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FINITE ELEMENT ANALYSIS OF PANTOGRAPH-CATENARY DYNAMIC INTERACTION

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Abstract: *Numerical simulations of the pantograph-catenary dynamic interaction allow for assessment of the current collection quality provided by this subsystem at the design stage of the pantograph or the catenary system. In this paper, the authors present numerical results of simulations of the dynamic interaction between a pantograph and a catenary. Adopted catenary and pantograph models are consistent with the reference model presented in the EN 50318:2002 document, which describes the validation procedure of the simulation for the dynamic interaction between pantograph and overhead contact line. Authors have used the Finite Element Method to analyze this complex system. Ten catenary spans of simple catenary (one messenger wire and one contact wire) are modelled, each of them is 60m long. The pre-sag of the catenary is 0mm and the model of the catenary includes the stagger which equals ±200mm. The pantograph model consists of lumped masses which are connected each other with spring-damper elements. First, the static initial configuration is obtained (under gravity and tensioning loads), after which the dynamic transient simulation is conducted. Obtained results for the contact force and uplifts at supports are within the reference ranges presented in the EN 50318:2002 document, therefore it can be considered that the adopted model correctly reproduces the dynamic behaviour of the pantograph-catenary dynamic interaction.*

Key words: *pantograph, catenary, interaction, EN50318, FEM.*

1. Introduction

Recently, important developments are noticed in many fields of the railway technology. In the passenger transport, the main trend is moving towards increasing the availability of high-speed railways for passengers, what is a relevant factor to make this transport type more competitive. Increasing the operating speed of trains involves adequate railway infrastructure. Proper interaction of components of trains and railway infrastructure is a crucial factor for reliability and safety of high-speed railways (Kumaniecka and Snamina, 2008). There are at least two interesting interfaces in a train, which are highly affected by vibrations: vehicle-track system (discussed e.g. by Sowiński 2003, 2006) and pantograph-catenary system. A pantograph mechanism is mounted on a roof of a train and is involved in transmission of the electric energy from the overhead contact line (OHL) to a train. The most basic form of the catenary consists of three important components: a contact wire, a messenger wire and droppers. Conspicuous scheme of this subsystem is presented in the picture below - Fig. 1. The contact wire touches the pantograph contact strip and takes part in supplying power to a train. The messenger wire is used to suspend the contact wire. Droppers connect contact and messenger wires. Pretension of contact and messenger wires and proper lengths of droppers provide required pre-sag of the contact wire and stiffness of the catenary structure.

Proper dynamic interaction between pantograph and catenary is particularly important because of a need for constant current supply to a train. Moreover, when operating speeds of trains are still increasing, there are more aspects crucial in the view of this phenomenon, which are not that important for conventional-speed trains (e.g. wave propagation in the catenary structure and aerodynamic forces exerted on the pantograph). Therefore, there is a need for detailed analysis and adjustment of the pantograph-catenary system to guarantee constant contact between the contact strip of the pantograph and the contact wire for high-speed trains. This should be done without relevant increase of wear of components in this system, which is related with the speed of a train, the contact force and the current density (Bucca and Collina, 2009). The contact force

