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# The Applications of Graph Algorithms to Modeling of Integrated Urban Water Management System

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## ABSTRACT

The application of methods using graphs to model a variety of engineering issues has been known for several decades, but the application of graph algorithms to model the urban water management issues is a completely new approach. The article reviews the scientific literature on integrated urban water management systems in terms of the use of graph theory algorithms in this topic. Such a review has not been done before and constitutes a completely novel study. Some of the algorithms presented are directly derived from graph theory, while others were developed from other sciences, including environmental engineering or genetics, to solve specific engineering problems. The paper presents a general scheme and a brief description of the most important components of an integrated urban water management system. The necessary concepts of graphs were defined, the origin and the principle of graph algorithms used in modeling water management issues (Loop-By-Loop Cutting Algorithm, Hanging Gardens Algorithm, Tree Growth Algorithm, Dijkstra's Algorithm, Genetic Algorithm, and Bayesian Networks Algorithm) were described. Their use in modeling the issues in stormwater, sanitary sewage and water distribution system was described. A complete list of scientific literature in this field was provided.

**Keywords:** graph modeling, graph algorithms, integrated analysis, network modeling, urban water.

## INTRODUCTION

Since the late 20th century, perceptions of engineering systems related to water supply and wastewater disposal of urbanized areas have been changing. These systems used to be considered separately as independent systems in which the physical phenomena associated with water, wastewater and sludge transport occur. Nowadays, there are trends of analyzing these systems holistically as well as treating individual systems and subsystems as one integrated urban water management system – integrated urban water management system (IUWMS). This can be seen in the works of many authors who postulate that efforts should be made to develop a comprehensive methodology for the design, analysis, and operation of IUWMS [1-6]. In such an approach, water intakes, water treatment and distribution systems, the area in which wastewater (rainwater, sanitary, industrial) is generated, the sewage network, the wastewater treatment plant, and the recipient of treated wastewater should be treated in a systemic (integrated) manner, together forming successive, interrelated elements of one integral system.

Therefore, the tools that will be able to analyze the afore-mentioned integrated system holistically as well as in the area of individual subsystems, are sought. Graphs constitute such mathematical tools. Some algorithms have been developed within the framework of graph theory, while others have been developed as a result of solving specific engineering problems.

The first applied science to use the tools of graph theory was electrical engineering (Kirchhoff's law, topological methods of circuit analysis). This was followed by chemistry, economic sciences, logistics, and transportation. In recent years, there has been intensive development of applications of graph theory in computer science, both in terms of methods (databases, artificial intelligence) and combinatorial algorithms [7]. Nowadays, a popular new trend is the application of graph theory tools in the social sciences. More scientific fields are using graphs as universal objects, which help to describe various issues with mathematical precision, model them using graphs, and then provide a solution using graph methods and algorithms.

A thorough analysis of the available literature has shown that although there are many papers that present particular algorithms based on graphs to solve specific engineering problems, including the analysis of elements of the urban water management system, according to authors' knowledge there is no review article that would present all the algorithms used so far and the possibility of their application in IUWMS. Hence, this paper aimed at reviewing the literature on the algorithms based on graph theory that can be applied to the analysis of network structures of elements included in IUWMS, both considering them independently and holistically.

## INTEGRATED URBAN WATER MANAGEMENT SYSTEM

Integrated urban water management system is complex network consisting of interconnected pipes and nodes of each subsystem [8]. IUWMS is essential urban infrastructure and has a direct influence on the public economy, health and environment [9, 3, 5]. IUWMS consists of two main parts: the water supply system and the sewage water system (Fig. 1).

### Water supply system

Water supply system contains three fundamental elements: supply source, the treatment or processing of the water and finally water distribution to the society as well as industry. The water that comes from the source is transferred into the treatment plant using aqueducts and pipes. This process occurs using flow in an open channel or pressure. After purification, the water is transported into distribution system immediately, also may be transferred to it by storage reservoirs [10-13].

### Source of supply

In the case of a social water supply, the source of water should ensure an amount adequate to satisfy all communal, corporate and manufacturing

