

Peter KAŠŠAY¹

COMPARISON OF PNEUMATIC FLEXIBLE SHAFT COUPLING STATIC LOAD CHARACTERISTICS OBTAINED EXPERIMENTALLY AND BY CALCULATION

Summary. Pneumatic flexible shaft couplings allow the change of their stiffness by the pressure change of gaseous medium in their pneumatic flexible elements, allowing their use as a device for continuous tuning of torsional oscillating mechanical systems directly during operation. The aim of this paper is to verify the accuracy of compression volume modeling method on existing pneumatic flexible coupling, by comparison of static load characteristics obtained experimentally with characteristics calculated from mathematical and physical model.

Keywords: pneumatic flexible shaft coupling, pneumatic springs, static properties, volume of pneumatic springs, mathematical and physical model

PORÓWNANIE STATYCZNYCH CHARAKTERYSTYK OBCIĄŻENIOWYCH PNEUMATYCZNYCH SPRZĘGIEŁ PODATNYCH UZYSKANYCH EKSPERYMENTALNIE I OBLICZENIOWO

Streszczenie. W Zakładzie Projektowania i Części Maszyn Wydziału Mechanicznego Uniwersytetu Technicznego w Koszycach prowadzone są prace dotyczące badań i rozwoju pneumatycznych sprzęgieł podatnych. Pneumatyczne sprzęgła podatne umożliwiają zmianę ich sztywności przez zmianę ciśnienia medium gazowego w podatnych elementach pneumatycznych. Celem niniejszego artykułu jest sprawdzenie dokładności metody modelowania objętości przestrzeni sprężania wybranego pneumatycznego sprzęgła podatnego przez porównanie statycznych charakterystyk obciążeniowych uzyskanych eksperymentalnie z obliczonymi na podstawie modelu matematyczno-fizycznego.

Słowa kluczowe: pneumatyczne sprzęgło elastyczne łączące wały, sprężyny pneumatyczne, właściwości statyczne, objętość sprężyn pneumatycznych, model matematyczno-fizyczny

1. INTRODUCTION

On our department we deal with the research and development of power train [1-15], especially pneumatic flexible shaft couplings. Pneumatic flexible shaft couplings allow the

¹ Department of Machine Design, Transport and Logistic, Faculty of Mechanical Engineering, The Technical University of Košice, Slovak Republic, e-mail: peter.kassay@tuke.sk

change of their stiffness by the pressure change of gaseous medium in their pneumatic flexible elements, allowing their use as a device for continuous tuning of torsional oscillating mechanical systems directly during operation.

The static properties of already manufactured pneumatic couplings are determined experimentally. The properties of flexible couplings on design are determined by a mathematical and physical model based on isothermal pressure changes in the compression space of pneumatic flexible elements. To create such a model, it is necessary to know the dependence of the pneumatic coupling's compression volume on the twist angle.

The aim of this paper is to verify the accuracy of compression volume modelling method on existing pneumatic flexible coupling, by comparison of static load characteristics obtained experimentally with characteristics calculated from mathematical and physical model.

2. INVESTIGATED PNEUMATIC FLEXIBLE SHAFT COUPLING

For description and the new method accuracy verification we will use pneumatic flexible shaft coupling type 3 – 1/110 – T – C manufactured by FENA company was used for the verification of the accuracy and description of the new method. This pneumatic flexible coupling (*fig. 1*) consists of driving hub (1) and a driven hub (2), connected by three pneumatic flexible elements (4). The compression spaces of pneumatic elements are interconnected by tubes (3). Because the flexible elements of this coupling are always simultaneously compressed or expanded, this coupling can transmit the torque only in one direction (in the direction of compression). The properties of this coupling were determined experimentally at our department [9]. The measured static load characteristics are presented on (*fig. 2*).

