities of the random quantity function of α_1 and α_2 parameters of bushes and shafts, respectively; x - is deviation of size; β - are the limits of spread of the sizes of mated parts.

The mathematical formulation of the task looks as follows [2]: to find

$$\min\{Z = \int_{-\beta}^{\beta} (p_1(x) - p_2(x))^2 dx\} \, , \tag{2}$$

keeping in mind the restrictions

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$$\begin{cases} \int_{-\beta}^{\beta} p_1(x)dx = 1; & (3) \\ \int_{-\beta}^{\beta} x \cdot p_1(x)dx = M_1; & (4) \\ \int_{-\beta}^{\beta} x^2 \cdot p_1(x)dx = \sigma_1^2 + M_1^2; & (5) \\ \int_{-\beta}^{\beta} p_2(x)dx = 1; & (6) \\ \int_{-\beta}^{\beta} x \cdot p_2(x)dx = M_2; & (7) \\ \int_{-\beta}^{\beta} x^2 \cdot p_2(x)dx = \sigma_2^2 + M_2^2. & (8) \end{cases}$$

where M_1, M_2 and σ_1^2, σ_2^2 - are mathematical expectations and dispersions defined by the $p_1(x)$ and $p_2(x)$ densities, respectively.

Functions (1) and (2) reach their extremes for the same meanings $p_1(x)$ and $p_2(x)$, which is why the equivalent substitution of the function (1) with the function (2) is proved. The given task is solvable if either $p_1(x)$ or $p_2(x)$ is known, or by the introduction of a new variable.

$$\Delta(x) = p_1(x) - p_2(x). \tag{9}$$

Thus let us subtract (6) from (3), (7) from (4), and let us subtract (8) from (5). Then the task is reduced to the following:

$$\min\{Z = \int_{-\beta}^{\beta} \Delta(x)^2 dx\},$$

$$\begin{cases} \int_{-\beta}^{\beta} \Delta(x)dx = 0, & (10) \\ \int_{-\beta}^{\beta} x \cdot \Delta(x)dx = M, & (11) \\ \int_{-\beta}^{\beta} x^2 \cdot \Delta(x)dx = \sigma^2, & (12) \end{cases}$$

where $M = M_1 - M_2$ is the difference of the mathematical expectations; $\sigma^2 = \sigma_{11}^2 - \sigma_{21}^2$ is the difference of the dispersions related to the non-zero mathematical expectation:

$\sigma_{11}^2 = \sigma_1^2 + M_1^2; \quad \sigma_{21}^2 = \sigma_2^2 + M_2^2$.

The task given of the search of the function extreme (2) is a variation task on a conditional extreme with integrated restrictions [1]. Solving Euler equation concerning $\Delta(x)$, we have:

$$\Delta(x) = -\frac{1}{2}\cdot\left(\lambda_1 + \lambda_2 \cdot x + \lambda_3 \cdot x^2\right). \tag{13}$$

The constants λ_1 and λ_2 are defined from (10), (11) and (12), if substituting (13) in them:

$$\lambda_1 = \sigma^2 \cdot \frac{15}{4\cdot\beta^3}, \qquad \lambda_2 = -\frac{3\cdot M}{\beta^3}, \qquad \lambda_3 = -\sigma^2 \cdot \frac{45}{4\cdot\beta^5}.$$

Then (13) will become

$$\Delta(x) = -\frac{15}{8\cdot\beta^3}\cdot\sigma^2 + \frac{3\cdot M}{2\cdot\beta^3}\cdot x - \frac{45\cdot\sigma^2}{8\cdot\beta^5}\cdot x^2. \tag{14}$$

For instance, it was supposed that the density function of parameters of bushes $p_1(x)$ has the truncated normal distribution and is defined on the interval $[-1;1]$ with the dispersion $\sigma_1^2 = 0{,}1$ and mathematical expectation $M_1 = 0{,}1$ of size deviation. The research of the amount of incomplete production was made with the help of the meanings variation of the dispersion σ_2^2 and mathematical expectation M_2. The received data are shown in Table 1. The view of the given density of sizes deviations function of bushes and the calculated density of sizes deviations function of shafts for the case $\sigma_2^2 = 0{,}2; \quad M_2 = 0{,}2$ are presented in Figure 1.

Table 1. Results of calculation of non-assembling probability

$\sigma_1^2 = 0{,}1; \quad M_1 = 0{,}1$				
	$\sigma_2^2 = 0{,}2$		$\sigma_2^2 = 0{,}3$	
M_2	0.1	0.2	0.1	0.2
P	0.14434	0.19668	0.28868	0.33701

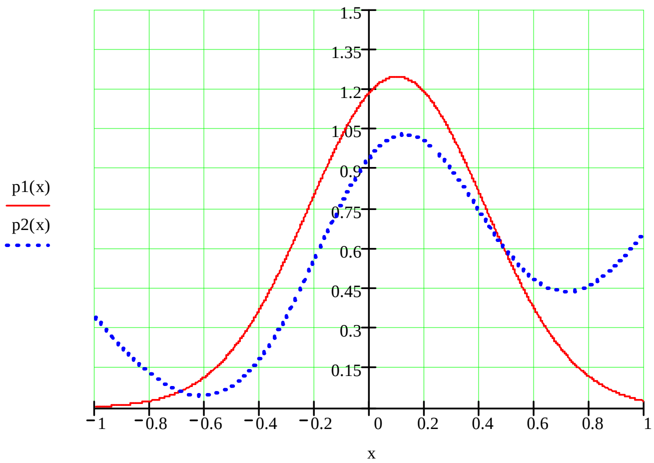

Fig. 1. Given density function of sizes deviations of bushes and calculated density function of sizes deviations of shafts for the case $\sigma_2^2 = 0{,}2; \quad M_2 = 0{,}2$

The approach described above is suitable for cases when the negative meanings for function $p_2(x)$ fail. If $p_2(x)$ is obtained as negative, it is necessary to use another approach.

