Constraint logic programming, constraint satisfaction problem, decision support system, knowledge engineering, projects management, multi-project environment

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RISK ASSESSEMENT IN MULTI-PROJECT ENVIRONMENT

Abstract:

Decision making supported by task-oriented software tools plays a pivotal role in a modern enterprise. That is because commercially available ERP systems are not able to respond in an interactive on-line/real-time mode. It opens a new generation of DSS that enable a fast prototyping of production flows in multi-project environment as well as an integrated approach to project execution evaluation. In that context our goal is to provide a knowledge base approach allowing one to be independent on a context or representation of particular data as well as allowing designing an interactive and task-oriented decision support system (DSS). The assumed knowledge base mode of specifying a production system leads to solving a logic-algebraic method (LAM) decision problem. The approach proposed complements the decision system with an additional module (evaluation module) and facilitates searching for possible solutions meeting company production programme execution evaluation criteria. The results obtained are implemented in a software package supporting project management in the SMEs. Illustrative example of the ILOG-based software application is provided.

1. INTRODUCTION

It is a tendency in modern trade that production is connected with the client's requirements. The quick appreciation of the market's needs or the fast reaction to the needs is one of the factors causing the firm's good position on the market [2]. The organizations themselves, customizations of production, technological development, the shortage of product's life cycle, are the cause of changes which occurred within recent years. The basis of the enterprise's activity are work orders, frequently analysed as projects [21].

Taking new production projects requires task planning in production system. The tasks deal with establishing final products production programmes, planning of positions (machinery) charge and determining surplus necessary for the execution of the programmes [4]. The

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problems regarding limited resources allocation (usually following from the company's limited resources and customers requirements) belong to a class of NP-hard problems [10].

Most of the publications on project management have been dedicated to a single project. In recent years there has been a growing interest in problems related to project scheduling in multi-project environment. A dominant criterion in single-project problem is the satisfaction of time constraints [3]. Scheduling of several projects with common constrained resources has to take into account other criteria such as idle resources, resource levelling, in process inventory, and projects splitting [14]. The problem of projects group management was discussed in the papers of [1, 6, 19]. The resources allocation in multi project environment were analysed among others in the papers of [12, 13, 14, 20, 23].

Available software applications supporting project management process facilitate searching for possible solutions for the production system capacity constraints (e.g. availability of production resources, transportation means warehouses capacity) and the customer requirements (e.g. the work order execution deadline). They facilitate defining conditions which guarantee calculation efficiency of a given procedure; however, they do not guarantee obtaining optimum solution. As a result a set of alternative possible schedules is obtained.

Project planning systems available in the market, support the planning expert in generating production programmes; they do not offer a possibility to evaluate, according to subjective company requirements concerning e.g. production programme execution deadline, resources charge distribution, processes execution cost [22].

There is, therefore, a need to elaborate and implement methods and tools facilitating both fast generation of project execution and a multi-criterion evaluation of choice decision on the basis of an expert's knowledge, included directly in the decision system.

Searching for solutions within venture portfolios variation is brought to a search for alternative resources allocations which guarantee safety of measures taken. Solving allocation problems in constraint conditions is a complex calculative problem, therefore a search for solutions should take place if we are certain that it exists. It means that conditions should be known which, if met, guarantee obtaining solutions in a searching space. An approach facilitating modelled object representation as a knowledge base seems to be a constraint satisfaction problem (CSP) description [25]. The problems considered are therefore seen as specific knowledge representations, with facts represented by CSP defined constraints. It gives a possibility to search in sufficient conditions, which guarantee possible considered decision problems solutions by applying logic-algebraic method (LAM) [7]. The method's formalism and available conclusion reaching methods allow searching for conditions which guarantee that the answer to routine question exists. The inference engine applied in the LAM is then easily implemented in a kind of constraint programming/constraint logic programming (CP/CLP) language [28].

The problem under analysis deals in its firs stage with knowledge management, understood as execution of three tasks:

- monitoring (knowledge base verification, taking into account appropriate conditions and relations),
- planning (prototyping of sufficient conditions),
- control (time efficient solutions searching strategies).

The approach proposed complements the decision system with an additional module (evaluation module) and facilitates searching for possible solutions meeting company production programme execution evaluation criteria. This facilitates selection of the generated schedules directly in the system, in accordance with unified arbitrarily chosen evaluation criteria. It gives a possibility to generate the "best" solution with regard to the subjective company requirements, to save time as well as to reduce the production programme preparation cost, with uninterrupted execution of all incoming orders, optimizing at the same time the production process as far as company extreme criteria are concerned. Final solutions evaluation has been done by means of Baas and Kwakernaak method [5] using Saaty's matrixes [17].

This paper is organized as follows. Assumptions concerning the considered class of systems and the main problem of the paper are formulated in the section 2. The considered decision problem consists in determination of triples (production and transportation operations commencement times and priority rules determining the projects execution sequence) guaranteeing a given projects portfolio is completed in arbitrarily assumed period of time. In the section 3 the introduction to the logic-algebraic method (LAM) is provided, and then its implementation to the knowledge generation and a decision problem resolution is presented. The section 4 introduces concept of a constraint satisfaction problem, and then its implementation to a knowledge base specification. An illustrative example of the approach provided to the projects portfolio prototyping and approach to imprecise data handling is shown in the Section 5. Moreover, section 5 includes and example of solution variants evaluation by means of Baas and Kwakernaak method. Conclusions and future research are presented in section 6.

2. PROBLEM STATEMENT

Given is a production system for the executed project portfolio. Production system (*PR*) includes: $PR=(\{R_i|\ i=1,...,r\},\ \{T_i|\ i=1,...,s\},\ \{L_i|\ i=1,...,p\})$, determining a set of production, transportation and human resources where:
R_i – *i*-th production resource,
T_i – *i*-th transportation resource,
L_i – *i*-th human resource.

Given is a set of projects: $P = \{P_1, P_2, P_3, \ldots, P_q\}$. Each project is a sequence of a finite number of operations, where: $q_k=(AP^k_{i,j}, AT^k_{i,j})$ – is a sequence of production and transportation operations executed on resources in k-th project. Production and transportation operations commencement time vectors are also defined:

$$ST^k_{i,j} = (ST^1_{1,1}, ST^1_{1,2}, \ldots, ST^1_{1,n}, ST^1_{2,1}, \ldots, ST^1_{m,n}, \ldots, ST^r_{m,n})$$

where:
$ST^k_{i,j}$ – *i*-th transportation operation commencement time at *j*-th resource in *k*-th project,

$$SP^k_{i,j} = (SP^1_{1,1}, SP^1_{1,2}, \ldots, SP^1_{1,n}, SP^1_{2,1}, \ldots, SP^1_{m,n}, \ldots, SP^r_{m,n})$$

where:
$SP^k_{i,j}$ – i-th production operation commencement time at j-th resource in k-th project,
$SP^k_{i,j}$ – the first production operation commencement time at *j*-th resource in *k*-th project,
$ST^k_{i,j}$ – the first transportation operation commencement time at *j*-th resource in *k*-th project,
$Q_k=(Q_1,Q_2,...,Q_p)$ – priority rules, determining project execution sequence.