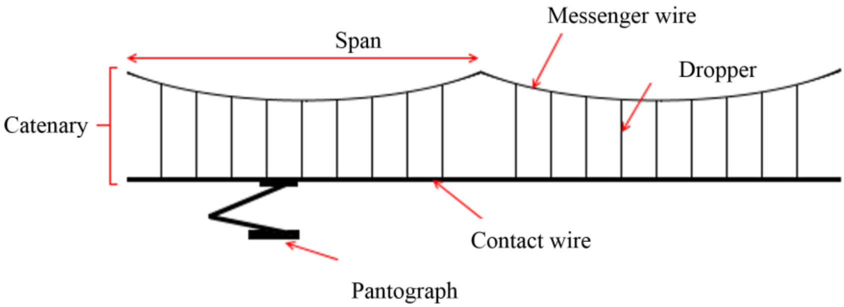

Fig. 1. Pantograph – catenary subsystem

is one of the parameters, which describes the quality of the pantograph-catenary interaction. Therefore, many researchers focus on numerical models of this phenomenon, that could help to determine the contact force in this vibrating subsystem. Numerical models help to determine the contact force and other output parameters (like uplift of contact wire), which are difficult and expensive to measure in real runs.
Simulations are used at the design stage of new catenaries and pantographs, but also at the modernization stage of existing systems, e.g. to increase the operational speed of trains with existing infrastructure, (Nåvik and Rønnquist, 2014). It is worth mentioning that numerical simulations of the dynamic interaction of the pantograph-catenary system are required by the Technical Specification of Interoperability (TSI) released by the European Commission for new catenary designs. The pantograph-catenary interaction is a complex, multi-physics phenomenon which involves following aspects: geometry nonlinearity, contact nonlinearity, slackening of droppers, vibrations under moving load, influence of electro-magnetic field (Poetsch, et al., 1997). All of these aspects have an influence on the final dynamic interaction between pantograph and catenary, therefore this phenomenon is difficult to describe in a proper mathematical manner. The simplest approach for modelling the pantograph-catenary interaction concerns lumped parameters models for pantograph and the catenary structure. Commonly known, simplified method simplifies the influence of catenary on the pantograph to a lumped mass and spring elements that represents the catenary (with various stiffness along the span), while a pantograph is modelled as a 2 or 3 degrees of freedom (DOF) lumped parameters model. This approach gives a general insight in the pantograph-catenary dynamic interaction and is very efficient in terms of computational time. Nevertheless, the simplified model does not consider very important phenomenon for the pantograph-catenary dynamic interaction – the wave propagation effect in the catenary structure under moving load, what is the biggest drawback of this approach. The simplified model is frequently encountered in the literature (Poetsch, et al., 1997; Lopez-Garcia, et al., 2007; Grajnert, 1978; Kaniewski, 2013; Kobielski, 1985; Zhou, et al., 2011; Wu, et al., 1997; Wu, et al., 1999), because of its simplicity, computational efficiency and the ability to get a general insight in the pantograph-catenary dynamic interaction. In recent years, many researchers have focused on more sophisticated methods to model the pantograph-catenary dynamic interaction, that allow to take into account wave propagation phenomenon. Most of them involve the Finite Element Method (FEM), therefore these models are more complex than the simplified model because the number of DOF of finite elements (FE) models depends on the amount and type of used FE. Models for the pantograph-catenary dynamic interaction based on the FEM are presented in literature: (Zhou, et al., 2011; Jimenez-Octavio, et al., 2015; Cho, 2015; Jönsson, et al., 2015; Ikeda, 2015). Comparisons between the simplified model and more complex FEM approaches are presented in reference (Jimenez-Octavio, et al., 2008; Carnicero, et al., 2011; Kia, et al., 2010). Comparison between different FE models for simulating the pantograph-catenary dynamic interaction, prepared by various research centres, for a described benchmark problem is presented in reference (Bruni, et al., 2015). The approach based

on the co-simulation between the FE model of the catenary and complex Multibody model of pantograph is presented in literature (Ambrósio, et al. 2011; Ambrósio, et al., 2012a; Ambrósio, et al., 2012b; Massat, et al., 2014). The co-simulation approach allows to take into account the dynamics of the pantograph in more realistic manner during its interaction with the catenary. The co-simulation approach for multi-pantographs interaction with the catenary is presented by Ambrosio and Pombo (Pombo and Ambrosio, 2012). In many papers, authors pay special attention to the method of determining the initial equilibrium of catenary spans, which are under the influence of gravity and tensioning forces, to satisfy required pre-sag of the catenary structure at the same time. One of the solution of this problem was presented by Lopez-Garcia el al. (Lopez-Garcia, et al., 2006; Lopez-Garcia, et al., 2007).

Zdziebko and Uhl (Zdziebko and Uhl, 2015, 2016) presented the use of the Modal Superposition Method for the catenary structure modelling. This method adopted for representing the flexibility of the catenary reduces the number of DOFs comparing to pure FE models of the catenary, because only several modes are considered. At the same time, the wave propagation phenomenon in the catenary structure can be taken into account in this approach. Simulation approach presented in reference (Zdziebko and Uhl, 2016) was validated according to the European Standard 50318:2002 and obtained results did not fulfil reference ranges. This fact was the motivation for authors to continue research in the field of numerical models for the pantograph-catenary dynamic interaction.