In this case the mathematical formulation of the task looks as follows:

$$\min\{Z = \int_{-\beta}^{\beta} (p_1(x) - p_2(x))^2 dx = \int_{-\beta}^{xm} (p_1(x) - p_2(x))^2 dx + \int_{xm}^{\beta} (p_1(x) - p_2(x))^2 dx\},$$

keeping in mind the restrictions:

$$\begin{cases} \int_{-\beta}^{xm} p_2(x)dx = P_{2xm}; & (15) \\ \int_{-\beta}^{xm} x \cdot p_2(x)dx = M_{2xm}; & (16) \\ \int_{-\beta}^{xm} x^2 \cdot p_2(x)dx = \sigma_{2xm}^2 + M_{2xm}^2; & (17) \\ \int_{xm}^{\beta} p_2(x)dx = 1 - P_{2xm}; & (18) \\ \int_{xm}^{\beta} x \cdot p_2(x)dx = M_2 - M_{2xm}; & (19) \\ \int_{-\beta}^{\beta} x^2 \cdot p_2(x)dx = \sigma_2^2 - \sigma_{2xm}^2 + M_2^2 - M_{2xm}^2, & (20) \end{cases}$$

where M_1, M_2 and σ_1^2, σ_2^2 are mathematical expectations and dispersions, defined by the densities $p_1(x)$ and $p_2(x)$ accordingly; P_{xm} - is the probability, defined by the density $p_2(x)$ on the interval $[-\beta; x_m]$; M_{2xm} is a part of mathematical expectation, falling in the interval $[-\beta; x_m]$; σ_{2xm}^2 is a part of the dispersion, falling in the interval $[-\beta; x_m]$; x_m - value of abscissa at which $p_2(x)$ vanishes.

Then $p_2(x)$ can be represented as:

$$p_2(x) = \begin{cases} p_2^{(1)}(x) = p_1^{(1)}(x) + \frac{1}{2} \cdot (\lambda_1 + \lambda_2 \cdot x + \lambda_3 \cdot x^2), & -\beta \le x < x_m; \\ p_2^{(2)}(x) = p_1^{(2)}(x) + \frac{1}{2} \cdot (\lambda_4 + \lambda_5 \cdot x + \lambda_6 \cdot x^2), & x_m \le x \le \beta, \end{cases} \tag{21}$$

and constants $\lambda_1 \ldots \lambda_6$ are defined keeping in mind the non-negativity of the function $p_2(x)$, the minimum of which is equal to null and also depends on parameters M_1, M_2, σ_1^2, σ_2^2, M_{2xm}, σ_{2xm}^2, P_{2xm}, x_m.

The simultaneous equations for the definition $\lambda_1 \ldots \lambda_6$ are obtained from the system (15) … (20) at substitution of (21) into it

$$\begin{cases} \lambda_1 = k_1 \cdot P_{xm} + k_2 \cdot M_{xm} + k_3 \cdot \sigma_{xm}^2, & (22) \\ \lambda_2 = k_2 \cdot P_{xm} + k_4 \cdot M_{xm} + k_5 \cdot \sigma_{xm}^2, & (23) \\ \lambda_3 = k_3 \cdot P_{xm} + k_5 \cdot M_{xm} + k_6 \cdot \sigma_{xm}^2, & (24) \\ \lambda_4 = -g_1 \cdot P_{xm} + g_2 \cdot (M - M_{xm}) + g_3 \cdot (\sigma^2 - \sigma_{xm}^2), & (25) \\ \lambda_5 = -g_2 \cdot P_{xm} + g_4 \cdot (M - M_{xm}) + g_5 \cdot (\sigma^2 - \sigma_{xm}^2), & (26) \\ \lambda_6 = -g_3 \cdot P_{xm} + g_5 \cdot (M - M_{xm}) + g_6 \cdot (\sigma^2 - \sigma_{xm}^2), & (27) \end{cases}$$

where

$P_{xm} = P_{2xm} - P_{1xm}$; $M_{xm} = M_{2xm} - M_{1xm}$; $M = M_2 - M_1$;

$\sigma_{xm}^2 = \sigma_{2xm}^2 - \sigma_{1xm}^2 + M_{2xm}^2 - M_{1xm}^2$; $\sigma^2 = \sigma_2^2 - \sigma_1^2 + M_2^2 - M_1^2$;

$k_i, g_i, (i = \overline{1,6})$ - constants inserted for simplification of the expression view.

There are ten unknowns in the given six equations: $\lambda_1 \ldots \lambda_6$, M_1, M_2, σ_1^2, σ_2^2, M_{x_m}, $\sigma_{x_m}^2$, P_{x_m}, x_m. It is necessary to use the following four conditions for their definition, assuming that the required function $p_2(x)$ is smooth, with an extreme equal to null. In this case the derivatives of the point x_m are equal to each other if they are equal to null:

$$\begin{cases} p_2^{(1)}(x_m) = 0, & (28) \\ \left. \dfrac{dp_2^{(1)}(x)}{dx} \right|_{x=x_m} = 0, & (29) \\ p_2^{(2)}(x_m) = 0, & (30) \\ \left. \dfrac{dp_2^{(2)}(x)}{dx} \right|_{x=x_m} = 0. & (31) \end{cases}$$

After the solution of the system (22)…(27) with conditions (28)…(31), retrieved unknown parameters are substituted in the expression (21). Using (21) the probability of non-assembling parts P is calculated. The results of the calculations are given in Table 2. The view of the given density function of a sizes deviation of bushes and the calculated density function of a sizes deviation of shafts is presented in Figure 2.

