Landslide Deformation Analysis Based on Robust M-estimations

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Abstract

There are often occurs a cases in surveying practice, when periodical observations of horizontal landslide areas displacements or different objects is necessary to evaluate in the local coordinate system. The reasons, for which some local coordinate systems are still using, even in the present time of global navigation satellite systems (GNSS), are different. It is above all the facts that till the put in practice of GNSS technology, the deformation analysis was carried out in national cartographic coordinate system. If the density and accuracy of geodetic bases not guarantee the required accuracy, deformation analysis is carried out in geodetic network structures located in the local coordinate systems. The advantage of this approach is that horizontal point displacements were evaluated in purpose-designed coordinate system with minimizing mathematical corrections of observation such as correction from height above sea level or from distortions of a map projection. Another reason for limiting the use of GNSS may be the fact that the signal reception can be considerably restricted inside of steep and narrow mountain valleys, near to rock walls or in areas with dense vegetation.

The most serious problem of periodical evaluation of positional points displacements in the local geodetic network are primarily risks associated with the change of geodetic networks datum in the local coordinate system as a result of positional instability of one or more reference points. The paper presents a way how can be the discussed problem, at least partially, eliminated by the use of robust M-estimates.

Keywords: transformation, deformation analysis, landslide, robust M-estimators

Introduction

In geodetic practice often occurs the task when it is necessary to perform a deformation analysis based on repeated surveying observations. The task is even more difficult if:

- there is not available covariance matrix of coordinates from a previous epoch,
- mathematical relationships to reduction of observations are not known; for example for reduction of observed distances to a local 2D coordinate system in the previous epoch t' (Sokol, Bajtala and Ježko, 2014),
- there are a limited number of reference points which, in addition may exhibit instability.
- the present contribution shows the way that can restrictive conditions mentioned above effective solve by applying some transformation models and robust methods.

Principle of transformation analysis of point stability.

The principle of transformation analysis of the stability of the set of points that are joined to a suitable geodetic network structure is based on a logical understanding of transformation tools, whose meaning and use can be described in the following paragraphs (Fotiou and Rossikopoulos, 1993; Teunissen, 1986; Sütti and Gašinec, 2001; Staňková, Černota and Novosad, 2012; (Yang, Song and Xu, 2002):

– There is a defined coordinate system S (XY) for the epoch t on the basis of adjustment of survey observations with the coordinates $C = [X\ Y]^T$.

– Based on repeated measurements in the same geodetic point field are determined coordinates $C' = [X'\ Y']^T$ in the implementation of the coordinate system S' for epoch t'.

– Coordinates C' are converted by the appropriate transformation method onto coordinates C_t in the coordinate system S valid for epoch t.

– From the coordinates C, C_t, which are not identical and which represent coordinate differences $dC = C_t - C$ formed on the points during the period $t'-t$ from various reasons (errors in observations, blunders, positional shifts, etc.).

Application of robust m-estimation methods for transformation models

For coordinate transformations C' on coordinates C_t it is possible to use various sorts of transformations, for example translational, similarity, affine and other. Each of them determined C_t with different values depending on transformation parameters what are determined (Welsch, 1982; Sütti and Gašinec, 2001):

- translations t_x, t_y in the direction of the coordinate axes x, y,
- rotation ω_{XY},
- maximum relative moves (normal) e1, e2 and the relative shearing strain γ.

There is a general linear (simplified) transformation relationship

$$C' = F\,C + t = (I+dF)\,C + t \tag{1}$$

between the coordinates C and C', where: F is the matrix of deformations, $t = [t_x\ t_y]^T$ are the translations components, I is identity matrix and dF is tensor of deformations. After modification of the equation (1) one receives deformation equation (for one point) in the form

$$\begin{gathered} C'-C = dC = \begin{bmatrix} dX \\ dY \end{bmatrix} = \\ \left[\begin{bmatrix} e_{xx} & e_{xy} \\ e_{yx} & e_{yy} \end{bmatrix} + \begin{bmatrix} 0 & \omega_{xy} \\ -\omega_{xy} & 0 \end{bmatrix}\right]\begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} \end{gathered} \tag{2}$$

