

Daria WOTZKA\*

## **THEORETICAL ANALYSIS OF OPTICAL RADIATION ABSORBED IN A LIGHTING DEVICE**

The article presents results of theoretical analysis performed using computer simulations based on the finite element method. The study involved analysis of propagation of optical radiation emitted by multiple LEDs installed in a lighting device.

The study considered calculation and analysis of optical radiation propagation in the structure of a lamp and its surroundings. Based on the achieved results, the source and amount of losses of optical radiation that is absorbed by the lamps structure elements were determined.

KEYWORDS: finite element method, numerical modeling, ray propagation

### **1. INTRODUCTION**

LED lamps are commonly know devices which are applied almost everywhere. In this paper LED obstruction lighting is considered. This kind of light becomes nowadays more and more important due to expansion of the aerial navigation [1]. Aircraft warning lights are various-intensity lighting devices that are attached to tall structures as e.g. transmission lines, wind turbines or towers, for making them more visible to aircraft. A suitable obstruction lights, which are constantly illuminated, prior to their application in the industry need to satisfy requirements according to national/international standards. Standards and air traffic rules are established by the International Civil Aviation Organization (ICAO) [2] and are usually applied worldwide by the National Civil Aviation of each country. These standards state the different patterns, colours, luminous intensity as well as placement of warning lights.

Application of computer simulations for complex systems modeling enables one to identify and understand their operation at a relatively low cost, due to iterative correction, testing, optimization without creating a real prototype [3]. In the research studies, result of which are presented in this paper, computer simulations were applied for theoretical studies of ray propagation phenomena and in particular for analysis of rays absorbed in the structure of a LED obstruction light. Information about the amount of rays lost is crucial when designing an energy efficient device.

---

\* Opole University of Technology.

## 2. MATERIALS AND METHODS

### 2.1. Solution idea and the object under study

A prototype LED obstruction lamp from BlueSoftSTC [4] was investigated. The lamp is powered by a number of LEDs, type LACP7P [5]. The LEDs emit light rays of vacuum wavelength 617 nm, which propagate in the lamp. The majority of rays is then reflected by the mirror and propagates parallel to the horizontal axis in space. But some of the rays are not reflected or are absorbed by the Plexiglas, which is part of the housing. The aim of this research was to define the sources of ray absorption and to calculate the amount of losses.

In order to minimize the computational effort a quarter of the cylindrical device was considered in calculations. In Fig 1 the domain under study: the LED lamp and the surrounding environment–air is depicted.

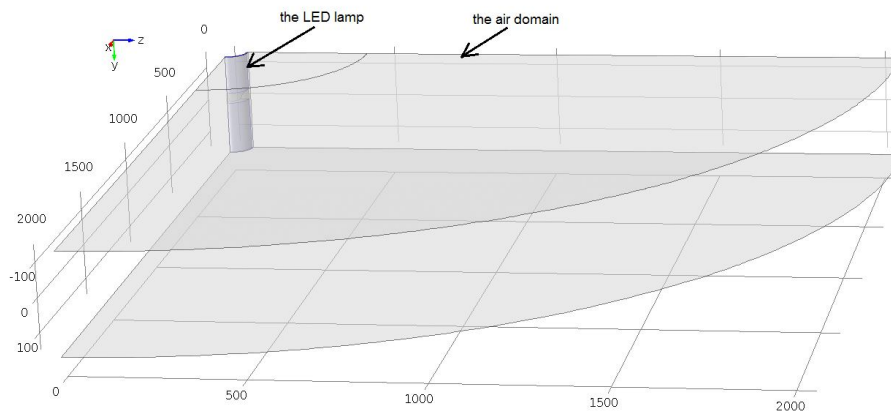


Fig. 1. The domain under study: the LED lamp and the surrounding air

The simplified CAD drawing of the LED lamp is depicted in Fig. 2 to the left. The lamp has a cylindrical shape of radius 54.47 mm. It is covered with a Plexiglas of width: 5 mm. Several LEDs are placed at the focus location of a mirror of parabolic shape. The mirror dimensions are: height: 31.64 mm, width: 24.47 mm.