Table 2. Results of calculation of the magnitude of incomplete production modelling.

$\sigma_1^2 = 0{,}1;\quad M_1 = 0{,}1$				
	$\sigma_2^2 = 0{,}2$		$\sigma_2^2 = 0{,}3$	
M_2	0.3	0.4	0.3	0.4
P	0.28151	0.38667	0.41673	0.52029

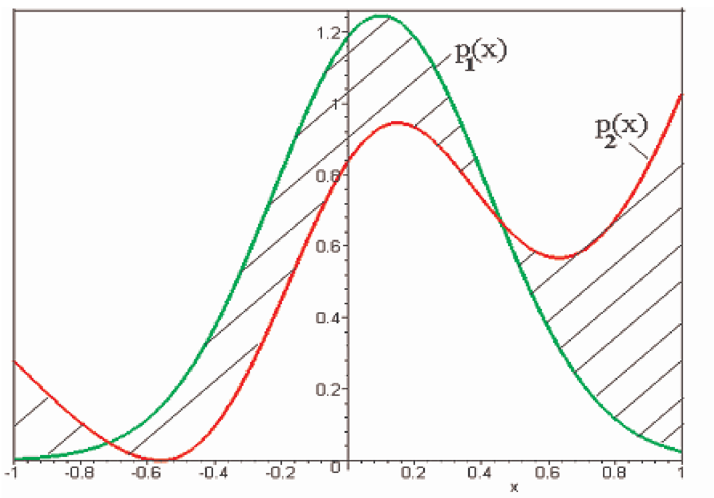

Fig. 2. Given density function of size deviation of bushes and calculated density function of size deviation of shafts ($\sigma_2^2 = 0{,}2$ and $M_2 = 0{,}3$)

The suggested method for definition of the probability of non-assembling allows finding the lower limit of incomplete production under the conditions mentioned above.

The suggested limits can be used for forecasting the volume of exhaustion of products and also for estimation of the quality of process monitoring of selective assembling as a comparison standard.

The idea of parameter control of selective assembling is offered in [1]. The procedure consists in displacement organization of the acceptance of shaft production that is proved by simpler manufacturing methods of outside surfaces. Therefore it is supposed to control their centres of an alignment.

All algorithms of controlled selective assembling can be divided into one-step and multi-step.

Firstly, let us consider the one-step algorithm which is organised as follows: the volume of the shafts deviations within the tolerance zone is divided into *n* generally unequal parts. Each group has its centre of alignment. The controlling parameters are the number of groups, their centres of alignment, and volumes. The indicated parameters are chosen so as to minimise the incomplete production, that is the task of multi-parameter optimisation is solved.

The task of the control of the selective assembling can be presented as follows: to find the meaning of the vector $Y(n,c_i,q_i)$ providing an extreme of the function

$$Y(n,c_i,q_i) \to \min\left\{\overline{Q} = 0{,}5 \cdot Q \cdot \int_{-\beta}^{\beta}\left|p_1(x) - \sum_{i=1}^{n} p_{2\,i}(x,n,c_i,q_i)\right|dx\right\}, \qquad (32)$$

$$Q_i = Q \cdot q_i \,,$$

where n is the number of groups; Q_i is the volume of parts in subgroup i; q_i is the probability of parts hit in subgroup i; $\sum_{i=1}^{n} q_i = 1$.

The view of the initial density function $p_1(x)$ and $p_2(x)$ for bushes and shafts, respectively, provided with existing inventory, is considered to be known. The optimum number of n_{opt} subgroups was defined by exhaustive search. Further calculations have shown that division into a number of subgroups exceeding 6…8 is inexpedient, as it does not result in substantial improvement. Therefore, the exhaustive search does not require considerable machining time and can be used as a method.

Thus, with the n fixed, the two-parameter optimisation on parameters c_i, q_i was carried out. Modelling was carried out for the symmetric laws $p_1(x)$ and $p_2(x)$. Thus, as it is shown in (9), (13), (14)

$$p_{2\,opt}(x) = p_1(x) + \frac{15}{8 \cdot \beta^3} \cdot \sigma^2 - \frac{45 \cdot \sigma^2}{8 \cdot \beta^5} \cdot x^2 \,.$$

The given expression is used as a comparison standard. Apart from this, it is necessary to define the mathematical expectations $m_{2H}(x)$ and the dispersions $\sigma^2_{2H}(x)$ corresponding to the new density function $p_{2н}(x)$, obtained after the division into n subgroups. Let us consider the expressions to define the indicated magnitudes while dividing into 2, 3, 4, 5 subgroups (33)…(37).

While dividing into 2 subgroups (in this case it is obvious that volumes of the subgroups are equal):

$$m_{2H}(x) = \int_{-(a+c)}^{a+c} x \cdot p_{2H}(x)dx = \frac{1}{2} \cdot \int_{-(a+c)}^{(a-c)} x \cdot p_2(x+c)dx + \frac{1}{2} \cdot \int_{-(a-c)}^{(a+c)} x \cdot p_2(x-c)dx = 0\,; \quad (33)$$

$$\sigma_{2H}^2(x) = \int_{-(a+c)}^{a+c} x^2 \cdot p_{2H}(x)dx = \sigma_2^2(x) + c^2 \quad (34)$$

While dividing into 3, 4, 5 subgroups, the mathematical expectations are also equal to null, therefore let us consider the expressions for dispersions only.