The main goal of this paper is to create the numerical model with the use of the FEM for the pantograph-catenary dynamic interaction, which is accurate like other models presented above, which represent the actual state of the art. As mentioned in the reference (Uhl and Pieczara, 2003), in the case of computation of loads, the model quality plays essential role in the quality of obtained results, so there is a need for model assessment by the benchmark test. To assess the correctness of obtained results in the adopted simulation approach, it is necessary to validate the model. The validation should be carried out according to the first step of validation process presented by the European Standard 50318:2002 to check the correctness of obtained results for the reference model. The paper is organized in the following way; chapter 2. presents the numerical model of catenary and pantograph according to the EN50318:2002 Standard, chapter 3. describes the validation method, chapter 4. presents results obtained from the numerical experiment and chapter 5. summarizes the paper.

2. Model of the pantograph-catenary system

The reference pantograph-catenary model according to the EN50318:2002 was adopted in this paper. It takes into account one dimensional, lumped parameters model of the pantograph and a multidimensional model of the catenary spans with appropriate mechanical parameters. The adopted model of pantograph consists of two lumped masses connected with spring-damper elements. The static contact force exerted by the collector strip of the pantograph on the contact cable is equal to 120N. Aerodynamic forces are neglected. OHL consists of the messenger wire, 9 droppers per each span, single contact wire and registration arms. 10 identical spans were modelled including the stagger of the OHL. The contact wire is lifted by droppers and registration arms. Nonlinear character of droppers is taken into account in the model (slackening of droppers). Precise description of the reference model is provided by the European Standard (EN50318, 2002). The Finite Element Method was adopted to analyze the dynamic behaviour of this system. The pantograph is considered as a two-level spring-damper-mass system as shown in Fig. 2.

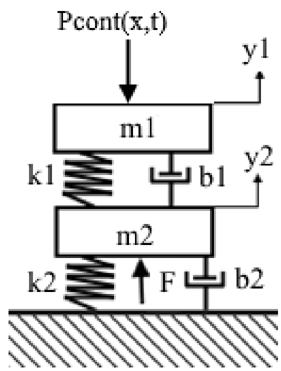

Fig. 2. Pantograph model

The dynamic equations of the pantograph model are (1):

$$\begin{cases} (m_1)\ddot{y_1} + b_1(\dot{y_1} - \dot{y_2}) + k_1(y_1 - y_2) = P_{cont}(x,t) \\ m_2\ddot{y_2} + b_1(\dot{y_2} - \dot{y_1}) + b_2\dot{y_2} + k_1(y_2 - y_1) + k_2 y_2 = F \end{cases} \quad (1)$$

where $P_{cont}(x,t)$ is the dynamic contact force between catenary and pantograph, $F$is the static uplift force of the pantograph and m, b, k represents the lumped mass, damping and stiffness of the collector head (subscript 1) and articulation frame (subscript 2).

The dynamic equation of the catenary (without damping) can be expressed as follows:

$$[M_C]\{\ddot{y_C}\} + [K_C]\{y_C\} = \{f(x,t)\} \quad (2)$$

where $[M_C]$ and $[K_C]$ are the mass and stiffness matrices of the catenary, $\{\ddot{y_C}\}$ and $\{y_C\}$ are the acceleration and displacement vectors of the catenary, $\{f(x,t)\}$ is the nodal force vector about the contact forces between pantograph and catenary at time *t*.

According to Equations (1) and (2), the dynamic equation of the pantograph-catenary system can be expressed in the following general form:

$$[M]\{\ddot{y}\} + [B]\{\dot{y}\} + [K]\{y\} = \{F(x,t)\} \quad (3)$$

Because of geometrical dimensions, cables of a catenary structure are usually modelled as 1-D elements. Force exerted by the collector strip of the pantograph introduces a bending moment in the contact wire, therefore beam elements (CBEAM) were used to model the contact wire. Despite the fact, that the bending component can be neglected in the messenger wire (which mainly operates in tension), beam elements were also used for the messenger wire modelling. Droppers were modelled as gap elements (CGAP), which have high stiffness for tension and almost zero for compression (nonlinear behaviour). Gap elements allow for shortening of elements, which is necessary to determine the initial equilibrium of the catenary structure (this was done in the iterative procedure), to provide zero pre-sag.