Production and transportation operation commencement times at j-th resource in k-th project have been presented in the following matrixes:

$$SP^k_{i,j} = \begin{bmatrix} SP^k_{1,1} & SP^k_{1,2} & \dots & SP^k_{1,n} \\ \vdots & & & \vdots \\ SP^k_{m,1} & SP^k_{m,2} & \dots & SP^k_{m,n} \end{bmatrix} \qquad ST^k_{i,j} = \begin{bmatrix} ST^k_{1,1} & ST^k_{1,2} & \dots & ST^k_{1,n} \\ \vdots & & & \vdots \\ ST^k_{m,1} & ST^k_{m,2} & \dots & ST^k_{m,n} \end{bmatrix}$$

Operation times of individual operations $T_{A^k{}_{i,j}}$ are described in the following way:

$$T_{A^k_{i,j}} = \begin{bmatrix} T_{A^k_{1,1}} & T_{A^k_{1,2}} & \dots & T_{A^k_{1,n}} \\ \vdots & & & \vdots \\ T_{A^k_{m,1}} & T_{A^k_{m,2}} & \dots & T_{A^k_{m,n}} \end{bmatrix}$$

Operation assignment to a resource in the *k*-th project is determined in accordance with the relation:

$$A^k_{i,j} = \begin{cases} 1 - \text{if operation } A_j \text{ is executed at } j-th \text{ resource in the } k-th \text{ project} \\ 0 - \text{otherwise} \end{cases}$$

Operations executed in the k-th project matrix are as follows:

$$\begin{bmatrix} A^k_{1,1} & A^k_{2,2} & \dots & A^k_{1,n} \\ \vdots & \vdots & & \vdots \\ A^k_{m,1} & A^k_{m,2} & \dots & A^k_{m,n} \end{bmatrix}$$

For such a defined projects portfolio it is necessary to answer a question: for what values of possible *i*-th operation commencement states on *j*-th resource in *k*-th project ($SP^k{}_{i,j}$, $ST^k{}_{i,j}$) and in what (*Q*) processes execution sequence is projects execution in arbitrarily set time possible?

Problem of multi-criterion projects portfolio execution efficiency evaluation is analysed. The solution includes a series of project portfolio executions, i.e. solutions of resources conflicts (making decisions which assure correct company work) together with evaluation of alternative project portfolio execution variants in accordance with the company specification and preferences. It is therefore necessary to answer the question: what order portfolio variant is best, taking into account company evaluation criteria. The answer to the question requires solving a series of subproblems e.g. can a project portfolio be executed in determined deadlines and at set cost in conditions of constrained availability to shared resources?

Due to complexity of the issue, solution searching commencement procedure should take place in conditions of certainty that the solution exists. It is therefore necessary to search for sufficient conditions (features and properties) which would guarantee an existing possible solution. In other words the values $SP^k{}_{i,j}$, $ST^k{}_{i,j}$, *Q* are sought, which guarantee an exiting possible solution.

The first subproblem of the analysed issue includes the answer to the question: is the *P* project execution in an arbitrarily set time possible for the $SP^k{}_{i,j}$, $ST^k{}_{i,j}$ and *Q* determined values? If so, what is the individual stages execution schedule? Are there alternative solutions variants for the execution of a group of projects?

Analysed issue: Project Portfolio Evaluation System (PPES) is based on a multistage analysis (Fig. 1).

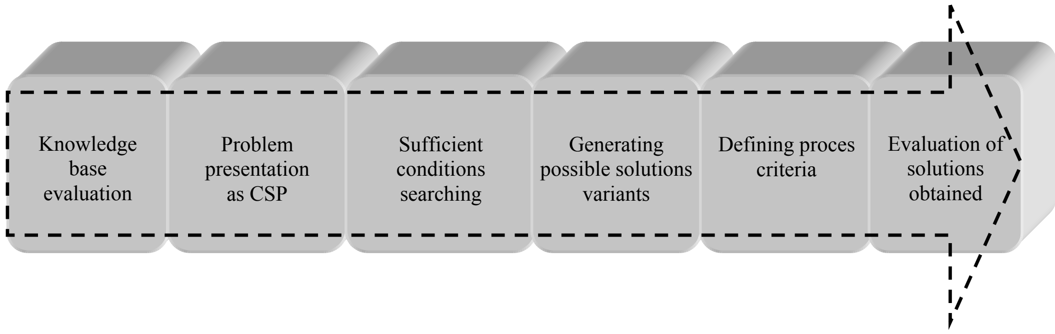


Fig. 1. Project portfolio execution evaluation states

Sufficient conditions in a form of order commencement times, initial resources assignment to orders and resources conflicts settlement rules are sought. Sufficient conditions are sought among operation times, initial states (operations assignment to resources), priority choice rules. It means that the approach proposed is brought to verifying knowledge base coherence using logic-algebraic method applied in CP/CLP techniques.

3. PPES KNOWLEDGE BASE

Elements of the considered systems class may be reflected as knowledge representation (*RW*). Knowledge representation is presented as *C*, *W*, *Y*, sets which determine the *c*, *y*, *w*, and variables domains describing certain system properties at quantitative level. Variables *c* are input variables, determining system input properties, variables *y*, are output variables determining system output properties, variables *w* are support variables. Knowledge determining properties of the system, is represented as a set of facts $F(c,w,y)$. Facts $F(c,w,y)$ are tasks which characterize (on a logical level) relations between variables *c*, *w*, *y*.

Information used for the construction of facts may be of various linguistic, algebraic expression form etc.

Triples *c*, *w*, *y,* for which all $F(c,w,y)$ facts are true, are presented as *RE* relations. Knowledge representation has therefore a form of:

$$RW = \langle C, W, Y, RE \rangle \tag{1}$$

where:

- $RE = \{(c,w,y): F(c,w,y) = 1\}$ – relation being the set of all triples (c,w,y), for which the facts *F* describing the system are true,
- $F(c,w,y) = (F_1(c,w,y)\ F_2(c,w,y), \dots, F_K(c,w,y))$ – is the sequence of the logic fact values being the functions of the variables *c, w, y*;
- $c = (c_1, c_2, \dots, c_m)$ – set of input variables;

$y = (y_1, y_2, ..., y_n)$ – set of output variables;
$w = (w_1, w_2, ..., w_o)$ – set of support variables;
$c \in C, y \in Y, w \in W$, C, Y, W – sets determining c, y, w variables domains.

