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# Resource allocation in MIMO systems specific to radio communication

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**Abstract:** The performance of the multi-input multi-output (MIMO) systems can be improved by spatial modulation. By using spatial modulation, the transmitter can select the best transmit antenna based on the channel variations using channel state information (CSI). Also, the modulation helps the transmitter to select the best modulation level such that the system has the best performance in all situations. Hence, in this paper, two issues are considered including spatial modulation and information modulation selection. For the spatial modulation, an optimal solution for obtaining the probability of selecting antenna is calculated and then Huffman coding is used such that the transmitter can select the best transmit antenna to maximize the channel capacity. For the information modulation, a multi quadrature amplitude modulation (MQAM) strategy is used. In this modulation, the modulation size is changed based on the channel state variations; therefore, the best modulation index is used for transmitting data in all channel situations. In simulation results, the optimal method is compared with Huffman mapping. In addition, the effect of modulation on channel capacity and a bit error rate (BER) is shown.

**Key words:** spatial modulation, multi quadrature amplitude modulation, modulation, Huffman coding, MIMO systems

## 1. Introduction

Due to the increasing number of wireless system devices, a spectrum band is being more and more scarce. Nowadays, several methods such as cognitive radio [1], spread spectrum [2] and etc., have been used by designers and researchers for improving the performance of communication systems. One of the powerful and attractive methods is MIMO. In the MIMO systems, both the transmitter and receiver have more than one antenna for sending and receiving data [3]. The MIMO systems increase channel diversity and therefore improve the system performance. Despite the huge advantages of the MIMO systems, these systems have the high cost of a radio

frequency chain [4]. Hence, to overcome this problem several methods were introduced by researchers. Spatial modulation is a powerful method that can reduce this cost. In the spatial modulation, the information bits are split into two parts including a signal symbol and spatial symbol, i.e. an antenna index [5]. The main advantage of using the spatial symbol is that the best transmit antenna can be selected based on the channel variations and therefore the system has the best performance in all situations. In fact, in conventional modern MIMO systems, all transmit antennas are selected to send data with equal probability. In the practical terms, transmit antennas have different channel situations i.e. channel fading gain for some antennas may be better than others. Therefore, selecting transmit antennas with equal probability is not an optimal solution [6]. Based on information theory, it can be concluded; more data should be transmitted over channels with higher channel fading gain and vice versa. The transmitter can be aware from the channel using channel state information (CSI). Hence, the CSI is a critical part of the spatial modulation [7].

Because of the importance of the spatial modulation, this topic has been studied in some articles such as [8, 9]. The optimal solution for the spatial modulation is investigated in [10]. An optimal spatial modulation problem is a convex optimization. So, in this research, the optimal solution is obtained by employing the Lagrange method. Although the optimal solution gives us the best results, due to its complexity, this method is not a suitable method for practical applications. For achieving a low-complexity solution, several algorithms such as an optimal aggregation algorithm [11], fuzzy logic [12, 13] and a harmony search algorithm [14] can be used instead of the Lagrange method. In some paper, suboptimal methods have been used for the spatial modulation. The main difference between optimal and suboptimal methods is that in the optimal method, the probability of selection of a transmit antenna can be any values between zero and one, while in the suboptimal methods, the probability of selection of antennas is restricted to defined values [15]. Huffman coding is one of the most attractive kinds of these suboptimal methods. Elias in [16] introduced Huffman coding for spatial modulation. Indeed, Huffman coding is a kind of optimal prefix code which is used for lossless data compression [17]. In this coding map, the probability of selecting a transmit antenna is proportional to the channel fading gain; i.e. the antenna with higher channel fading gain has more chance than other antennas to be selected by an antenna switch. This paper investigates both the optimal solution and Huffman mapping for calculating the selection probability for all antennas to improve the performance of the spatial modulation.