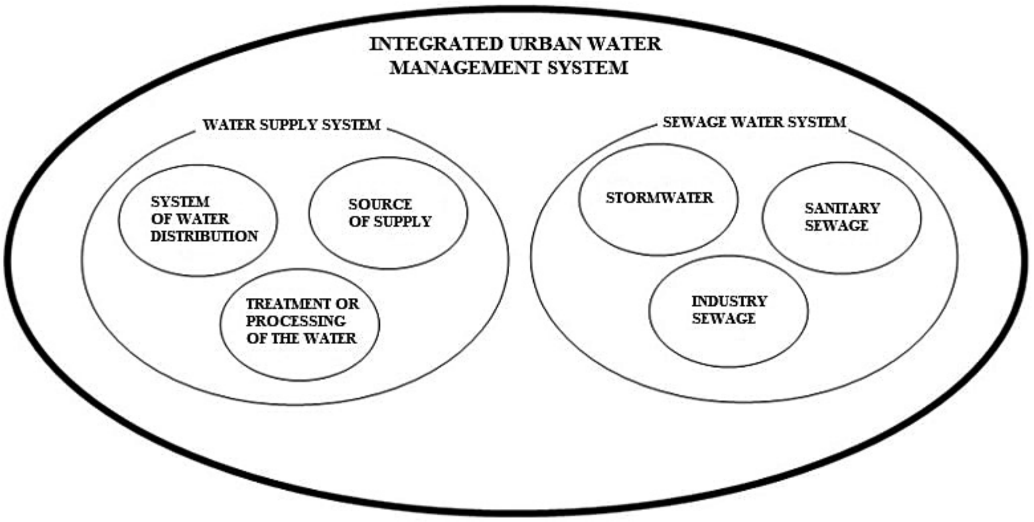

**Fig. 1**. Integrated urban water management system

necessities, also including the firefighting requirement. The surface water and groundwater can be applied. Large water reservoirs, such as lakes and rivers can be a source of water supply. Groundwater is usually derived using sinking wells into the saturated area which is placed below the groundwater level [10, 14-16].

### Treatment or processing of the water

The basic role of water treatment is improving the quality of water in order to be suitable to a specific end-use, more precisely drinking, industry water supply, irrigation, water recreation or many other uses. Water treatment removes contaminants and undesirable elements, or decreases their concentration. As a consequence, the water becomes fit for its desired end-use [11, 17, 18].

### System of water distribution

It involves the elements which move drinkable water from treatment plants or wells to society so as to satisfy the residential, commercial, industrial and firefighting requirements. This system includes pipelines, storage facilities, pumps and other elements [12, 14, 16].

### Sewage water (wastewater) system

The second part of IUWMS is the sewage water (wastewater) system. The sewage water system is a network of pipes, manholes, pumping stations, and related facilities. They transfer sewage from its points of origin to a point of treatment and disposal [19, 20]. The sewage flow rate differs from place, according to economic factors and public nature, characteristics of industrial companies in the studied area, use of water, weather conditions and kind of sewage systems.

There are three types of sewage system:

- sanitary sewage (from bathrooms, toilets, kitchens, etc.),
- industry sewage,
- stormwater.

### Sanitary sewage

Sanitary sewage involves effluents derived from toilets, washbasins, sinks and so on. They come out of the apartments, as well as business or corporate buildings. In general, sanitary sewerage is highly polluted, mainly due to the content of human excreta and food leftovers [21].

### Industry sewage

Industry sewage mainly is composed of discharge derived from the manufacturing procedures of different branches of industry, for instance, paper making, brewing, varnishing, heavy industry and textile industry. In qualitative terms, the industry sewerage is connected with the kind of manufacturing and the chemical substances as well as physical and biological processes which are applied. These may be extremely polluted and demand thorough purification prior to discharge to communal sewerages [22].

### Stormwater

Storm sewers accumulate and transfer rainwater, snow melt and irrigation runoff into storm drains in parking lots, streets and sewer manholes. These drains are linked by an underground pipe network which in most cases transports water immediately to rivers, lakes and other water reservoirs, quite often without treatment at a wastewater treatment plant. The storm sewer system, in contrast to the sanitary sewer system (which transports polluted wastewater to a treatment plant), transports untreated runoff water immediately into environment [23]. As it was observed, stormwater systems are becoming less efficient under changing climatic conditions. Thus, strong flooding can occur in urban regions. Moreover, considerable sanitary issues can appear due to the improper regulation of sewage coming out from network to the surface [24-27].

## DESCRIPTION OF SOME ALGORITHMS BASED ON THE GRAPH THEORY

Traditional integrated urban water management systems rest on centralized network-based infrastructures. Lately, the subject of centralized drainage networks in the cities has more often been disputed. The recent research suggests replacing the centralized systems with the decentralized or hybrid systems. Consequently, tools and methods are needed to estimate and optimize water and wastewater networks of any degrees of centralization (DC). A lot of algorithms based on the graph theory are used for this reason.