While dividing into 3 subgroups

$$\sigma_{2H}^2(x) = \int_{-(a+c)}^{a+c} x^2 \cdot p_{2H}(x)dx = \sigma_2^2(x) + (q_1 + q_3) \cdot c_1^2\,, \quad (35)$$

$$|c_1| = |c_2|\,.$$

While dividing into 4 subgroups

$$\sigma_{2H}^2(x) = \int_{-(a+c)}^{a+c} x^2 \cdot p_{2H}(x)dx = \sigma_2^2(x) + (q_2 + q_3) \cdot c_2^2 + (q_1 + q_4) \cdot c_1^2\,, \quad (36)$$

$$|c_2| = |c_3|\,,\ |c_1| = |c_4|\,.$$

While dividing into 5 subgroups

$$\sigma_{2H}^2(x) = \int_{-(a+c)}^{a+c} x^2 \cdot p_{2H}(x)dx = \sigma_2^2(x) + (q_2 + q_4) \cdot c_2^2 + (q_1 + q_5) \cdot c_1^2 \quad (37)$$

$$|c_1| = |c_5|\,,\ |c_2| = |c_4|$$

The numbers i of subgroups are counted starting from the left boundary of size deviations.

The calculations show that while dispersing the allocation of sizes of the bushes $\sigma_2^2 = 0{,}1$ and shafts $\sigma_1^2 = \frac{13}{21}\sigma_2^2$ the incomplete production in the case of controlled selective assembling while dividing into two subgroups makes 0.01642 at the incontrollable boundary estimation of 0.05499. Thus the displacement is $c = \pm 0{,}18$. The given example visually shows the expediency of controlled selective assembling since the incomplete production has decreased by more than 3 times.

Analogous researches have been carried out for 3, 4 and 5 subgroups. The following results (table 3) are received. The views of curves are presented in the figure 3.

Table 3. Results of the calculations

n	2	3		4		5		
c_i	±0,18	0	±0,3	±0,3	±0,01	0	±0,31	±0,31
q_i	2×0,5	0,59	2×0,21	2×0,21	2×0,29	0,42	2×0,1	2×0,19
P	0,01642	0,00204		0,00201		0,00178		

For 3 subgroups, IP is 0.00204 with the displacement $c_0 = 0$, $c_1 = \pm 0{,}3$ and probabilities of parts hit in subgroups $q_1 = 0{,}21$, $q_2 = 0{,}58$. For 4 subgroups, IP is 0.00201 with the displacement $c_1 = \pm 0{,}3$, $c_2 = \pm 0{,}01$ and probabilities of parts hit in subgroups $q_1 = 0{,}21$, $q_2 = 0{,}29$. These values are 27-28 times less than those obtained with an incontrollable selective assembling.

For 5 subgroups, IP is 0,00178 with the displacement $c_0 = 0$, $c_1 = \pm 0{,}09$, $c_2 = \pm 0{,}31$ and probabilities of parts hit in subgroups $q_0 = 0{,}42$, $q_1 = 0{,}1$, $q_2 = 0{,}19$. It is 30 times less than in the case of incontrollable selective assembling.

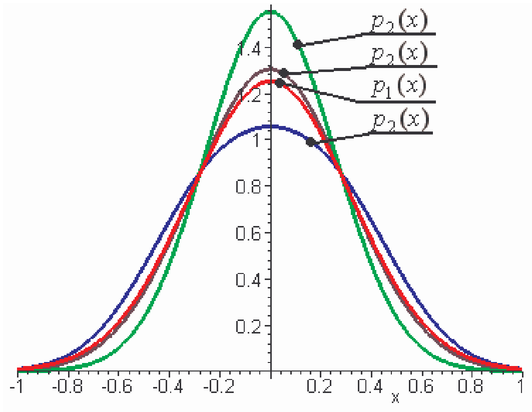

Fig. 3. Curves of size deviations $p_1(x)$ and $p_2(x)$ while dividing into 3, 4 and 5 subgroups

Further division into subgroups has not given any considerable results.

For the analysis of expediency of controlled assembling usage if n is fixed, one-parameter optimisation on parameter c_i has also been carried out. As before, modelling was carried out for the symmetric laws $p_1(x)$ and $p_2(x)$. Besides that, the mathematical expectations $m_{2H}(x)$ and the dispersions $\sigma^2_{2H}(x)$ corresponding to new density function $p_{2н}(x)$ obtained have been spotted while dividing into n subgroups (special cases of expressions (33) … (37)). Let us consider the expressions to define the indicated magnitudes while dividing into 2, 3, 4, 5 subgroups.

While dividing into 2 subgroups:

$$m_{2H}(x)=\int_{-(a+c)}^{a+c} x\cdot p_{2H}(x)dx=\frac{1}{2}\cdot\int_{-(a+c)}^{(a-c)} x\cdot p_2(x+c)dx+\frac{1}{2}\cdot\int_{-(a-c)}^{(a+c)} x\cdot p_2(x-c)dx=0\,;$$

$$\sigma_{2H}^2(x)=\int_{-(a+c)}^{a+c} x^2\cdot p_{2H}(x)dx=\sigma_2^2(x)+c^2\,.$$

While dividing into 3, 4, 5 subgroups, mathematical expectations are also equal to null, therefore let us consider expressions for dispersions only.

While dividing into 3 subgroups

$$\sigma_{2H}^2(x)=\int_{-(a+c)}^{a+c} x^2\cdot p_{2H}(x)dx=\sigma_2^2(x)+\frac{2}{3}\cdot c_1^2\,.$$

$$|c_1|=|c_2|\,.$$

While dividing into 4 subgroups

$$\sigma_{2H}^2(x)=\int_{-(a+c)}^{a+c} x^2\cdot p_{2H}(x)dx=\sigma_2^2(x)+\frac{1}{2}\cdot c_2^2+\frac{1}{2}\cdot c_1^2\,,$$

$$|c_1|=|c_4|\,,\ |c_2|=|c_3|\,.$$

While dividing into 5 subgroups

$$\sigma_{2H}^2(x)=\int_{-(a+c)}^{a+c} x^2\cdot p_{2H}(x)dx=\sigma_2^2(x)+\frac{2}{5}\cdot c_2^2+\frac{2}{5}\cdot c_1^2\,.$$

$$|c_1|=|c_5|\,,\ |c_2|=|c_4|\,.$$

Numbers of subgroups i are read off from the left-hand boundary line of deviations of size.

