Modeling of Vibrating Angular Motion Sensor in Matlab/SIMULINK

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Abstract—**This paper presents different models and results of simulations of MEMS vibrating gyroscope in Matlab/SIMULINK environment. Each model is created with different approach, using different blocks and different physics. Therefore author presents his proposal of mathematical, electrical equivalent and physical models. Results obtained from these models can give some hints regarding design and dimensions of some crucial parts of MEMS gyroscope. Here, decoupled vibrating sensor is considered, which minimize drive and sense motion mutual influence.**

Index Terms—**microelectromechanical systems, simulation model, microaccelerometer, MATLAB, SIMULINK, MEMS modeling, Simscape toolbox.**

I. Introduction

SIMULINK is part of Matlab software which is used for modeling, simulation and analysis linear or non-linear system dynamics. Its important advantage is graphical area for schema design and representing functional parts of system with blocks. It is possible to model many aspects of specified phenomena. Thanks to the so-called toolboxes, it is very eagerly used in many academic and research centers. The huge success of the software comes from the scope of application for both in micro and macro scale. Here, I will pay attention on modeling and simulation of MEMS angular velocity sensors - gyroscopes.

Although few end users have any idea about these small devices that are used in popular smartphones or in cars, ensuring high comfort of driving, their presence is more and more marked nowadays both in commonly available cheap devices as well as in advanced technologies used in transport, medicine or in the army. This is due to low weight and low energy consumption. Application of MEMS devices requires on early creation process, research of parameters behaviors and evaluates them under specified case. The aim of this research work is to present modeling MEMS motion sensors approach in Matlab/SIMULINK environment and discuss response characteristics obtained from these methods.

II. Theory Background

We consider mostly two kinds of motion – linear and rotational. They give us six degrees of freedom. Position can be described with static parameters like position and orientation; dynamical physics quantities measuring these simple parameters in time are velocity and acceleration.

Small dimensions is crucial factor leverages on complexity and fabrication methods. It is commonly known, there are very restricted amount of physical phenomena that can be used in such small devices. Electrostatic physics is the most often used in sensing operation [2] besides of piezoresistive [3] and the tunneling current method [1].

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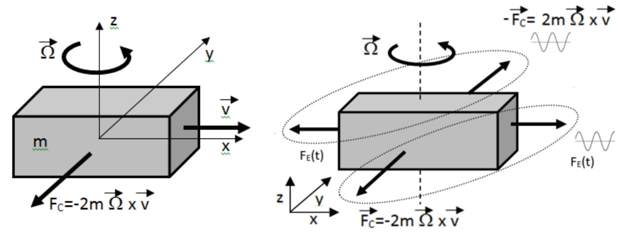

Fig. 1. Gyroscope operation principal.

Principle of capacitive gyroscope operation is not complex however it combines both actuator and sensor. It uses Coriolis effect which appears in rotating objects (fig.1) and behaves as a damped mass on a spring [4-5].

Coriolis phenomena consists in that when object rotates around z axis and additionally moves along axis which is orthogonal to rotating one (in this example x) additional force appears called Coriolis force. This phenomena appears in rotating reference frames in MEMS gyroscopes. In case of clockwise rotation, force acts to the left of the object motion. In case of anticlockwise rotation, the force acts to the right direction. This kind of force is called Coriolis force and equals [1, 3]:

$$F_C = -2m(\vec{\Omega} \times \vec{v}) \tag{1}$$

where Ω is angular velocity, v – linear velocity, m – mass of object.

The "minus" sign is related to opposite force vector to motion direction. The huge problem it to get linear velocity. In reality object have only one linear or rotational motion, so consequently, this velocity needs to be enforced. However, to reduce linear motion on position in Cartesian coordinate system, linear motion is enforced by vibration with specified frequency.

Actuator (in drive direction) is implemented by enforcing with comb structures [7] located on opposite sides of device. When rotational motion is non-zero, inertial mass vibrates also along y (sense) axis, where on opposite sides another pair of comb drives are located. This motion causes capacitance change because one (static) comb structure is anchored to substrate, and other one is movable (It is a part of inertial mass). Changes of capacitance can be measured and transformed to other physical quantity.

The mathematical model which describes dynamics of any system is presented with following second order differential equation:

$$ma_x = m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx - F_{el} \tag{2}$$

where k - spring constant, b - damping coefficient, m – proof mass, x - displacement.