### 2.2. Discretisation mesh

For calculations the FEM method was applied. The object under study was divided into a finite number of elements with the preset size  $\Delta x$ , which create the so-called mesh. The FEM method enables for approximation of the dependent variable with a function described with a specific number of

parameters called degrees of freedom (DoF) in each element of the mesh. For every node  $x_j$  in the mesh a  $DoF_j = p(x_j)$  and a polynomial basis function  $\varphi_j(x)$  are defined. In this way a set of  $p(x)$  functions creating the function space called the finite element space is expressed by (1).

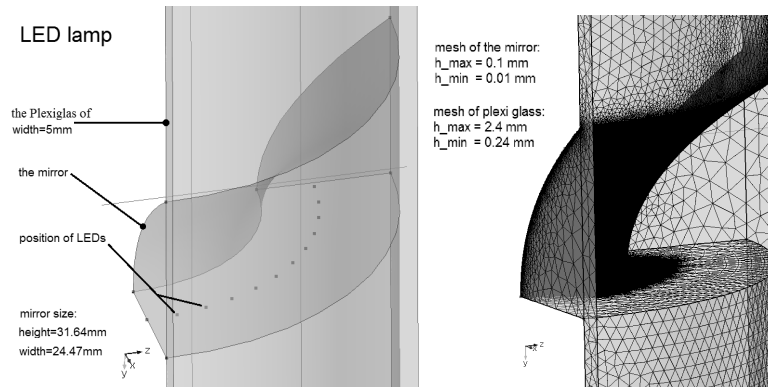


Fig. 2. Left: CAD drawing of the LED lamp, right: the object divided into finite elements

$$p(x) = \sum_{v_j} DoF_j \varphi_j(x) \quad (1)$$

The vector **DoF** includes all  $DoF_j$  elements and constitutes the solution, that is the calculation result. The no of finite elements within the object under study depends from the elements size and is the key parameter in numerical modeling, which determines the resulting model accuracy. Taking into account available computer resources, one needs to optimize the mesh size since a growing number of elements is directly related to the increase in RAM consumption and computation duration.

## 2.3. Mathematical equations used for numerical calculations

### 2.3.1. The equations used for ray calculation

The wave vector and frequency are analogous to the generalized momentum  $\mathbf{p}$  and Hamiltonian  $H$  of a solid particle [6]. Based on this analogy the ray trajectory can be computed by solving six coupled first-order ordinary differential equations for the components of  $\mathbf{q}$  and  $\mathbf{k}$ , as defined by (2) and (3).

$$\frac{d\mathbf{q}}{dt} = \frac{\partial \omega}{\partial \mathbf{k}} \quad (2)$$

where  $\mathbf{q} \in \{q_x, q_y, q_z\}$  – position of the ray in space,  $\mathbf{k} \in \{k_x, k_y, k_z\}$  – vector of wave number components in space,  $\omega$  – angular frequency [rad/s],  $t$  – time [s].

$$\frac{d\mathbf{k}}{dt} = \frac{\partial \omega}{\partial \mathbf{q}} \quad (3)$$

$$\frac{ds_{i,0}}{ds} = -2|\mathbf{k}|\kappa s_{i,0} \quad \text{for } i \in \{0,1,2,3\} \quad (4)$$

where:  $\kappa$  is the imaginary part of the refractive index [-],  $s_{i,0}$  – the initial Stokes parameters [W/m<sup>2</sup>],  $s_i$  – the Stokes parameters [W/m<sup>2</sup>].

$$s_i = \frac{r_{1,0}r_{2,0}}{r_1r_2} s_{i,0} \quad (5)$$

where:  $r_{i,0}$  – the initial radius of curvature [m],  $r_1, r_2$  – the radius of curvature [m].

$$\frac{dr_1}{ds} = -1 \quad (6)$$

$$\frac{dr_2}{ds} = -1 \quad (7)$$

$$\frac{d\psi}{ds} = |\mathbf{k}| \quad (8)$$

where:  $\psi$  – the phase of the ray [rad].

$$\frac{dL}{ds} = n \quad (9)$$

where:  $L$  – the optical path [m],  $n$  – refractive index, real part [-].

The four Stokes parameters characterize the intensity and polarization of a ray [7]. They can be interpreted as indicators of the ray intensity that would be measured when sending a ray through various arrangements.

