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Empirical and theoretical models for prediction of soil thermal conductivity: a review and critical assessment

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Abstract: The paper discusses existing models used to estimate the thermal conductivity of the soil medium. The considerations are divided into three general sections. In the first section of the paper, we focus on the presentation of empirical models. Here, in the case of Johansen method, different relations for Kersten number are also presented. In the next part, theoretical models are considered. In the following part, selected models were used to predict measured thermal conductivities of coarse- and fine-grained soils, at different water contents. Based on these predictions as well as on the authors' experience, a critical assessment of the existing models is provided. The remarks as well as advantages and disadvantages of those models are summarized in a tabular form. The latter is important from a practical point of view; based on the table content, one can simply choose a model that is suitable for the particular problem.

Keywords: Heat flow; thermal conductivity; soil; Kersten number.

1 Introduction

The use of modern geotechnical solutions, such as geothermal piles, requires a new approach to design from engineers. In order to correctly design a thermal pile that combines the classical foundation features with the idea of recovering heat from the ground, one must determine the deformation state implied not only by mechanical loads, but also by the thermal loads resulting from the cyclical fluctuation of temperature in the pile and in the surrounding soil medium. Therefore, to determine the overall state of pile deformation, one must know not only the mechanical parameters of the concrete and the soil, but also their thermal characteristics, especially regarding their thermal conductivity. Thus, the correct determination of the thermal conductivity characteristics of the soil medium is of primary importance for the design of structures for which the surrounding ground constitutes a source of thermal energy.

There exists a large number of models that can be used to predict soil thermal conductivity. In the most general way, they can be divided into two groups (Różański, 2018): empirical and theoretical models. The former are models whose parameters are determined based on the results of laboratory tests (e.g., Kersten, 1949; Johansen, 1975; Donazzi et al., 1979; Côté & Konrad, 2005b; Lu et al., 2007; Chen, 2008; Lu et al., 2014). Their main disadvantage is that the scope of their applicability is usually very limited and gives the possibility of determining the thermal conductivity only for a particular type of soil, e.g., a certain model can work very well for sand, whereas when applied to clay, it produces enormous error. Hence, the empirical models usually are not 'general' ones. The main defect of the models included in the second group (theoretical or semi-theoretical ones, e.g., Mickley, 1951; Gemant, 1952; de Vries, 1963; Gori, 1983; Tong et al., 2009; Haigh, 2012; He et al., 2017) is their assumption of the 'grain-water-air' microstructure – in general, the geometry of the soil microstructure ('grain-air-water' mixture) is rather very simple. As a result, they are rarely able to correctly describe the complex thermal characteristics of partially saturated soils, that is, the change of thermal conductivity with the change in water content. This problem mainly affects fine-grained soils, which are characterized by a specific dependence of thermal conductivity on very low water contents, that is, to a certain limit of water content in the pores (usually quite small), the addition of water does not result in a significant increase in thermal conductivity (e.g., Johansen, 1975).

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The current paper is a classical review of existing empirical and theoretical models used for the prediction of soil thermal conductivity. In the first part, selected models are briefly described. In particular, five empirical and six theoretical models are presented. Special attention is paid to the Johansen model (Johansen, 1975), which uses the concept of normalized conductivity, referred to as the Kersten number. Recent modifications of Kersten number are discussed. In Section 4, a critical assessment of thermal conductivity models is provided. Selected models were used to predict measured thermal conductivities of coarse- and fine-grained soils, at different water contents (results are taken from Lu et al., 2007). Based on these predictions as well as on the authors' experience critical remarks with respect to individual models are formulated. At the end of this section, a tabular summary of the research is provided. The summary contains remarks, advantages and disadvantages of each considered model. The latter is important from a practical point of view, that is, based on the table content, one can simply choose a model that is suitable for the particular problem.

2 Empirical models

2.1 Kersten's empirical equations

On the basis of the results of thermal conductivity measurements of 19 soils, Kersten (1949) proposed empirical equations to determine the thermal conductivity of the soil medium on the basis of moisture content w [%] and the dry density ρ_d [g/cm^3]. For fine-grained soils (silt, clay, etc.) the thermal conductivity of the soil medium can be estimated using the following expression:

$$\lambda = 0.1442(0.9 \log w - 0.2)10^{0.6243\rho_d} \tag{1}$$

The analogous formula for coarse-grained soils is as follows:

$$\lambda = 0.1442(0.7 \log w + 0.4)10^{0.6243\rho_d} \tag{2}$$

2.2 Johansen method with its subsequent modifications

Johansen (1975) proposed the determination of thermal conductivity of partially saturated soil medium on the basis of thermal conductivity of dry soil λ^{dry} and in the state of full water saturation λ^{sat}. For that purpose, a normalized (dimensionless) thermal conductivity, also known as the Kersten number K_e, was introduced. According to Johansen's (1975) approach, thermal conductivity of the soil is governed by the following equation:

$$\lambda = (\lambda^{sat} - \lambda^{dry}) \cdot K_e + \lambda^{dry} \tag{3}$$

Based on the data contained in Kersten's work (1949), Johansen proposed the correlation formulas between the Kersten number K_e and the degree of saturation S_r. The conductivity of the dry soil was proposed to be evaluated using an empirical equation based on the dry density of the soil. The value of λ^{sat} was proposed to be computed using the geometrical mean approach, which involves the conductivity of water and of soil solids. All the equations constituting Johansen method are summarized in Table 1.