The received values were compared to the boundary estimation of incontrollable selective assembling. The results are given in Table 4.

With dispersions of size allocation of the bushes $\sigma_2^2=0{,}1$ and shafts $\sigma_1^2=\frac{13}{21}\sigma_2^2$, incomplete production takes place if it is controlled selective assembling with one-parameter optimisation concerning the parameter c_i for 3 subgroups of 0.01642 at incontrollable boundary estimation of 0.05499. Thus the displacement is $c_0=0$, $c_1=\pm0{,}18$. In this case the incontrollable incomplete production has decreased 3 times more. For 4 subgroups, IP is 0.00583 at a boundary estimation of 0.05499. Thus the displacement is $c_1=0$, $c_2=\pm0{,}27$. For 5 subgroups IP is 0.00515. Thus the displacement is $c_0=0$, $c_1=0$, $c_2=\pm0{,}27$.

Table 4. Results of the calculations

n	2	3		4		5		
c_i	±0,18	0	±0,23	0	±0,27	0	±0,31	±0,29
P	0,01642	0,01196		0,0058		0,0051		

The given instances visually show the expediency of using one-step algorithm of controlled selective assembling. Further division into subgroups will not give any considerable decrease of IP result.

Multi-step controlled selective assembling is planning the indicated process minding the incomplete production generated after accomplishment of the previous step. Not collected parts participate in the following stage of assembling, and the following batch is made minding the volume and density function of size deviations of the remaining non-collected parts. Thus, the task is reduced to research of effect of parameters of control of the selective assembling in dynamics, i.e. planning of production is carried out minding the incomplete production of accomplishment of the previous step.

Control of the process of selective assembling in dynamic conditions is possible with the help of sampling of the number of groups into which the volume of agglomerated parts at an observed step is divided, coordinates of the centres of an alignment of inventory, and volumes of groups of made parts.

Mathematically the task can be presented as finding the meaning of vector $Y(n_j, c_{ij}, q_{ij}, Q_j)$ providing an extreme of the function

$$Y(n_j, c_{ij}, q_{ij}, Q_j) \to \min\left\{ \overline{Q} = 0{,}5 \cdot \sum_{j=1}^{m} [Q_j \cdot \int_{-\beta}^{\beta} \left| p_{1j}(x) - \sum_{i=1}^{n_j} p_{2j}(x,\ n_j, c_{ij}, q_{ij}) \right| dx] \right\}; \qquad (38)$$

$$P_j = \int_{-\beta}^{\beta} \left| p_{1\Sigma j}(x) - \sum_{i=1}^{n} p_{2\Sigma j}(x, c_{ij}, q_{ij}, n_j) \right| dx; \qquad (39)$$

$$\overline{Q}_j = 0{,}5 \cdot Q_j \cdot P_j = 0{,}5 \cdot P_j \cdot \sum_{i=1}^{n_j} Q_{ij}; \qquad Q_{ij} = Q_j \cdot q_{ij},$$

where n_j - number of subgroups on a step j; c_{ij} - coordinate of the centre of alignment of subgroups i at step j; q_{ij} - probability of hit of parts in subgroup i at step j ($\sum_{i=1}^{n_j} q_{ij} = 1$); Q_j - volume of parts agglomerated at step j; Q_{ij} - volume of subgroup i at step j; j - the number of a step at realization of multi-step assembling $j = \overline{1,m}$; m - maximum number of steps ($m = \overline{2,\infty}$); $Q = \sum_{j=1}^{m} Q_j$ - total amount of agglomerated parts; $p_{1\Sigma j}(x)$ - density function of bushes agglomerated at step j, depending on magnitude of not assembling at step ($j-1$); $p_{2\Sigma j}(x, c_{ij}, q_{ij}, n_j)$ - density function of shafts agglomerated at step j, depending on parameters c_{ij}, q_{ij}, n_j and magnitude of not assembling at step ($j-1$); P_j - probability of not assembling parts at step j ; $\overline{Q}_j$ - volume of not assembling parts at step j .

If agglomerated volumes of parts at each step are equal to ($Q_k = Q_l = \frac{1}{m} \cdot Q;\ k \neq l,\ k = \overline{1,m};\ l = \overline{1,m}$), then the expressions (38, 39) will become:

$$Y\left(n_j, c_{ij}, q_{ij}, Q\right) \rightarrow \min\left\{\overline{Q} = 0{,}5 \cdot Q \cdot \frac{1}{m}\sum_{j=1}^{m}\int_{-\beta}^{\beta}\left|p_{1\Sigma\, j}(x) - \sum_{i=1}^{n} p_{2\Sigma\, j}\left(x,\ n_j, c_{i\,j}, q_{i\,j}\right)\right| dx\right\}; \quad (40)$$

$$P = \frac{1}{m}\sum_{j=1}^{m}\int_{-\beta}^{\beta}\left|p_{1\Sigma\, j}(x) - \sum_{i=1}^{n} p_{2\,\Sigma j}\left(x,\ n_j, c_{i\,j}, q_{i\,j}\right)\right| dx, \quad (41)$$

where P - average overall probability of non-assembling, corresponding to the whole process.

Let us mark that, as a whole, controlled selective assembling includes one- and multi-step algorithms in various combinations. The one-step algorithm was described earlier, and the multi-step can be divided into two aspects:

- multi-step selective assembling with a given volume of agglomerated parts;
- multi-step selective assembling with unlimited volume of agglomerated parts within m months ($m = \overline{2,\infty}$), with the presence or absence of scheduled monthly tasks.