Above equation (2) is greatly applied to MEMS gyroscope. Model of such device and requires application two equations (2): one which describes motion in drive direction and one in sense direction:

$$m\frac{d^2x}{dt^2} + b_x\frac{dx}{dt} + k_x x = F_D\, sin(\,\omega t) \qquad (3)$$

$$m\frac{d^2y}{dt^2} + b_y\frac{dy}{dt} + k_y y = -2m\frac{dx}{dt}\Omega \qquad (4)$$

where: m – vibrating mass, b_x, b_y – damping coefficient, k_x, k_y – spring coefficient, Ω – measured angular velocity, F_D – force generated by comb drive actuator.

Matlab/SIMULINK with its toolboxes enable to implement model of MEMS gyroscope with use dedicated blocks.

III. Models in Matlab/SIMULINK

Mathematical model based on equations (3) and (4) is presented in fig. 2. This model consists of two main parts related to drive direction and sense direction [7]. Both drive and sense directions are represented with double integration blocks multiplied by damping coefficient and spring constant quantities. Actuator is presented as sinus-based force source. Model was tested with module including capacitances working in opposite phases to get as much as possible amplitude of motion. Sinusoidal generator has possibility to input frequency what allows to create characteristics in frequency domain. In fig. 3 physical structure of MEMS vibrating gyroscope considered in this paper is presented.

Model includes subsystems responsible for particular specific parts of vibratory gyroscope. Thermal calculation factor is calculated and then applied for all dimensions as well as Young's modulus.

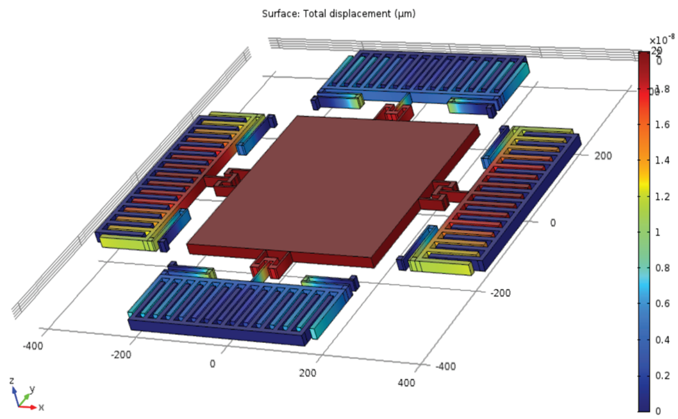

Fig. 3. Physical structure considered in this paper.

Both drive and sensing direction subsystems of model include double integrations. There are related to Coriolis force with linear velocity quantity. Presented model is very flexible, because particular subsystems (drive and sense) have spring constants and damping coefficients separately loaded for both directions.