The project portfolio knowledge representation will therefore be as follows:

$$RW = \langle SP_{i,j}^k, ST_{i,j}^k, Q, X, RE \rangle \tag{2}$$

where:

$RE=\{(SP^k_{i,j}, ST^k_{i,j}, Q, x, RE):F(SP^k_{i,j}, ST^k_{i,j}, Q, x, RE)=1\}$ – relation between individual variables:
$F(SP^k_{i,j}, ST^k_{i,j}, Q, x, RE)=1$ – facts determining relations between variables.

An input relation RE_x, is sought for project portfolio described by RW knowledge representation, which would facilitate meeting a known output relation RE_y. Relations RE_x and RE_y are defined as follows:

$RE_x = \{(SP^k_{i,j}, ST^k_{i,j}, Q): F_x(SP^k_{i,j}, ST^k_{i,j}, Q) = 1\}$
set of $SP^k_{i,j}, ST^k_{i,j}, Q$ values, with met system input property $F_x(SP^k_{i,j}, ST^k_{i,j}, Q)$.
$RE_y = \{x: F_y(x) = 1\}$
set of x, values with met system output property $F_y(x)$.
where:
$F_x(SP^k_{i,j}, ST^k_{i,j}, Q)$ – is a set of logical sentences describing system input properties depending on the initial state of operation execution and priority rules, determining project execution sequence.
$F_y(x)$ – is a set of logical statements describing system output properties depending on x sequence value.

Determining RE_x relation (and at the same time $F_x(SP^k_{i,j}, ST^k_{i,j}, Q)$) takes place on the basis of logical-algebraic method [9]. Re_x relation is sought on the basis of previously determined sets S_{x1} and S_{x2}:

$$S_{x1}= \{(SP^k_{i,j}, ST^k_{i,j}, Q): F (SP^k_{i,j}, ST^k_{i,j}, Q) = 1, F_y(x) = 1\}; \tag{3}$$

$$S_{x2}= \{(SP^k_{i,j}, ST^k_{i,j}, Q): F (SP^k_{i,j}, ST^k_{i,j}, Q) = 1, F_y(x) = 0\}; \tag{4}$$

$$RE_x = S_{x1} \backslash S_{x2} \tag{5}$$

RE_x set includes input parameters values which constitute sufficient conditions; if met – they guarantee that a non-empty solutions space for the analysed decision problem exists.

4. PPES MODEL OF KNOWLEDGE BASE

Every RW knowledge representation of the portfolio project execution could be presented as a CSP – constraint satisfaction problem.

The reasons for choosing to represent and solve a decision problem at hand as a CSP one is that the representation as a CSP is often much closer to the original problem: the variables of the CSP directly correspond to problem entities, and the constraints can be expressed without having to be translated into linear inequalities. This makes the formulation simpler, the solution easier to understand, and the choice the best solution.

CSP problem =((X, D), C) is defined in the following way:

Given is:

- a finite discreet decision variables set $X = \{x_1, x_2, \ldots, x_n\}$,
- a family of finite variables domains $D = \{D_i \mid D_i = \{d_{i1}, d_{i2}, \ldots, d_{ij}, \ldots, d_{im}\}, i = 1..n\}$
- and a finite constraints set $C = \{C_i \mid i = 1..L\}$ limiting the values of decision variables.

A solution is such an assignment of the variable values that all the constraints are satisfied.

In case of a *CSP* problem transforming *RW* knowledge representation, facts which are included in $F(SP^k_{i,j}, ST^k_{i,j}, x, Q)$ perform the function of C constraints and variables values $SP^k_{i,j}$, $ST^k_{i,j}$, Q perform the function of X variables. Variables domains have a form of sets D. *CSP* problem takes the following form:

$$CSP=(((SP^k_{i,j}, ST^k_{i,j}, x, Q), D), \{F(SP^k_{i,j}, ST^k_{i,j}, x, Q) = 1\}) \quad (6)$$

Solution of so understood decision problem with regard to *CSP* is related with solving the following problems:

$$CSP_{Sx1}=((SP^k_{i,j}, ST^k_{i,j}, x, Q), D), \{F(SP^k_{i,j}, ST^k_{i,j}, x, Q)=1, F_y(x)=1\}), \quad (7)$$

$$CSP_{Sx2}=((SP^k_{i,j}, ST^k_{i,j}, x, Q), D), \{F(SP^k_{i,j}, ST^k_{i,j}, x, Q)=1, F_y(x)=0\}). \quad (8)$$

The solution of problems presented (searching result of all possible solutions) are sets S_{x1} i S_{x2}.

Sufficient conditions existence substantiates commencement of solutions searching process, a search for alternative resources allocations which guarantee project execution factors balance. In that context the considered problem can be implemented using the concept of Constraint (Logic) Programming (C(L)P). C(L)P techniques can be applied in decision support systems, both in production and in service enterprises, e.g. at the goods transportation planning stage in distribution networks, projects management, production planning [15].

C(L)P is an emergent software technology for declarative description CSP and can be considered as a pertinent framework for the development of DSS.

The most important issues that contribute to the efficiency of CP/CLP techniques are the procedures of a feasible solution selection: constraints propagation, variable distribution (Fig. 2). Constraint propagation procedures deal with eliminating of decision variables, which do not meet the constraints. This is supplemented with a mechanism (variables distribution), which assigns certain values to the variables. Linking of constraint propagation with variables distribution facilitates setting a feasible solution or indicating lack of such solution.

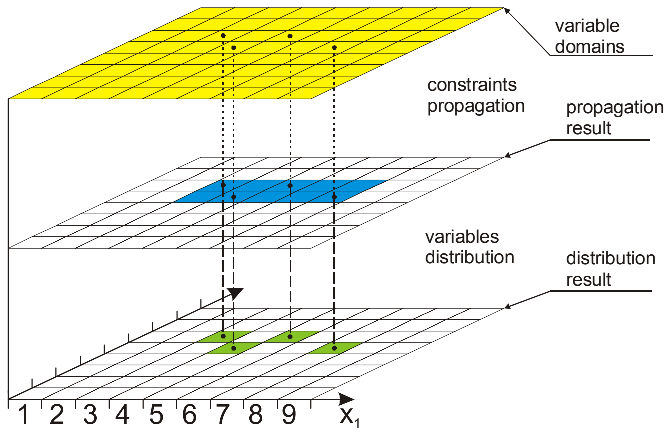
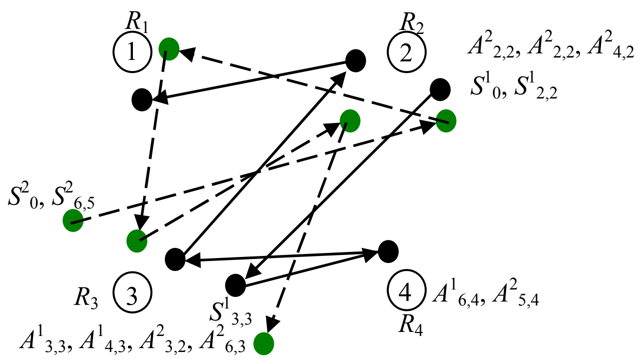


Fig. 2. Stages of the constraints propagation and variables distribution

CP/CLP techniques constitute an alternative (facilitating on-line work) for the currently available systems. It may refer especially to the construction of task oriented interfaces (which facilitate making decisions without necessary operator's interference).