For this purpose, let us recall some basic facts and definitions related to graph theory. By a graph $G$ we mean a triple which includes a set of vertices $V(G)$, a set of edges $E(G)$, and a relation which links any two vertices (not necessarily different) named endpoints with each edge. A simple graph is a kind of a graph that does not have more than one edge between any two vertices and no edge starts and ends at the same vertex, i.e. a graph without multiple edges and loops. A path is a simple graph whose vertices can be ordered in a such way that two arbitrary vertices are connected by an edge if they are consecutive in the list. A subgraph of a given graph $G$ is a kind of graph $H$ satisfying inclusions $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$ and the allocation of endpoints to edges in the graph $H$ is identical to the graph $G$. A graph $G$ is called connected if any two vertices of the graph are connected by a path; otherwise $G$ is disconnected. A cycle is a path in which only the first and the last vertices are equal. A graph is called acyclic when it does not contain any cycle. A connected acyclic graph is called a tree. A spanning subgraph of $G$ is a subgraph with vertex set $V(G)$. A spanning tree is a spanning subgraph that is a tree [28, 29]. Figure 2 presents an example of a connected graph $G$, its example spanning subgraph $H$ which is disconnected, and its example spanning tree $T$.

Some of the most popular algorithms will be presented below. The first two of algorithms were developed in the practical context of urban water systems and the other algorithms are more theoretical, derived from pure mathematics, namely from the graph theory.

## Loop-by-loop cutting algorithm

The loop-by-loop algorithm was introduced in 2013 by A. Hanghighi to design feasible sewer layouts from the base graph [30]. The first step of this algorithm is to introduce an undirected base graph to the configuration that involves all feasible connections and pipes. On the base graph, number of edges (number of pipes) are denoted by $m$, letter $n$ denotes number of vertices (manholes) and number $NL$ is the number of loops. In the other words, in order to build a spanning tree from the initiative base graph which has $NL$ loops, $NL$ edges should be cut (one edge in every loop). According to Hanghighi by loops we mean cycles in the graph. All vertices, edges, and loops included in the base graph are arbitrarily marked using numbers. This graph is described by the $B$-matrix, which contains $m$ rows and $NL+3$ columns.

The structure of $B$-matrix:

- the first column consists of the names of sewerage,
- the columns from 2 to $NL+1$ consist of the sewer-in-loop indices that point out if a sewerage is in a loop or not,
- the final columns from $NL+2$ to $NL+3$ consist of the manhole names (the name of sewerage ends).

In order to build a tree-like layout, in the base graph one sewerage of particular loop should be cut. After choosing a pipe to cut, cutting may be carried out both in the upper and lower manhole. Consequently, two decision parameters for starting every loop can be mentioned, containing the name of pipe which is cut and also the position of cut. They are denoted by $\alpha$ and $\beta$, respectively. Following loop opening, the base graph is modified, whereas the $B$-matrix needs to be appropriately modified (which is described in detail in [30, 31]). After implementing to above-mentioned modifications, another loop is considered and the process carries on till all $NL$ loops are opened. Finally, a possible sewerage setup including $m$ sewers, $n+NL$ manholes and without loop is created on the basis of determined $\alpha$ and $\beta$ parameters. Next, the sewerage directions are established towards the outlet manhole on the basis of the rule that "except for the outlet exactly one sewer leaves every manhole". The direction of every sewerage is changed in the

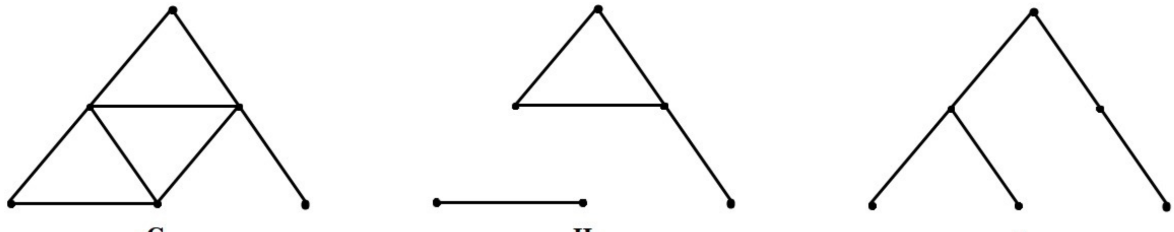

**Fig. 2**. Examples of graphs

$B$-matrix so that the upper manholes are placed in Column $NL+1$ and the lower ones are placed in Column $NL+2$. It is worth noting that each input data set and position of the outlet often imply a possible directed tree using this methodology [30-32].

## Hanging gardens algorithm

The hanging gardens algorithm was introduced in 2019 by A. Hanghighi, A.E. Bakhshipour and others to create all feasible sewerage layouts and to examine various decentralization degrees [31]. The name "hanging gardens algorithm" is derived from the fact that this algorithm cuts the main hanging tree into numerous hanging trees of smaller size. It produces decentralized layouts using the following elements:

- pathfinder – to look for a distance among the suggested extra new root and the existing root(s),
- separator – to choose the position to cut this distance and to divide the graph into two sections,
- matrix constructor – to look for significant nodes and pipes in all parts and create a $H$-matrix (which relies on nodes whereas the B-matrix relied the pipes) for each of the new trees.