In which e_{xx}, e_{yy}, e_{xy} the elements of the tensor of deformation (strain). Modification of the relationship (2) we can obtain relations between point shifts dC and deformation parameters Θ

$$dC = C' - C = \mathrm{H}\Theta \tag{3}$$

where:

$$H = \begin{bmatrix} 1 & 0 & X' & X & Y' & 0 \\ 0 & 1 & -Y' & 0 & X' & Y' \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & X' & X & Y' & 0 \\ 0 & 1 & -Y' & 0 & X' & Y' \end{bmatrix} \tag{4}$$

$$\Theta = [t_x t_y \omega_{xy} e_{xx} e_{xy} e_{yy}]^T \tag{5}$$

If only some components of the vector of deformation parameters are used, positional changes of points (3) can be expressed by one of the models:

$$dC = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} t_x \\ t_y \end{bmatrix} \tag{6}$$

$$dC = \begin{bmatrix} 1 & 0 & X' \\ 0 & 1 & -Y' \\ \vdots & \vdots & \vdots \\ 1 & 0 & X' \\ 0 & 1 & -Y' \end{bmatrix} \begin{bmatrix} t_X \\ t_Y \\ \omega_{XY} \end{bmatrix} \tag{7}$$

or

$$dC = \begin{bmatrix} 1 & 0 & X' & -Y' \\ 0 & 1 & Y' & X' \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & X' & -Y' \\ 0 & 1 & Y' & X' \end{bmatrix} \begin{bmatrix} t_X \\ t_Y \\ \cos\omega_{XY} \\ \sin\omega_{XY} \end{bmatrix} \tag{8}$$

where estimates of deformation vector can be determined with the Least Squares Method based on the processing of redundant identical points

$$\hat{\Theta} = (H^T P H)^{-1} H^T P dC \tag{9}$$

where P is the weight matrix of coordinates C'. Transformed coordinates C_t and residuals v shall be determined on the basis of relations

$$C_t = -H\hat{\Theta} + C' \tag{10}$$

$$dC_t = v = -H\hat{\Theta} + dC \tag{11}$$

There are often occur situations in the processing of the measurement that one or more blunders penetrated into vector of observations, as a result of inaccurate determination of their weights (wrongly-set stochastic model), or incomplete formation of geometrical and physical relationships between measurements and unknown parameters (wrongly-set functional model) which leads to the fact that observations do not have a normal distribution N(μ,σ²) with mean μ and variance σ² (Ižvoltová and Koťka, 2014; Brejcha, Staňková and Černota, 2016; Vrublová et al., 2014). Therefore, robust M-estimates, which are the generalized form of the maximum likelihood estimations, introduced by has developed particularly by courtesy the development and availability of computer technology.

In general, these estimates minimize not only the expression $v^T P v$ but also the more general term $f((v,C,C'),P) \rightarrow min$.

The diagonal elements p_i of the weighting matrix

$$\underset{n,n}{P} = \begin{pmatrix} p_1 & 0 & \cdots & 0 \\ 0 & p_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & p_n \end{pmatrix} = \sigma_0^2 \begin{pmatrix} \frac{1}{\sigma_1^2} & 0 & \cdots & 0 \\ 0 & \frac{1}{\sigma_2^2} & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\sigma_n^2} \end{pmatrix} \tag{12}$$