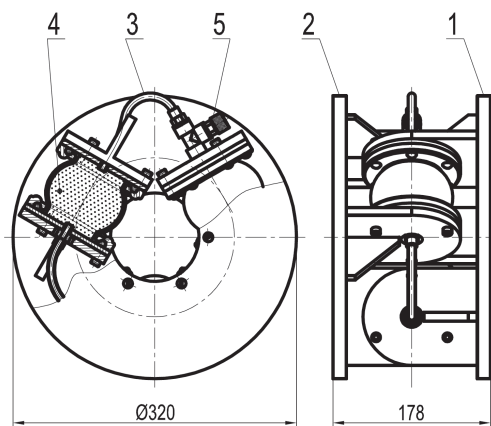


Fig. 1. Pneumatic flexible shaft coupling type 3 – 1/110 – T – C.

Rys. 1. Elastyczne sprzęgło pneumatyczne łączące wały typu 3 – 1/110 – T – C

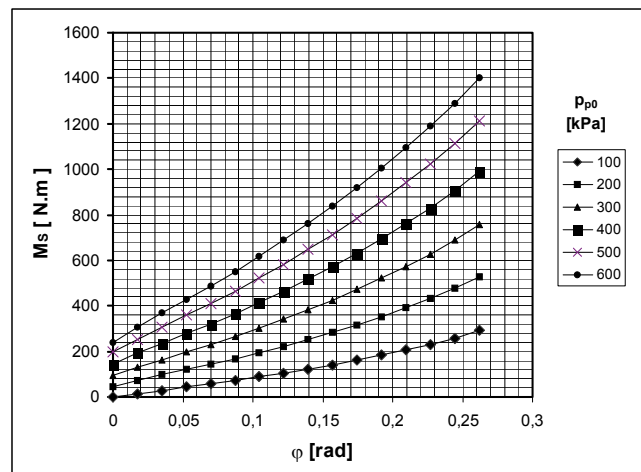


Fig. 2. Static load characteristics

Rys. 2. Charakterystyki statyczne obciążenia

The torque of pneumatic flexible shaft coupling M_S is a sum of pneumatic flexible element rubber shell torque M_G and torque M_V from the overpressure of gaseous medium enclosed in the compression space of coupling [9]. For the comparison with mathematically obtained static load characteristic we will use only torque M_V [9].

3. MATHEMATICAL MODEL OF PNEUMATIC FLEXIBLE SHAFT COUPLING

Investigated pneumatic flexible shaft coupling is shown on *fig. 3*.

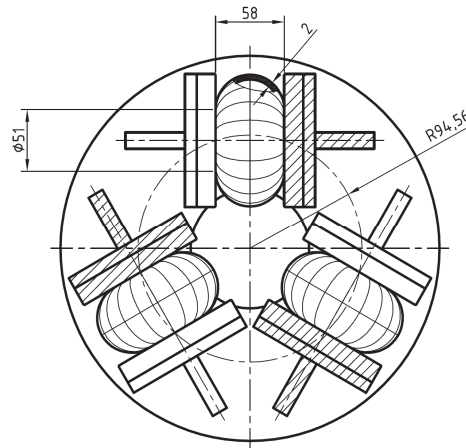


Fig. 3. Flexible shaft couplings type 3 – 1/110 – T – C

Rys. 3. Sprzęgło elastyczne pneumatyczne typu 3 – 1/110 – T – C

All dimensions necessary to calculate the static load characteristics are shown on *fig. 3*.

For static load characteristics computations the following conditions were considered:

- compression volumes of wedge elements are interconnected,
- neutral surface of the bellow lies in the middle of the tire's thickness,
- the length of meridial fibres of neutral surface was considered constant [8],
- the contact surface between elements and hubs is planar,
- in the part where flexible elements do not touch the supporting surfaces, meridial fibres of neutral surface are circular arcs [8], touching the equidistants of supporting surfaces,
- under static loading, the gas compresses and expands isothermally [8],
- equal absolute values of loading torque work and mechanical work of compressing air.