5. EXAMPLE - PROJECT PORTFOLIO EVALUATION SYSTEM (PPES)

Example of a situation, when two projects with 6 operations are to be executed have been included in Fig. 3. Operations are executed, using 4 resources. Three of them are shared by two projects - they are resources R_1, R_2 and R_3. R_1, R_3 are human resources, R_3 is a transportation resource, R_4 a transportation resource. It was assumed that resources R_1 i R_3 are alternative resources and may mutually be used for the project tasks execution.



$T_{A^k_{i,j}}$ – A_j operation times in *k*-th project, $A^k_{i,j}$ – operation assignment to *j*-th resource in *k*-th project, $SP^k_{0,j}$ – the first operation commencement time at *j*-th resource in *k*-th project, $\{R_i| i=1,...,r\}$ – a set of resources, R_1, R_2 and R_3 – shared resources.

Fig. 3. Shared resources processes

Operation assignment to a given resource in project planned to be executed have been presented as matrixes:

$$A^{1}{}_{i,j} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A^{2}{}_{i,j} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Operation execution time on resources and operation assignment to a given resource within a project has been presented in the following matrixes:

$$T_{A^{1}{}_{i,j}} = \begin{bmatrix} 10 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 12 & 0 \\ 0 & 0 & 15 & 0 \\ 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 \end{bmatrix} \quad T_{A^{2}{}_{i,j}} = \begin{bmatrix} 20 & 0 & 0 & 0 \\ 0 & 15 & 0 & 0 \\ 0 & 0 & 14 & 0 \\ 0 & 8 & 0 & 0 \\ 0 & 0 & 0 & 10 \\ 0 & 0 & 12 & 0 \end{bmatrix}$$

The R_1 resource use cost is 8 c.u. (cost units), R_2 resource - 9 c.u., R_3 resource - 11 c.u., R_4 resource - 7 c.u. Assumed project execution deadline is 35 t.u. (time units).

Priority rules have been defined. Vector of executed operations sequences for projects has been determined:

$$P_1 = \left(A^1_{2,2}, A^1_{3,3}, A^1_{6,4}, A^1_{4,3}, A^1_{5,1}, A^1_{1,1}\right)$$

$$P_2 = \left(A^2_{6,3}, A^2_{4,2}, A^2_{1,1}, A^2_{3,3}, A^2_{2,2}, A^2_{5,3}\right)$$

Examples of facts defined in the knowledge base:

- $F_1(S_0, Q)$: $A^1{}_{2,2} \Rightarrow (S^1{}_{2,2} = S_0)$
- $F_2(S_0, Q)$: $A^1{}_{3,3} \Rightarrow (S^1{}_{3,3} \geq S^1{}_{2,2} + T_{A^1{}_{2,2}})$
- $F_3(S_0, Q)$: $A^1{}_{6,4} \Rightarrow (S^1{}_{3,3} \geq S^1{}_{3,3} + T_{A^1{}_{3,3}})$
- …..

We need to answer the question: is a project execution for a determined resources number at a scheduled project execution deadline possible? If so, what possible solution variant is to be chosen (taking into account a company specific features and demand, especially in chosen company departments e.g. marketing, production, logistics departments)?

To answer the questions posed by the decision maker it is necessary to work out a computer system facilitating generation of possible project execution variants and their evaluation directly in the system, without a necessity of user interference.

According to the approach presented in chapter 3, the CSP problem has been implemented by means of CP techniques in Ilog.

Examples of schedules generated in the system have been presented in Fig. 4. There are three different schedules, they have different costs, deadlines and use alternative resources.

a)

```
--- Solution #1 ---
Proces 1[1 -- 68 --> 69]
P1-A1[59 -- 10 --> 69]
P1-A2[1 -- 6 --> 7]
P1-A3[7 -- 12 --> 19]
P1-A4[39 -- 15 --> 54]
P1-A5[54 -- 5 --> 59]
P1-A6[19 -- 20 --> 39]
Proces 2[69 -- 79 --> 148]
P2-A1[89 -- 20 --> 109]
P2-A2[123 -- 15 --> 138]
P2-A3[109 -- 14 --> 123]
P2-A4[81 -- 8 --> 89]
P2-A5[138 -- 10 --> 148]
P2-A6[69 -- 12 --> 81]
Alternative resources:
P1-A6 => R4
P2-A6 => R3
P2-A5 => R3
P2-A3 => R3
P1-A4 => R3
P1-A3 => R3
P2-A4 => R2
P2-A2 => R2
P1-A5 => R2
P1-A2 => R2
P2-A1 => R1
P1-A1 => R1
 Koszty P1: 616 P2: 763
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b)

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--- Solution #2 ---
Proces 1[1 -- 68 --> 69]
P1-A1[59 -- 10 --> 69]
P1-A2[1 -- 6 --> 7]
P1-A3[7 -- 12 --> 19]
P1-A4[39 -- 15 --> 54]
P1-A5[54 -- 5 --> 59]
P1-A6[19 -- 20 --> 39]
Proces 2[1 -- 83 --> 84]
P2-A1[21 -- 20 --> 41]
P2-A2[59 -- 15 --> 74]
P2-A3[41 -- 14 --> 55]
P2-A4[13 -- 8 --> 21]
P2-A5[74 -- 10 --> 84]
P2-A6[1 -- 12 --> 13]
Alternative resources:
P2-A6 => R1
P2-A5 => R1
P2-A3 => R1
P2-A1 => R1
P1-A4 => R3
P1-A3 => R3
P1-A1 => R3
P1-A6 => R4
P2-A4 => R2
P2-A2 => R2
P1-A5 => R2
P1-A2 => R2
 Koszty P1: 646 P2: 655
```

c)

```
--- Solution #3 ---
Proces 1[1 -- 68 --> 69]
P1-A1[59 -- 10 --> 69]
P1-A2[1 -- 6 --> 7]
P1-A3[7 -- 12 --> 19]
P1-A4[39 -- 15 --> 54]
P1-A5[54 -- 5 --> 59]
P1-A6[19 -- 20 --> 39]
Proces 2[1 -- 83 --> 84]
P2-A1[21 -- 20 --> 41]
P2-A2[59 -- 15 --> 74]
P2-A3[41 -- 14 --> 55]
P2-A4[13 -- 8 --> 21]
P2-A5[74 -- 10 --> 84]
P2-A6[1 -- 12 --> 13]
Alternative resources:
P2-A6 => R1
P2-A5 => R1
P2-A3 => R3
P2-A1 => R3
P1-A4 => R1
P1-A3 => R3
P1-A1 => R1
P1-A6 => R4
P2-A4 => R2
P2-A2 => R2
P1-A5 => R2
P1-A2 => R2
 Koszty P1: 571 P2: 757
```

Fig. 4. Solutions obtained in Ilog system

To choose one of the three generated project execution programmes it is necessary to take into account the opinion and preferences of chosen company department managers.

