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EFFECT OF ASYMMETRY ON RADII OF CURVATURE FOR SPUR GEARS WITH NONSYMMETRICAL TEETH

Summary. This article discusses the involute gears with asymmetric teeth wheels and resolves radii of curvature for different parameter values of gearing. The article deals with the reduced radii of curvature in the pitch point and the extreme points of the engagement, demonstrating the effect of angle change on contact stresses.

Keywords: asymmetric tooth, radii of curvature, line of action, contact stress.

WPLYW ASYMETRII NA PROMIEN KRZYWIZNY W KOŁACH ZĘBATYCH PROSTYCH Z ZĘBAMI NIESYMETRYCZNYMI

Streszczenie. Niniejszy artykuł przedstawia koła zębate z zazębieniem ewolwentowym i zębami asymetrycznymi oraz zajmuje się różnymi promieniami krzywizny w przypadku różnych wartości paramentów zazębienia. Artykuł analizuje zredukowane promienie krzywizny w punkcie tocznym oraz punktach skrajnych przyporu oraz wykazuje wpływ zmiany kąta przyporu na naprężenia styczne.

Słowa kluczowe: ząb asymetryczny, promienie krzywizny, odcinek przyporu, naprężenia styczne.

1. INTRODUCTION

Gearing with involute gears can be designed for specific purposes. If gearing is produced in large series, or if the minimization of cost is not a decisive criterion, it is possible to design gearing with asymmetric teeth. Such gears allow you to meet different requirements, for example: related to minimizing size, weight reduction, reducing vibration [3].

2. DESIGN GEAR WITH ASYMMETRIC TEETH

Classical design of gears is based on the number of teeth and design module, the shape of the basic rack is defined by a standard module and a pressure angle 20° [1]. Minimum number of teeth, for a normalized pressure angle also depends on the value of the coefficient addendum. When designing gearing, we refer to strength calculations, and after choosing the number of teeth, we propose a modulus of bending and of contact stress calculation.

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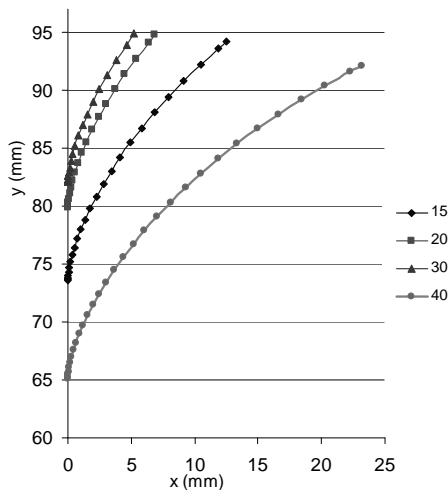
When designing asymmetric teeth, there is a number of options. These wheels can be designed mainly for one direction of the rotation [1]. When designing asymmetric teeth, the requirements may be considered in an advance. Minimum number of teeth may be reduced due to different angles for functional and non-functional tooth flanks. Proposal for asymmetric teeth must be defined so that its parameters meet all the basic requirements. Asymmetric tooth is characterized by:

- angle of gear on both sides of the tooth,
- root curve,
- other values of power ratios, when used in a reverse motion.

Even asymmetric tooth must satisfy the following criteria:

- minimum number of teeth,
- a sufficient tooth thickness on the head,
- a sufficient duration coefficient contact ratio.

Involute changes its shape depending on the angle of the profile, as the angle decreases, the curve gets steeper. Base circle diameter significantly decreases with increasing angle image.



The larger the difference between the angles of profile, the more pronounced is the asymmetry, and hence there is a significant difference between the diameters of the base circle.

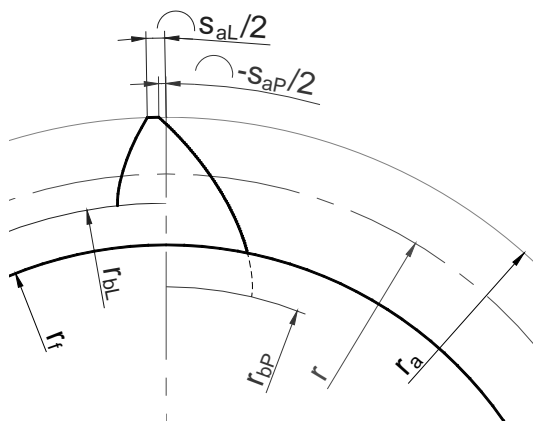
In Fig. 1 are involute profiles for wheels with numerous teeth $z = 17$ and $m = 10$ mm module and selected angles α profile are 15° , 20° , 30° , 40° . The first point is for the base circle and the last for the tip circle. The greater the difference between the angles of engaging (driving) and not engaging side of the tooth, the greater the difference between the base circles; and root (transition) curves must continuously combine these parts with the accomplishment of the requirement to transition most conveniently.