To improve the information modulation performance, MQAM modulation is used for modulating the signal. QAM is a type of the modulation that is widely used in the modern communication systems. In an adaptive MQAM modulation, the modulation level of the modulation is changed adaptively based on the channel state information. The idea behind the adaptive modulation is that the transmitter has the best performance regardless of channel variations [18]. In fact, in the adaptive MQAM modulation, transmitters estimate channel variations using CSI and then calculate transmit power based on the channel gain. A modulation level is calculated based on the allocated power.

The rest of this paper is organized as follows; in section 2, a system model is introduced and the transmission rate for the MQAM modulation strategy is calculated. The optimal solution is described in section 3. In section 4, Huffman coding is introduced. Simulation results are presented in section 5.

## 2. System Model

The MIMO system which is used in this research has  $N_t$  transmit antennas and  $N_r$  receive antennas. The transmitter has an antenna switch to select the transmit antenna. In addition, a single radio frequency chain is connected to transmit antennas using this switch. In the receiver side, each receive antenna has its corresponding radio frequency chain as shown in Figure 1.

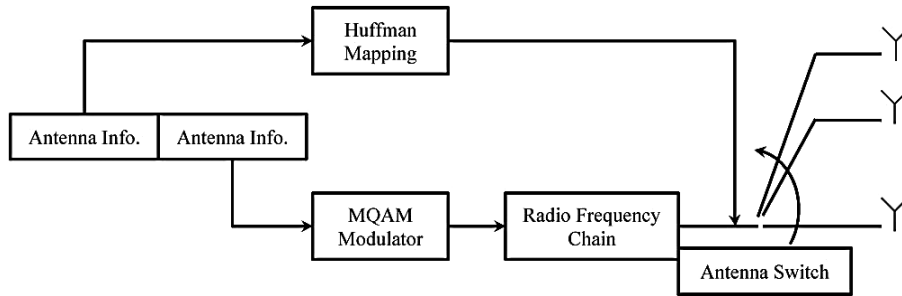


Fig. 1. The MIMO system

At the receiver side, the received signal can be written mathematically as follows:

$$\mathbf{y} = \sqrt{\rho} \mathbf{H} \mathbf{x} + \mathbf{n}, \quad (1)$$

where  $\rho$  is the signal to noise ratio (SNR),  $\mathbf{H} = [h_1, h_2, \dots, h_{N_t}]$  is the flat-fading MIMO channel,  $\mathbf{n} \sim N(0, IN_r)$  is the additive white Gaussian noise (AWGN) and  $\mathbf{x}$  is the transmitted signal. The probability of selecting the  $i^{\text{th}}$  transmit antenna is obtained by Equation (2):

$$\text{prob}(r = e_i) = p_i, \quad i = 1, 2, \dots, N_t, \quad (2)$$

where  $e_i$  is a  $N_t \times 1$  vector with the  $i^{\text{th}}$  element being 1 and all other elements 0. In addition, the vector  $\mathbf{p} = [p_1, p_2, \dots, p_{N_t}]$  indicates the vector of probabilities. Also, it is obvious that  $p_i$  should satisfy Equation (3):

$$\sum_{i=1}^{N_t} p_i = 1. \quad (3)$$

In the information theory, the channel capacity is defined as follows [19]:

$$\mathbf{C} = \mathbf{I}(\mathbf{x}; \mathbf{y}) = \mathbf{H}(\mathbf{y}) - \mathbf{H}(\mathbf{y}|\mathbf{x}) = \mathbf{I}(s; \mathbf{y}|r) + \mathbf{I}(r; \mathbf{y}), \quad (4)$$

where  $\mathbf{I}(s; \mathbf{y}|r)$  is the signal information and  $\mathbf{I}(r; \mathbf{y})$  is the antenna information. Equation (4) indicates, for maximizing channel capacity, both spatial modulation and information modulation should be maximized. Hence, in this section, the spatial modulation and information modulation are expressed mathematically. For the spatial modulation, the mathematical formula is introduced in [10]. This formula expresses the spatial information which is related to the selection probability of the transmit antenna, i.e.:

$$\mathbf{I}(r; \mathbf{y}) = \sum_{i=1}^{N_t} p_i \log_2 \frac{1}{p_i}. \quad (5)$$

For the information modulation, the adaptive transmission rate is calculated based on the MQAM modulation scheme.