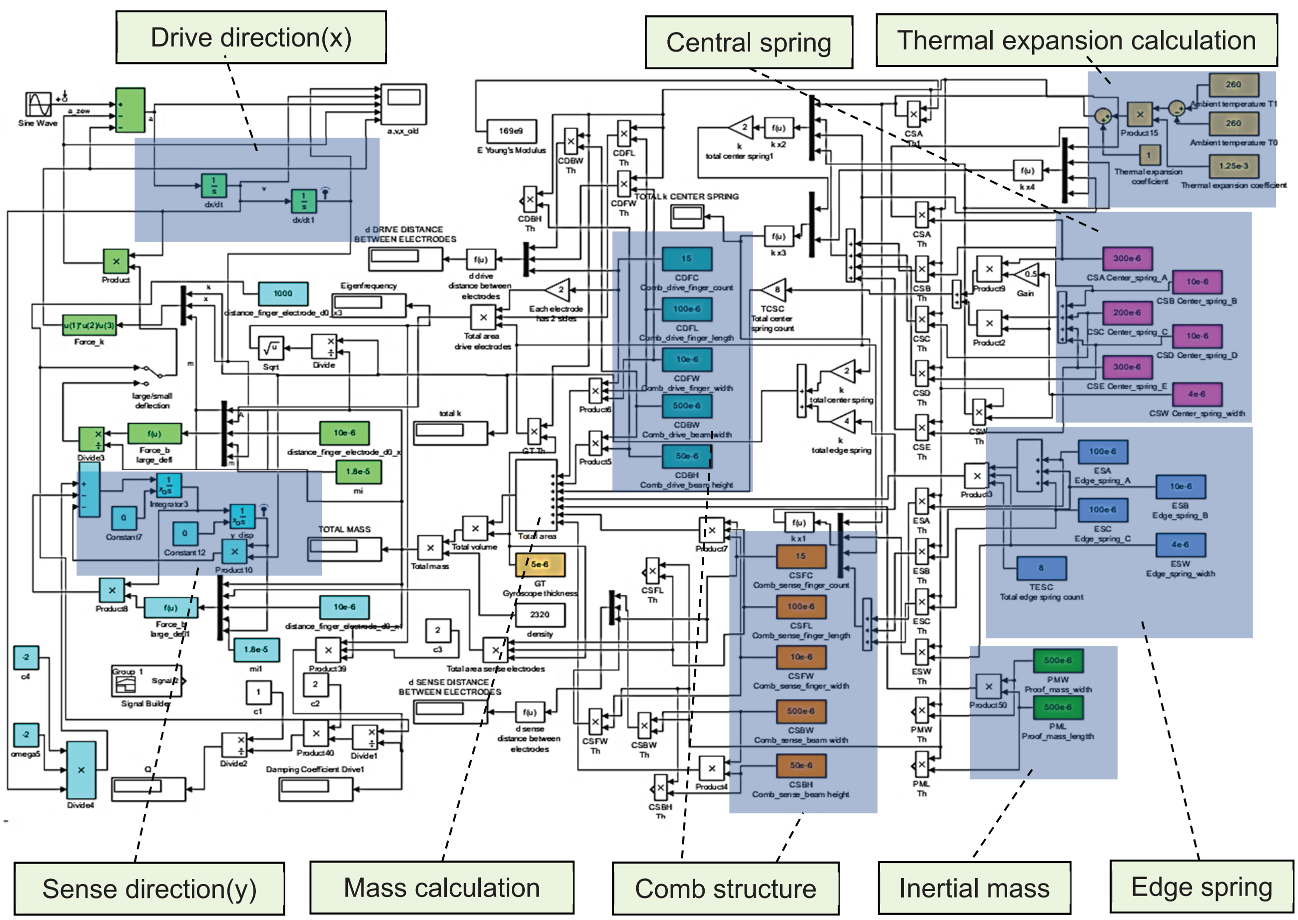

Fig. 2. Mathematical model of Gyroscope implemented in SIMULINK

A. Electrical equivalent model of MEMS Device

Electrical model created in Matlab/SIMULINK was created based on electrical analogy of mechanical and electrical quantities. Table I. shows these analogies. Particular forces in mechanical domain can be transformed to electrical domain, therefore mass can be replaced with capacitance, spring constant with inverse of inductance and damping coefficient with inverse of resistance.

TABLE I.
LIST OF ELECTRICAL ANALOGIES FOR MODEL.

Mechanical Quantity	**Electrical Quantity**
v - velocity	V - voltage
F – force	I - current
m - mass	C - capacitance

Referring to (2) one can conclude that velocity can be replaced with voltage, to obtain displacement, integration of voltage must be carry out. Transformed equation is presented below. Next equation is just transformed previous one – flux ϕ in electrical domain is equivalence of displacement x:

$$I = C\frac{dV}{dt} + \frac{1}{R}V + \frac{1}{L}\int V dt$$

$$I = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi$$

Model presented in fig. 4 is transformation of the mathematical model with application above equations.

Again, electrical equivalent model consists of two parts – drive and sense; force is implemented with electrical sources, spring constant with reverse inductance and damping coefficient with reverse of resistance. Mass equivalent is capacitance. There is no need to rescale any input data - they are dimensionless.

B. Physical model of MEMS Device

Third model presented in Matlab/SIMULINK uses toolbox to represent physical models of devices – Simscape™. Thanks to it, it allows to connect particular parts of system as they exist and operate in real world. These particular parts are represented with blocks only and do not reflect geometrical shapes, however these shapes are reflected with mass module implementation. Here it was built physical models within SIMULINK environments based on physical connections which integrate with block diagram. Simscape™ offers blocks either for linear or rotational motion operation. This is why it is possible to model simply these motion MEMS sensors.

Here again, vibrating MEMS gyroscope consists of two linear devices – actuator (along drive direction) and sensor (along sense direction). In fig. 5 there is presented physical gyroscope model, created with linear motion blocks.

Model consists of two parts: first is input data/output results subsystem and second is the physical model subsystem. There is not possible to manipulate data directly – it is necessary to use specific Simulink-PS Converter blocks. Here, in physical model subsystem there are physical mass, spring, damper, force source and motion sensor blocks included, separate for both directions.

C. Mass calculation

Fig. 6 presents subschema for mass calculation (in electrical equivalent this reflects to capacitance) used for all presented models. This subschema calculates total mass based on sum of masses particular parts: inertial mass, edge and center springs.