Rod elements (CROD) were used to model registration arms. Length of elements used for the catenary modelling is important factor in terms of obtained results. Detailed description of above mentioned elements is presented in reference (MSC.Nastran, Reference Manual, 2004). Too large elements are insufficient for proper consideration of the wave propagation and reflection phenomenon, while too small elements introduce large number of DOFs of the model and lead to high computational costs. In the adopted simulation model, elements with the length of 500-650mm were used to model carrier and contact wires, and single elements were used for droppers and registration arms. The interaction between the pantograph strip and the contact wire was implemented by means of the beam-to-beam contact (MSC.Nastran, Nonlinear User's Guide SOL400, 2014). The view of prepared FE model is presented in Fig. 3. The Generalized Alpha Operator (MSC.Nastran, Nonlinear User's Guide SOL400, 2014) was adopted as an method for solving the equations of motion of a dynamic system. Following parameters were used: $\alpha_f = -0.412$ and $\alpha_m = -0.235$ which corresponds to spectral radius $s = 0.7$. The step size equal to 0.001s was adopted in the transient step of the simulation.

In general, the configuration of the model is similar to others presented in the introduction, but it is worth to mention that nonlinear droppers are present here. They are modelled using not typical "gap" elements, which support the shortening of the element's length. This is useful at the initial configuration defining stage, to introduce appropriate length of the droppers (to achieve zero pre-sag).

3. Validation of simulation

The validation of adopted simulation method for the pantograph-catenary dynamic interaction was carried out according to the European Standard (EN50318, 2002). This document was released by the European Commission in the reference to the Interoperability Directive 96/48/EC.

The 50318 Norm describes the validation of the simulation of the dynamic interaction between pantograph and overhead contact line. The first step of validation consists of assessment of results obtained in the adopted simulation approach for the reference pantograph-catenary model, which is described in this norm.

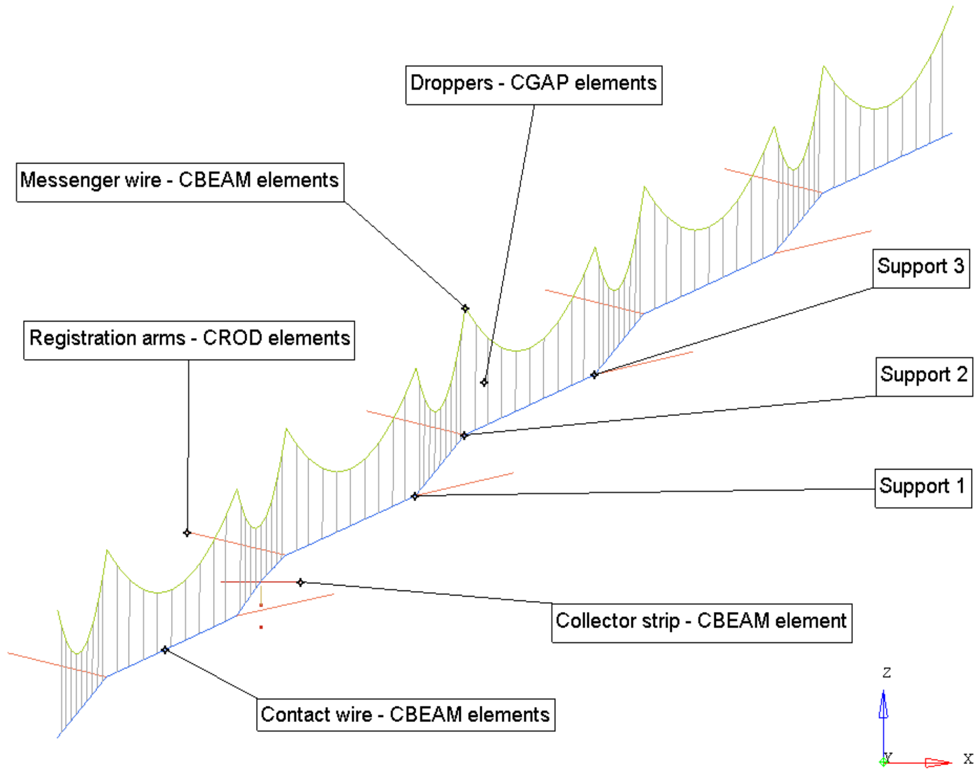

Fig. 3. The view of pantograph-catenary FE model

Simulations shall be carried out for two speeds of the pantograph run: 250km/h and 300km/h. To check the accuracy of the model, it is necessary to determine the following output results: arithmetic mean contact force $see\ eq.1$ (F_m , see eq. 1) standard deviation of contact force (σ , see eq. 2), statistical maximum of contact force ($F_m + 3\sigma$), statistical minimum of contact force ($F_m - 3\sigma$), actual minimum of contact force – minimum contact force while the pantograph passes over the analysis section, actual maximum of contact force – maximum contact force while the pantograph passes over the analysis section, percentage loss of contact – part of simulation time when the contact force is zero, and finally maximum uplift of the contact wire at supports.