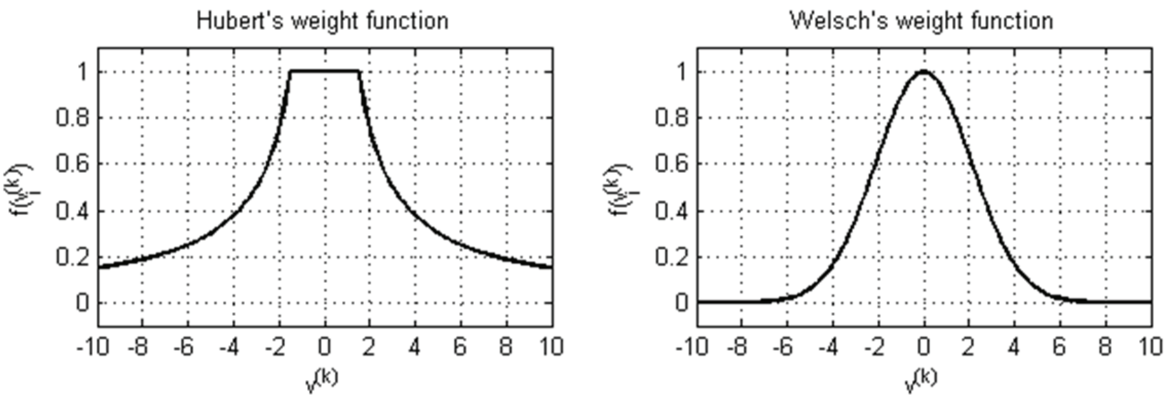

Fig. 1. Graphical representation of the Hubert's and Welsch's weight functions

Rys. 1. Reprezentacja graficzna funkcji wagi Huberta i Welscha

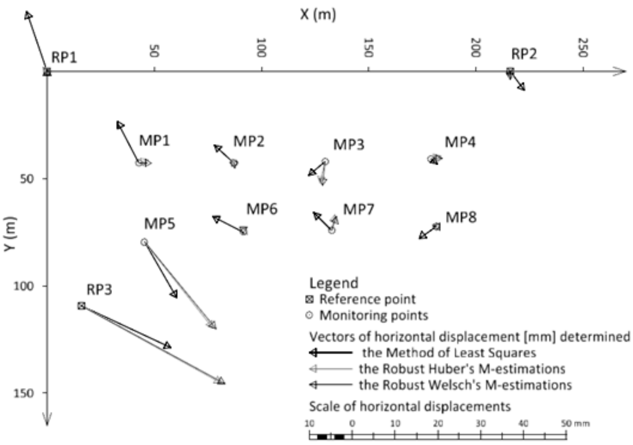

Fig. 2. Graphical interpretations of horizontal displacement

Rys. 2. Reprezentacja graficzna poziomych przesunięć

are changed in each subsequent iteration (*k+1*) according to the rule (13). Therefore appellation iteratively re-weighting the LS is also used for these methods.

$$(p_i^{(k+1)}) = p_1^{(k)} f(v_1^{(k)}) \tag{13}$$

where σ_i^2 are variances of observations and σ_0^2 is the a priori variance of unit weight.

There are a number of theoretically derived and published forms for weighting function $f(v_i)$, based on M-estimators such as the Huber, Hampel, Andrews, Geman-McClure, Tukey and Welsch estimators or the Danish method (Andrews, 1974; Hampel, Ronchetti and Rousseeuw, 1986; Huber and Ronchetti, 2011; Jäger et al., 2005).

Positive features of robust M-estimators will be demonstrated on the example of their practical application on the transformational analysis of positional shifts monitoring. Robust M-estimation by Huber and Welsch were used to solution of mentioned problem.

Weight function of Huber's M-estimation (Huber and Ronchetti, 2011; Jäger et al., 2005) is given by equation

$$f(v_i^{(k)}) = \begin{cases} 1 \quad if\ v_i^{(k)} \le c \\ \dfrac{c}{v_i^{(k)}} if\ v_i^{(k)} > c \end{cases} \tag{14}$$

where k is the number of iterations and c = 1.5 σ_{Li} = 1.5 $\sigma_0/\sqrt{p_i}$.

For the robust estimation by Welsch is derived relationship of weighting function in the shape:

$$f(v_i^{(k)}) = e^{-(v_i^{(k)}/a)^2} \tag{15}$$

where the constant a takes the value a = 2.985 σ_{Li} (Jäger et al., 2005; Valero and Moreno, 2005). Shapes of weighting functions are shown in charts Fig. 1.

Practical application

Verification of points positional stability based on applying transformation analysis was realized

Tab. 1. Coordinates of points at epoch *t* and *t′*

Tab. 1. Współrzędne punktów w epoce *t* i *t'*

Epoch	***t***		***t′***	
Point	***X [m]***	***Y [m]***	***X′ [m]***	***Y′ [m]***
RP1	0.000	0.000	0.001	0.000
RP2	215.922	0.000	215.922	0.001
RP3	16.025	109.180	16.060	109.198
MP1	42.826	42.692	42.829	42.692
MP2	87.176	42.692	87.177	42.693
MP3	129.678	42.075	129.678	42.080
MP4	178.956	40.842	178.959	40.841
MP5	45.290	79.680	45.308	79.700
MP6	91.487	74.755	91.488	74.755
MP7	132.757	74.138	132.759	74.134
MP8	181.419	72.288	181.419	72.287