For the MQAM modulation, the BER can be written as follows [1]:

$$\text{BER}_i = 0.2 \exp\left(-1.5 \frac{\rho \|\mathbf{h}_i\|^2}{M_i - 1}\right), \quad (6)$$

where  $M$  is a modulation index or constellation size. Based on Equation (6), the modulation index is calculated as follows:

$$M_i = 1 + \Gamma \rho \|\mathbf{h}_i\|, \quad (7)$$

Equation (7) indicates that the modulation index is changed based on the channel variations; in other words, the transmitter can select a modulation size adaptively to achieve the best performance in all situations. For noisy channels, a low modulation size is selected while for channels with high gain, a modulation size can be increased. In fact, a higher modulation size gives a higher transmission rate and by defining a target BER,  $\text{BER}_0$ , the minimum quality of services (QoS) is guaranteed for the system. In the above equation,  $\Gamma$  is an SNR gap and it is a constant value:

$$\Gamma = \frac{-1.5}{\ln(\text{BER}_0)}. \quad (8)$$

The number of bits per symbol that can be transmitted is calculated as follows:

$$b_i = \log_2(M_i) = \log_2(1 + \Gamma \rho \|\mathbf{h}_i\|^2). \quad (9)$$

If the transmitter transmits its data by symbol duration,  $T_s$ , the transmission rate (signal information) can be calculated as follows:

$$I_i(s; y|r) = \frac{1}{T_s} \log_2(1 + \Gamma \rho \|\mathbf{h}_i\|^2). \quad (10)$$

By combining Equation (5) and Equation (10) and considering the probability of selecting transmit antenna, the transmission rate can be rewritten as follows:

$$C = \frac{1}{T_s} \sum_{i=1}^{N_t} p_i \log_2(1 + \Gamma \rho \|\mathbf{h}_i\|^2) + \sum_{i=1}^{N_t} p_i \log_2 \frac{1}{p_i}. \quad (11)$$

## 2.1. Optimal problem formulation

Channel capacity is defined in the previous section. In this section, we maximize the channel capacity in the MIMO system. The optimization is defined based on the selection probability for a transmit antenna; i.e., the aim of this section is to calculate the optimal values for selection probability to maximize the channel capacity. In the optimal solution it is assumed that the probability of selecting a spatial antenna at the transmitter can be any positive value between 0 and 1. Also, the total of the probabilities should be equal to 1. Therefore, the problem formulation and its constraints can be written as the following equation:

$$\max_{p_i} C.$$

Subject to:

$$\sum_{i=1}^{N_t} p_i = 1, \quad p_i \geq 0, \quad i = 1, 2, \dots, N_t. \quad (12)$$

This problem is a convex optimization. By employing the convex optimization method and applying Karush-Kuhn-Tucker (KKT) conditions the optimal value for  $p_i$  can be obtained as follows:

$$p_i = \frac{\|\mathbf{h}_i\|^2}{\sum_{j=1}^{N_t} \|\mathbf{h}_j\|^2}, \quad i = 1, 2, \dots, N_t. \quad (13)$$

### Proof

By using convex optimization method, optimally allocated  $p_i$  at each transmit antenna can be calculated. The KKT conditions can be written as:

$$\mathbf{L} = \mathbf{C} - \alpha \left( \sum_{i=1}^{N_t} p_i - 1 \right) + \gamma_i p_i, \quad (14)$$

$$\frac{\partial \mathbf{L}}{\partial p_i} = 0 \rightarrow \frac{\partial \mathbf{C}}{\partial p_i} - \alpha + \gamma_i = 0, \quad (15)$$

$$\alpha \left( \sum_{i=1}^{N_t} p_i - 1 \right) = 0, \quad (16)$$

$$\gamma_i p_i = 0, \quad (17)$$

$$\gamma_i \geq 0, \quad i = 1, 2, \dots, N, \quad \alpha_i \geq 0, \quad (18)$$

where  $\gamma$  and  $\alpha$  are the Lagrange parameters. By removing  $\gamma_i$  from Equation (15) and considering Equation (17), the optimal solution is obtained as follows:

$$p_i = \frac{1 + \rho \|\mathbf{h}_i\|^2}{\sum_{j=1}^{N_t} (1 + \rho \|\mathbf{h}_j\|^2)}, \quad i = 1, 2, \dots, N_t. \quad (19)$$

