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State estimation in a decentralized discrete time LQG control for a multisensor system

ZDZISLAW DUDA

In the paper a state filtration in a decentralized discrete time Linear Quadratic Gaussian problem formulated for a multisensor system is considered. Local optimal control laws depend on global state estimates and are calculated by each node. In a classical centralized information pattern the global state estimators use measurements data from all nodes. In a decentralized system the global state estimates are computed at each node using local state estimates based on local measurements and values of previous controls, from other nodes.

In the paper, contrary to this, the controls are not transmitted between nodes. It leads to nonconventional filtration because the controls from other nodes are treated as random variables for each node. The cost for the additional reduced transmission is an increased filter computation at each node.

Key words: multisensor system, LQG problem, Kalman filter

1. Introduction

Multisensor systems find applications in many areas such as aerospace, robotics, image processing, military surveillance, medical diagnosis. The advantage of using these systems over systems with a single sensor results from e.g. improved reliability, robustness, extended coverage, improved resolution e.t.c. In the systems a state estimation problem is one of the critical concerns.

Theoretically, state estimates can be determined by using a conventional Kalman filter in a centralized structure where all process measurements are sent to a central station.

The centralized architecture produces an optimal estimate in a minimum mean square error (MMSE) sense, but it may imply low survivability and requires high processing and communication loads.

Due to limited communication bandwith or reliability constraints fusion algorithms and appropriate architectures (from hierarchical to fully decentralized) are proposed.

The Author is with Silesian University of Technology, Gliwice, Poland, e-mail: Zdzislaw.Duda@polsl.pl

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In these architecture each node carries out Kalman filtering upon its own measurements and then transmits the local processed data to a fusion center [4, 5, 10] (hierarchical structure) or to other nodes [9, 8] (fully decentralized structure). In fusion nodes a global state estimate is calculated. It may be equivalent to the optimal centralized estimate [4, 5, 10] or suboptimal [1, 2, 3].

In majority papers autonomous systems [1, 2, 3, 6, 12] are considered. A control, if introduced, is a known input [5] or depends on local state estimate [7], only.

In [11] a decentralized Linear-Quadratic-Gaussian (LQG) control problem involving M nodes is considered. Local controls depend on global state estimates. At each node local state estimates are computed using measurement data obtained at that node and values of previous controls from other nodes. The local control law at each node is a linear combination of local state estimates and previous controls transmitted from other nodes.

In this paper a Linear-Quadratic-Gaussian (LQG) problem [11] is considered. Contrary to [11], controls are not transmitted between nodes. It leads to nonconventional local filtration because controls from other nodes should be treated as random variables for each node. The cost for this additional reduced transmission is increased filter computation at each node.

2. Problem formulation

Consider a linear multisensor system described by the equations

$$x_{n+1} = Ax_n + \sum_{j=1}^{M} B^j u_n^j + w_n \tag{1}$$

$$y_n^j = C^j x_n + r_n^j, \quad j = 1, \ldots, M \tag{2}$$

where x_n is a state vector, u_n^j is a control vector at node j, y_n^j is a measurement vector at node j ; A, B^j, C^j are the system and observation models, w_n, r_n^j are the state and measurement noises, respectively. It is assumed that $x_0 \sim N(\bar{x}_0, X_0)$, $w_n \sim N(0, W_n)$, $r_n^j \sim N(0, R_n^j)$ and $x_n \in R^k$, $w_n \in R^k$, $y_n^j \in R^{p^j}$, $r_n^j \in R^{p^j}$; $A \in R^{k \times k}$, $C^j \in R^{p_j \times k}$. Additionally, w_n, $r_m^j, j = 1, \ldots, M$ are gaussian white noise processes independent of each other and of the gaussian initial state x_0.