The $H$-matrix contains $n$ rows and $n+2$ columns. The structure of the $H$-matrix:

- the first column consists of the levels of node,
- the second column consists of the names of node,
- the columns from 3 to $n+2$ consist of the information about the node connection, pointing out if a node is linked to others or not.

Applying this method, one may divide each directed tree into two also directed trees. Setting a new root into the procedure, the method first looks for the part comprising the new root; then, the presented parts are employed in order to separate it. The process may be repeated till all feasible roots (and all feasible connections) are contained in the ultimate setup. The last stage is to use all the modifications in the actual setup within the base graph [31-33].

The loop-by-loop cutting algorithm produces a centralized layout. On the other hand, the hanging gardens algorithm applies this created layout to generate a decentralized one. This process is presented in Figure 3.

## Tree growth algorithm

Tree growth algorithm (TGA) is an approach that was developed in 2017 by A. Cheraghalipour and M. Hajiaghaei-Keshteli to solve complex optimization problems [34]. This algorithm is introduced aiming at global optimization. TGA includes two stages: the first is intensification, whereas the second is diversification of the

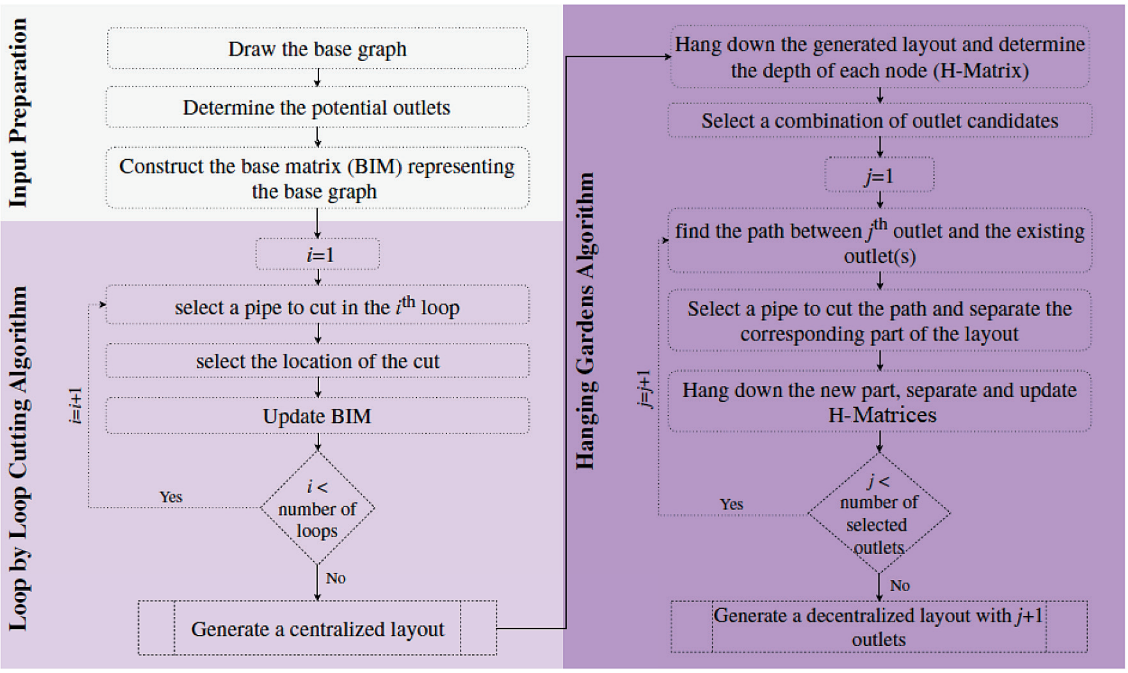

**Fig. 3**. Suggested structure to create decentralized layouts [31]

algorithm. Generally, intensification relies on restarting from high quality solutions or modifying choice rules to favor the inclusion of attributes to these solutions. In this phase, the best trees, which satisfy of light absorbing, competition on food source, are allowed. Moreover, movement occurs only towards better food source or the local optimum. During the diversification stage, some of trees may compete for light absorbing and move towards new places (solutions). The equilibrium between intensification and diversification may be obtained by means of modifying the parameters [35]. In TGA, these two stages are shared in the following sets:

- In the first group, named the group of best trees, particular better trees, according to advantageous factors for rise, will continue to increase and when the quantity of obtained light is fulfilled, their will compete for food. The slow rise of trees contributes to smoothness and tall growth of the good trees; most importantly, they are older than the rest. The older a tree is, the slower the rate of its growth (in comparison with the young tree). The major contest of these trees concentrates on food in roots.
- In the second group, named the competition for light group, certain trees in order reach the sun, shift by length among the nearest best trees from various angles.
- In the third group, called "remove and replace", a few feeble trees that do not grow sufficiently, are cut by foresters and replaced with new trees.
- In the last group, named the reproduction group, the best trees, due to favorable growth, start to reproduce and form fresh plants. Since they grow close to the mother tree, they take over certain aspects of that position.