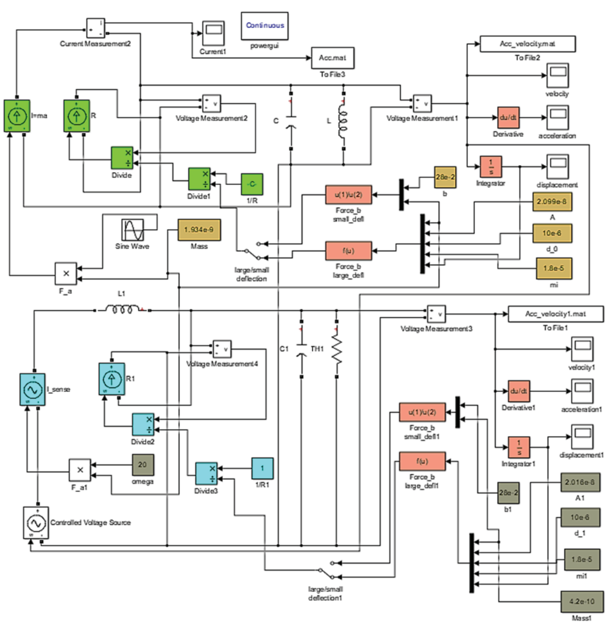

Fig. 4. General schema of the presented model.

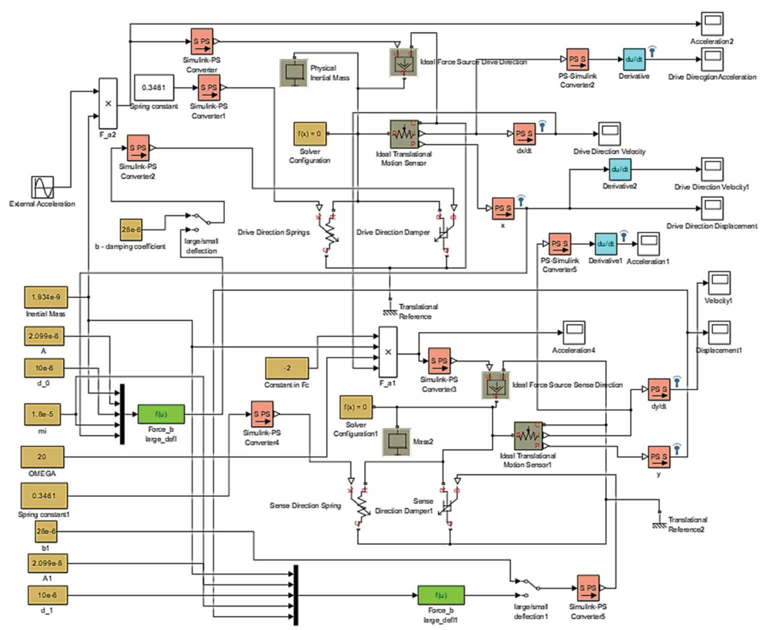

Fig. 5. Gyroscope model implemented with linear blocks coming from Simscape™ toolbox.

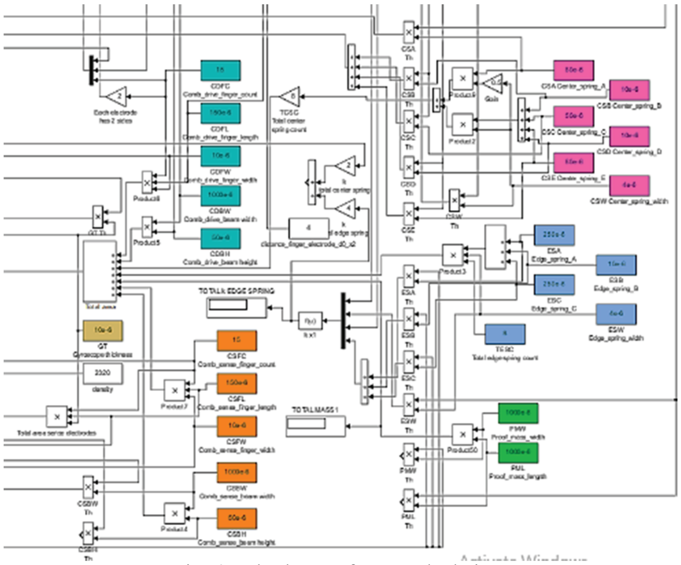
Fig. 6. Subschema of mass calculation.