Multi-step selective assembling with a given volume of agglomerated parts is fulfilled by the following algorithm. Division of the agglomerated total number of parts Q into m groups for production and assembling at m steps is carried out. Production of parts at the indicated steps is carried out in succession in time, i.e. each group is made keeping in mind the previous assembling at support of a minimum of incomplete production. Let us consider implementation of control by multi-step selective assembling on the example of a two-step process.

Multi-step selective assembling occurs for a given volume of agglomerated parts according to the following algorithm:

1. Two semi-batches of parts are made, in volumes Q'.
2. They are assembled in units, as a result of which incomplete production is made, in volume q'.
3. Two semi-batches of parts are made, in volumes $Q'' = Q' - q'$, the number of parts in the new batches being less than required in q'. Displacement of distribution functions of these batches is selected so that the level of incomplete production at assembling of batches $\overline{Q''}$ is minimal. That is, on assembling the required number of parts, as parts are added in a batch, dead assembling at the previous step comes in action.

Thus, there is a process of control by selective assembling in dynamic conditions.

The view of partition law of size deviation of bushes and shafts at the second step is definable. At this step a mix of detail parts remaining from the previous (first) step and manufactured at the current (second) step is formed.

Let us mark out density functions of size deviations of two semi-batches of parts (bushes and shafts) as $p_1(x)$ and $p_2(x)$, respectively. In practice, it is possible to plot them by building up histograms. At the presence of sorting machines, this process is executed automatically. As a result of assembling at the previous step, there appeared incomplete production.

It is necessary to find distribution functions $p_{1\Sigma}(x)$ and $p_{2\Sigma}(x)$ of size deviations of shafts and bushes participating in assembling at the current (second) step, previously having spotted allocations of size deviations of parts $p_{11}(x)$ and $p_{21}(x)$, included in the incomplete production which has remained after the previous step.

Let event A be the hit of bushes in the given interval $[-\beta; x]$ at the next step of assembling process. Then $P(A)$ is the probability of that the bush has got in the given interval $[-\beta; x]$.

By the formula of composite probability

$$P(A) = \sum_{j=1}^{2} P(H_j) \cdot P(A/H_j), \tag{42}$$

event A can occur under the condition of implementation of one of the hypotheses H_1, H_2. The hypothesis H_1 will be that at the current step of assembling, bushes manufactured at the given step are used. The hypothesis H_2 will be that at the current step of assembling, bushes are used that were manufactured on previous step and appeared not assembled. Thus $P(H_1)$ - probability that in assembling bushes manufactured at the current step participate. $P(H_2)$ - probability that in assembling bushes which were manufactured at the previous step and did not find mating parts participate. $P(A/H_1)$ - probability of hit of bushes in an interval $[-\beta; x]$ under the condition of implementation of hypothesis H_1 equal distribution functions $F_1(x)$ of size deviation of parts (bushes) at the given step. $P(A/H_2)$ - probability of hit of bushes in an interval $[-\beta; x]$ under the condition of implementation of hypothesis H_2 equal distribution functions $F_{11}(x)$ of size deviation of parts (bushes) which have remained from the previous step. The densities corresponding to indicated distribution functions are marked out as $p_1(x)$, $p_{11}(x)$.

Thus, the formula (42) will become

$$P(A) = P(H_1) \cdot P(A/H_1) + P(H_2) \cdot P(A/H_2) = P(H_1) \cdot F_1(x) + P(H_2) \cdot F_{11}(x) \tag{43}$$

that fully refers to parts 1 (bushes) and to parts 2 (shafts). According to the function and density function of shafts manufactured at the previous and given steps, these are marked out as $F_2(x)$, $F_{21}(x)$, $p_2(x)$, $p_{21}(x)$.

The probability of implementation of hypothesis H_2 is equal to the relative share of parts not assembled at the previous step in the total the amount M of agglomerated parts at the given step. Keeping in mind the above and formula (1), $P(H_1)$ and $P(H_2)$ are equal to:

$$P(H_2) = \frac{\overline{Q}}{M} = k_2; \tag{44}$$

$$P(H_1) = 1 - P(H_2) = k_1. \tag{45}$$

Then the functions and density function of size deviation of bushes and shafts, obtained from (43) … (45), are accordingly equal to:

$$F_{1\Sigma}(x) = k_1 \cdot F_1(x) + k_2 \cdot F_{11}(x)\,; \tag{46}$$

$$F_{2\Sigma}(x) = k_1 \cdot F_2(x) + k_2 \cdot F_{21}(x)\,; \tag{47}$$

$$p_{1\Sigma}(x) = k_1 \cdot p_1(x) + k_2 \cdot p_{11}(x)\,; \tag{48}$$

$$p_{2\Sigma}(x) = k_1 \cdot p_2(x) + k_2 \cdot p_{21}(x)\,. \tag{49}$$

Expressions for $p_{11}(x)$ and $F_{11}(x)$ are defined as follows:

$$p_{11}(x) = \begin{cases} \dfrac{p_1(x) - p_2(x)}{0{,}5 \cdot P}, \; p_1(x) > p_2(x); \\ 0, \; p_1(x) < p_2(x) \end{cases} \tag{50}$$

$$F_{11}(x) = \begin{cases} \dfrac{F_1(x) - F_2(x)}{0{,}5 \cdot P}, \; p_1(x) > p_2(x); \\ 0, \; p_1(x) < p_2(x), \end{cases} \tag{51}$$

where P is determined according to (39), and in special cases to (41).