This method is presented in Figure 4. More information about TGA can be found in [34, 35].

## Dijkstra's algorithm

This algorithm was introduced in 1959 by Dr. Edsger W. Dijkstra to find the shortest paths between vertices in a graph [36]. This algorithm begins in the chosen vertex (source node) and it explores the graph in order to search the shortest path connecting this vertex and all the remaining vertices within the graph. The procedure follows the shortest identified path between any vertex and source vertex, then it changes these quantities when a shorter distance is discovered. When the method has identified the shortest path from the source vertex to other vertex, then this vertex is signed by "visited" and attached to the path. The procedure lasts till each vertex in the graph has been inserted into the path. It yields a path which links source vertex to all remaining vertices following the shortest path capable of achieving every vertex [37].

The restriction of Dijkstra's algorithm is that it may run solely with the graphs which possess positive weights. It follows from the fact that throughout the procedure, the edge weights must be added in order to search for the shortest path. The method will not operate correctly when there is any negative weight in the graph. When a vertex has been signed by "visited", the actual distance leading to this vertex is signed by "the shortest path" to achieve this vertex. It may be modified by negative weights, if the whole weight may be decreased following this step.

The scheme of the Dijkstra's algorithm is presented in Figure 5. In the diagram set $S$ is used to record the vertices of which the shortest path has not been found, namely the set $S$ contains only the starting point. The set $U$ contains vertices other than the starting point. More detailed information about the Dijkstra's algorithm can be found in [7, 37, 38].

## Genetic algorithm

In 1975, John Holland and his co-workers proposed the genetic algorithm (GA). Its main purpose was to investigate the adaptive procedure of some natural methods and to devise the methods which simulate the adaptive process of natural methods [39, 40]. Genetic algorithms are common methodologies of searching for artificial evolution, the essence of which are evolution and population genetics mechanisms. These methods follow natural, highly efficient optimization of evolution technologies, which are rested on preferred experience and reproduction of the best suited participants of population, the preservation of a population with various members, the inheritance of genetic information from parents and the occasional mutation of genes [41]. The idea of GA is aimed to follow the natural transformations that appear in living ecosystems, which is social systems, evaluate the psychological consequences, and model the variable methods.