The Johansen method is recognized in the literature as providing one of the best predictions of the thermal conductivity of soil medium (Farouki, 1981; Dong et al. 2015; Różański, 2018; Zhang & Wang, 2017). In the following years, various researchers modified the Johansen method. The changes mainly concern the introduction of a slightly improved form of the function describing the K_e - S_r relationship and the proposal of new approach for determining the thermal conductivity of dry soils.

Côté and Konrad (2005a; 2005b) modified the Kersten number by introducing an additional parameter κ, reflecting the type of soil. In addition, a modified formula for calculation of λ^{dry} value was proposed – it was assumed to be dependent not only on the porosity but also on two additional parameters reflecting the type of soil and grain shape (χ, η). Another modification of Kersten number and conductivity of dry soil was proposed by Lu et al. (2007). According to the work of Lu et al. (2007), the Kersten number is governed by the exponential function with parameter α whose value depends on whether we are dealing with coarse- or fine-grained soil. The empirical formula for calculation of λ^{dry} is the linear function of soil porosity n. The most recent proposal for a functional dependence between Kersten number and S_r is the one proposed in the work of He et al. (2017). All formulas for the Johansen method and its subsequent modifications are collected in Table 1. The sign '-' means that the authors did not modify the parameter or they did not refer to it at all in their work.

2.3 Donazzi et al. model

Donazzi et al. (1979) studied the thermal and hydrological characteristics of soils surrounded by underground

Table 1: Different approaches used for evaluation of the Kersten number K_e and the conductivity of dry and saturated soil.

Model	Kersten number	Thermal conductivity of dry soil	Thermal conductivity of saturated soil
Johansen (1975)	Coarse-grained soil: $K_e \cong 0.7 \log S_r + 1.0$ Fine-grained soil: $K_e \cong \log S_r + 1.0$	$\lambda^{dry} = \frac{0.135\rho_d + 64.7}{2700 - 0.947\rho_d}$	$\lambda^{sat} = \lambda_w^n \lambda_s^{(1-n)}$
Côté and Konrad (2005a; 2005b)	$K_e = \frac{\kappa S_r}{1 + (\kappa - 1)S_r}$	$\lambda^{dry} = \chi 10^{-\eta n}$	-
Lu et al. (2007)	$K_e = exp\{\alpha[1 - S_r^{\alpha - 1.33}]\}$	$\lambda^{dry} = 0.51 - 0.56n$	-
He at al. (2017)	$K_e = \begin{cases} 0 & S_r = 0 \\ \frac{1}{A\, exp[(S_r n)^{-B}]} & S_r > 0 \end{cases}$	-	-

where:
λ_w – thermal conductivity of water,
λ_s – thermal conductivity of soil solids,
n – porosity,
κ – parameter reflecting the type of soil: 4.60 for gravel and coarse sand; 3.55 for medium and fine sands; 1.90 for silt and clay; 0.60 for organic soils,
χ, η – parameters reflecting the type of soil and grain shape: $\chi = 0.30$ and $\eta = 0.87$ for organic soils; $\chi = 0.75$ and $\eta = 1.20$ for mineral soils,
α – parameter depending on the type of soil: 0.96 or 0.27, respectively for coarse- or fine-grained soils,
A, B – fitting parameters.

installations. Based on the results of the laboratory tests, they proposed the following correlation formula to determine the thermal conductivity of the soil, which can be applied to all types of soil:

$$\lambda = \lambda_w^n \lambda_s^{(1-n)} \exp(-3.08n(1 - S_r)^2). \tag{4}$$

2.4 Chen model

Chen (2008) carried out tests on the thermal conductivity of quartz sands in different saturation states. On this basis, the empirical equation of thermal conductivity as a function of porosity and degree of saturation was proposed:

$$\lambda = \lambda_w^n \lambda_s^{(1-n)} [(1 - b)S_r + b]^{cn} \tag{5}$$

Based on the results of thermal conductivity measurements, the model parameters were determined as: $b = 0.0022$ and $c = 0.78$. Equation (5) has a good accuracy in estimating the conductivity of sandy soils with relatively high quartz content.

2.5 Lu et al. model

In the work of Lu et al. (2014), the following empirical equation for thermal conductivity at various saturation states was proposed:

$$\lambda = \lambda^{dry} + \exp[\beta - (S_r n)^{-\alpha}] \qquad S_r > 0 \tag{6}$$

The conductivity of dry soil is evaluated using the approach proposed by Lu et al. (2007) – see Table 1. Parameters α and β were estimated to be dependent on the contents of sand and clay separates as well as on the soil dry density, that is:

$$\alpha = 0.67\phi_{Cl} + 0.24 \tag{7}$$

$$\beta = 1.97\phi_{Sa} + 1.87\rho_d - 1.36\phi_{Sa}\rho_d - 0.95 \tag{8}$$

where ϕ_{Cl} and ϕ_{Sa} are the contents of, respectively, clay and sand separates. The value of the density ρ_d should be taken in units [g/cm^3]. Note, that this model gives the possibility to estimate the thermal conductivity of the same soils, but in different states of compaction.