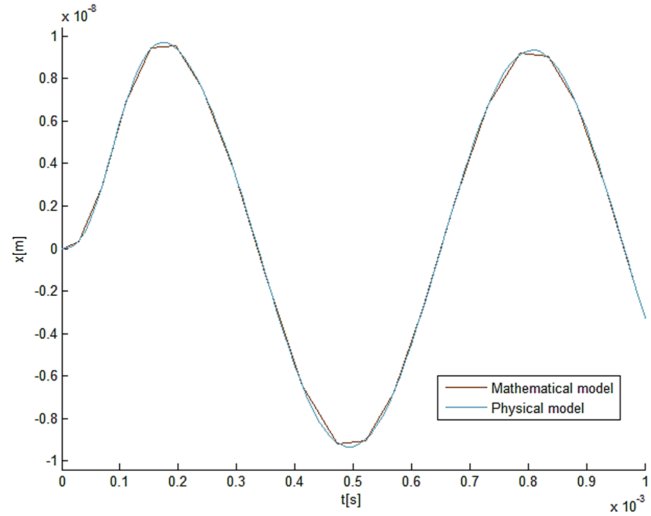

Fig. 7. Displacement results for mathematical and physical MEMS Gyroscope models along drive direction for frequency equals to 10000Hz.

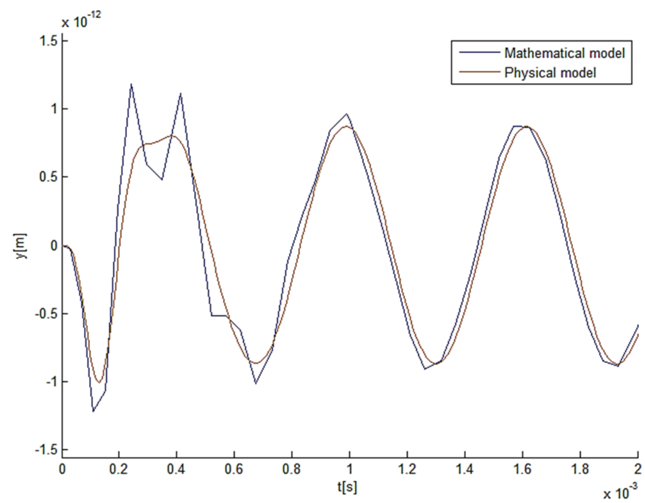

Fig. 8. Displacement results for mathematical and physical MEMS Gyroscope models along sense direction for frequency equals to 10000Hz.

IV. Results of simulation

Test simulations were performed to see what are differences between results from all models. Input parameters are presented in Table II.

Results for physical and mathematical model are very similar with small differences in x direction (fig. 7). These differences one can observe also in case of y (sense) direction (fig. 8). Consequently, all models are properly created and each of them can be used for simulation presented model in fig. 3.

TABLE II.
List of input parameters.

Symbol	Quantity	Value
μ	Air viscosity	$1.8 \cdot 10^{-5}$Ns/m^2
V	Voltage load	2.5V
ε_o	Permittivity coefficient	$8.854 \cdot 10^{12}$F/m,
ρ	Density (Polysilicon)	2328 kg/m^3
a	Acceleration	1g
W_{el}	Static electrode width	$4 \cdot 10^{-6}$m
L_{el}	Static electrode length	$160 \cdot 10^{-6}$m
d_0	Initial distance between electrodes	$4 \cdot 10^{-6}$m
L_{spr}	Spring length	$500 \cdot 10^{-6}$m
W_m	proof mass width	$150\text{-}500 \cdot 10^{-6}$m
L_m	Proof mass length	$150\text{-}500 \cdot 10^{-6}$m
t	Device thickness	$4 \cdot 10^{-6}$m
W_f	Movable electrode width	$4 \cdot 10^{-6}$m
L_f	Movable electrode length	$160 \cdot 10^{-6}$m

First tests were performed for frequency 10000Hz. On initial period of oscillations one can see that amplitudes (y direction) are meaningfully different, what comes directly from geometry shape consequently influence on sensitivity estimation and range of operation (it is required to fit comb structure distances to avoid short circuits).

Next step was to perform simulations which were concerned with influence drive on sense motion in frequency domain (fig. 9). This is crucial, because it allows to adjust optimal oscillations of external force source. Before these tests were performed, it was necessary to compute spring coefficient, mass and then resonant frequency. Results were for W_m=$450 \cdot 10^{-6}$m, L_m=$450 \cdot 10^{-6}$m) as following: k=0.3461N/m, m=$1.189 \cdot 10^{-9}$m, ω_0=17060 1/s.