A choice of one, within the numerous solutions variants, taking into account various evaluation criteria may be done by means of methods such as: a method of weighted criteria, hierarchic optimization method, limited criteria method, global criterion method. The methods are efficient when the criteria evaluation values are of deterministic character. The assumption does not always allow reflecting actual conditions in which evaluation information is frequently of approximated, subjective character. One of the methods using approximated evaluation is Baas and Kwakernaak method in which criteria evaluation and importance are in a fuzzy form. The method may be applied in a choice of optimum solution from a finite set of possible solutions (e.g. production schedules variants):

$U = \{U_1, \ldots, U_2, U_N\}$ by means of K criteria set, where $K=\{ K^{(1)}), K^{(2)}),\ldots, K^{(M)})\}$, and every criterion importance is given in a fuzzy form. The method uses subjective pointwise decision makers' evaluations. Each of the decision makers is responsible for creating criteria importance evaluation matrix by means of Saaty's method based on comparing criteria pairs. The stages of multi-criterion solution variants in Baas and Kwakernaak method are as follows:

1. Creating criteria importance matrix by means of Saaty's method.
2. Creating collective criteria importance matrix.
3. Determining criteria importance by means of power method.
4. Pointwise evaluation with regard to assumed criteria.
5. Reducing pointwise evaluations to normalized values.
6. Creating collective normalized evaluations by average evaluations given by individual experts.
7. Creating decision function.
8. Choice of the best solution variant.

5.1 Project Portfolio Evaluation

The analysed example refers to the choice of the best possible solution among the production schedules generated in machinery industry. Out of the three possible solutions obtained in Ilog (Fig. 4), it is necessary to choose the best solution taking into account criteria chosen by a company.

Using CP techniques, three variants of projects execution were generated in Ilog:

1. U_1 - solution *a*
2. U_2 - solution *b*
3. U_3 - solution *c*

The following evaluation criteria were used in the analysis:

1. $K^{(1)}$ – time evaluation,
2. $K^{(2)}$ – cost evaluation,
3. $K^{(3)}$ – production resources charge evaluation.

Four company employees (E_1, E_2, E_3, E_4) took part in the criteria significance evaluation and the solutions evaluation. The employees were: the president, production manager, marketing department manager and logistics department manager. Although the strategic aims of the company are clearly defined and connected with generating possible highest profit, frequently, in a company the aims of individual units or departments are divergent. It is therefore necessary to take into account the evaluation of individual employees from chosen company departments in the production programme evaluation.

Production programme evaluation and the approach presented in this section has been implemented in Ilog system. A decision support system has been established concerning the evaluation of various projects portfolio execution variants. The system is not easy to operate and gives a possibility to generate solutions without user's interference.

Every employee evaluated solutions (production programmes) with regard to the analysed criteria, using an evaluation scale of 1 to 5 points (the higher the number of points the better the evaluation). Figures 5 and 6 include input data. Figure 5 includes evaluations of possible solutions with regard to individual criteria. Fig. 6 presents the criteria importance evaluations.

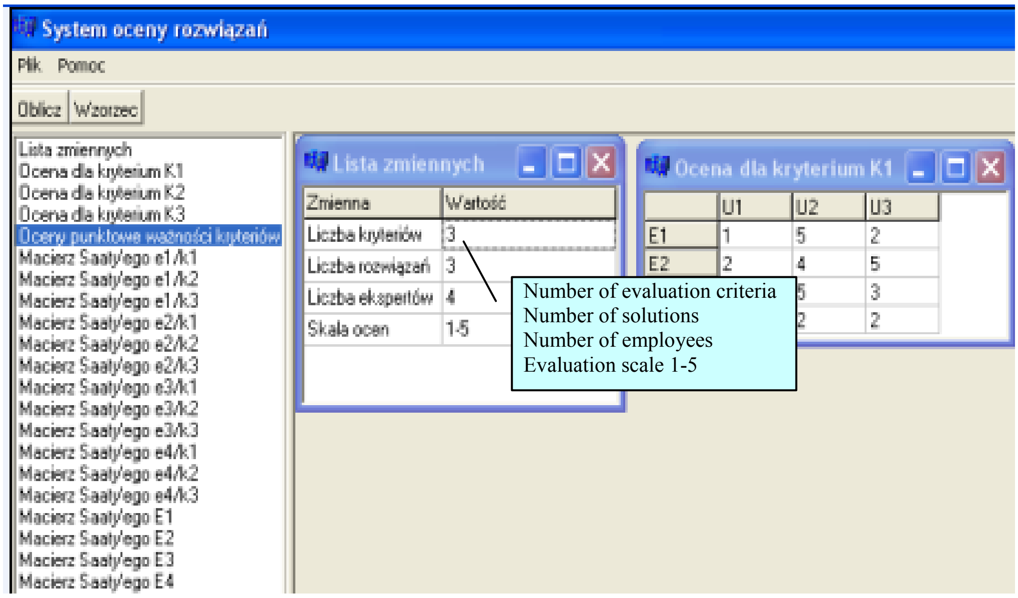
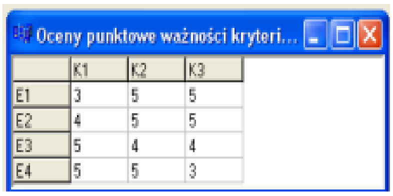


Fig. 5. Possible solutions evaluations, with regard to individual criteria.



	K1	K2	K3
E1	3	5	5
E2	4	5	5
E3	5	4	4
E4	5	5	3

Fig. 6. Points' evaluation of criteria importance

Multi-criterion solutions variants evaluation stages have been included in Fig. 7.