3 Theoretical models

3.1 Wiener bounds

In 1912, Wiener (Wiener, 1912) proposed an upper and lower bound for the thermal conductivity of a porous medium consisting of three separate phases: solid, water and air (Fig. 1).

The lower (λ^L) and upper (λ^U) bounds of thermal conductivity are obtained, respectively, for the series and parallel arrangement of the individual phases, relative to the direction of the heat flux, that is:

$$\lambda^L = \left(\sum_i \frac{\phi_i}{\lambda_i}\right)^{-1} \quad (9)$$

$$\lambda^U = \sum_i \phi_i \lambda_i \quad (10)$$

where ϕ_i and λ_i are respectively the volume fraction and thermal conductivity of phase *i*. It is commonly known (e.g., Różański, 2018) that these bounds can only be treated as a rough estimate of soil thermal conductivity. It is due to the fact that for large contrast in conductivities, the bounds are 'very wide'. In the case of unsaturated soil, the contrast between skeleton, water and air is of the two orders of magnitude.

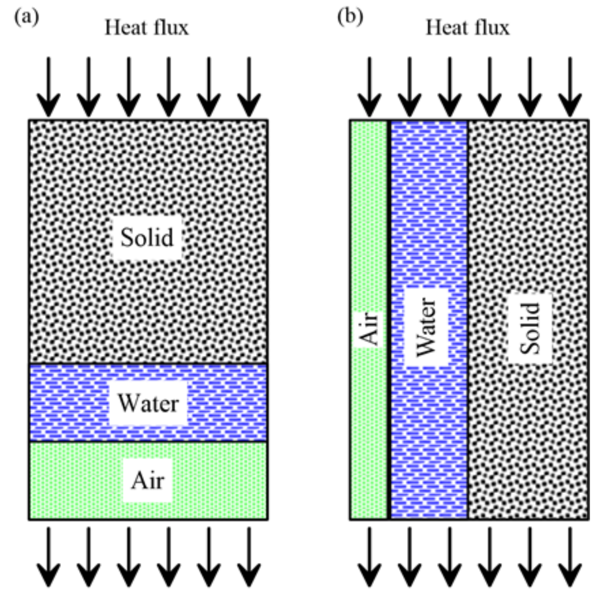

Figure 1: Wiener model scheme (Wiener, 1912): (a) series model - lower limit; (b) parallel model - upper limit.

3.2 Mickley's model

In the work of Mickley (1951), it was assumed that elementary volume of a soil is a unit cube with separated 'channels', filled with air and/or water (Fig. 2). In the case of dry soil (S_r=0), after assuming the direction of heat flow (Fig. 2a), the layer system proposed by Mickley implies that heat flows simultaneously through four separate columns: (1) the air-filled space between the outer walls of a cube of sectional area a^2 and unit length, (2) the soil skeleton between the outer walls of a cube of sectional area (1-a^2) and unit length, and (3) and (4) two columns (air and soil skeleton) in series, each with sectional area a(1-a^2) and length a and (1-a), respectively, for air and soil skeleton.

The thermal conductivities at different states of saturation are evaluated using the following relations (Mickley, 1951):

– dry soil

$$\lambda^{dry} = \lambda_a a^2 + \lambda_s (1-a)^2 + \frac{\lambda_s \lambda_a (2a - 2a^2)}{\lambda_s a + \lambda_a (1-a)}, \quad (11)$$

where λ_a is the thermal conductivity of air,

– fully saturated soil

$$\lambda^{sat} = \lambda_w a^2 + \lambda_s (1-a)^2 + \frac{\lambda_s \lambda_w (2a - 2a^2)}{\lambda_s a + \lambda_w (1-a)}, \quad (12)$$

– partially saturated soil

$$\lambda = \lambda_a c^2 + \lambda_s (1-a)^2 + \lambda_w (a-c)^2 + \frac{2\lambda_w \lambda_a c(a-c)}{\lambda_w c + \lambda_a (1-c)} + \\ + \frac{2\lambda_s \lambda_w \lambda_a c(1-a)}{\lambda_w \lambda_s c + \lambda_s \lambda_a (a-c) + \lambda_w \lambda_a (1-a)} + \frac{2\lambda_s \lambda_w (a-c)(1-a)}{\lambda_s a + \lambda_w (1-a)} \quad (13)$$

where c=a-b.

The geometric variables a and c are to be determined from the soil porosity n and the degree of saturation S_r by solving the equations below:

$$3a^2 - 2a^3 = n \quad (14)$$

and

$$3c^2 - 2c^3 = n(1 - S_r). \quad (15)$$

3.3 Gemant model

Gemant (1952) proposed a model based on an idealized grain shape – a cube with a side a, the three walls of which form 'pyramids' with a square base. It was assumed that grains are in contact with each other only by the top