4. GEOMETRY OF PNEUMATIC FLEXIBLE ELEMENT

For the computation of static properties of pneumatic flexible coupling it is necessary to know the geometric properties of pneumatic flexible elements bellow depending on twist angle [8], [9].

On *fig. 4* is shown a scheme of couplings supporting surfaces in plane Θ , which is the elements plane of symmetry and it is perpendicular on the couplings rotation axis. Planes Γ and Γ' are planes which passes trough the bellows neutral surface in the contact area between the bellow and supporting surface. On *fig. 5* is shown the location of planes Γ and Γ' in neutral position and by twist angle φ . Plane A is the plane of symmetry between planes Γ and Γ' .

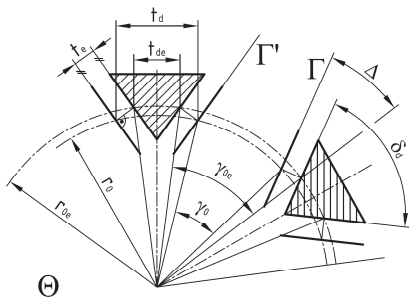


Fig. 4. Determining the dimensions of neutral surface from external dimensions

Rys. 4. Określenie rozmiarów powierzchni neutralnej na podstawie powierzchni zewnętrznej

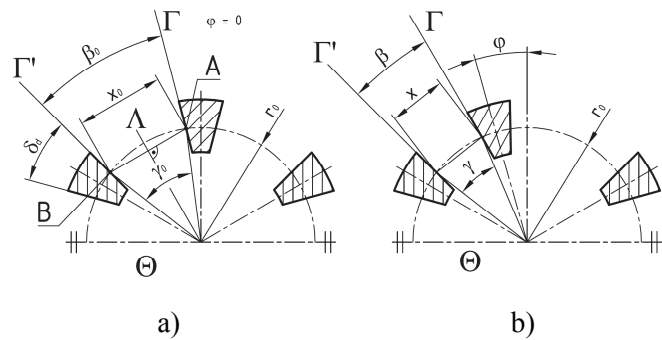


Fig. 5. Determining the dimensions of neutral surface in neutral position (a) and by maximum twist angle (b)

Rys. 5. Określenie rozmiarów powierzchni neutralnej w pozycji neutralnej (a), przy maksymalnym skręceniu (b)

For known number of elements i , thickness of supporting surface t_{de} , pitch radius of elements r_0 and supporting surface angle δ_d we can compute angles Δ and β_0 in the neutral position of coupling:

$$\beta_0 = \frac{2\pi}{i} - \delta_d \quad (1)$$

$$\Delta = \frac{\delta_d}{2} - \arcsin\left(\frac{t_{de}}{2 \cdot r_{0e}}\right) \quad (2)$$

Then for the known bellows external thickness t_e , t_d and r_0 for the neutral surface are:

$$t_d = t_{de} + 2 \cdot t_e \cdot \cos\left(\frac{\delta_d}{2}\right) \quad (3) \quad r_0 = \sqrt{(r_{0e} - t_e \cdot \sin \Delta)^2 + (t_e \cdot \cos \Delta)^2} \quad (4)$$

Then we determine the angles γ_0 and γ_{0e} :

$$\gamma_0 = \frac{2\pi}{i} - 2 \cdot \arcsin\left(\frac{t_d}{2 \cdot r_0}\right) \quad (5) \quad \gamma_{0e} = \frac{2\pi}{i} - 2 \cdot \arcsin\left(\frac{t_{de}}{2 \cdot r_{0e}}\right) \quad (6)$$

For angles β and γ by twist angle φ applies:

$$\gamma = \gamma_0 - \varphi \quad (7) \quad \beta = \beta_0 - \varphi \quad (8)$$