The optimal control problem is to find

$$I^o = \min_{\{a_n^j(\vec{y}_n), n=0,\ldots,N, j=1,\ldots,M\}} E[\frac{1}{2} \sum_{n=0}^{N} (x_n^T Q_n x_n + \sum_{j=1}^{M} u_n^{jT} H^j u_n^j)_{u_n^j = a_n^j(\vec{y}_n)}] \tag{3}$$

subject to the stochastic system (1) where Q_n and H^j are positive semidefinite and positive definite, respectively, symmetric matrices and $\vec{y}_n = \{y_0, \ldots, y_n\}, \quad y_i = [y_i^{1T}, \ldots, y_i^{MT}]^T$ is the measurement available history.

It is the control problem formulated in a classical information pattern, because control laws are functions of all measurement data.

The solution to the control problem is known [11]. Control laws $a_n^j(\vec{y}_n),\ \ j=1,...,M$, depend on global state estimates based on measurements obtained from all nodes. In [11] a decentralized system is proposed. The global state estimate is obtained as a linear combination of local state estimates based on local measurement information and values of previous controls from all nodes.

The problem formulated in the paper is to compute local state estimates using measurements only at that node. It leads to a decentralized system with an additional reduced data transmission.

3. Solution to the centralized LQG control problem

The solution to the LQG problem [11] has the form

$$u_n^{jo} = S_n^j \hat{x}_{n|n} \tag{4}$$

where the global state estimate $\hat{x}_{n|n}$ is defined as

$$\hat{x}_{n|n} = E(x_n|\vec{y}_n) \tag{5}$$

The quantity y_n is a stacked measurement vector resulting from the eqn. (2) and written in the form

$$y_n = Cx_n + r_n \tag{6}$$

where $y_n = [y_n^{1T},...,y_n^{MT}]^T,\ C = [C^{1T},...,C^{MT}]^T,\ r_n = [r_n^{1T},...,r_n^{MT}]^T,\ R_n = Er_nr_n^T = diag\{R_n^1,...,R_n^M\}$.

The control gain S_n^j is

$$S_n^j = -(H^j + B^{jT}\Lambda_{n+1}B^j)^{-1}B^{jT}\Lambda_{n+1}A \tag{7}$$

where Λ_n is propagated backwards in time as

$$\Lambda_n = A^T\Lambda_{n+1}A - \sum_{j=1}^{M} S_n^{jT}(H^j + B^{jT}\Lambda_{n+1}B^j)S_n^j + Q_n \tag{8}$$

The global state estimate $\hat{x}_{n|n}$ is propagated as

$$\hat{x}_{n+1|n+1} \quad = \hat{x}_{n+1|n} + K_{n+1}(y_{n+1} - C\hat{x}_{n+1|n}) \tag{9}$$

where

$$\hat{x}_{n+1|n} = E(x_{n+1}|\vec{y}_n) = A\hat{x}_{n|n} + \sum_{j=1}^{M} B^j u_n^j \tag{10}$$

The Kalman gain K_{n+1} is given as

$$K_{n+1} = P_{n+1|n}C^T(CP_{n+1|n}C^T + R_{n+1})^{-1} \tag{11}$$

where $P_{n+1|n} = E(x_{n+1} - \hat{x}_{n+1|n})(x_{n+1} - \hat{x}_{n+1|n})^T$ has the form

$$P_{n+1|n} = AP_{n|n}A^T + W_n, \quad P_{0|-1} = X_0 \tag{12}$$

The covariance matrix $P_{n+1|n+1} = E(x_{n+1} - \hat{x}_{n+1|n+1})(x_{n+1} - \hat{x}_{n+1|n+1})^T$ is propagated as

$$P_{n+1|n+1} = (\mathbf{1} - K_{n+1}C)P_{n+1|n} \tag{13}$$

or in the information form

$$P^{-1}_{n+1|n+1} = P^{-1}_{n+1|n} + \sum_{j=1}^{M} C^{jT}(R^j_{n+1})^{-1}C^j \tag{14}$$

where **1** is the identity matrix.