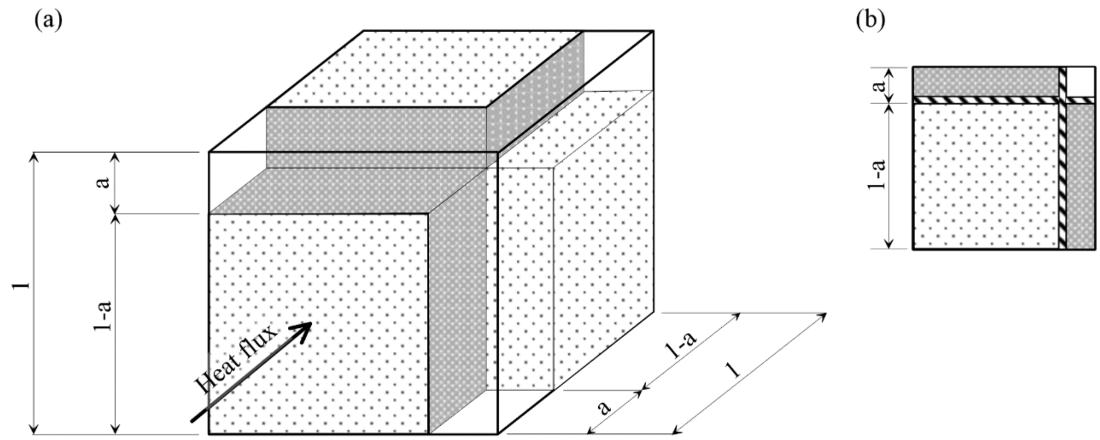

Figure 2: The unit cube considered by Mickley (Farouki, 1981): (a) isometric view; (b) lateral view showing the water layer.

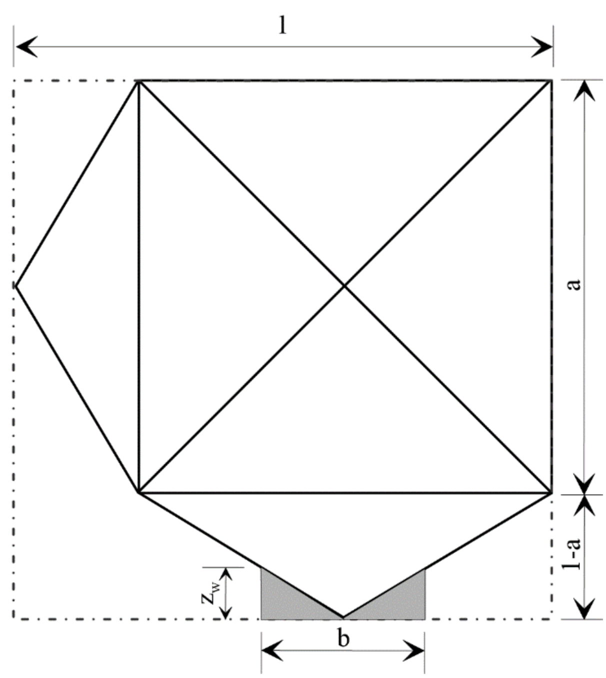

Figure 3: An idealized grain model according to Gemant (Farouki, 1981).

of 'pyramids' and the water accumulates around these contact points forming the so-called thermal bridges between the particles (Fig. 3).

Gemant (1952) proposed the following formula for the conductivity of the soil:

$$\frac{1}{\lambda}=\frac{\left[\frac{(1-a)}{a}\right]^{\frac{4}{3}} arctan\left[\frac{\lambda_s-\lambda_w}{\lambda_w}\right]^{\frac{1}{2}}}{\left(\frac{h}{2}\right)^{\frac{1}{3}}\left[\lambda_w(\lambda_s-\lambda_w)\right]^{\frac{1}{2}}}+\frac{1-z_w}{\lambda_s a}f\left\{\frac{b^2}{a}\right\} \tag{16}$$

where λ_s and λ_w $[Wm^{-1}K^{-1}]$ are the thermal conductivities of the soil skeleton and water, respectively. Remaining parameters are expressed by following relations:

$$a=4.869\sqrt{\rho_d} \tag{17}$$

$$h=9.988\cdot 10^{-3}\rho_d w-h_0 \tag{18}$$

$$z_w=\left(\frac{1-a}{a}\right)^{\frac{2}{3}}\left(\frac{h}{2}\right)^{\frac{1}{3}} \tag{19}$$

$$b^2=\left(\frac{a}{1-a}\right)^{\frac{2}{3}}\left(\frac{h}{2}\right)^{\frac{2}{3}} \tag{20}$$

In equations above, h[-] represents the amount of water accumulated around contact points and h_0 is the water adsorbed as a film on the surfaces of solid particles. Gemant assumed that h_0 decreases as the temperature increases following the theory of water adsorption of glass (Farouki, 1981), for example: for T=20°C, h_0=0.01. Analogically, namely using a separate nomogram, the values of $f\{b^2/a\}$ are estimated. In the work of Różański (2018), the following approximation of this function was introduced:

$$f\left\{\frac{b^2}{a}\right\}\approx\left(0.976+0.2355\ln\left(\frac{b^2}{a}\right)\right)^{-1} \tag{21}$$

3.4 De Vries model

De Vries model (de Vries, 1963) is based on the Maxwell equation defining the effective electrical conductivity of a mixture of spherical particles dispersed in a continuous fluid (Eucken, 1932). De Vries assumed that the soil medium could be modelled as a mixture of N ellipsoidal particles dispersed in a continuous matrix (being water or air, depending on the degree of saturation). The thermal conductivity of the soil medium is expressed as:

$$\lambda = \frac{\phi_0 \lambda_0 + \sum_{i=1}^{N} F_i \phi_i \lambda_i}{\phi_0 + \sum_{i=1}^{N} F_i \phi_i} \quad (22$$

where the subscripts 0 and i indicate the parameters characterizing, respectively, a continuous matrix or ellipsoidal inclusions dispersed within it. Moreover, the weighting parameters F_i are calculated using following formula:

$$F_i = \frac{2}{3}\left[1 + \left(\frac{\lambda_i - \lambda_0}{\lambda_0}\right) g_a\right]^{-1} + \frac{1}{3}\left[1 + \left(\frac{\lambda_i - \lambda_0}{\lambda_0}\right)(1 - 2g_a)\right]^{-1} \quad (23)$$

where g_a is the parameter defining the shape of the ellipsoidal inclusion (de Vries proposed g_a=0.125).