Next simulations were concerned with influence drive on sense motion in frequency domain (fig. 12) with electrical and physical models. For vibrating gyroscope, such analysis is critical, because it allows to adjust optimal oscillations of external force source.

These tests were performed for angular velocity Ω=5rad/s. Results obtained from simulations in frequency domain shows that in case of resonance frequency (calculated ω_0 is applied as external frequency of force source) there is dramatically growth of amplitude displacement in sense direction. Simulation results shown in fig. 9 are presented for different inertial mass dimensions. It can be observed, that geometrical dimensions (especially inertial mass, which is dominant part of total mass) have meaningful impact of characteristics in frequency domain.

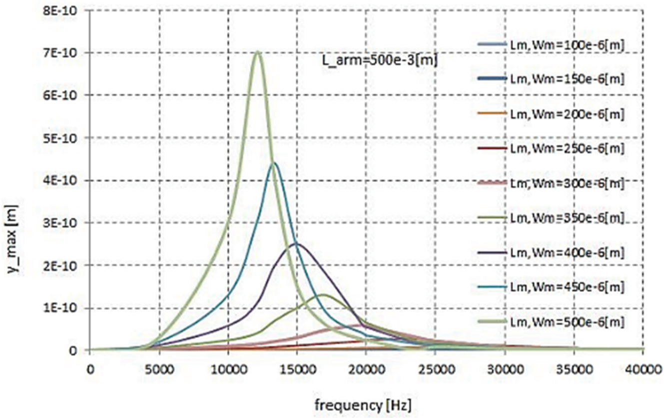

Fig. 9. Displacement in *y* (sense) direction in frequency domain in dependency of inertia mass dimensions.

Simulation performed confirms, that large size of inertia mass (Lm=450·10-6m) gives much better results of y displacement than in small size case (Lm=100·10-6m). Along with mass growth, resonant frequency obviously decreases and amplitude enormously grows because of resonance phenomena. For high resonant frequencies (large mass), plot in frequency domain become more smooth. To obtain the best result (amplitude), frequency applied in drive direction must be precisely fit (because plot is very steep).

In fig 10. We also observe that amplitude in sense direction is exponentially dependent on frequency applied (drops).

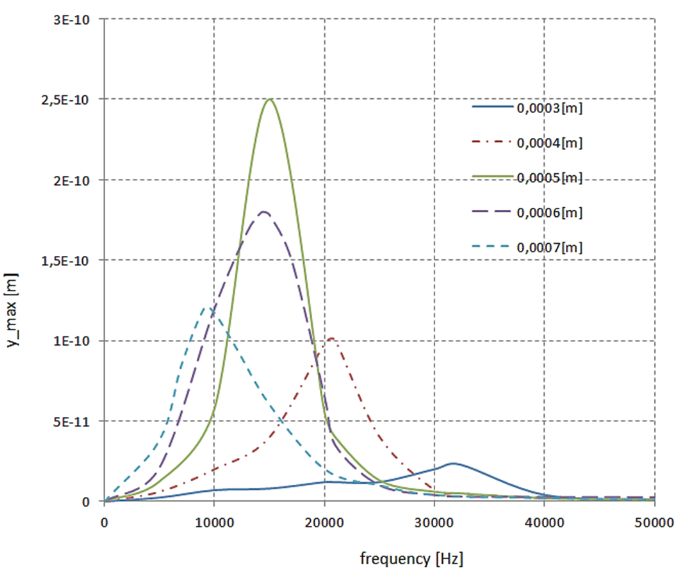

Fig. 10. Displacement in *y* (sense) direction in frequency domain in dependency of spring length.

For different spring length (fig. 10) we can observe there is strong dependency between sense displacement and frequency. However, results show that there is no continuous growth sense amplitude along with frequency drop. For different edge spring length maximum is for different resonant frequency. For model considered here optimal spring length L=0.

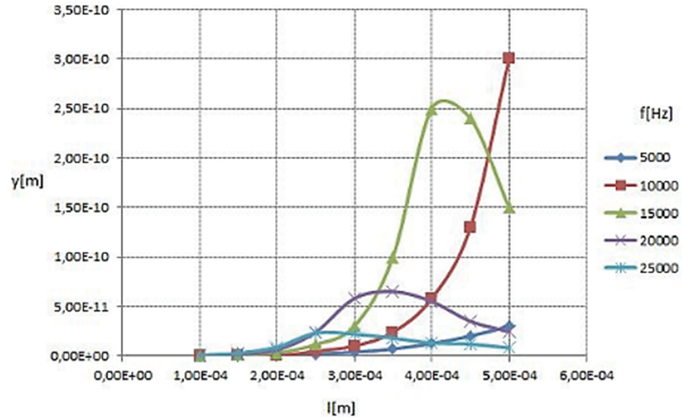

Fig. 11. Displacement in *y* (sense) direction in frequency domain in dependency of inertia mass dimensions.