Using (11) and (13) gives

$$K_{n+1} = P_{n+1|n+1}C^T R^{-1}_{n+1} \tag{15}$$

Then the propagation of the estimate $\hat{x}_{n|n}$ described by (9) can be expressed in the form

$$\hat{x}_{n+1|n+1} = \hat{x}_{n+1|n} + P_{n+1|n+1}\sum_{j=1}^{M} C^{jT}(R^j_{n+1})^{-1}(y^j_{n+1} - C^j\hat{x}_{n+1|n}) \tag{16}$$

The eqn. (4)-(16) form the solution to the LQG problem.

Let us notice that the state estimate (16) with (10) depends on measurements and controls from all nodes. In decentralized systems state estimates and consequently controls should be calculated by each node using measurements available only at that node.

4. Solution to the decentralized LQG problem

In [11] a solution to the decentralized LQG problem is presented. The control laws have the form (4).

The state estimate $\hat{x}_{n|n}$ is divided into two parts

$$\hat{x}_{n|n} = \hat{x}^D_{n|n} + x^C_n \tag{17}$$

where

$$x^C_{n+1} = Ax^C_n + \sum_{j=1}^{M} B^j u^j_n, \quad x^C_0 = \bar{x}_0 \tag{18}$$

and

$$\hat{x}^D_{n+1|n+1} = \sum_{j=1}^{M} [P_{n+1|n+1}(P^j_{n+1|n+1})^{-1}\hat{x}^{Dj}_{n+1|n+1} + h^j_{n+1}] \tag{19}$$

At each node an additional vector h^j_n is calculated as

$$h^j_{n+1} = F_{n+1}h^j_n + G^j_{n+1}\hat{x}^{Dj}_{n+1|n}, \;\; h^j_0 = 0 \tag{20}$$

where

$$\begin{aligned} &\hat{x}^{Dj}_{n+1|n} = A\hat{x}^{Dj}_{n|n}, \quad F_n = P_{n|n}P^{-1}_{n|n-1}A \\ &G^j_{n+1} = P_{n+1|n+1}[(P_{n+1|n})^{-1}AP_{n|n}(P^j_{n|n})^{-1}A^{-1} - (P^j_{n+1|n})^{-1}] \end{aligned} \tag{21}$$

and $P^j_{n|n} = E(x^D_n - \hat{x}^{Dj}_{n|n})(x^D_n - \hat{x}^{Dj}_{n|n})^T$ and $P^j_{n|n-1} = E(x^D_n - \hat{x}^{Dj}_{n|n-1})(x^D_n - \hat{x}^{Dj}_{n|n-1})^T$.

The state estimate $\hat{x}^{Dj}_{n|n}$ determined by each node has the form

$$\hat{x}^{Dj}_{n+1|n+1} = A\hat{x}^{Dj}_{n|n} + P^j_{n+1|n+1}C^{jT}(R^j_n)^{-1}(y^j_{n+1} - C^jx^C_{n+1} - C^jA\hat{x}^{Dj}_{n|n}), \;\; \hat{x}^{Dj}_{0|0} = 0 \tag{22}$$

where a covariance matrix $P^j_{n+1|n+1}$ has a classical form.

Using (7), (17) and (19) in (4) the $j-th$ local optimal control law becomes

$$u^{jo}_n = -(H^j + B^{jT}\Lambda_{n+1}B^j)^{-1}B^{jT}\Lambda_{n+1}\{\sum_{l=1}^{M}[P_{n|n}(P^l_{n|n})^{-1}\hat{x}^{Dl}_{n|n} + h^l_n] + x^C_n\} \tag{23}$$

Let us notice that in order to determine the value of the optimal control u^{jo}_n at the $j-th$ node, the p_j dimensional vectors α^{lj}_n defined as

$$\alpha^{lj}_n = B^{jT}\Lambda_{n+1}[P_{n|n}(P^l_{n|n})^{-1}\hat{x}^{Dl}_{n|n} + h^l_n], \quad l = 1, ..., M, \;\; l \neq j \tag{24}$$

should be transmitted from other nodes to the node j. Moreover the controls u^l_{n-1} from other nodes must be transmitted too, so that to form (18), (22) and finally (23).