### 2.3.1. The equations used for specular reflection at the mirror surface

$$\mathbf{n}_r = \mathbf{n}_i - 2 \cos(\theta_i) \mathbf{n}_s \quad (9)$$

$$k_{u,r} = k_{u,i} - 2k_{u,s} \cos(\theta_i) \mathbf{n}_s \quad (10)$$

$$k_{uv,r} = -k_{uv,i} + 2k_{uv,s} \quad (11)$$

$$k_{v,r} = k_{v,i} - \frac{2}{\cos(\theta_i)} k_{v,s} \quad (12)$$

$$\psi_r = \psi_i + \arg(r) \quad (13)$$

where:  $\mathbf{n}_r$  – reflected wavefront normal,  $\mathbf{n}_i$  – normalized ray direction vector [1],  $\mathbf{n}_s$  – material discontinuity normal [1],  $k_{u,r}, k_{uv,r}, k_{v,r}$  – reflected curvature tensor components [1/m],  $k_{u,s}, k_{uv,s}, k_{v,s}$  – surface curvature tensor components [1/m],  $k_{u,i}, k_{uv,i}, k_{v,i}$  – incident wavefront curvature tensor comp. [1/m],

$\psi_r$  – phase of the reflected ray [rad],  $\psi_i$  – phase reinitialization variable [rad],  $\theta_i$  – angle of incidence [rad],  $r = 0.98$ , reflection coefficient [–].

### 2.3.2. The equations used for wall accumulating the rays

$$\mathbf{k} = \mathbf{k}_c \tag{14}$$

where:  $\mathbf{k}_c$  is the ray wave vector when striking the wall. All rays are "frozen" after reaching the wall.

The time derivative of the accumulated variable depends on the instantaneous ray positions, as given by (15).

$$\frac{\partial(ACU_x)}{\partial t} = \sum_{j=1}^{N_t} R_j \tag{15}$$

Where:  $ACU_x$  is accumulated variable for one of the following:  $ACU_1$  – rays absorbed by the Plexiglas and forwarded to the upper part of the lamp, boundary: *plexi\_up*,  $ACU_2$  – rays absorbed by the Plexiglas and forwarded to the lower part of the lamp, boundary: *plexi\_dawn*,  $ACU_3$  – rays not reflected by the mirror and forwarded to the upper part of the surrounding air domain, boundary: *not\_refl\_up*,  $ACU_4$  – rays reflected by the mirror and forwarded outside of the lamp direction the upper part of the surrounding air domain, boundary: *refl\_up*,  $ACU_5$  – rays reflected by the mirror and forwarded outside of the lamp direction straight to the surrounding air domain at a distance of 2 m, boundary: *refl\_straight*,  $R_j$  – source equal to 1 for the  $j$ -th mesh element [cd],  $N_t$  – total nr of rays.

### 2.4. Realization of ray source

Rays are emitted by LEDs, which are situated as depicted in Fig. 2 left. Each LED is represented by 24 conical light sources (various cones of angles between 0 – 70 deg) of various intensity in order to satisfy parameters of the real device [5]. The intensity was determined based on the datasheet [5] as no of rays emitted for particular cone shaped source of specified angle, as depicted in Fig. 3.

Each of the LED source components emitted rays of power equal to 0.7 W. The light was unpolarized. The equation applied for calculation of the initial wave vector is given by (16).

$$\mathbf{k} = k_0 \frac{\mathbf{L}_0}{|\mathbf{L}_0|} \tag{16}$$

where:  $\mathbf{L}_0$  – ray direction vector [–],  $k_0 = 2\pi / f$  – wave number in free space,  $f = 617$  [nm].