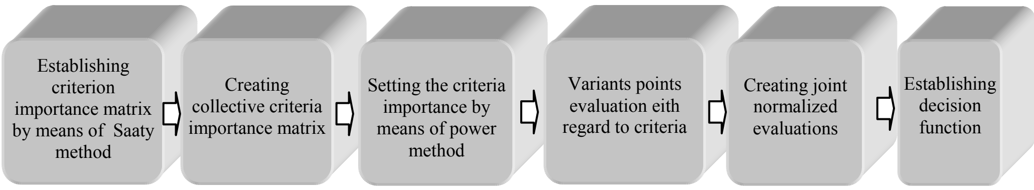


Fig. 7. Multi-criterion stages of solution variants evaluation

In a chosen $E_l/K^{(k)}$ element, Saaty's matrix is calculated as follows:

$$S_{i,j} = \frac{K_{l,i}^{(k)}}{K_{l,j}^{(k)}} \tag{9}$$

Saaty's matrixes have been formed on the basis of points evaluations. Saaty's matrix of solutions evaluation for the first decision maker has been included in Fig. 8. Fig. 9 includes Saaty's criteria importance matrix for the employee E_1.

a)

E_2/K^1

	U_1	U_2	U_3
U_1	1,000	0,500	0,400
U_2	2,000	1,000	0,800
U_3	2,500	1,250	1,000

b)

E_2/K^2

	U_1	U_2	U_3
U_1	1,000	0,667	0,400
U_2	1,500	1,000	0,600
U_3	2,500	1,667	1,000

c)

E_2/K^3

	U_1	U_2	U_3
U_1	1,000	0,800	1,000
U_2	1,250	1,000	1,250
U_3	1,000	0,800	1,000

Fig. 8. Saaty's solutions evaluation matrix for the employee E_2

	K^1	K^2	K^3
K^1	1,000	0,600	0,600
K^2	1,667	1,000	1,000
K^3	1,667	1,000	1,000

Fig. 9. Saaty's citeria importance matrixes for employee E_1

The next stage is calculating vector for each Saaty's matrix, corresponding to the highest own value. Due to considering the principle of consistence when creating a matrix, the vector is included in the first matrix column. The principle of normalizing vector coordinates is assumed so that the number of their squares equals 1. Eigenvectors of individual matrixes have been calculated from the following:

$$\sum_{j=1}^{l_U} (K_{ij}^{(k)})^2 = 1 \quad i = 1,2,...,l_e, k = 1,2,...,l_e \tag{10}$$

$$\sum_{j=1}^{l_U} (W_{ij})^2 = 1 \tag{11}$$

Eigenvectors of individual matrixes calculated on the basis of the above mentioned relation with regard to criterion K^1 has been presented in Fig 10. The coordinates of eigenvectors for criteria importance matrixes have been included in Fig. 11.

K^1	U_1	U_2	U_3
E_1	0,183	**0,913**	0,365
E_2	0,298	0,596	0,745
E_3	0,324	0,811	0,487
E_4	0,728	0,485	0,485

Fig. 10. The coordinates of eigenvectors for solutions evaluation matrixes

	K^1	K^2	K^3
E_1	0,391	0,651	0,651
E_2	0,492	0,615	0,615
E_3	**0,662**	0,530	0,530
E_4	0,651	0,651	0,651

Fig. 11. The coordinates of eigenvectors for criteria importance matrixes

The next step is to normalize coordinates of eigenvectors according to the following relation:

$$\hat{K}_{ij}^{(k)} = \frac{K_{ij}^{(k)}}{\max\limits_{i,j} K_{ij}^{(k)}} \qquad i = 1,2,...,l_e;\ j = 1,2,...,l_u;\ k = 1,2,...,l_k \tag{12}$$

$$\hat{w}_{ij} = \frac{w_{ij}}{\max\limits_{i,j} w_{ij}} \qquad i = 1,2,...,l_e;\ j = 1,2,...,l_k \tag{13}$$

Normalized eigenvectors are presented in Fig. 12 and Fig. 13.

K^2	U_1	U_2	U_3
E_1	0,330	0,989	0,659
E_2	0,400	0,600	1,000
E_3	0,634	0,634	0,846
E_4	0,761	0,951	0,190

Fig. 12. Normalized eigenvectors' coordinates for solution evaluations matrixes

	K^1	K^2	K^3
E_1	0,590	0,983	0,983
E_2	0,743	0,929	0,929
E_3	1,000	0,800	0,800
E_4	0,983	0,983	0,983

Fig. 13. Normalized eigenvectors' coordinates for criteria importance matrixes

In the next stage coordinates of characteristic assignment functions for fuzzy criteria importance and solutions evaluations were determined. Coordinates characteristic for the fuzzy solutions evaluation assignment functions diagrams have been presented in Fig. 14. The coordinates have been determined on the basis of the following relations:

$$\hat{K}R^{(k)}_{\min,j} = \min_i \hat{K}^{(k)}_{ij} \tag{14}$$

$$\hat{K}R^{(k)}_{\max,j} = \max_i \hat{K}^{(k)}_{ij} \tag{15}$$

$$\hat{K}R^{(k)}_{\mathrm{mod},j} = \frac{1}{l_e}\sum_{i=1}^{l_e} \hat{K}^{(k)}_{ij} \tag{16}$$

a)

$\hat{K}R^{(1)}$

K^1	U_1	U_2	U_3
MIN	0,200	0,531	0,400
MOD	0,420	0,768	0,570
MAX	0,797	1,000	0,816

b)

$\hat{K}R^{(2)}$

K^2	U_1	U_2	U_3
MIN	0,330	0,600	0,190
MOD	0,531	0,794	0,674
MAX	0,761	0,989	1,000

c)

$\hat{K}R^{(3)}$

K^3	U_1	U_2	U_3
MIN	0,333	0,471	0,333
MOD	0,549	0,740	0,549
MAX	0,707	1,000	0,707

Fig. 14. Coordinates characteristic for the fuzzy solutions evaluation assignment functions diagrams

Coordinates characteristic for the criteria importance assignment function diagrams:

$$\hat{W}R_{\min,j} = \min_i \hat{W}_{ij} \tag{17}$$

$$\hat{W}R_{\max,j} = \max_i \hat{W}_{ij} \tag{18}$$

$$\hat{W}R_{\mathrm{mod},j} = \frac{1}{l_e} \sum_{i=1}^{l_e} \hat{W}_{ij} \tag{19}$$

WR	K^1	K^2	K^3
MIN	0,590	0,800	0,800
MOD	0,829	0,924	0,924
MAX	1,000	0,983	0,983

Fig. 15. Coordinates characteristic for the criteria importance assignment function diagrams

Solutions evaluation assignment function diagrams were established on the basis of the above mentioned data (Fig. 16, Fig. 17, Fig. 18).