At each node the vector h^j_n must be calculated. Since h^j_n depends on measurements, it should be calculated on-line. The operations in (19) and (20) can be done in parallel.

5. New approach to filtration in the decentralized LQG problem

Let us consider the eqn. (1) in which the control is described by the eqn. (4) i.e.

$$x_{n+1} = Ax_n + B_n\hat{x}_{n|n} + w_n \tag{25}$$

where $B_n = \sum_{j=1}^{M} B^jS^j_n$.

The objective of the local filtration at the $j-th$ node is to compute local estimates using measurements available at that node.

The system (25) can be written in the form

$$x_{n+1} = A_n x_n - B_n \widetilde{x}_{n|n} + w_n \tag{26}$$

where $A_n = A + B_n$ and $\widetilde{x}_{n|n} = x_n - \hat{x}_{n|n}$.

Let a local estimate at the $j-th$ node has the form

$$\hat{x}_n^j = E(x_n|\vec{y}_n^j) \tag{27}$$

where $\vec{y}_n^j = \{y_0^j, ..., y_n^j\}$.

Then the estimate $\hat{x}_{n+1|n+1}^j$ is propagated as

$$\hat{x}_{n+1|n+1}^j = \hat{x}_{n+1|n}^j + K_{n+1}^j (y_{n+1}^j - C^j \hat{x}_{n+1|n}^j) \tag{28}$$

The term $\hat{x}_{n+1|n}^j$ in (28) becomes

$$\hat{x}_{n+1|n}^j = E(x_{n+1}|\vec{y}_n^j) = A_n \hat{x}_{n|n}^j - B_n E(\widetilde{x}_{n|n}|\vec{y}_n^j) \tag{29}$$

We find that

$$\begin{aligned} E(\widetilde{x}_{n|n}|\vec{y}_n^j) &= E[(x_n - \hat{x}_{n|n})|\vec{y}_n^j] = E(x_n|\vec{y}_n^j) - E(\hat{x}_{n|n}|\vec{y}_n^j) = \\ &= E(x_n|\vec{y}_n^j) - E\{[E(x_n|\vec{y}_n)]|\vec{y}_n^j\} = E(x_n|\vec{y}_n^j) - E(x_n|\vec{y}_n^j) = 0 \end{aligned} \tag{30}$$

Thus the last term in (29) is equal to zero and $\hat{x}_{n+1|n}^j$ in (28) has the form

$$\hat{x}_{n+1|n}^j = A_n \hat{x}_{n|n}^j \tag{31}$$

The Kalman gain K_{n+1}^j is

$$K_{n+1}^j = P_{n+1|n}^j C^{jT} (C^j P_{n+1|n}^j C^{jT} + R_{n+1}^j)^{-1} \tag{32}$$

The covariance matrix $P_{n+1|n}^j$ is defined as

$$P_{n+1|n}^j = E(\widetilde{x}_{n+1|n}^j \widetilde{x}_{n+1|n}^{jT}) \tag{33}$$

where $\widetilde{x}_{n+1|n}^j = x_{n+1} - \hat{x}_{n+1|n}^j$.

From (26) and (31) we have

$$\widetilde{x}_{n+1|n}^j = A_n x_n - B_n \widetilde{x}_{n|n} + w_n - A_n \hat{x}_{n|n}^j = A_n \widetilde{x}_{n|n}^j - B_n \widetilde{x}_{n|n} + w_n \tag{34}$$

where $\widetilde{x}^j_{n|n} = x_n - \hat{x}^j_{n|n}$.