Points A , B are the centres of elements flanges on the neutral surface of elements bellows. Their flowline is the axis of element bellows neutral surface. The distance x of points A , B is determined as:

$$x = 2 \cdot r_0 \cdot \sin\left(\frac{\gamma}{2}\right) \quad (9)$$

The element bellows geometry is determined in the section of plane Π , which passes through the flowline of points A and B . The location of plane Π is determined by angle ξ (fig. 6)

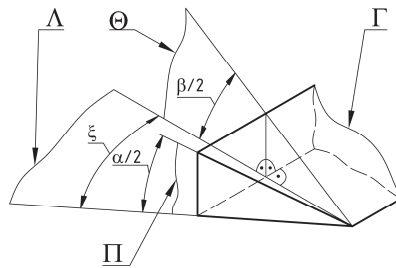


Fig. 6. Relative position of flexible elements bellows planes
 Rys. 6. Wzajemne położenie powierzchni płaszcza elementu elastycznego

The angle between support surfaces α in plane Π is described by formula:

$$\alpha = 2 \cdot \arctg \left[\cos(\xi) \cdot \tg \left(\frac{\beta}{2} \right) \right] \quad (10)$$

On *fig.7* is shown the form of meridial fibre of bellows neutral surface in plane Π , on *fig.8a* is shown the bellow in section, on *fig.8b* is shown the cross-sectional of area between one side of internal surface and the axis of element bellows neutral surface.

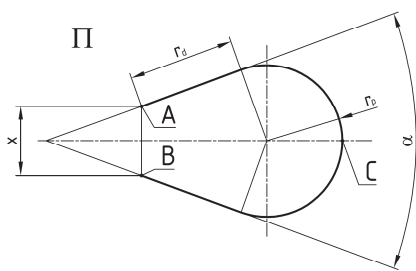


Fig. 7. Meridial fibre of neutral surface in plane Π
 Rys. 7. Włókno południkowe powierzchni neutralnej w płaszczyźnie Π

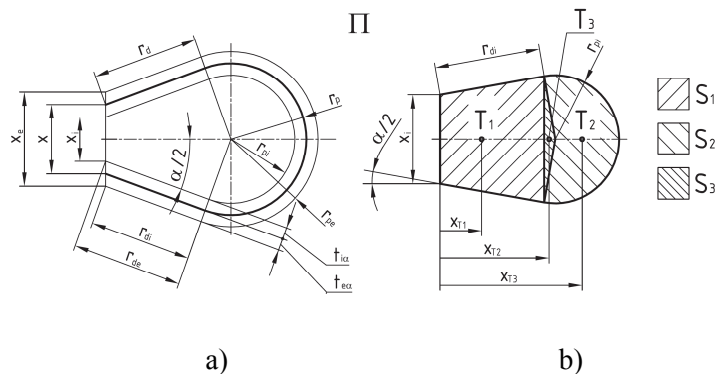


Fig. 8. Determining the characteristics of internal surface in plane Π , dimensional and angular (a) sectional (b)
 Rys. 8. Określenie właściwości długości i kątów (a) przekrojów (b) powierzchni wewnętrznej w płaszczyźnie Π

For the condition of meridial fibres constant length l_{vp} depending on twist angle φ then follows:

$$r_d = \frac{l_{vp} \cdot \cos\left(\frac{\alpha}{2}\right) - \frac{\alpha + \pi}{2} \cdot x}{2 \cdot \cos\left(\frac{\alpha}{2}\right) + (\alpha + \pi) \cdot \sin\left(\frac{\alpha}{2}\right)} \quad (11)$$

$$r_p = \frac{x}{2 \cdot \cos\left(\frac{\alpha}{2}\right)} + r_d \cdot \tg\left(\frac{\alpha}{2}\right) \quad (12)$$