Obtained results of the contact force and uplift at support have to be within the reference ranges presented in Table 1.

Table 1. Reference ranges of results for the reference model

Speed (km/h)	250	300
Mean contact force [N] F_M	110-120	110-120
Standard deviation of contact force [N] σ	26-31	32-40
Statistical maximum of contact force: $F_M + 3\sigma$	190-210	210-230
Statistical minimum of contact force: $F_M - 3\sigma$	20-40	(-5)-20
Actual maximum of contact force [N]	175-210	190-225
Actual minimum of contact force [N]	50-75	30-55
Maximum uplift at supports [mm]	48-55	55-65
Percentage contact loss [%]	0	0

Source: EN50318 (2002).

If the results obtained for the adopted simulation approach are not within the reference ranges given in Table 1, then the simulation method shall be rejected.

4. Results and discussion

In order to validate the adopted simulation model, results obtained for the model presented in the chapter 2. were compared with reference ranges (from EN 50318). According to EN 50318 document, all above mentioned parameters were designated for filtered course of contact force. Ideal low-pass filter with cutoff frequency 20Hz was used in this case. Output parameters were analyzed for the subset of the total overhead contact line model – called the analysis section, which consists of the span no. 5 and 6. (where the course of contact force is not influenced by initial transients and end effects of the model). Uplift of the contact wire at supports was calculated for three registration arms inside the analysis section - *Support 1, Support 2, Support 3* in Fig. 3.

Following figures present the received output results for the analysis section. Results for the pantograph run with the speed of 250 km/h are presented in Fig. 4 and results for the pantograph run with the speed of 300km/h are presented in Fig. 5. The statistical study of received contact force and uplift of the contact cable for the speed of 250km/h and 300km/h are presented in Table 2 and respectively. Results were compared with the reference ranges for reference mode, which was adopted in this study, according to EN50318. Outcomes were studied for prescribed analysis section (span no. 5. and 6.), and all results were filtered as mentioned above. All designated output parameters are within reference ranges. It can be observed, that minimum and maximum values of the contact force were noted nearby registration arms, where usually maximum static stiffness of the catenary is noted.