Expressions for $p_{21}(x)$ and $F_{21}(x)$ are defined analogously:

$$p_{21}(x) = \begin{cases} \dfrac{p_2(x) - p_1(x)}{0{,}5 \cdot P}, \; p_1(x) < p_2(x); \\ 0, \; p_1(x) > p_2(x) \end{cases} \tag{52}$$

$$F_{21}(x) = \begin{cases} \dfrac{F_2(x) - F_1(x)}{0{,}5 \cdot P}, \; p_1(x) < p_2(x); \\ 0, \; p_1(x) > p_2(x); \end{cases} \tag{53}$$

Let us give an instance of use of the received expressions. If permissible, densities of allocations of size deviation of the bushes and shafts correspond to Figure 4.

In this case the number of intersection points of curves $p_1(x)$ and $p_2(x)$ is equal to 2, and $p_1(0) < p_2(0)$ (that is the ordinate of the density function of size deviation of bushes at that value is more than the ordinate of the density function of shafts, as shown in the graph). The view of densities $p_{11}(x)$ and $p_{21}(x)$ received according to expressions (50), (52) is presented in Figures 5 and 6. The view of densities $p_{1\Sigma}(x)$ and $p_{2\Sigma}(x)$ received according to expressions (48), (49) is presented in Figure 7. Magnitude of non-assembling, calculated according to formulas (38), (39), (48) … (53) is equal 0,00395 at displacement $c_2 = \pm 0{,}28$ and equal volumes of subgroups.

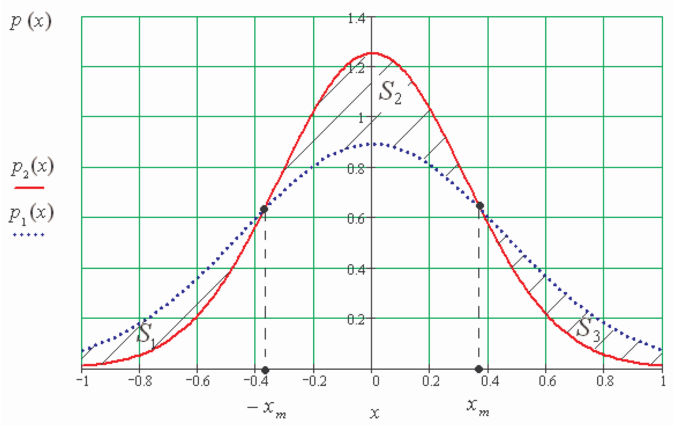

Fig. 4. Densities of allocations of size deviations of shafts and bushes:
$p_1(x)$ and $p_2(x)$

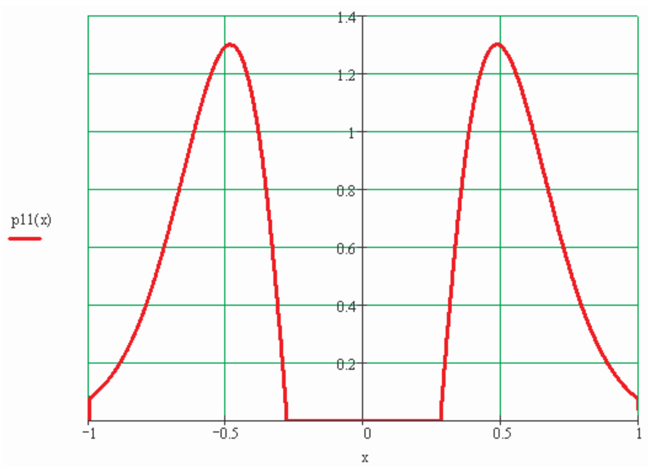

Fig. 5. Density function of bushes which have remained from the previous step

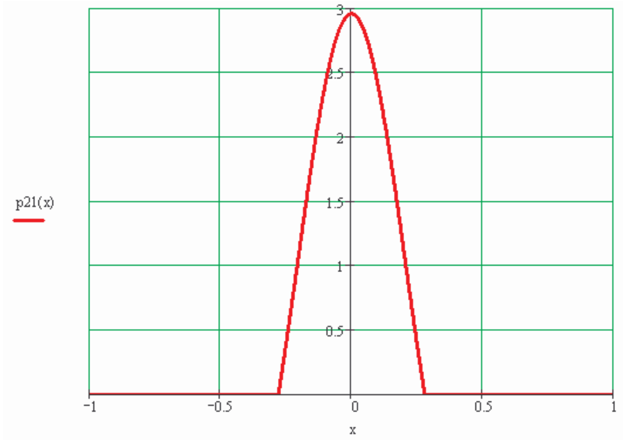

Fig. 6. Density function of shafts which have remained from the previous step

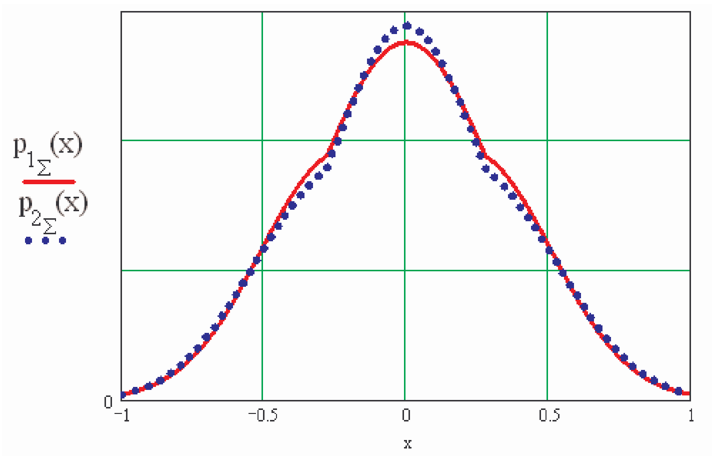

Fig. 7. Density function $p_{1\Sigma}(x)$ and $p_{2\Sigma}(x)$ size deviations of bushes and shafts, respectively

The results of modelling are exemplified in Table 5.