In fig. 11 there are shown maximum displacement in sense direction in dependency of spring length for specified 5 different frequencies. Maximum displacement increases along with spring length; optimal frequency of actuator increases, too. For given spring length there is maximum sense displacement threshold which cannot be exceeded. In fig. 12 plots for drive and sense maximum displacement are presented (note that *y* axis is logarithmic). We see that for both drive and sense direction amplitude dependency on spring length are non-linear. Difference between amplitudes for both directions lessens along with spring length, but for higher spring length this difference is smaller than for short springs. Concluding, to get maximum sense displacement it is necessary to extend spring as much as possible.

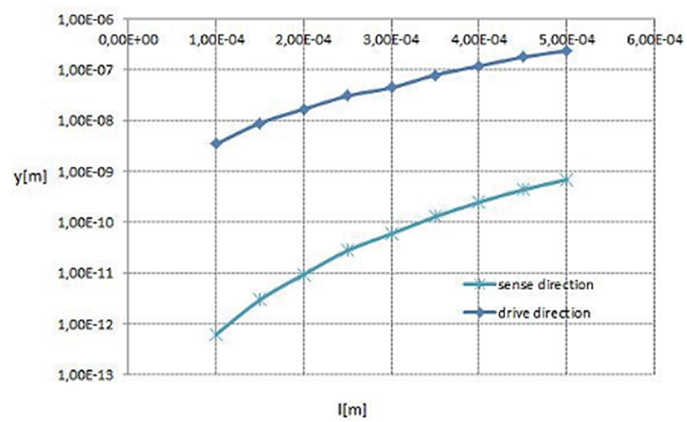

Fig. 12. Comparison of displacement in drive and sense directions in dependency of spring length.

Next tests are considered response in *y* direction on dynamical changes angular velocity for large inertial mass (500-2000·10^{-6}m), because of good results (see fig. 9). List of geometrical parameters are presented in Table III. Created models of MEMS gyroscope allowed us to use signal generator instead of constant value of angular velocity. This generator was very useful for model testing in case of fast changes of angular velocity of the object.

TABLE III.
LIST OF GEOMETRICAL DIMENSIONS.

Quantity	Value
Distance between electrodes	10^{-5}m
Spring length	$250\cdot10^{-6}$m
proof mass width	$500\text{-}2000\cdot10^{-6}$m
Proof mass length	$500\text{-}2000\cdot10^{-6}$m
Device thickness	10^{-5}m
Central spring length	$140\cdot10^{-6}$m
Edge spring length	$250\cdot10^{-6}$m

In fig. 13 there are data presented for two cases. In both cases shape of input omega signal is similar, however values of omega changes meaningfully. In first case frequency applied was 7519Hz. That was eigenfrequency for gyroscope with inertial mass $2000\text{x}2000*10^{-6}$m and edge spring length $500*10^{-6}$m. For this case frequency has too small value, because angular velocity changes too fast and vibrations have not enough time to stabilize for correct measurement. Many inaccuracies appear in such configuration. When we shorten edge springs, frequency will meaningfully grow, therefore in fig. 13b more oscillations we can observe, what has positive influence on accuracy. In case two shape of signal (angular velocity) is similar, however there is meaningfully difference between values. In case two particular angular velocities are 20 times more than in case one. In fig. 14a there are results for sense direction presented. We observe that shapes of plots are the same, however amplitudes differ 100 times. In case 2 displacements in sense direction are 10^{-9} order, while in case 1 10^{-11} order only.

In fig. 15 and 16 there are results for other input signals. There are 3 cases considered. In the first case angular velocity is 5 rad/s, next increases to 10 rad/s (after 6ms) and then it drops to -20 rad/s (after next 6ms).

We see on fig. 6 that sense amplitude changes slightly and for change rotation motion to opposite direction, switching time takes more time (here between 5 and 6 ms) and oscillations change their phase 180 degrees. Additionally, depend on point in time in which angular motion changes, some artefacts or unexpected fluctuations can appear, but they last short in most cases. In fig. 16 they can be observed at 3rd ms. The optimal response is when angular velocity switching point is for 0 amplitude in sense direction.