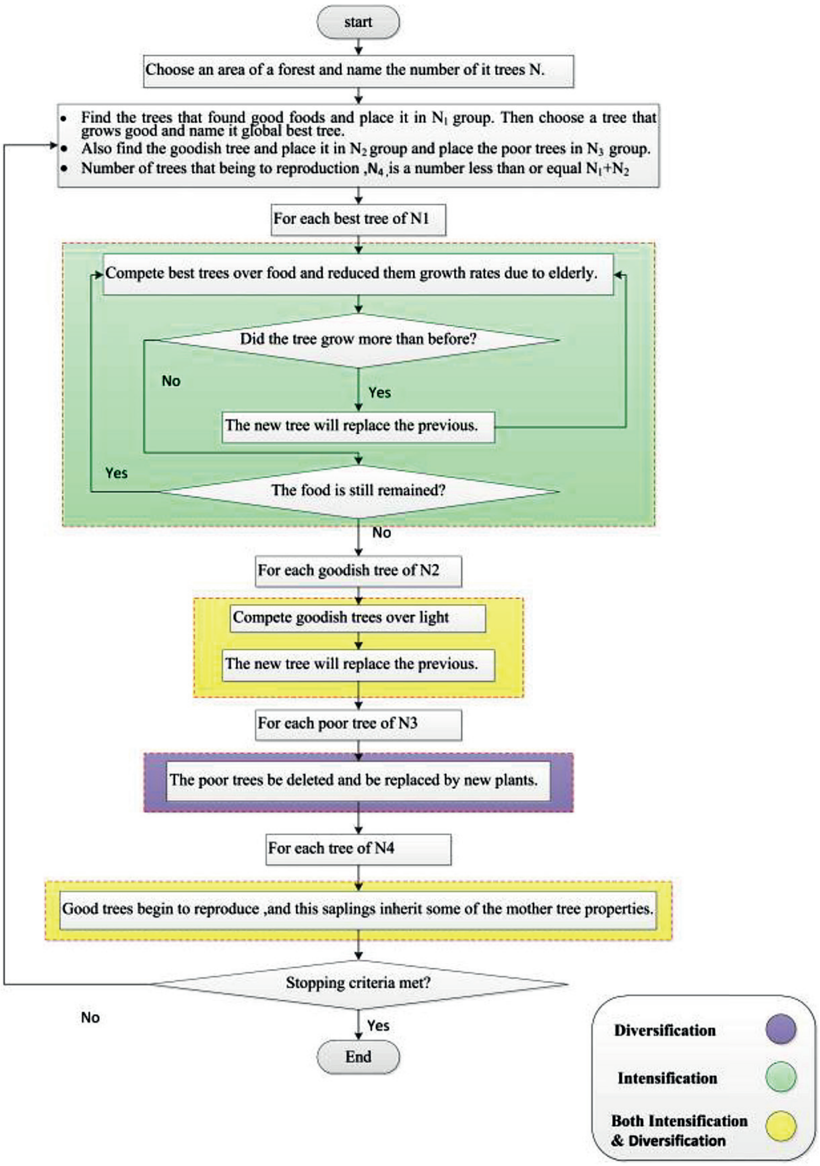

**Fig. 4**. Tree growth algorithm’s flowchart [34]

In general, a fundamental genetic algorithm consists of five stages: initialization, selection, crossover, mutation and termination [42, 43].

- Initialization – a set of genes consists of a population of proposed solutions. This method produces random sequences from the particular solutions to create a preliminary population. The initialization takes place in a random way in order to account for the whole array of feasible solutions in the area of searching.
- Selection and fitness value – particular genomes are chosen from living population in order to raise the next generation. Single solutions, called fitter solutions, are chosen by using a fitness function inside fitness-based procedure. Methods of choice are used in order to evaluate the fitness of all solutions. For the purpose of giving feedback the fitness value is created, so the GA may discover the optimal solution.
- Crossover – this genetic operator adopts at least one parent solution and produces a child solution. Parent chromosome genes are taken, and more offspring is generated. Particularly, the operator chooses a random point of intersection. By taking points of intersection, the efficiency of the GA can be enhanced by using certain intersection on appropriate issues.
- Mutation – this operator modifies a singular bit in the random way in the actual offspring created from the crossover operator. The method necessitates forming a new guess by mutating the available one. The parent string is transformed into the table consisting of parent strings. After replacement of one letter in

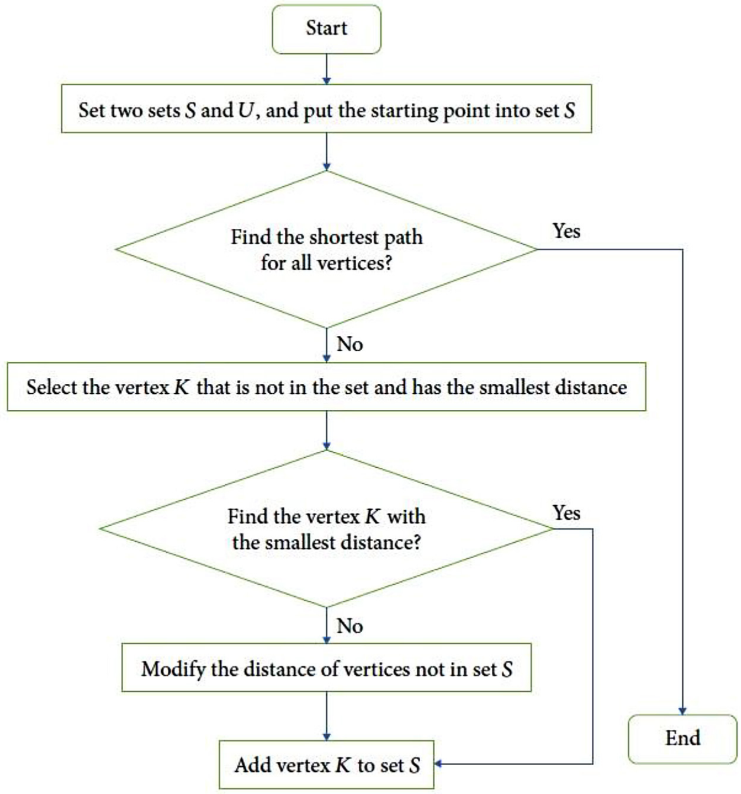

**Fig. 5**. Flowchart of the Dijkstra's algorithm [37]

this table with a letter chosen randomly from the set of genes, the outcome is recombined forming a new string.

- Termination – total process is repeated till the noted solution has been discovered, the population of $n$-iterations has not modified, or some time and generations passed.

This algorithm is presented in Figure 6. More information about GA can be found in [43].

## Bayesian networks algorithm

Bayesian networks were introduced in 1985 by J. Pearl to make a point of the individual character of the input data, the dependence on Bayes' conditioning qua the fundamental for upgrading data as well as the difference among causative and evidentiary ways of thinking [44]. Bayesian networks (BNs), called Bayesian Belief Networks, constitute a type of statistic methods known as progressive graphic models. These models may present probabilistic connections among parameters [45]. The structure of Bayesian network contains two sections: a qualitative section in terms of a directed graph and a quantitative section, by means of tables of conditional probability [46]. More precisely, a directed graph includes edges and nodes which are directed. The parameters in this algorithm are expressed in the terms of nodes and directed edges among the nodes show informative or causative relationships between the parameters [47]. The most important constraint in Bayesian network is the assumption of acyclicity of directed graph, this means that the edges can neither form loops nor cycles inside network [48].