Fig. 1. Involute tooth profiles for a variety of angles

Rys. 1. Ewolwentowe profile zęba dla różnych kątów przyporu

3. THE RADII OF CURVATURE AND SPECIFIC SLIDING

In Fig. 2 is an asymmetric tooth wheel with a number of teeth 17, the module $m = 10$, left-hand side $\alpha_L = 20^\circ$, right $\alpha_P = 35^\circ$. The right side of the tooth, $\alpha_P = 35^\circ$, the base circle below the root circle.



Root curves must be designed in a different way as in a standardized rack, because they will not be correct by using standard parameters clearance (superstructure addendum tool). Root curve shall not cause stress concentrations. The possibility of creating a correct profile [2], depends on the asymmetry of the tooth and on its dependent values.

Fig. 2. Asymmetric involute tooth, left side $\alpha_L = 20^\circ$, right $\alpha_P = 40^\circ$

Rys. 2. Asymetryczny ząb ewolwentowy, strona lewa $\alpha_L = 20^\circ$, prawa $\alpha_P = 40^\circ$

The change of angle α leads to changes in the radii of curvature (Fig. 3), which affect the Hertz pressures. *Tab. 1* shows the values of the radii of curvature, mesh points *A*, *C*, *E*, a view from the left and right sides, taking into account the minimum number of teeth with permissible undercut (mesh points for certain parameters are after the correct last mesh point, which is not an obstacle in a part of a functionless tooth). Hertz pressures are a function of radius of curvature.

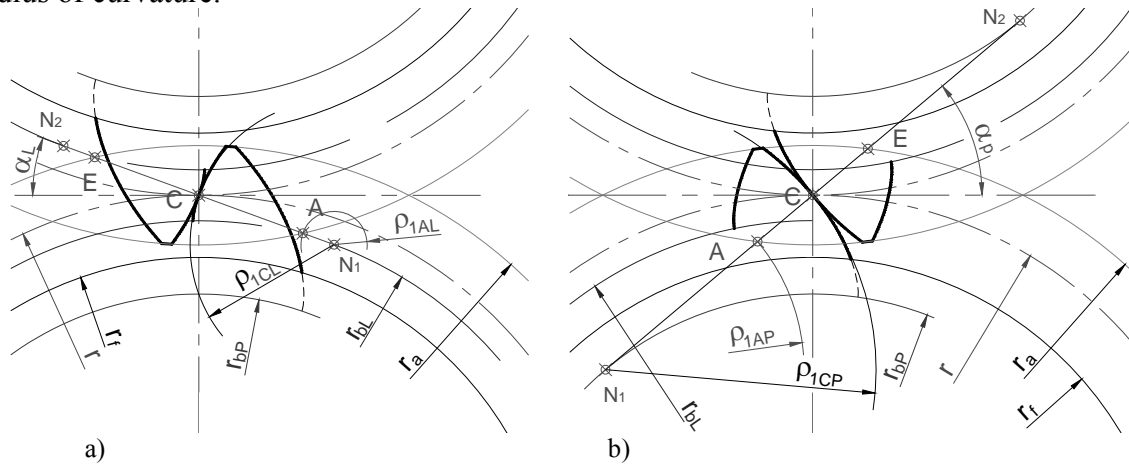


Fig. 3. Mesh asymmetrical tooth: a) driving side $\alpha_L=20^\circ$, b) driving side $\alpha_p=35^\circ$

Rys. 3. Zazębianie zęba asymetrycznego: a) strona napędowa dla $\alpha_L=20^\circ$, b) strona napędowa dla $\alpha_p=35^\circ$

Table 1

Length of mesh line segments, radii of curvature and reduced radii of curvature

z_1	z_2	α ($^\circ$)	$N_1 N_2$ (mm)	Point C			Point A		Point E	
				ρ_{1C} (mm)	ρ_{RC} (mm)	$1/\sqrt{(\rho_R \cdot \cos \alpha_t)}$ ($\text{mm}^{-1/2}$)	ρ_{1A} (mm)	ρ_{RA} (mm)	ρ_{1E} (mm)	ρ_{RE} (mm)
10	20	25	63,393	21,131	14,087	0,2799	1,054	1,037	39,326	14,930
		30	75,000	25,000	16,667	0,2632	7,177	6,49	41,533	18,533
	50	25	126,785	21,131	17,609	0,2503	-0,741	-	39,326	27,128
		30	150,000	25,000	20,833	0,2354	6,038	5,795	41,533	30,033
14	28	20	71,824	23,941	15,961	0,2582	-0,237	-	45,532	16,668
		35	120,451	40,150	26,767	0,2136	23,765	19,076	55,786	29,949
	70	20	143,648	23,941	19,951	0,2310	-2,741	-	45,532	31,100
		35	240,902	40,150	33,459	0,1910	23,184	20,953	55,786	42,867
17	34	20	87,215	29,072	19,381	0,2343	4,265	4,057	51,431	21,102
		35	146,262	48,754	32,503	0,1938	32,210	25,117	64,629	36,071
	85	20	174,430	29,072	24,226	0,2096	2,009	1,986	51,431	36,267
		35	292,524	48,754	40,628	0,1733	31,710	28,273	64,629	50,350