Hence $P^j_{n+1|n}$ in (32) has the form

$$\begin{aligned} P^j_{n+1|n} &= E(A_n\widetilde{x}^j_{n|n} - B_n\widetilde{x}_{n|n} + w_n)(A_n\widetilde{x}^j_{n|n} - B_n\widetilde{x}_{n|n} + w_n)^T = \\ &= A_nP^j_{n|n}A_n^T - A_nP^{j*}_{n|n}B_n^T - B_nP^{*j}_{n|n}A_n^T + B_nP_{n|n}B_n^T + W_n \end{aligned} \tag{35}$$

where

$$P^j_{n|n} = E(\widetilde{x}^j_{n|n}\widetilde{x}^{jT}_{n|n}), \qquad P^{j*}_{n|n} = E(\widetilde{x}^j_{n|n}\widetilde{x}^T_{n|n}), \qquad P^{*j}_{n|n} = E(\widetilde{x}_{n|n}\widetilde{x}^{jT}_{n|n}) = (P^{j*}_{n|n})^T \tag{36}$$

should be determined.

The covariance matrix $P^j_{n+1|n+1} = E(\widetilde{x}^j_{n+1|n+1}\widetilde{x}^{jT}_{n+1|n+1})$ in (35) can be found by a classical way and has the form

$$P^j_{n+1|n+1} = (\mathbf{1} - K^j_{n+1}C^j)P^j_{n+1|n} \tag{37}$$

or in an information form

$$(P^j_{n+1|n+1})^{-1} = (P^j_{n+1|n})^{-1} + C^{jT}(R^j_{n+1})^{-1}C^j \tag{38}$$

Using (32) and (37) gives

$$K^j_{n+1} = P^j_{n+1|n+1}C^{jT}(R^j_{n+1})^{-1} \tag{39}$$

By subtracting both sides of (28) from the identity $x_{n+1} = x_{n+1}$ we obtain

$$\widetilde{x}^j_{n+1|n+1} = (\mathbf{1} - K^j_{n+1}C^j)\widetilde{x}^j_{n+1|n} - K^j_{n+1}r^j_{n+1} \tag{40}$$

and similarly to (9)

$$\widetilde{x}_{n+1|n+1} = (\mathbf{1} - K_{n+1}C)\widetilde{x}_{n+1|n} - K_{n+1}r_{n+1} \tag{41}$$

The covariance matrix $P^{j*}_{n+1|n+1} = E(\widetilde{x}^j_{n+1|n+1}\widetilde{x}^T_{n+1|n+1})$ in (35) may be expressed in the form

$$\begin{aligned} P^{j*}_{n+1|n+1} &= E[(\mathbf{1} - K^j_{n+1}C^j)\widetilde{x}^j_{n+1|n} - K^j_{n+1}r^j_{n+1}][(\mathbf{1} - K_{n+1}C)\widetilde{x}_{n+1|n} - K_{n+1}r_{n+1}]^T = \\ &= (\mathbf{1} - K^j_{n+1}C^j)P^{j*}_{n+1|n}(\mathbf{1} - K_{n+1}C)^T + K^j_{n+1}R^{j*}_{n+1}K^T_{n+1} \end{aligned} \tag{42}$$

where $P^{j*}_{n+1|n} = E(\widetilde{x}^j_{n+1|n}\widetilde{x}^T_{n+1|n})$ and $R^{j*}_{n+1} = [R^{j1}_{n+1}, \ldots, R^{jl}_{n+1}, \ldots, R^{jM}_{n+1}]$.

The matrices R^{jl}_{n+1} are defined as $R^{jl}_{n+1} = E(r^j_{n+1}r^{lT}_{n+1}) = 0$ for $j \neq l$ and $R^{jj}_{n+1} = E(r^j_{n+1}r^{jT}_{n+1}) = R^j_{n+1}$

Using (13), (37) and (15), (39) in (42) gives

$$P^{j*}_{n+1|n+1} = \tag{43}$$
$$P^{j}_{n+1|n+1}(P^{j}_{n+1|n})^{-1}P^{j*}_{n+1|n}P^{-1}_{n+1|n}P_{n+1|n+1} + P^{j}_{n+1|n+1}C^{jT}(R^{j}_{n+1})^{-1}\overbrace{R^{j*}_{n+1}R^{-1}_{n+1}C}^{C^j}P_{n+1|n+1}$$