3.5 Gori model

Gori (1983) proposed a theoretical model to determine the thermal conductivity of soils in different saturation states. Gori assumed that the soil (as a multi-phase medium) consists of periodic cubic cells with a ‘centrally’ located skeleton (solid) surrounded by air (dry soil) or water (saturated soil) or both air and water (partially saturated soil). A geometry of the representative unit cell, at various states of saturation, is presented in Fig. 4.

Due to the fact that the calculation procedure using this model, compared to most other proposals, is relatively difficult, the use of this model in engineering practice is quite limited. In addition, the disadvantage of the model is that while the formulation itself results directly from theoretical considerations, the water content values determining the geometry of the periodicity cell (Fig. 4) were determined empirically depending on the type of soil. In the computational sense, determination of the thermal conductivity of dry soils seems to be relatively simple (Fig. 4a). This prediction depends only on the porosity of the soil and the thermal conductivities of the air and soil skeleton and is given by the following equation:

$$\frac{1}{\lambda^{dry}} = \frac{\psi - 1}{\lambda_a \psi} + \frac{\psi}{\lambda_a(\psi^2 - 1) + \lambda_s} \quad (24)$$

where

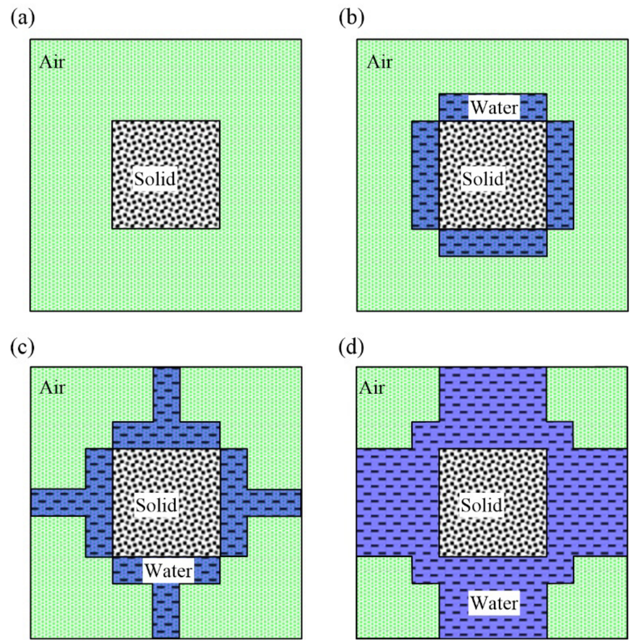

Figure 4: Periodic cell according to the Gori model (Gori, 1983): (a) dry soil; (b) low water content – thin film around the grain; (c) partially saturated soil – formed water bridges; (d) the state close to full saturation.

$$\psi = \sqrt[3]{\frac{n}{1 - n}} \quad (25)$$

3.6 Tong et al. model

In the work of Tong et al. (2009), the thermal conductivity of a multicomponent porous medium was considered. A new model based on Wiener as well as Hashin-Shtrikmann[1] bounds was proposed. The value of thermal conductivity was shown to be affected by such parameters as mineral composition, temperature, saturation degree and porosity. As proposed by Tong et al. (2009), the thermal conductivity of a three-component porous medium (solid, water, air) is determined by the following expression:

$$\lambda = \eta_1(1-n)\lambda_s + (1-\eta_2)[1-\eta_1(1-n)^2]\left[\frac{(1-n)(1-\eta_1)}{\lambda_s} + \frac{nS_r}{\lambda_w} + + \frac{n(1-S_r)}{\lambda_a}\right]^{-1} + \eta_2[(1-n)(1-\eta_1)\lambda_s + nS_r\lambda_w + n(1-S_r)\lambda_a] \quad (26)$$

where η_1 is the coefficient dependent on soil porosity, while the value of parameter η_2 depends on porosity,

1 For a macroscopically isotropic medium the bounds of Hashin-Shtrikman provide the narrowest range of estimates, which are expressed only in terms of volume fractions of constituents (e.g., Łydżba, 2011).

saturation and temperature. Both parameters are in the range from 0 to 1 which is a direct consequence of fulfilling the Wiener bounds. Determination of the coefficients η_1 and η_2 requires a series of laboratory tests for samples of porous material with different porosities and saturation degrees and subjected to several different temperatures.

3.7 Haigh model

Haigh (2012) model is used to determine the thermal conductivity of sandy soils in different water saturation states. The equation describing thermal conductivity was determined on the basis of the assumed simplified geometry of the soil microstructure. A problem of 1D heat flow between two identical grains of spherical shape and radius R was considered (Fig. 5). It was assumed that grains are placed in a cylindrical cell of radius R and length $R(1 + \xi)$ – the contact between grains is only when $\xi = 0$.