The internal and external distances of elements centres x_i and x_e we compute as:

$$x_i = x - 2 \cdot \frac{t_i}{\cos\left(\frac{\beta}{2}\right)} \quad (13)$$

$$x_e = x + 2 \cdot \frac{t_e}{\cos\left(\frac{\beta}{2}\right)} \quad (14)$$

As plane Π is perpendicular on plane Γ only for $\xi = 0$ and $\xi = 180^\circ$, so the internal and external thickness of bellow (t_{ia} and t_{ea}) depends on the angle ξ (α):

$$t_{ia} = \frac{x - x_i}{2} \cdot \cos\left(\frac{\alpha}{2}\right) \quad (15)$$

$$t_{ea} = \frac{x_e - x}{2} \cdot \cos\left(\frac{\alpha}{2}\right) \quad (16)$$

The internal and external dimensions of bellow in plane Π we determine by formulas:

$$r_{pi} = r_p - t_{ia} \quad (17)$$

$$r_{pe} = r_p + t_{ea} \quad (18)$$

$$r_{di} = r_d + t_{ia} \cdot \operatorname{tg}\left(\frac{\alpha}{2}\right) \quad (19)$$

$$r_{de} = r_d - t_{ea} \cdot \operatorname{tg}\left(\frac{\alpha}{2}\right) \quad (20)$$

Then we compute the areas S_1 to S_3 :

$$S_1 = \left[x_i + r_{di} \cdot \sin\left(\frac{\alpha}{2}\right) \right] \cdot r_{di} \cdot \cos\left(\frac{\alpha}{2}\right) \quad (21)$$

$$S_2 = \frac{1}{2} \cdot r_{pi} \cdot \sin\left(\frac{\alpha}{2}\right) \cdot \left[x_i + 2 \cdot r_{di} \cdot \sin\left(\frac{\alpha}{2}\right) \right] \quad (22)$$

$$S_3 = \frac{1}{2} \cdot (\pi + \alpha) \cdot r_{pi}^2 \quad (23)$$

and the locations of their centres of area x_{T1} to x_{T3} :

$$x_{T1} = \frac{3 \cdot x_i + 4 \cdot r_{di} \cdot \sin\left(\frac{\alpha}{2}\right)}{6 \cdot \left[x_i + r_{di} \cdot \sin\left(\frac{\alpha}{2}\right) \right]} \cdot r_{di} \cdot \cos\left(\frac{\alpha}{2}\right) \quad (24)$$

$$x_{T2} = r_{di} \cdot \cos\left(\frac{\alpha}{2}\right) + \frac{1}{3} \cdot r_{pi} \cdot \sin\left(\frac{\alpha}{2}\right) \quad (25)$$

$$x_{T3} = r_{di} \cdot \cos\left(\frac{\alpha}{2}\right) + r_{pi} \cdot \sin\left(\frac{\alpha}{2}\right) + \frac{4}{3} \cdot \frac{r_{pi} \cdot \sin\left(\frac{\alpha + \pi}{2}\right)}{\alpha + \pi} \quad (26)$$

Now we can determine the internal volume of element V_e by using the static moments of area M_S :

$$M_S = \sum_{i=1}^3 (x_{Ti} \cdot S_i) \quad (27)$$

$$V_e = \int_0^{2\pi} M_S \cdot d\xi \quad (28)$$

5. COMPUTATION OF STATIC LOAD CHARACTERISTICS

For the computation of pneumatic couplings compression volume V we have to sum up the compression volumes of all elements and the volumes of filling and interconnecting tubes. The compression volume V twist angle φ graph is shown on *fig. 9*.