In order to determine $P^{j*}_{n+1|n} = E(\widetilde{x}^{j}_{n+1|n}\widetilde{x}^{T}_{n+1|n})$ in (43) we have from (1) and (10)

$$\widetilde{x}_{n+1|n} = A\widetilde{x}_{n|n} + w_n \tag{44}$$

and from (34)

$$\begin{aligned} P^{j*}_{n+1|n} &= E[A_n\widetilde{x}^{j}_{n|n} - B_n\widetilde{x}_{n|n} + w_n][A\widetilde{x}_{n|n} + w_n]^T = \\ &= A_nP^{j*}_{n|n}A^T - B_nP_{n|n}A^T + W_n \end{aligned} \tag{45}$$

Equations (28), (31), (32), (35), (37), (43) and (45) form the solution to the local filtration problem at the jth node.

Inserting (39) to (28) gives

$$\hat{x}^{j}_{n+1|n+1} - \hat{x}^{j}_{n+1|n} = P^{j}_{n+1|n+1}C^{jT}(R^{j}_{n+1})^{-1}(y^{j}_{n+1} - C^{j}\hat{x}^{j}_{n+1|n}) \tag{46}$$

Multiplying the both sides of the eqn. (46) by $(P^{j}_{n+1|n+1})^{-1}$ we have that

$$(P^{j}_{n+1|n+1})^{-1}(\hat{x}^{j}_{n+1|n+1} - \hat{x}^{j}_{n+1|n}) = C^{jT}(R^{j}_{n+1})^{-1}(y^{j}_{n+1} - C^{j}\hat{x}^{j}_{n+1|n}) \tag{47}$$

Thus

$$C^{jT}(R^{j}_{n+1})^{-1}y^{j}_{n+1} = \tag{48}$$
$$(P^{j}_{n+1|n+1})^{-1}\hat{x}^{j}_{n+1|n+1} - [\overbrace{(P^{j}_{n+1|n+1})^{-1} - C^{jT}(R^{j}_{n+1})^{-1}C^{j}}^{(P^{j}_{n+1|n})^{-1}(38)}]\hat{x}^{j}_{n+1|n}$$

Then the propagation of the estimate $\hat{x}_{n|n}$ described by (16) can be expressed in the form

$$\begin{aligned} \hat{x}_{n+1|n+1} &= [\mathbf{1} - P_{n+1|n+1}\sum_{j=1}^{M}C^{jT}(R^{j}_{n+1})^{-1}C^{j}]\hat{x}_{n+1|n} + \\ &+ P_{n+1|n+1}\sum_{j=1}^{M}(P^{j}_{n+1|n+1})^{-1}\hat{x}^{j}_{n+1|n+1} - P_{n+1|n+1}\sum_{j=1}^{M}(P^{j}_{n+1|n})^{-1}\hat{x}^{j}_{n+1|n} \end{aligned} \tag{49}$$

According to the eqn. (26) the term $\hat{x}_{n+1|n}$ in (49) becomes

$$\hat{x}_{n+1|n} = E(x_{n+1}|\vec{y}_n) = A_n\hat{x}_{n|n} - B_nE(\widetilde{x}_{n|n}|\vec{y}_n) = A_n\hat{x}_{n|n} \tag{50}$$