Table 5. Results of modelling of the process of multi-step controlled selective assembling with given volume of agglomerated parts

n	c_i		q_i			P
2	±0,28	–	–	0,5	0,5	0,0395
3	0	±0,3	0,1	2×0,45	–	0,00184
4	±0,25	±0,4	–	2×0,4	2×0,1	0,00141

If it is necessary to provide a set volume of assembled units, it is required to define the detail parts volume made at the second step. This procedure is executed through iteration (by selection) and does not involve any difficulty.

Multi-step selective assembling with unlimited volume differs from the previous one in the absence of restriction for the total amount of parts Q made during n months, and n is not defined beforehand. At each step (monthly program is accepted for a step) it is either one-step algorithm that is used, or a multi-step with a given volume of agglomerated parts given above. Generally, monthly programs can be various. At each step parts participate in assembling either left from the previous step, or produced during the current one. Their amount is selected for the given monthly program through iteration, as mentioned above. Density and distribution functions of size deviations of parts either left from the previous step, or generating a mix on a given one, are defined with the help of the formulas (46)…(53). The monitoring of the assembling process is made with the help of the same parameters as in multi-step selective assembling with given volume of agglomerated parts. The basic difference between the given process and the previous one is that its duration and, consequently, the volume of agglomerated parts are not defined beforehand.

Let us put the following instance of use of the built model. It is admissible that firmness of allocations of size deviation of bushes and shafts after step i will match Figure 8. Here the kind of module of difference of density function of size deviations of bushes and shafts $q(x)$ is adduced. At the indicated step of assembly, the number of cross points of curves $p_1(x)$ and $p_2(x)$ is equal to four, and $p_1(0) > p_2(0)$ (that is the ordinate of density function of size deviations of bushes at value $x = 0$ is more than ordinate of density function of shafts, Fig. 8).

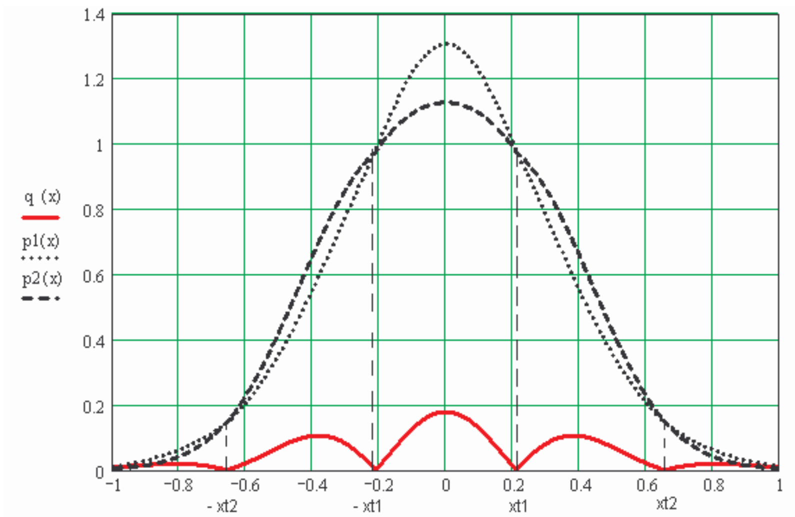

Fig. 8. Density functions $p_1(x)$ and $p_2(x)$ of size deviations of bushes and shafts, respectively, $q(x) = |p_1(x) - p_2(x)|$ - module of difference of density function, xt_j - coordinates of abscissae of cross points, where $i = 4$

Let us mark out, through $p_{1,i-1}(x)$ and $p_{2,i-1}(x)$, the density function of size deviations of bushes and shafts remaining after non-assembling at step $i-1$, and through $p_{1,i\Sigma}(x)$ and $p_{2,i\Sigma}(x)$ - the density function of size deviations of bushes and shafts which are forming a mix at step i. These curves are shown in Figures 7, 8. Coordinates of cross points xt_j of curves $p_{1,i\Sigma}(x)$ and $p_{2,i\Sigma}(x)$ were determined numerically with the help of a special procedure.

The complexity of use of the built model consists in the volume that the process of an operation with $p_{1,i\Sigma}(x)$ and $p_{2,i\Sigma}(x)$, already after two steps, is extremely labour-consuming. Therefore, in the model their approximation by degree polynomials not below the tenth order is used by the least-squares method. Thus

$$p_{k,i\Sigma}(x) = \sum_{i=0}^{l} a_i (x - x_0)^i,\ k = 1,2\ ,$$

where x_0 is a point of decomposition which is equal to null in the cases observed above.

The results of modelling are exemplified in Table 6.

Table 6. Results of modelling of multi-step controlled selective assembling process with unlimited volume of agglomerated parts while dividing the monthly batch of parts into two subgroups.

Number of months	1	2	3	4	5	6
c	±0,18	±0,18	±0,18	±0,15	±0,17	±0,18
P	0,153	0,00435	0,00164	0,00935	0,00181	0,00358

The researches carried out show that process monitoring of selective assembling allows to lower the level of incomplete production, and in some cases, as can be observed from the Tables, by 1,5 times. Thus the usage expediency of the suggested methods of selective assembling control is obvious, as these methods do not require any additional apparatus expenses, and consequently, any financial expenditure.

References

[1] BULOVSKIJ P. I., KRYLOV P. I., LAPUCHIN W. A.: Avtomatizacja selektivnoj sborki priborov (in Russian). Mashinostroenie, Leningrad 1978, 232 s.

[2] KOPP W. J., SEROWA N. B.: Variacionnyj podchod k ocenke nezaverszonnogo proizvodstva pri selektivnoj sborke pri nesovpadajuszczich toczecznych ocenkach sluczainych vieliczin (in Russian). Sb. Naucznych rabot SNIJEiP/10, Sewastopol 2004, s. 41-48.