Tab. 2. Determining the transformed coordinates *C′* by the LS, Huber's and Welsch's methods

Tab. 2. Określenie przekształconych współrzędnych C'' za pomocą metody Hubera i Welscha

Least Square Method									
Point	***X [m]***	***Y [m]***	***v [mm]***	***P***	***v [mm]***	***P***	***d_t [mm]***	***ω [gon]***	***T***
RP1	-0.0049	-0.0141	-4.88	1.00	-14.09	1.00	14.91	278.75	0.22
RP2	215.9253	0.0045	3.32	1.00	4.49	1.00	5.58	59.47	2.23
RP3	16.0459	109.1899	20.91	1.00	9.88	1.00	23.13	28.09	0.10
MP1	42.8215	42.6832	-4.53	1.00	-8.78	1.00	9.88	269.65	0.19
MP2	87.1714	42.6878	-4.64	1.00	-4.17	1.00	6.24	246.58	**6.65***
MP3	129.6742	42.0783	-3.78	1.00	3.26	1.00	5.00	154.69	0.90
MP4	178.9574	40.8432	1.42	1.00	1.22	1.00	1.87	45.23	0.15
MP5	45.2976	79.6930	7.56	1.00	13.00	1.00	15.04	66.47	0.60
MP6	91.4799	74.7515	-7.07	1.00	-3.45	1.00	7.87	228.92	0.15
MP7	132.7527	74.1339	-4.26	1.00	-4.12	1.00	5.93	248.94	0.12
MP8	181.4150	72.2908	-4.04	1.00	2.76	1.00	4.89	161.79	0.10
Robust Huber's M-estimations									
RP1	-0.0010	-0.0009	-0.98	1.00	-0.93	0.84	1.35	248.17	0.26
RP2	215.9217	0.0012	-0.32	1.00	1.16	1.00	1.20	117.33	0.21
RP3	16.0576	109.1980	32.59	0.01	17.99	0.02	37.23	32.11	0.02
MP1	42.8271	42.6916	1.13	1.00	-0.38	1.00	1.19	379.11	0.32
MP2	87.1755	42.6928	-0.53	1.00	0.84	1.00	0.99	135.81	2.33
MP3	129.6768	42.0800	-1.20	1.00	5.05	0.28	5.19	114.84	2.40
MP4	178.9582	40.8413	2.19	1.00	-0.71	1.00	2.30	379.94	0.24
MP5	45.3060	79.6999	15.96	0.03	19.91	0.02	25.52	56.98	0.03
MP6	91.4863	74.7551	-0.66	1.00	0.11	1.00	0.67	189.72	0.05
MP7	132.7577	74.1343	0.66	1.00	-3.69	0.52	3.75	311.32	0.12
MP8	181.4180	72.2875	-0.95	1.00	-0.46	1.00	1.06	228.54	0.23
Robust Welsch's M-estimations									
RP1	0.0000	-0.0005	0.00	0.37	-0.46	0.00	0.46	300.00	-0.00
RP2	215.9220	0.0019	0.00	0.84	1.88	0.00	1.88	100.00	-0.03
RP3	16.0584	109.1981	33.39	0.00	18.14	0.00	38.00	31.68	-0.00
MP1	42.8279	42.6920	1.93	0.00	0.00	0.10	1.93	0.00	-0.16
MP2	87.1761	42.6933	0.14	0.00	1.28	0.00	1.28	93.15	-0.06
MP3	129.6773	42.0805	-0.66	0.00	5.54	0.00	5.58	107.57	-0.03
MP4	178.9586	40.8418	2.57	0.00	-0.16	0.00	2.58	396.03	-0.25
MP5	45.3067	79.7002	16.71	0.00	20.19	0.00	26.21	55.97	-0.00
MP6	91.4870	74.7555	-0.04	0.00	0.45	0.00	0.45	105.87	-0.37
MP7	132.7582	74.1347	1.15	0.00	-3.29	0.00	3.49	321.44	-0.12
MP8	181.4184	72.2880	-0.61	0.00	-0.00	0.60	0.61	200.00	-0.48

in the deformation network consisting of 11 points. Coordinates of monitoring points (MP1, MP2,… MP8) are determined from the reference points RP1, RP2 and RP3 (Tab. 1, Fig. 2) by total station at every epoch in the local coordinate system.