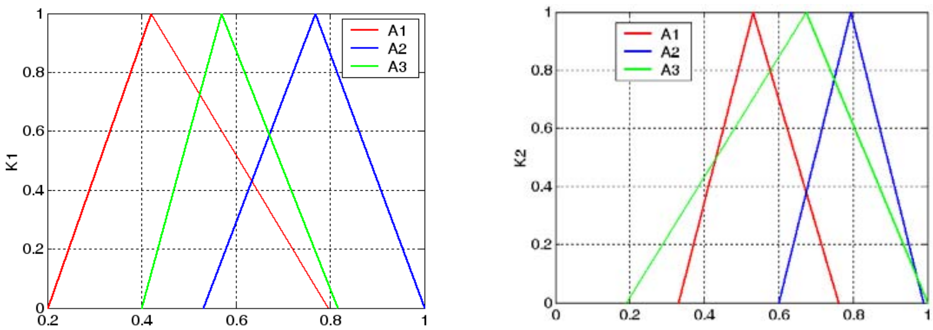


Fig. 16. Solutions fuzzy evaluations with regard to criteria K^1 and K^2

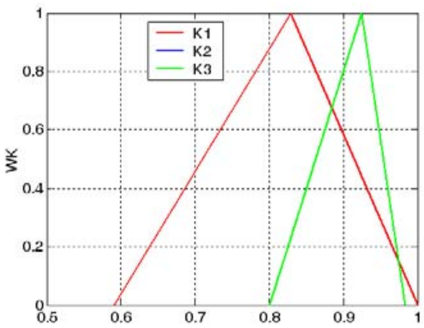


Fig. 17. Solutions fuzzy evaluations with regard to evaluations, criterion K^3

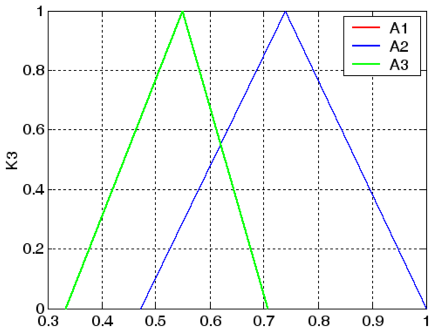


Fig. 18. Criteria importance fuzzy

After joint criteria importance and solutions evaluation have been established, the assignment function of substitution criteria of individual solutions was determined, using the following relationship:

$$\mu_{U_i}(U_j) = \frac{\sum_{i=1}^{l_k} (\mu_{KR_j^{(k)}}(KR_j^{(i)}) \cdot \mu_{WR}(WR_i))}{\sum_{i=1}^{l_k} \mu_{WR}(WR_i)} \tag{20}$$

Using α-cross sections of fuzzy numbers: $x_2 = \begin{cases} 1\ dla\ \mu_\beta(x) \ge \alpha \\ 0\ dla\ \mu_\beta(x) < \alpha \end{cases}$

$\mu_{U_j}(U_j)=\alpha$ gdy

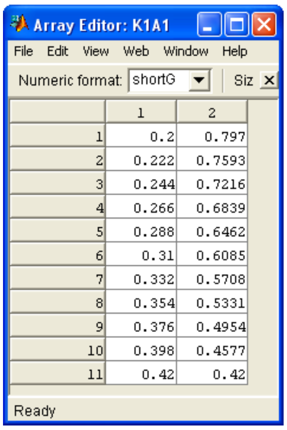
$$K^{(k)}{}_{L\alpha}=KR^{(k)}{}_{\min}+(KR^{(k)}{}_{\mathrm{mod}} - KR^{(k)}{}_{\min})\cdot\alpha$$
$$K^{(k)}{}_{P\alpha}=KR^{(k)}{}_{\max}+(KR^{(k)}{}_{\max} - KR^{(k)}{}_{\mathrm{mod}})\cdot\alpha$$
$$W^{(k)}{}_{P\alpha}=WR^{(k)}{}_{\max}+(WR^{(k)}{}_{\max} - WR^{(k)}{}_{\mathrm{mod}})\cdot\alpha$$
$$W^{(k)}{}_{L\alpha}=WR^{(k)}{}_{\min}+(WR^{(k)}{}_{\mathrm{mod}} - WR^{(k)}{}_{\min})\cdot\alpha$$

and for a given α

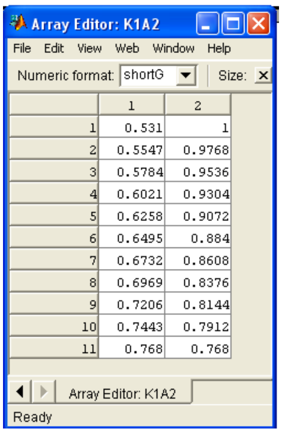
$$\mu_{LU_i}(U_j) = \frac{\sum_{i=1}^{l_k} (\mu_{K_{L\alpha}^{(i)}}(K_{L\alpha}^{(i)}) \cdot \mu_{WL\alpha}(W_{L\alpha i}))}{\sum_{i=1}^{l_u} (\mu_{P\alpha}(W_{P\alpha i}))} \tag{21}$$

$$\mu_{PU_i}(U_j) = \frac{\sum_{i=1}^{l_k} (\mu_{K_{P\alpha}^{(i)}}(K_{P\alpha}^{(i)}) \cdot \mu_{WP\alpha}(W_{P\alpha i}))}{\sum_{i=1}^{l_u} (\mu_{L\alpha}(W_{L\alpha i}))} \tag{22}$$

To illustrate the analysed calculation method, in Fig. 19 we included a partial fuzzy importance criteria multiplication sheet by fuzzy solutions U_1, U_2, U_3 with regard to criterion K^1 , their sum and the importance criteria sum with regard to the above mentioned relationships.

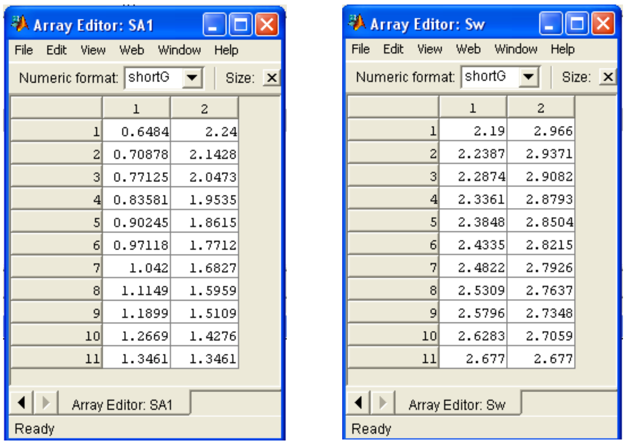


	1	2
1	0.2	0.797
2	0.222	0.7593
3	0.244	0.7216
4	0.266	0.6839
5	0.288	0.6462
6	0.31	0.6085
7	0.332	0.5708
8	0.354	0.5331
9	0.376	0.4954
10	0.398	0.4577
11	0.42	0.42