Reduced radius of curvature ρ_R for the mesh points:

$$\rho_R = \frac{\rho_1 \cdot \rho_2}{\rho_1 + \rho_2} \quad (1)$$

where: ρ_1 – radius of curvature with respect to the pinion, ρ_2 – radius of curvature with respect to the wheel.

Hertz pressure:

$$\sigma_H = z_M \cdot \sqrt{\frac{F_t}{2 \cdot b_w \cdot \rho_R \cdot \cos \alpha_t}} = z_M \sqrt{\frac{F_t}{2 \cdot b_w}} \cdot \frac{1}{\sqrt{\rho_R \cdot \cos \alpha_t}} = K \cdot \frac{1}{\sqrt{\rho_R \cdot \cos \alpha_t}} \quad (2)$$

where: z_M – material factor, F_t – tangential force, b_w – axial face width, α_t – pressure angle in a transverse plane

Hertz pressures and specific sliding speeds are directly related to the asymmetry. If we consider the coefficient values of the material, width of the teeth and tangential forces for the proposed gearing as constant, then variable values are angle α and radius of curvature. Hertz pressure can be expressed as a multiple of the constant K and the fraction $1 / \sqrt{(\rho_R \cdot \cos \alpha_t)}$. The values of reduced radii of curvature have a favorable effect on contact stress even with a decreasing value of the cosine of the pressure angle.

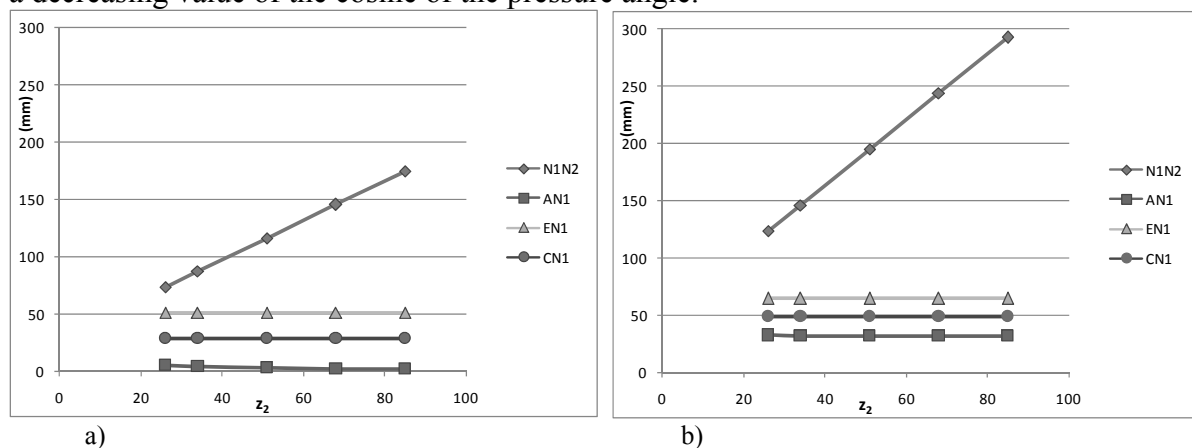


Fig. 4. Length of action line and radii of curvature for $z_1=17$, $m=10$: a) $\alpha_L=20^\circ$, b) $\alpha_p=35^\circ$

Rys. 4. Długość odcinka przyporu i promień krzywizny dla $z_1=17$, $m=10$: a) $\alpha_L=20^\circ$, b) $\alpha_p=35^\circ$

4. CONCLUSION

Asymmetry allows to reduce the number of teeth of the wheels by the use of drive side pursuits with bigger pressure angle that improve the value of reduced radii of curvature and thus may reduce contact stresses. Larger angles decrease tooth thickness on the addendum, but the asymmetry will in many cases establish correct tooth with good overall parameters. Significant contribution may be the reduction of number of teeth and thus the size reduction with the same module gearing.

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