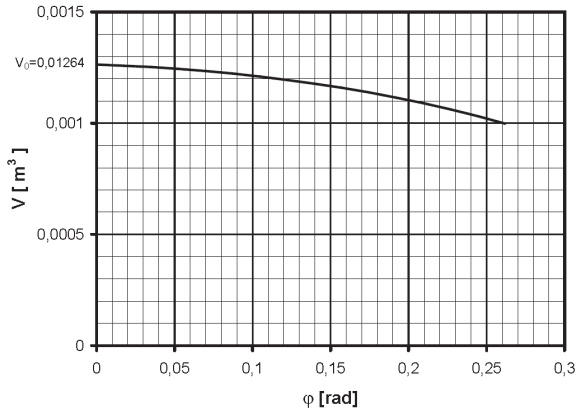


Fig. 9. Volume V twist angle graph
Rys. 9. Zależność objętości V od kąta skrętu

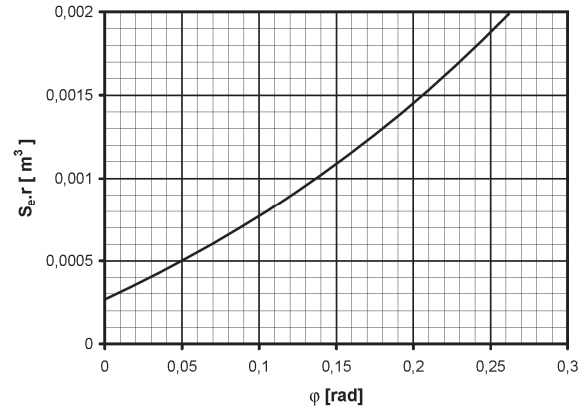


Fig. 10. Static moment of effective area
Rys. 10. Statyczny moment efektywnej powierzchni

Then it is necessary to know the dependency of couplings static moment of effective area $S_e \cdot r$ on twist angle φ (29) shown on *fig. 10*. And finally it is possible to define the static torque of coupling M_V from the overpressure of gaseous medium enclosed in the compression space of coupling (30) depending on the initial overpressure of gaseous media inside of elements for isothermic compression [8], [9]:

$$S_e \cdot r = -\frac{dV}{d\varphi} \quad (29) \quad M_V = \left[(p_{p0} + p_a) \cdot \frac{V_0}{V_0 - \int_0^\varphi S_e \cdot r \cdot d\varphi} - p_a \right] \cdot S_e \cdot r \quad (30)$$

where:

p_{p0} – initial overpressure of gaseous media by couplings neutral position [Pa];

p_a – atmospheric pressure [Pa];

V_0 – compression volume by couplings neutral position [m^3].

Static load characteristics of the flexible coupling determined from the mathematical model of flexible couplings and also the experimental values are shown on *fig. 11*. The values of percentual difference δM_V between computed and measured values are shown on *fig. 12*.

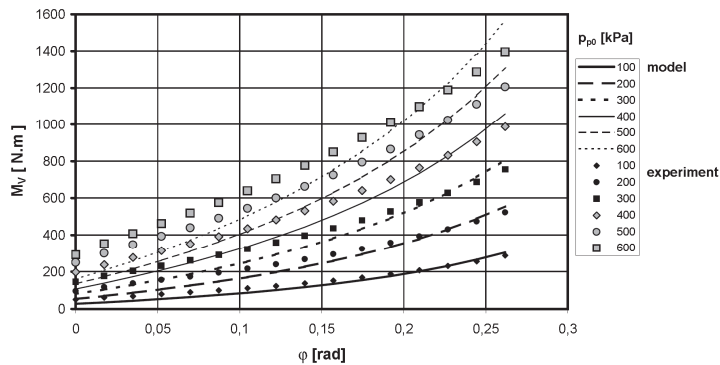


Fig. 11. Static load characteristics
Rys. 11. Charakterystyki statyczne obciążenia

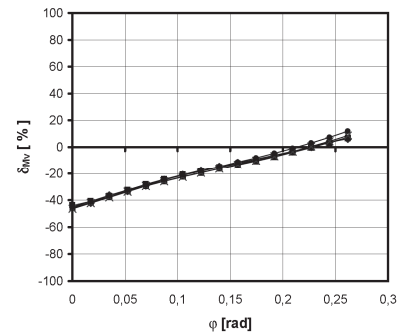


Fig. 12. Percentage difference between measured and computed values
Rys. 12. Procentowa różnica między zarejestrowanymi a obliczonymi wartościami

6. CONCLUSION

We can see that percentual difference between experimental and computed values lying in the range $\delta_{M_s} \cong (-46\% \text{ to } +12\%)$ is relatively large. However, we must remember that the greatest differences are at small twist angles, where flexible coupling in service does not work, because it would be too oversized. For maximum of twist angle the mentioned mathematical model gives quite good results. This model we consider suitable for rough preliminary estimation at the design stage of pneumatic flexible shaft couplings.