The equation describing the thermal conductivity of the soil is expressed by the following formula:

$$\frac{\lambda}{\lambda_s} = 2(1+\xi)^2 \left\{ \frac{\alpha_w}{(1-\alpha_w)^2} \ln\left[\frac{(1+\xi)+(\alpha_w-1)x}{\xi+\alpha_w}\right] + \frac{\alpha_a}{(1-\alpha_a)^2} \ln\left[\frac{(1+\xi)}{(1+\xi)+(\alpha_a-1)x}\right] \right\} + \\ + \frac{2(1+\xi)}{(1-\alpha_w)(1-\alpha_a)}\left[(\alpha_w-\alpha_a)x-(1-\alpha_a)\alpha_w\right] \tag{27}$$

where $\alpha_w=\lambda_w/\lambda_s$ and $\alpha_a=\lambda_a/\lambda_s$. In addition, ξ and x are the values depending on porosity and saturation degree and they should be computed using the formulas below:

$$\xi = \frac{3n-1}{3(1-n)} \tag{28}$$

$$x = \left(\frac{1+\xi}{2}\right)(1 + cos\vartheta - \sqrt{3}\, sin\vartheta) \tag{29}$$

$$cos\vartheta = \frac{2(1+3\xi)(1-S_r)-(1+\xi)^3}{(1+\xi)^3} \tag{30}$$

4 Critical assessment of the models

Fig. 6 shows the predictions obtained using the considered models for coarse-grained soil, that is, sand with following soil separates: 92% of sand, 7% of silt and 1% of clay. The dry density and the specific density are ρ_d=1.58 [g/cm^3] and ρ_s=2.65 [g/cm^3], respectively. The results of thermal conductivity at different saturation states are taken from the work of Lu et al. (2007). The value of thermal conductivity is assumed analogically as in the work of Lu et al. (2007), that is, λ_s=6.91 [W m^{-1} K^{-1}].

Due to the high contrast between the conductivity of soil skeleton and air/water, Wiener bounds were omitted

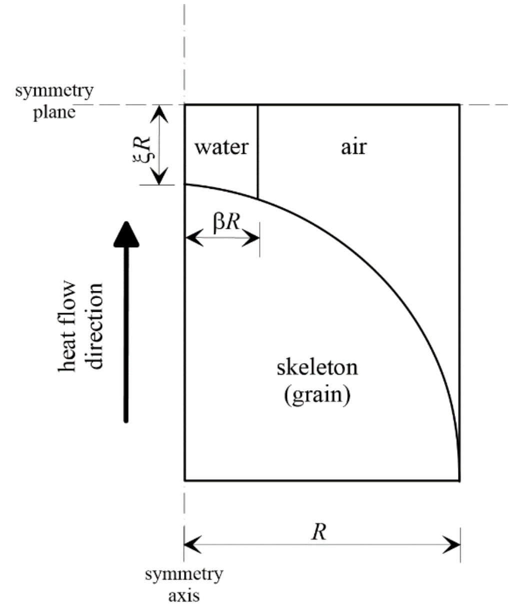

Figure 5: Assumed geometry of a three-phase medium (Haigh, 2012).

in the analysis since they provide a very wide range of possible values of λ. The results for Gori model (1983) as well as Tong et al. (2009) and He et al. (2017) models have not been presented due to the fact that there is no clear relation for determination of model parameters.

Kersten model (1949) quite well describes, in a qualitative way, the relationship λ-S_r. From a quantitative point of view, Kersten equation underestimates the thermal conductivity values for relatively high values of S_r. It is probably the consequence of the fact that Kersten did not test fully saturated soils, and the thermal conductivity for S_r=1 was determined by extrapolating the results.

Mickley (1951) and de Vries (1963) models do not reflect correctly, neither qualitatively nor quantitatively, the thermal characteristic of the analysed soil. Relatively high porosity (about 40%) may be a problem for Mickley model. De Vries model (1963), due to the assumption of no interaction between the particles, underestimates the thermal conductivity of sand for small values of S_r.

Gemant model (1952) overestimates the thermal conductivity of the analysed soil. This is probably a consequence of the fact that originally Gemant proposed a formula for determining the thermal conductivity of the soil skeleton based only on the clay fraction. Using Gemant proposal, the thermal conductivity of the soil skeleton would be 5.81[$Wm^{-1} K^{-1}$]. This value would improve the prediction λ-S_r in relation to the value 6.91[$Wm^{-1} K^{-1}$], which was adopted for the analysis.