When an SNR is very high, the optimal solution can be rewritten as:

$$p_i = \frac{\|\mathbf{h}_i\|^2}{\sum_{j=1}^{N_t} \|\mathbf{h}_j\|^2}, \quad i = 1, 2, \dots, N_t. \quad (20)$$

## 2.2. Huffman coding

In the previous section, the optimal value for  $p_i$  is calculated. However, due to practical complexity, it is difficult to use the optimal solution for calculating the probability of selecting transmit antenna. In fact, practical systems prefer to use some pre-defined values for  $p_i$  to reduce

the complexity of the system. Huffman coding is a technique that gives spatial values for  $p_i$ . Huffman coding maps the antenna information bits to a transmit antenna using a constructed Huffman code. In this coding, the longer code word is assigned to the transmit antenna that has less chance to select the antenna switch and vice versa. In fact, the transmit antenna in which its corresponding channel is worse than other channels have less opportunity to be selected by the transmitter

In [10] and [20], the Huffman coding for selection probability is defined as follows. Indeed, a different Huffman coding can be defined. In other words, any values that satisfy Equation (21) can be used as Huffman coding. To clarify it, we compare the performance of the system in the simulation results section, by two different values of probabilities.

$$\sum_{i=1}^{N_t} p_i = 1, \quad p_i = \{0, 1, \dots, 2^{-\beta}\}, \quad (21)$$

where  $0 \leq \beta \leq N_t - 1$  is the integer number and it is related to a transmission codebook size. By comparing Equation (21) with Equation (12), it is observed that in the optimal solution, the value of  $p_i$  can be any values between 0 and 1, while in Huffman coding; this value should be only specified values. For explaining Huffman code, we provide an example when the transmitter uses 5 antennas.

**Example:** we consider the vector probability  $p = [0.25, 0.25, 0.25, 0.125, 0.125]$  for transmit antennas. This vector indicates that the first, the second and the third antennas are selected by a probability of 0.25, while the fourth and the fifth antennas are selected with a probability of 0.125. Figure 2 shows the corresponding Huffman tree for this vector probability. The transmitter maps incoming bits into different transmit antenna indices. The antenna TX-1 is selected when the first and the second incoming bits are equal to 0. The antenna TX-2 is selected when the first incoming bit is equal to 0 and the second bit is 1. The TX-3 is selected when the first coming bit is equal to 1, and the second bit is equal to 0. When the first and the second bits both are equal to 1, the third bit determines the transmit antenna. If the third bit is equal to 0, the TX-4 is selected and otherwise, the TX-5 is selected by the transmitter. The corresponding Huffman mapping scheme is shown in Table 1.

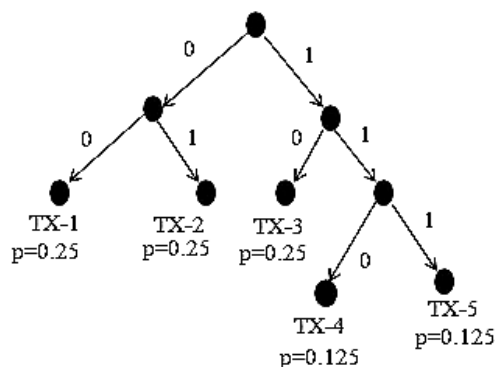


Fig. 2. Huffman tree

Table 1. Huffman mapping scheme

Bit sequence	Selected antenna	Probability
00	TX-1	0.25
01	TX-2	0.25
10	TX-3	0.25
110	TX-4	0.125
111	TX-5	0.125