	1	2
1	0.531	1
2	0.5547	0.9768
3	0.5784	0.9536
4	0.6021	0.9304
5	0.6258	0.9072
6	0.6495	0.884
7	0.6732	0.8608
8	0.6969	0.8376
9	0.7206	0.8144
10	0.7443	0.7912
11	0.768	0.768

Fig . 19. Multiplication of criterion K^1 fuzzy importance and U_1, U_2, U_3 fuzzy solution evaluation



SA1:

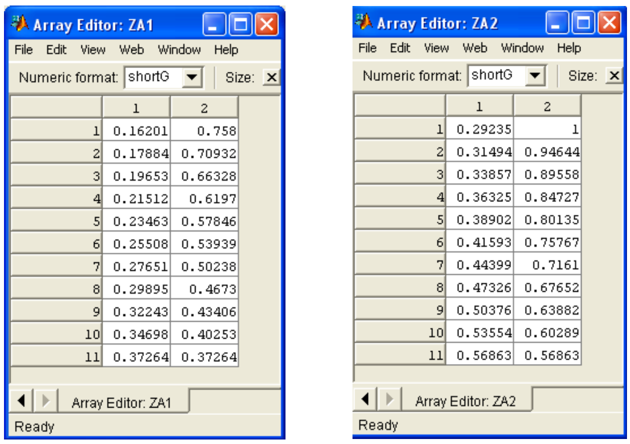
	1	2
1	0.6484	2.24
2	0.70878	2.1428
3	0.77125	2.0473
4	0.83581	1.9535
5	0.90245	1.8615
6	0.97118	1.7712
7	1.042	1.6827
8	1.1149	1.5959
9	1.1899	1.5109
10	1.2669	1.4276
11	1.3461	1.3461

Sw:

	1	2
1	2.19	2.966
2	2.2387	2.9371
3	2.2874	2.9082
4	2.3361	2.8793
5	2.3848	2.8504
6	2.4335	2.8215
7	2.4822	2.7926
8	2.5309	2.7637
9	2.5796	2.7348
10	2.6283	2.7059
11	2.677	2.677

Fig. 20. Fuzzy sums of solution evaluations

Finally, substitute evaluation criterion for the U_1, U_2, U_3 (Fig. 21) solution was calculated. Then normalized substitute criterion evaluation assignment forms for all solutions under analysis were calculated and on this basis assignment functions diagrams have been established (Fig. 22).



ZA1:

	1	2
1	0.16201	0.758
2	0.17884	0.70932
3	0.19653	0.66328
4	0.21512	0.6197
5	0.23463	0.57846
6	0.25508	0.53939
7	0.27651	0.50238
8	0.29895	0.4673
9	0.32243	0.43406
10	0.34698	0.40253
11	0.37264	0.37264

ZA2:

	1	2
1	0.29235	1
2	0.31494	0.94644
3	0.33857	0.89558
4	0.36325	0.84727
5	0.38902	0.80135
6	0.41593	0.75767
7	0.44399	0.7161
8	0.47326	0.67652
9	0.50376	0.63882
10	0.53554	0.60289
11	0.56863	0.56863

Fig. 21. Normalized substitute criterion evaluation assignment functions for all analysed solutions.

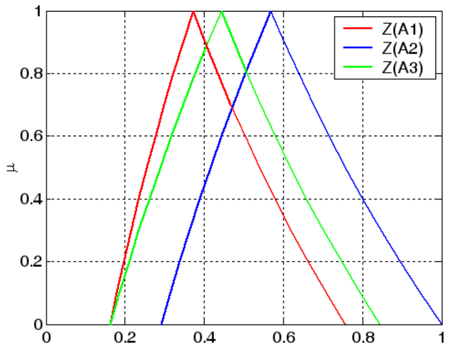


Fig. 22. Diagrams of solutions evaluation assignment functions with regard to substitute criterion

In the last stage a defusification process of substitute fuzzy criterion evaluations was executed by means of determining fields' centre of gravity under the assignment functions diagrams. The analysis was carried out according to the following relationship:

$$Z_{Uj} = \frac{\int u_j \cdot \mu_{U_j}(u_j)du_j}{J\mu_{U_j}(u_j)du_j} \tag{23}$$

$$Z_{U_j} = \frac{\frac{1}{2}\sum_{i=1}^{N}(\mu_{uj}(u_j(i)) \cdot u_j(i) + \mu_{uj}(u_j(i+1)) \cdot u_j(i+1)) \cdot (u_j(i+1) - u_j(i))}{\frac{1}{2}\sum_{i=1}^{N}(\mu_{uj}(u_j(i)) + \mu_{uj}(u_j(i+1)) \cdot (u_j(i+1) - u_j(i))}$$

N – number of α sections

Three solutions evaluations were obtained as a result of defusification:

1. The first solution evaluation – $U_1 = 0.4180$
2. The second solution evaluation – $U_2 = 0.6064$
3. The third solution evaluation – $U_3 = 0.4698$

In accordance with the approach adopted (taking into consideration evaluation criteria of a given company and its departments' preferences and needs) solutions evaluations for various projects variants execution were obtained. The closest to the value one is the best evaluation; therefore, as a result of calculations we may state that the second solution is the best one. It meets the company various departments' managers' expectations which means that the solution generated constitutes a compromise between frequently divergent various departments needs.

6. CONCLUDING REMARKS

Decision makers, choosing the right projects portfolio, face the problem of making optimal decision that meets an organisation's objectives and priorities in different situation under given constraints with various sources of knowledge.

The currently commercially available software packages do not offer a possibility to plan the projects execution in multi-project environment, characteristic for medium and small size enterprises. It gives rise to an increased demand for decision taking support packages for these companies. Such tools should facilitate answering the question: is a project execution for a determined resources number at a scheduled project execution deadline possible? If so, what possible solution variant is to be chosen (taking into account a company specific features and demand?

Proposed approach to projects portfolio prototyping provides the framework allowing one to take into account both: generating sufficient conditions (which guarantee that a non empty possible solution set exists) and choosing the best solution on the basis of chosen evaluation criteria. System properties are presented in formalism of the logic-algebraic method (LAM), which is then easily implemented in a kind of the constraint programming (CP) language. The analysed issue deals with ventures efficiency evaluation in multi-project environment in constraint conditions e.g. resource, time, sequence and cost constraint conditions.

Further research is aimed at developing the approach proposed by a possibility of decision support systems design for fuzzy problems, by linking constraint logic programming including decomposition methods, which are currently used in solving logic algebraic method problems.

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