Donazzi model (1979) significantly overestimates thermal conductivity for low water contents. Moreover,

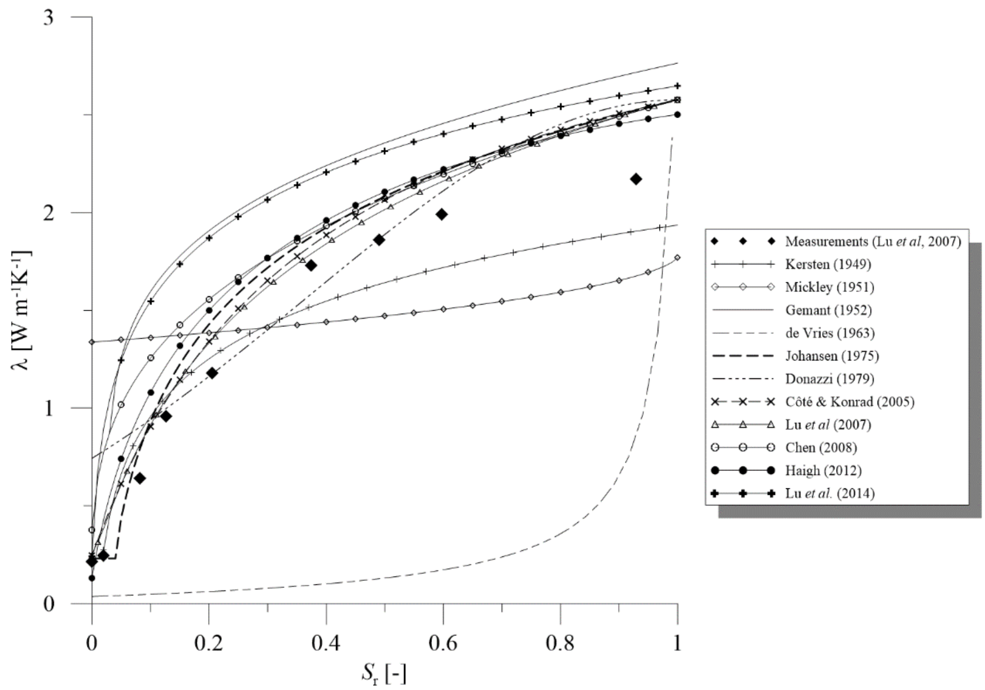

Figure 6: Prediction of thermal conductivity for coarse-grained soil at different saturation states (expressed as saturation degree S_r) and measurement results (Lu et al., 2007)

the convexity of the curve λ-S_r is not entirely consistent with the characteristic observed in the literature (e.g., Johansen, 1975; Lu et al., 2007, 2014). Chen (2008) and Haigh models (2012) represent similar characteristics both in quantitative and qualitative terms. Predictions obtained with Haigh model (2012), as suggested by the author of the model, were increased a fixed factor of 1.58.

Weak prediction was obtained by Lu et al. (2014) model. Almost within the entire range of S_r, the prediction is heavily higher in relation to the test results. This may be due to the fact that the correlation formulae for parameters α and β were determined on the basis of studies on soils with relatively low volume fraction of sand separate (Lu et al., 2014). The soil analysed in this study is characterized by high volume fraction of sand separate – 92%.

Similar predictions, qualitatively and quantitatively, were obtained using models of Johansen (1975), Côté and Konrad (2005b) and Lu et al. (2007). This is not surprising, as compared to Johansen's proposal, the other two formulations introduce only slight modifications to the original relationship defining the Kersten number (see Table 1).

Similar analysis was also conducted for fine-grained soil, that is, clay with following content of soil separates: 32%, 38% and 30% respectively for sand, silt and clay; the dry density ρ_d=1.29[g/cm^3] and the specific density ρ_s=2.65[g/cm^3]. The results of thermal conductivity measurements and thermal conductivity of the soil skeleton (λ_s=3.08 [Wm^{-1} K^{-1}]) for the soil under consideration were taken from Lu et al. (2007). Fig. 7 presents the obtained predictions and measurement data. The analysis omitted those models that are dedicated to sandy soils, that is, Chen (2008) and Haigh (2012) models.

Again, the prediction based on Lu et al. model (2014) is overestimating the measurement data almost in the entire range of S_r. Johansen model (1975) fits very well into the results of laboratory measurements with the exception of thermal conductivity corresponding to low S_r values (it also slightly underestimates the thermal conductivity of the soil in the dry state). Côté and Konrad model (2005b) behaves analogously to how it modifies the Kersten number in relation to Johansen proposal. Lu et al. (2007) model definitely best reflects the thermal characteristics of the fine-grained soil under study. This is a consequence of introducing a properly defined relation K_e - S_r.

Based on the predictions presented above as well as on the authors' experience, some critical remarks with respect to individual models are formulated and presented in a tabular form (Table 2 and Table 3). Tables contain remarks, advantages and disadvantages of each considered model.