The construction of a Bayesian network is determined schematically. In this method the variables (nodes) are linked through unidirectional arrows (arcs). The BN algorithm is created as a causative construction in which A-node influences B-node that consequently can influence C-node. Then, A is called a parent of B, while B is called a child of A. Next B will become a parent of C, usually called an intermediate node, see Figure 7. This BN structure may be described by a conceptual or influence "box and arrow" chart. In this case, the network involves a group of probabilities, more precisely one probability for every node, determining if node will be in a special state established the states of those nodes which influence it immediately, creating a complete BN. These sets of probabilities are known as tables of conditional probability and they are applied in order to represent as well as compute the relationships between nodes [49, 50].

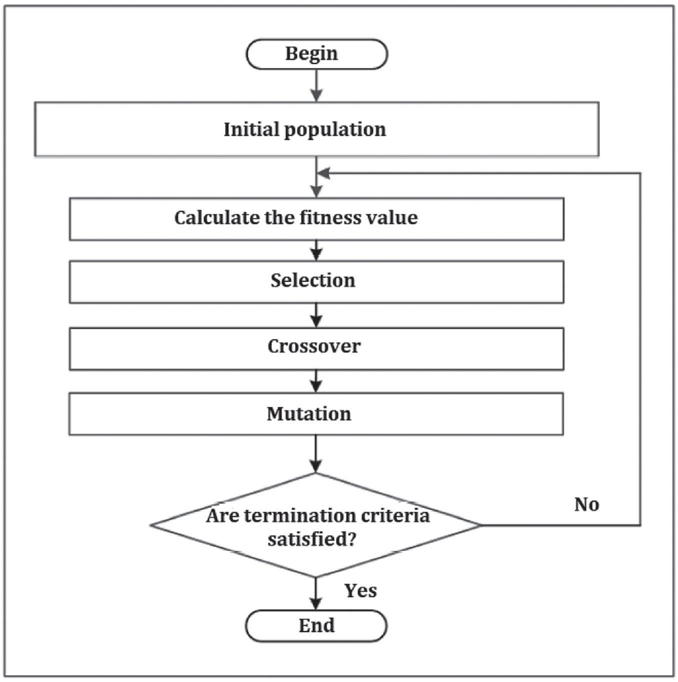

**Fig. 6**. Flowchart of the standard genetic algorithm [43]

## THE ALGORITHMS OPTIMIZING THE INTEGRATED URBAN WATER MANAGEMENT SYSTEM

### Optimization of stormwater management system

The hanging gardens algorithm is fitted to fulfill all restrictions of urban drainage pipe network design. This algorithm may contain suitable optimizing algorithms searching for the optimal setup combined with optimal DC. The hanging gardens algorithm (HGA) was introduced in order to produce and optimize (de)centralized drainage systems in the cities for steep and flat areas [31]. This algorithm was used by Hanghighi and others in comparison with Ahvaz, a city in Iran. Despite of the fact that their outcomes presented that the suggested method may both create real structures and produce nearly optimal answers, the researchers merely took into account the construction costs as the objective function [33, 32].

The loop-by-loop cutting algorithm is introduced in order to form possible sewerage layouts using the base graph. By means of this method, total restrictions of the sewerage setup subproblem are regularly solved. The optimum layout is attained by using an objective function and applying a straight-forward genetic algorithm. Next, the sewerage configuration is established, whereas the descriptions for pumps and pipes are projected using discrete differential dynamic programming model. The loop-by-loop cutting algorithm is particularly helpful for the project of urban drainage systems in flat areas. Moreover, this algorithm is computationally effective, as well as simple to apply and incorporate to the optimization solvers [31, 30].

The genetic algorithm was introduced in order to optimize the designing of rainwater networks. In this model, by the decision variables, the nodal elevations of the sewage network are assumed. The code of simulation of steady-state is applied to examine the experimental solutions given by the GA optimizer. These algorithms are fundamentally built for unlimited optimization issues. The use of GA for restricted optimization issues, namely stormwater networks, demands a transformation of the fundamental restricted issue to an unrestricted optimization issue [42, 51, 52].

**Fig. 7**. Basic casual structure of a Bayesian network [49]

Because of storms, large amount of water comes out from the sewage system through a combined sewage overflow. Then, a path is established towards to certain overflow (aim) in the sewages for every source (node). Every path is of a specific length or "costs". For each node, the cheapest path to the combined overflow structure is established by the Dijkstra's algorithm. Implementation of this method demands allocating the cost among any two nodes. As sewage systems flow primarily via gravity, negative inclinations definitely raise the cost of extraction. Consequently, a modified length may be applied to express the cost among arbitrarily two nodes [53, 54].