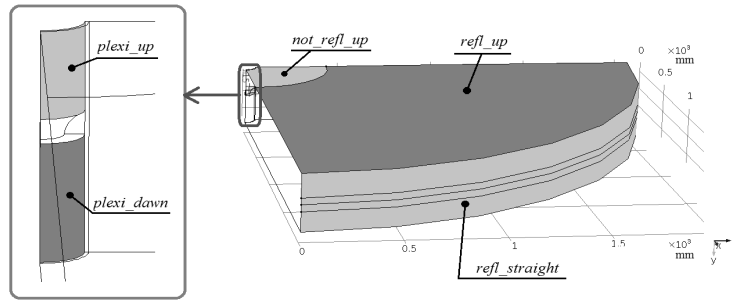


Fig. 3. Designation of the boundary area: *not\_refl\_up*, *refl\_up*, *refl\_straight*, *plexi\_up* and *plexi\_down*

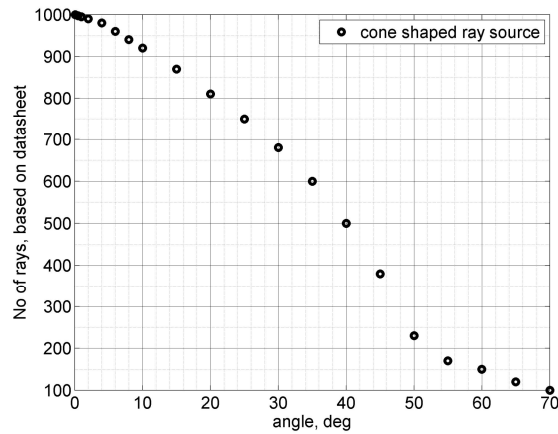


Fig. 3. No of rays emitted by cone shaped ray sources

## 2.5. Model parameters

The lamp is filled with air as well as the surrounding is air. The lamp cover is made from Plexiglas. The parameters important for ray tracing calculation are summarized in Table 2.1.

Table 2.1. The parameters used in calculations

Lp.	Material type	Name of the parameter	Value
1.	air	Refractive index, real part	$n=1$
2.	air	Refractive index, imaginary part	$\kappa_i=0$
3.	PMMA	Refractive index, real part	$n=1.49$
4.	PMMA	Refractive index, imaginary part	$\kappa_i=0$

### 3. RESULTS AND DISCUSSION

For the specified numerical model computer simulations in the time domain were carried out. The time steps were specified in ns from 0 to 10 with step of 0.05 ns. A stop condition was initiated after no more active rays remained in the model, what happened when emitted rays reached the most distant wall. In Fig. 4 and 5 ray trajectories at time steps  $t = 0.01$  ns and  $t = 7$  ns are depicted respectively. From Fig. 4 one can clearly recognize the rays emitted by the particular LEDs. After 7 ns the stop condition was activated and simulation stopped. From Fig. 5 it is no more possible to determine how many and where the rays fall, thus the domain walls were divided into parts and the accumulated variables were there determined.

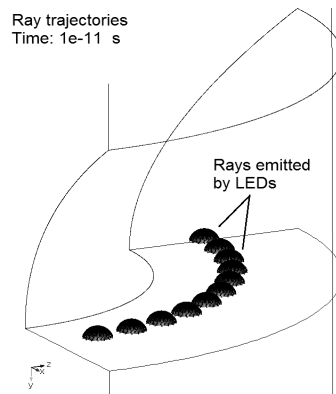


Fig. 4. Calculation result depicting ray trajectories at time step  $t = 0.01$  ns

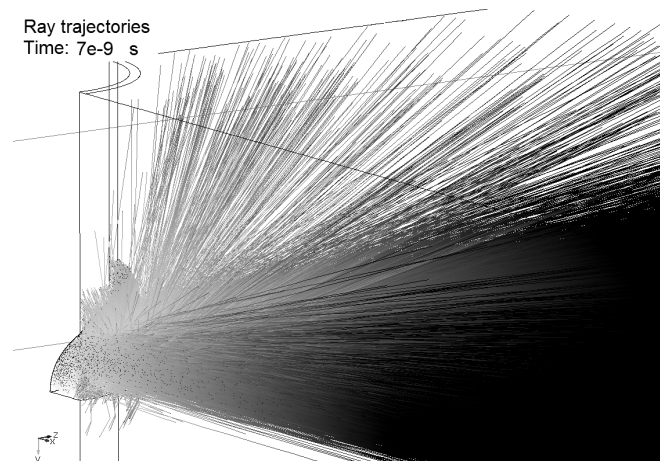


Fig. 5. Calculation result depicting ray trajectories at time step  $t = 7$  ns

In Fig. 6 results of calculations in the first 7 ns of subsequent time steps are presented: the four considered variables, associated with the number of rays reaching the defined boundary area are depicted. No of rays absorbed in the plexiglass that reach the upper part of lamp is relatively high as compared to the no of rays reaching the upper part of the domain without reflection.