In case 2 signal is similar but particular signal levels last shorter. In this case again, amplitudes of vibrations switches slightly, and there is enough time to switch and achieve targeted particular amplitudes. Although signal is similar to this discussed in previous case we observe that artefacts do not appear again. We observe also that changes rotation direction causes changes vibration phase.

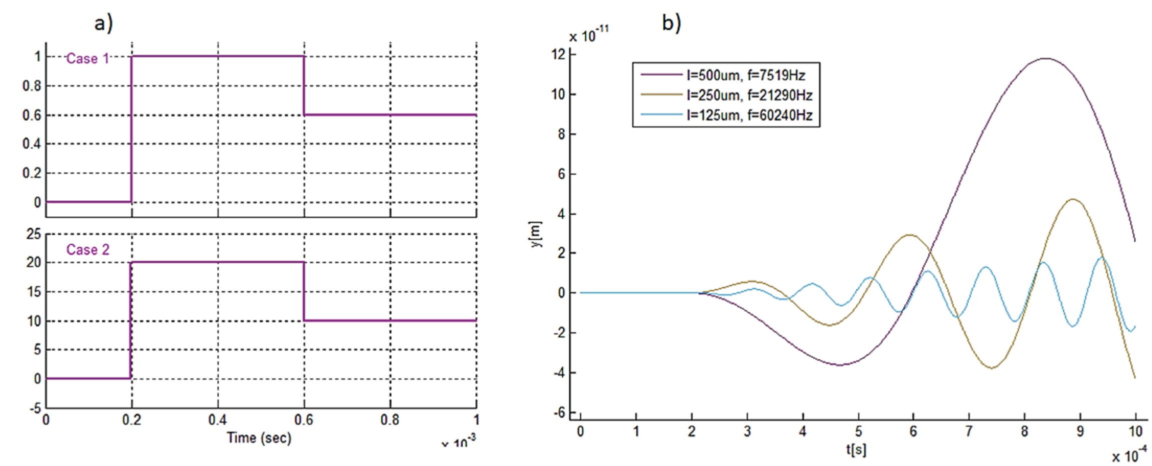

Fig. 13. Signal input (angular velocity, for small and large values) (a) and displacement in y (sense) direction changes in time domain in dependency of spring length (for small angular velocity values) (b).

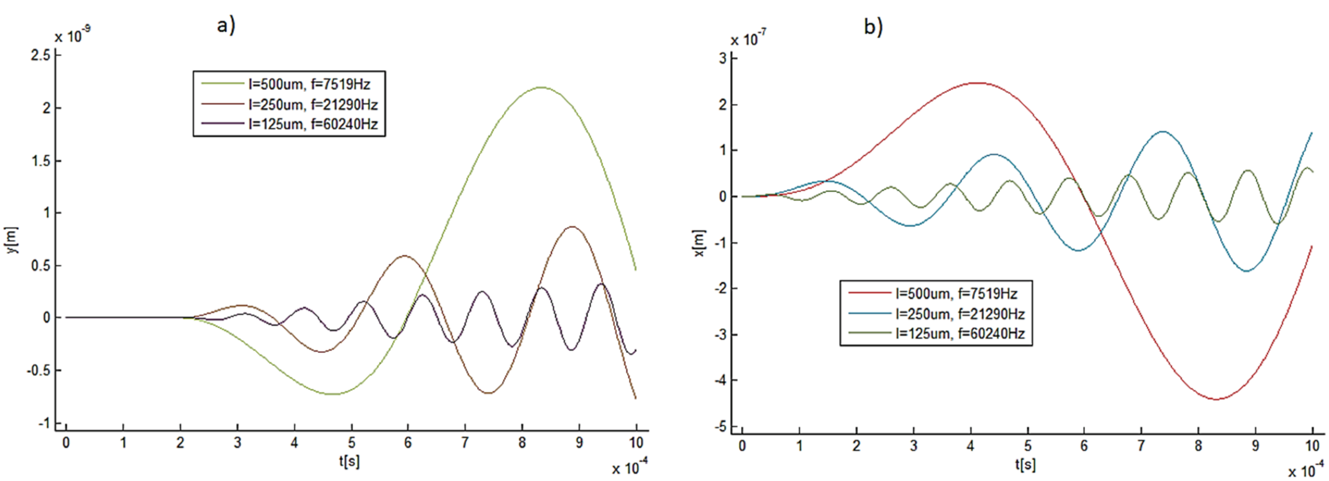

Fig. 14. Displacement in y (a) (sense) and x (b) directions changes in time domain in dependency of spring length (for large angular velocity values).