## Optimization of sanitary sewage

The loop-by-loop cutting algorithm produces a possible sanitary sewerage structure that fulfils all the constraints totally by applying graph theory ideas and methodologies with no extra assumptions. This algorithm is a method which requires two variables. They are necessary to establish the sewerage connection which should be cut in particular loop and as a consequence the manhole which should be cut. In accordance with the values of variables, the various structures are achieved. This can be done using the trial-and-error method. Another way is to use an optimizing algorithm to which GA is applied. The loop-by-loop cutting algorithm is applied to the layout production to minimize the system construction cost. By means of this algorithm, the restrictions and bounds of the sewer networks layout are immediately met by, step-by-step, opening the loops of an initial base graph [21, 55].

The Dijkstra's algorithm is employed to obtain the layout of a sanitary sewerage system by applying the shortest-path spanning tree and minimum spanning tree algorithms [21].

The genetic algorithm in sanitary sewage is applied as the optimization tool. The target is to minimize the total cost, to minimize the residence time of the water within the system and also to maximize the network reliability metric [21, 56].

The layout of sanitary sewage network resembles a tree, thus the method called Tree Growing Algorithm is applied to build a tree layout using the base graph which describes the network named a feasible layout. TGA is applied to successfully solve the sanitary sewer network layout and size optimization issue with pipe diameters taken as decision variables [56, 57].

A Bayesian network algorithm is applied to evaluate the risk of public health connected to sanitary sewage overflows. Especially, the ability for the method to account for the uncertainty inherent in sewer overflow events and subsequent impacts through the use of probabilities is a valuable function. Moreover, there are advantages of the probabilistic inference function of the Bayesian network concerning prioritizing the regulation possibilities to minimize the risk of public health related to sanitary sewage overflows [58].

## Optimization of water distribution system

Genetic algorithm is the method used to optimization of water supply system. This model is an essential part of intelligent diagnostic configuration applied to the local water supply system. Water leakage detection and localization constitute the major role of the afore-mentioned process. In the case of inputs, the structure employs the data from sensors of flow or sensors of pressure, installed on the pipeline network, whereas the output is the part of data concerning leakage detecting as well as the position. The major advantage of this system is an opportunity to estimate leakage localization only by a finite number of assembled sensors [59, 60].

The idea to determine a fixed water measurement of a water supply network consistent with the hydraulic parameters of the system relies on the rules of the graph theory and, particularly, on the recognition of minimal dissipated power paths, namely the shortest paths and evaluated via Dijkstra's algorithm. This idea, which starts with the shortest paths, at the beginning determines the major network layout and subsequently draws the major graph with specific properties [61, 62].

## Optimization in IUWMS

The application of all characterized algorithms in particular elements of integrated urban water management system is included in Table 1. The analysis shows that the described algorithms are primarily used both in sanitary sewage and stormwater. However, they are less often applied in water distribution system. It is worth mentioning that these algorithms have not yet been applied in source of supply, processing or water and wastewater treatment. On the other hand, the most popular algorithms used in IUWMS are genetic algorithm and Dijkstra's algorithm. Both of them are adapted in stormwater, sanitary sewage and water distribution systems.

**Table 1**. The application of algorithms in IUWMS

| Algorithm | Application |
|---|---|
| Hanging Gardens Algorithm | stormwater [31-33] |
| Loop-By-Loop Algorithm | stormwater [31, 30], sanitary sewage [21, 55] |
| Genetic Algorithm | stormwater [42, 51, 52], sanitary sewage [21, 56], water distribution system [59, 60] |
| Bayesian Networks Algorithm | sanitary sewage [58] |
| Dijkstra's Algorithm | stormwater [53, 54], sanitary sewage [21], water distribution system [61, 62] |
| Tree-Growing Algorithm | sanitary sewage [56, 57] |

## CONCLUSIONS

The presented review of the scientific literature on the use of algorithms based on graph theory in the modeling of the issues concerning integrated urban water management systems organizes and classifies many issues in this field, both in terms of their subject matter and the algorithm used. Thus, the novelty of this article may constitute a starting point for further exploration of the possible application of existing graph algorithms in the issues of urban water management systems, as well as the development of new algorithms modeling selected issues based on other sciences. In the latter case, it would also be interesting to generalize such algorithms and describe them using the universal language of graph theory, enabling their application in other fields of science. Another possible direction for further research may become the use of objects even more universal than graphs for modeling issues of water management systems. These are hypergraphs, the properties and advantages of which are already used in modeling complex systems, including in chemistry, electrical engineering and mechanics.

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