Let, similarly to [11]

$$\hat{x}_{n+1|n+1} = \sum_{j=1}^{M}[P_{n+1|n+1}(P^j_{n+1|n+1})^{-1}\hat{x}^j_{n+1|n+1} + h^j_{n+1}] \tag{51}$$

where

$$h^j_{n+1} = F_{n+1}h^j_n + G^j_{n+1}\hat{x}^j_{n+1|n} \tag{52}$$

When we substitute (50) and next (51) into the eqn. (49) , we find

$$\overbrace{\sum_{j=1}^{M}[P_{n+1|n+1}(P^j_{n+1|n+1})^{-1}\hat{x}^j_{n+1|n+1} + h^j_{n+1}]}^{\hat{x}_{n+1|n+1}} = \tag{53}$$

$$= [\mathbf{1} - P_{n+1|n+1}\sum_{j=1}^{M}C^{jT}(R^j_{n+1})^{-1}C^j]A_n\overbrace{\sum_{j=1}^{M}[P_{n|n}(P^j_{n|n})^{-1}\hat{x}^j_{n|n} + h^j_n]}^{\hat{x}_{n|n}} +$$

$$+P_{n+1|n+1}\sum_{j=1}^{M}(P^j_{n+1|n+1})^{-1}\hat{x}^j_{n+1|n+1} - P_{n+1|n+1}\sum_{j=1}^{M}(P^j_{n+1|n})^{-1}\hat{x}^j_{n+1|n}$$

Thus

$$\begin{aligned} h^j_{n+1} &= P_{n+1|n+1}\overbrace{[(P_{n+1|n+1})^{-1} - \sum_{j=1}^{M}C^{jT}(R^j_{n+1})^{-1}C^j]}^{P^{-1}_{n+1|n}(14)}A_n h^j_n + +P_{n+1|n+1}\{[(P_{n+1|n+1})^{-1} \\ &- \sum_{j=1}^{M}C^{jT}(R^j_{n+1})^{-1}C^j]A_nP_{n|n}(P^j_{n|n})^{-1}A_n^{-1} - (P^j_{n+1|n})^{-1}\}\hat{x}^j_{n+1|n} = \\ &\overbrace{P_{n+1|n+1}(P_{n+1|n})^{-1}A_n}^{F_{n+1}}h^j_n + \\ &+\overbrace{P_{n+1|n+1}[(P_{n+1|n})^{-1}A_nP_{n|n}(P^j_{n|n})^{-1}A_n^{-1} - (P^j_{n+1|n})^{-1}]}^{G^j_{n+1}}\hat{x}^j_{n+1|n} \end{aligned} \tag{54}$$

Using (51) and (7) in (4) gives

$$u^{jo}_n = -(H^j + B^{jT}\Lambda_{n+1}B^j)^{-1}B^{jT}\Lambda_{n+1}\{\sum_{l=1}^{M}[P_{n|n}(P^l_{n|n})^{-1}\hat{x}^l_{n|n} + h^l_n]\} \tag{55}$$

Let us notice that in order to determine the value of the optimal control u_n^{jo} at the $j-th$ node the p_j dimensional vectors α_n^{lj} defined as

$$\alpha_n^{lj} = B^{jT}\Lambda_{n+1}[P_{n|n}(P_{n|n}^{l})^{-1}\hat{x}_{n|n}^{l} + h_n^l], \quad l = 1,...,M, \; l \neq j \tag{56}$$

should be transmitted from other nodes to the $j-th$ node .

The control laws (23) and (55) have very similar forms and properties discussed in [11].

The main difference is that the latter does not depend on controls from other nodes. Thus contrary to (23) controls need not be transmitted from node to node. However, the cost for this additional reduced transmission is increased filter computation at each node.

6. Conclusions

In the paper the decentralized filtration in LQG control problem is presented. It is shown that the state estimates calculated at each node are updated with current measurement obtained only at that node. Additionally, at each node the vector dependent on past data must be determined. But, the controls need not be transmitted from node to node.

The local state estimation is nonclassical in this sense that it requires some additional calculations to obtain error covariance matrix $P_{n|n}^{j}$. Fortunately, these calculations do not depend on measurement information and can be done off-line.

The control laws are linear combinations of data calculated at each node. To calculate controls at the jth node only the transmission of the vectors α_n^{lj} from other nodes is needed.

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