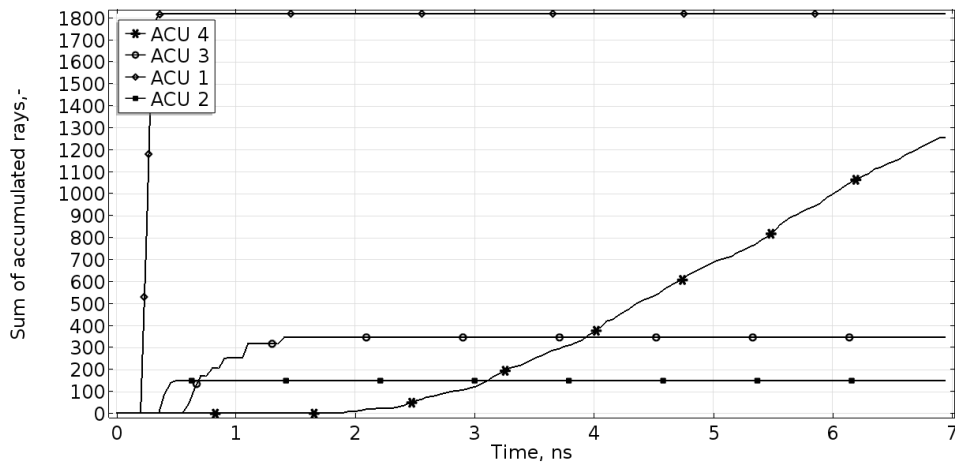


Fig. 6. The overall no of rays accumulated at various boundaries

In Table 3.1 values gathered from simulation at the last time step  $t = 6.95\text{ns}$ , when no more rays were active in the domain.

Table 3.1. The calculated no of rays at the considered boundaries

Boundary name	<i>plexi_up</i>	<i>plexi_dawn</i>	<i>t_refl_up</i>	<i>refl_up</i>	<i>not_refl_up</i>
Variable name	<i>ACU 1</i>	<i>ACU 2</i>	<i>ACU 5</i>	<i>ACU 4</i>	<i>ACU 3</i>
Value [-]	1 820.1	150.01	71 871	1 254	347

Relatively much radiation ( $plexi\_up = 1\,820$ ) is reflected by the mirror and then gently broken by the plexiglass to the upward direction of the domain. A small amount of rays ( $not\_refl\_up = 347$ ) propagates directly, without any reflection, in the upper region of the lamp, the smallest volume ( $plexi\_dawn = 150$ ) propagates dawn. By far the most number of rays fall after reflection from the mirror directly on the wall ( $t\_refl\_up = 71\,871$ ) and relatively significantly less radiation falls after reflection from the mirror in the upper area of the lamp ( $refl\_up = 1\,254$ ).



**REFERENCES**

- [1] Ochoa C.E., Aries M.B.C., Hensen J.L.M., State of the art in lighting simulation for building science: a literature review, *Journal of Building Performance Simulation*, Vol. 5, np. 4, pp. 209 – 233, 2012, DOI: 10.1080/19401493.2011.558211.
- [2] International Civil Aviation Organization, <http://www.icao.int/about-icao/Pages/default.aspx>, Jan 2017.
- [3] Pérez-Ocóna F., Pozoa A.M., Rabazab O., New obstruction lighting system for aviation safety, *Engineering Structures*, Vol. 132, pp. 531 – 539, 2017, DOI:10.1016/j.engstruct.2016.11.054.
- [4] BlueSoft, <http://www.bsstc.pl/>, Jan 2017.
- [5] OSLOM SSL 80, LA CP7P Datasheet.
- [6] Landau L.D., Lifshitz E.M., *The Classical Theory of Fields*, 4th ed., Elsevier, 2005.
- [7] Born M., Wolf E., *Principles of Optics*, 7th ed., Cambridge, 1999.

(Received: 20. 01. 2017, revised: 14. 02. 2017)