### 3. Numerical results

In this section, our method is explained by using numerical examples. It  $T_s$  is assumed to be  $4 \mu s$ . The channel has a Rayleigh probability distribution function (PDF) with the unit mean. The target BER is equal to  $10^{-3}$ . The results are achieved from 10 000 independent simulation iteration runs. In Figure 3, it is assumed that the transmitter has 5 transmit antennas. In addition, the receiver has 3 antennas. Two Huffman coding scenarios including  $p1 = [1/2, 1/4, 1/8, 1/16, 1/16]$  and  $p2 = [1/4, 1/4, 1/4, 1/8, 1/8]$  are considered. In Figure 3, the Huffman mapping 1 curve is corresponding to  $p1$  and the Huffman mapping 2 curve is corresponding to  $p2$ . It is observed that the capacity is increased by increasing an SNR. This is an obvious result based on Equation (11). However, the performance of the optimal method is better than Huffman mapping. Furthermore, both Huffman maps have same results. On the other hand, the performance of the system is independent of the probability vector. Figure 4, shows the capacity vs. SNR when  $N_t = 4$ . The receiver has 3 receive antennas. Similarly to Figure 3, two Huffman mappings, including  $p1 = [0.25, 0.25, 0.25, 0.25]$  and  $p2 = [0.25, 0.125, 0.125, 0.25]$ , are calculated. In addition,

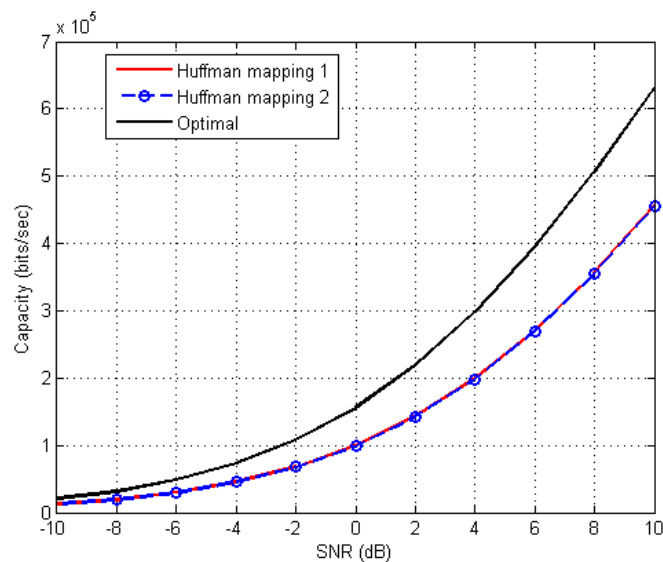


Fig. 3. Capacity versus the SNR for  $N_t = 5$

Huffman mapping 1 and Huffman mapping 2 curves are corresponding to probability vectors  $p_1$  and  $p_2$ , respectively. The optimal method for calculating the probability of selecting the transmit antenna gives the best result. In addition, both probability vectors have the same results.

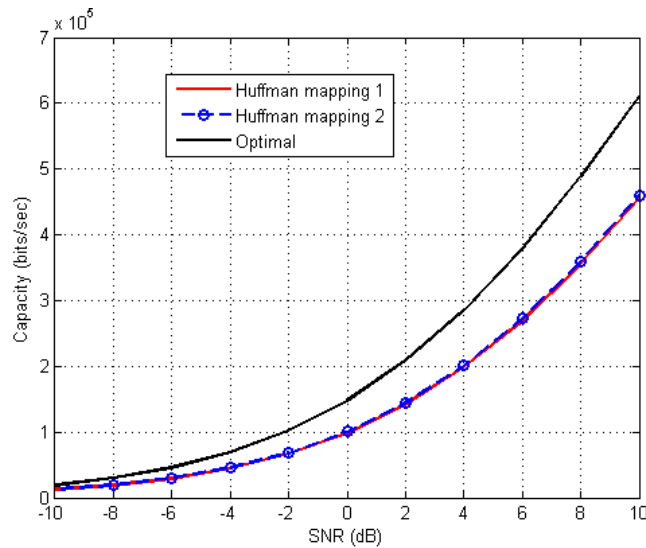


Fig. 4. Capacity versus the SNR for  $N_t = 4$

The capacity versus the SNR for different  $\beta$  is illustrated in Figure 5. It is observed that the capacity of the system for different values of  $\beta$  is approximately constant. Therefore, based on Figure 5, the value of  $\beta$  and the types of the Huffman mapping do not have the serious effect on