Assess the statistical significance of the transformation displacement was carried out on the basis of the L-test. The test statistic is variable in form (Sütti and Gašinec, 2001)

$$T_i = \frac{n-k-d}{d}\frac{R_i}{v^T v - R_i} \approx F(f_1, f_2) \quad (16)$$

with the critical value

$$T_{kr} = F(1-\alpha, f_1, f_2) \quad (17)$$

which for the chosen risk level α = 0.05 takes the value $F(0.95,2,16) = 3.63$. As can be seen in chart Tab. 2 the test statistic T_i exceeds the critical value T_{kr} of L-test only on the point MP2 in applying the least squares method. Huber's and Welsch's methods clearly show that it is a mistaken conclusion and the points at which transformation horizontal shifts

$$dh_t = \sqrt{dX_t^2 + dY_t^2} \quad (18)$$

reach outliers are points RP3 and MP5. The symbols in the relations (16) and (17) represent:
n – the number of coordinates of identical points,
k – number of transformation parameters (k = 4 for model (8)),
d – dimension of tested variable (d=2),

$$R_i = v_i^T Q_{vi}^{-1} v_i$$

$$Q_v = I - H(H^T P H)^{-1} H$$

– cofactor matrix of residuals.

Conclusion

From the obtained results presented in the table Tab. 2 and figure Fig. 2 follows, that robust methods present in deformation analysis (Šíma, Ižvoltová and Seidlová, 2011; Labant and Weiss, 2012) based on application of transformation models relatively simple but very effective tool for suppression the negative impact of those points who are suspected, that their position was significantly changed between epochs *t* a *t'*. In the example of real observed deformation network was from a set of robust M-estimation methods chosen Huber's and Welsch's method. Iterative procedure (14), (15) was stopped after 98 (Huber) and 151 (Welsch) cycles after fulfilling the conditions, that the maximum difference of absolute values of belonging weights in consecutive cycles does not exceed a pre-determined value 1e-5. At the same time it is appropriate to prevent the degeneration of the iteration cycle by setting a maximum number of cycles. It should also be noted that probability1-α = 0.95 of test statistics (16) does not exceed the critical value (17) at any point in case of robust methods, that results from suppression of unstable points by reduce of their weights.

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Analiza deformacji powierzchni za pomocą estymacji dynamicznych Robust M

Często w praktyce badawczej obserwuje się okresowo zmiany poziome powierzchni lub różnych obiektów aby ocenić ich położenie w lokalnym systemie współrzędnych. Powody, dla których niektóre lokalne systemy współrzędnych są wciąż w użyciu, nawet w obecnym satelitarnym systemie nawigacji (GNSS) są różne. Wiadomo, że zanim wdrożono w praktyce technologię GNSS analizy deformacji były przeprowadzane w narodowym kartograficznym systemie współrzędnych. Jeżeli gęstość i prawidłowość bazy geodezyjnej nie gwarantuje odpowiedniej jakości, analiza deformacji przeprowadzana jest w strukturach sieci geodezyjnej zlokalizowanej w lokalnym systemie współrzędnych. Przewagą tego podejścia jest to, że przesunięcia poziome punktu były oceniane w systemie współrzędnych stworzonym w konkretnym celu przy minimalizowaniu korekt matematycznych obserwacji takich jak korekta z wysokości powyżej poziomu morza lub ze zniekształceń projekcji mapy. Innym powodem ograniczonego zastosowania GNSS może być to, że recepcja sygnału może być znacząco ograniczona wąskimi dolinami górskimi, zlokalizowanymi niedaleko skalnych ścian lub na terenach o dużej wegetacji. Najbardziej poważnym problemem okresowej oceny przesunięć pozycji punktów w lokalnej sieci geodezyjnej są ryzyka związane ze zmianą danych sieci geodezyjnej w lokalnym systemie współrzędnych jako wynik niestabilności jednego lub wielu punktów referencyjnych. Artykuł prezentuje jak można, choćby częściowo, wyeliminować omawiany problem za pomocą estymacji dynamicznych robust M.

Słowa klucze: transformacja, analiza deformacji, ukształtowanie terenu, estymatory robust M