Acknowledgements

This paper was written in the framework of Grant Project VEGA: „1/0688/12 – Research and application of universal regulation system in order to master the source of mechanical systems excitation”.

Bibliography

1. Bednár R., Sága M., Vaško M.: Study of the Nelder-Mead optimizing algorithm usability in the case of uncertain large-scale structures analyses. Transactions of the Universities of Košice, no. 3 (2012), p. 25-32.
2. Czech P.: Vibroacoustic diagnostics of gasket under engine head damage with the use of probabilistic neural networks. Transactions of the Universities of Košice, no. 3 (2012), p. 33-38.
3. Grega R., Krajňák J.: The application of pneumatic flexible coupling in conveyor drive. Technológ, Roč. 5, no. 4 (2013), p. 51-54.
4. Hal'ko J., Pavlenko S.: Analytical Suggestion of Stress Analysis on Fatigue in Contact of the Cycloidal – Vascular Gearing System. Zeszyty Naukowe Politechniki Śląskiej, Vol. 76, no. 1864 (2012), p. 63-66.
5. Homišin J.: Nové typy pružných hriadeľových spojok: vývoj, výskum, aplikácia. TU, SjF, Košice 2002.

6. Homišin J.: Intentions of grant project: „research and application of universal regulation system in order to master the source of mechanical systems excitation”. *Zeszyty Naukowe Politechniki Śląskiej*, Vol. 1865, no. 76 (2012), p. 7-12.
7. Jakubovičová L., Sága M., Vaško M.: Contribution to discrete optimising of beam structures subjected to fatigue damage. *Transactions of the Universities of Košice*, no. 2 (2011), p. 137-142.
8. Jurčo M.: Stanovenie matematického modelu pneumatických pružných hriadeľových spojok: doktorandská dizertačná práca, Košice 1999.
9. Kaššay P., Homišin J., Urbanský M.: Formulation of Mathematical and Physical Model of Pneumatic Flexible Shaft Couplings. *Zeszyty Naukowe Politechniki Śląskiej*, seria: Transport, Vol. 76, no. 1864 (2012), p. 25-30.
10. Medvecká-Beňová S., Vojtková J.: Stress in the Eccentric Elliptical Gearing Tooth. *Zeszyty Naukowe Politechniki Śląskiej*, Vol. 76, no. 1864 (2012), p. 37-42.
11. Peruń G., Łazarz B.: Diagnosing combustion engine using signal analysis. *Transactions of the Universities of Košice*, no. 3 (2012), p. 97-104.
12. Sapietová A., Dekýš V., Budinský M.: Utilizing of Sensitivity Analysis in Preparation of Optimizing Procedure. *Zeszyty Naukowe Politechniki Śląskiej*, Vol. 76, no. 1864 (2012), p. 113-118.
13. Pešík L., Skarolek A.: Heat Transfer Effects on Vibration Isolation Differential Pneumatic System. *Transactions of the Universities of Košice*, no. 3 (2012), p. 109-114.
14. Urbanský M.: Presentation of continuous tuning of mechanical systems. *Inżynier XXI wieku: 2. międzynarodowa konferencja studentów oraz młodych naukowców. 30 listopada 2012, Bielsko-Biała. Wydawnictwo Naukowe Akademii Techniczno-Humanistycznej, Bielsko-Biała 2012*, p. 195-200.
15. Wojnar G.: Detecting local defects in toothed gears. *Transactions of the Universities of Košice*, no. 3, 2012, p. 135-138.