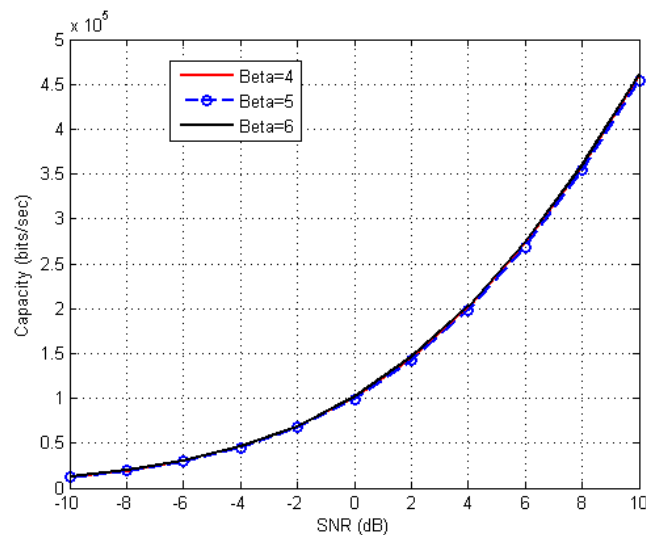


Fig. 5. Capacity versus the SNR for different values of  $\beta$



channel capacity. It is worth to note that the MQAM modulation is used for modulating signal information. Therefore, the constellation size of the modulation is changed based on channel variation to guarantee the system performance. Hence, the BER versus the SNR is illustrated in Figure 6. It is seen that the BER is decreased by increasing the SNR.

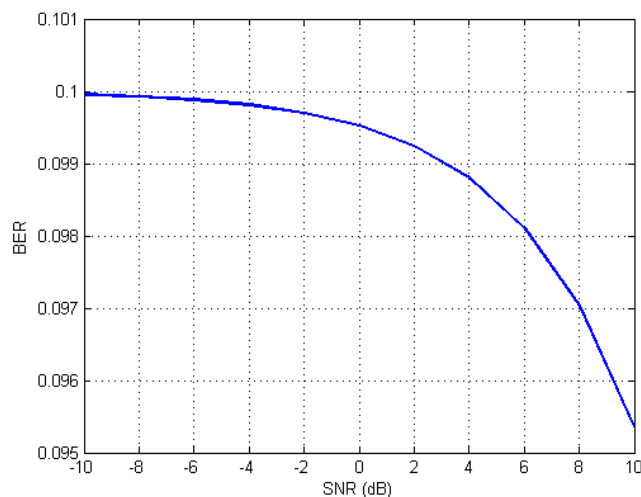


Fig. 6. BER versus the SNR

#### 4. Conclusion

In this paper, two problems; including spatial modulation and information modulation for a single radio frequency chain MIMO to maximize the channel capacity are investigated. In the proposed modulation scheme, the modulation level is changed based on channel variations and a target BER. Therefore, the system has the best performance in all channel situations. In the spatial modulation, the transmit antenna is selected based on the probability,  $p_i$ . This probability determines the chance for each transmit antenna to be selected by the antenna switch. This probability can be calculated by an optimal solution, where it can be any values between 0 and 1. Due to the complexity of the optimal method, suboptimal methods such as Huffman coding are used where this probability is restricted by some pre-defined values. Simulation results indicate that by using adaptive modulation and spatial modulation, the performance of the MIMO systems is increased.

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