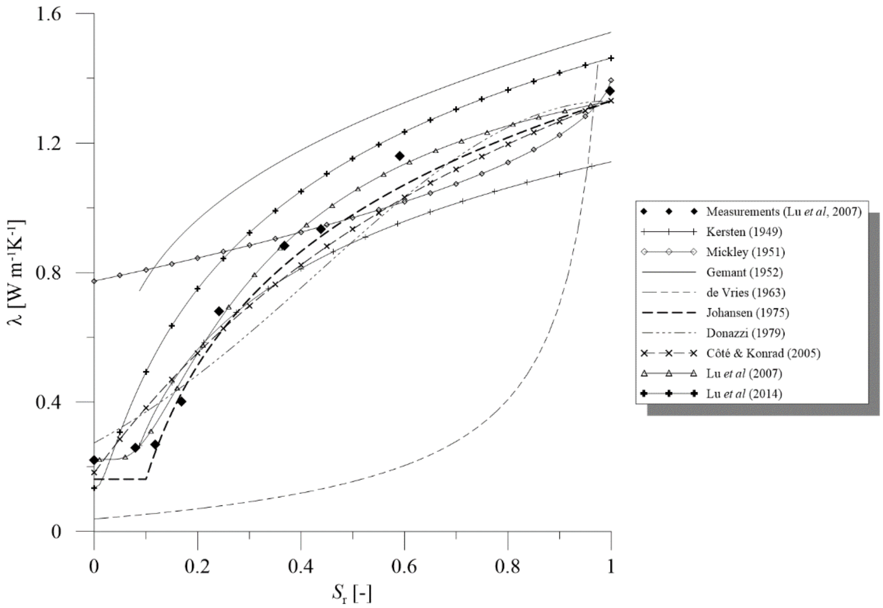

Figure 7: Prediction of thermal conductivity for fine-grained soil at different saturation states (expressed as saturation degree S_r) and measurement results (from Lu et al., 2007).

Table 2: Advantages, disadvantages and comments on considered empirical models (Różański, 2018).

Empirical models			
Model	Advantages	Disadvantages	Comments
Kersten (1949)	Simple formula; for every type of soil	Equations do not take into account the quartz content which has the largest contribution in overall value of λ	Thermal conductivity in dry state cannot be determined
Johansen (1975)	Can be used for frozen soils; relatively high quality of prediction; for every type of soil	Possible inaccuracy for dry soil (±20%)	Empirical relations valid for S_r>0.05 (coarse-grained soil) and S_r>0.01 (fine-grained soil)
Donazzi et al. (1979)	Simple formula; for every type of soil	Weak prediction for soils with low water content	The shape of the λ - S_r curve does not fully comply with the test results and with most other models presented in literature
Côté and Konrad (2005b)	For every type of soil; includes the type of soil and the shape of grains	The course of the K_e - S_r curve is not entirely consistent with common knowledge for fine-grained soils with low water content	Modification of the Johansen method (1975) with respect to the Kersten number and dry soil conductivity
Lu et al. (2007)	For every type of soil; very good reflection of thermal conductivity for fine-grained soils with very low water content; the type of soil is taken into account	Unknown influence of the type of soil on the conductivity in the dry state	Modification of the Johansen method with respect to the Kersten number and dry soil conductivity
Chen (2008)	Simple formula; good quality of prediction	Limited applicability	Only for sands with a high quartz content
Lu et al. (2014)	Simple formula; for every type of soil; includes the effect of the dry density on thermal conductivity	For the analysed soils, the model clearly overestimated the values of λ in the entire range of water content	Dry soil conductivity should be computed using empirical relation proposed in Lu et al. (2007); possible weaker prediction for soils with high content of sand separate
He at al. (2017)	Simple formula; for every type of soil; good quality of prediction	Lack of correlation formulas for determining model parameters	Modification of the Johansen method with respect to the Kersten number

Table 3: Advantages, disadvantages and comments on the considered theoretical models (Różański, 2018).

Theoretical models			
Model	**Advantages**	**Disadvantages**	**Comments**
Wiener (1912)	Determination of the range of possible thermal conductivity values of porous media (soils); simple formula	Rough estimate	For coarse soils, due to the contrast between the thermal conductivity of the components, these bounds are very wide
Mickley (1951)	For every type of soil	Weak prediction for dry soils or with low water content	Should not be applied to the soils with relatively high porosity
Gemant (1952)	For every type of soil	Complicated formula; need to use nomograms	Not applicable to dry soils; possible overestimation of thermal conductivity results if Gemant formula is not used to determine the thermal conductivity of the soil skeleton λ_s
de Vries (1963)	For every type of soil; can be used for partially or fully frozen soils	Need to assume values of shape factors g_a; weak prediction for dry soils; weak reflection of real λ - S_r characteristic	For good predictions, one should incorporate in Eq. (22) heterogeneity of solid phase and at least five minerals should be taken into account (Tarnawski & Wagner, 1992, 1993); do not use if the volume fraction of water is less than 0.03 (coarse-grained soil) or 0.05-0.10 (fine-grained soil)
Gori (1983)	For every type of soil; can be used for different temperatures	Very complex formula; Certain parameters should be determined on the basis of laboratory tests' results	Possible underestimation of thermal conductivity for dry soils
Tong et al. (2009)	Includes the impact of many factors on the thermal conductivity of the porous media	Complex formula; a series of laboratory tests have to be performed	Lack of formulas from which model parameters can be computed
Haigh (2012)	High accuracy of prediction	Complex formula; Underestimation of results by a constant factor, about 1.58	Only for sandy soils with a porosity higher than 0.333

5 Conclusions

The paper presented a classical review of the existing empirical and theoretical models used for prediction of soil thermal conductivity. Five empirical and six theoretical models were widely discussed. Special attention was paid to the Johansen model, which uses the concept of the Kersten number. Selected modifications of Kersten number, presented in different papers, were discussed. Critical assessment of thermal conductivity models was provided in Section 4. Based on model predictions, measurement results (from Lu et al., 2007) for two types of soils, as well as on the authors' experience, the critical remarks with respect to individual models were formulated. These were presented in a tabular form. From the practical point of view, the discussion provided in Section 4 is very important. Based on the tables' content, one can simply choose a model that is suitable for the particular engineering problem.

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