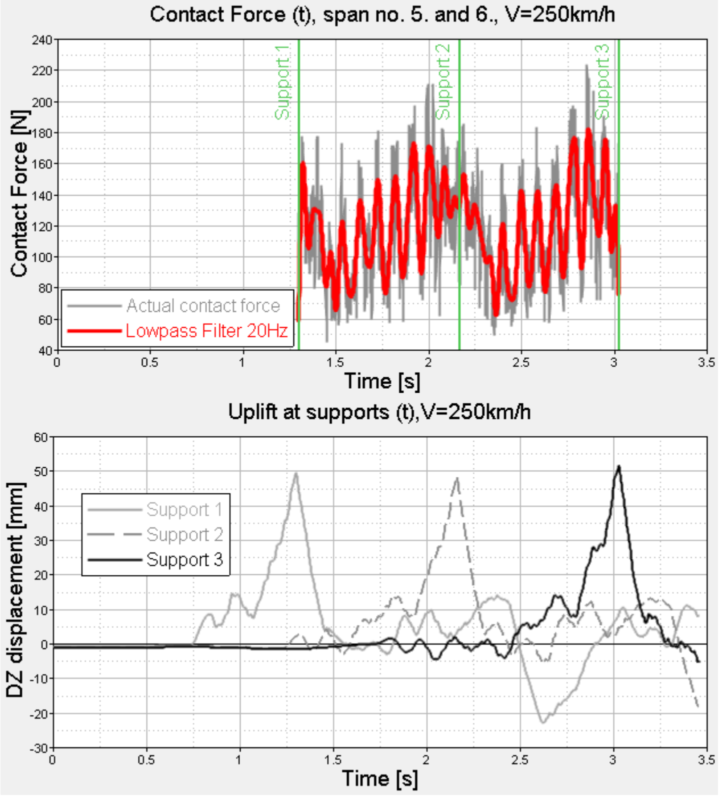

Fig. 4: Contact force and uplift at supports, V=250km/h

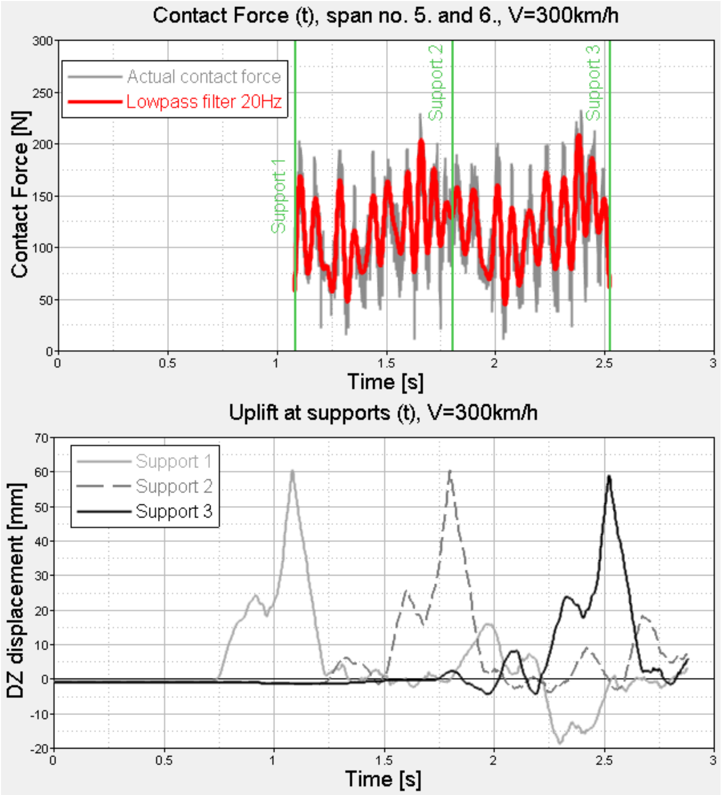

Fig. 5. Contact force and uplift at supports, V=300km/h

Table 2. Simulation results, V=250km/h

		Simulation results	Reference
Speed (km/h)		250	250
Mean contact force [N] F_M		✓ 118.9	110-120
Standard deviation of contact force [N] σ		✓ 27.5	26-31
Statistical maximum of contact force: $F_M + 3\sigma$		✓ 201.4	190-210
Statistical minimum of contact force: $F_M - 3\sigma$		✓ 36.4	20-40
Actual maximum of contact force [N]		✓ 181.6	175-210
Actual minimum of contact force [N]		✓ 59.5	50-75
Maximum uplift at supports [mm]	Support 1	✓ 49.2	48-55
	Support 2	✓ 48.3	
	Support 3	✓ 51.9	
Percentage contact loss [%]		✓ 0	0

Table 3. Simulation results, V=300km/h

		Simulation results	Reference
Speed (km/h)		300	300
Mean contact force [N] F_M		✓ 119.1	110-120
Standard deviation of contact force [N] σ		✓ 34.4	32-40
Statistical maximum of contact force: $F_M + 3\sigma$		✓ 222.3	210-230
Statistical minimum of contact force: $F_M - 3\sigma$		✓ 15.9	(-5)-20
Actual maximum of contact force [N]		✓ 208.7	190-225
Actual minimum of contact force [N]		✓ 44.8	30-55
Maximum uplift at supports [mm]	Support 1	✓ 59.1	48-55
	Support 2	✓ 59.1	
	Support 3	✓ 58.0	
Percentage contact loss [%]		✓ 0	110-120

On the basis of the results (Fig. 4 and Fig. 5), some typical trend of variability of the contact force can be noticed. Starting from the first support inside the analysis section (*Support 1* in Fig. 3), the contact force reaches its minimum in about ¼ of the length of the span, and then gradually increases while the pantograph passes along the span, then reaches a maximum value in about ¾ of the length of the span. Next, the contact force again decreases progressively and pantograph reaches the next registration arm and starts the run along another span, and cycle starts again.

5. Conclusions

Within the above work, the numerical model for the dynamic interaction between the pantograph and catenary was developed. The model is based on the use of the Finite Element Method, which has proved to be a good tool to analyze above mentioned system. Simulation method was validated according to the European Standard 50318 to check the accuracy of the model. Results obtained from the numerical experiment fulfill reference ranges presented in EN 50318, which indicates the accuracy and reliability of the proposed method of simulation and suggests the ability of the proposed model to properly simulate the dynamic cooperation of the catenary-pantograph system.

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