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Analysis of Dynamic Characteristics of Selected Pneumatic Systems with Fractional Calculus Used in Telematics

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ABSTRACT
The section of the paper on simulation studies presents the application of fractional calculus to describe the dynamics of pneumatic systems used in telematics. In the construction of mathematical models of the analyzed dynamic systems, the Riemann-Liouville definition of non- integer order was used. For the analyzed model, transfer function of integer and non-integer order was determined. Functions describing characteristics in time and frequency domains were determined, whereas the characteristics of the analyzed systems were obtained by means of computer simulation. The section of the paper on laboratory research presents the results of the laboratory tests of the injection system of the internal combustion engine with special attention to the verification of simulated tests of selected pneumatic systems described with the use of fractional calculus.

KEYWORDS: combustion engine, fractional calculus, pneumatic systems, telematics

1. Introduction

The dynamic development of research in recent years on the use of fractional calculus for the analysis of dynamic systems has prompted the authors to use it in the analysis and modeling of pneumatic systems that have been described so far with "classical" mathematical analysis [1, 2, 4, 5, 11, 12, 13, 15, 16].

The authors of the paper have developed a method for describing the dynamic properties of pneumatic systems, based on fractional calculus which allows to analyze the properties of a wide range of pneumatic systems of any order [6, 7, 8, 9, 10]. The simulation tests of the membrane pressure transducer, presented in the paper, were performed with the use of classical differential calculus and fractional calculus. In the construction of the mathematical model of the analyzed dynamical system, the Riemann-Liouville definition of the differintegral of non-integer order was used. For simulation studies, Microsoft Office Excel and MATLAB were used [14]. In the laboratory tests, which were the verification of the simulation tests of the membrane pressure transducer, the following assumptions were made: the analyzed pneumatic systems were modeled as a linear system; the pneumatic system was described with a transfer function characterizing the dynamics of this system and the components contained therein, assuming constant physical parameters and omission of aging of its components; an assessment was accepted of the dynamic properties of the pneumatic systems in terms of amplitude and phase; pneumatic systems with a pressure of up to 1MPa were analyzed while operating in the frequency range up to 500Hz; with the variability of the thermodynamic parameters of the air as a working medium, it can be treated as an ideal gas; in the analysis of the pneumatic systems, an adiabatic process was assumed whereas the pressure distribution in the whole volume of the measuring chamber was homogeneous [7, 8].

2. Membrane pressure transducer

Simulation tests of the membrane pressure transducer were performed using a classical and fractional differential calculus. The tested transducer was made from a pressure chamber and an inlet pipe, which supplied a working medium (air). To determine how the connection of the intake pipe to the transducer chamber affected its dynamic properties, the acoustic system shown schematically in Fig. 1 is considered [6, 7, 9, 10].

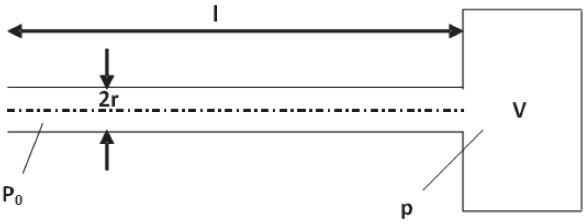

Fig. 1. Pressure chamber with inlet pipe: r,l- tube dimensions, p0- input pressure (force), p0 - pressure in transducer chamber [3]

The relationship that binds the output signal p (t) (pressure inside the chamber) to the signal p0 (t) (pressure at the open end) can be represented as:

$$\frac{d^2p(t)}{dt^2}+\frac{R_p}{L_p}\frac{dp(t)}{dt}+\frac{1}{L_pC_p}p(t)=\frac{1}{L_pC_p}p_0(t) \quad (1)$$

Constants occurring in equation (1) can be represented as:

$$C_p=\frac{V}{\rho c^2}\left[Ns^2m^{-5}\right] \quad (2a)$$

$$L_p=\frac{4l\rho}{3\pi r^2}\left[m^5N^{-1}\right] \quad (2b)$$

$$R_p=\frac{8\eta l}{\pi r^2}\left[Nsm^{-5}\right] \quad (2c)$$

where: r[kgm-3] – density of gas; η[kgm-1s-1] – dynamic viscosity; *Cp* [Ns2m-5] – pneumatic capacity (gas compressibility); *p(t)*[Pa] – pressure in transducer chamber; *p0(t)* [Pa] – input pressure; *V*[m3] – transducer chamber volume; *Lp* [m3N-1] – pneumatic induction (gas inertia); *Rp* [Nsm-5] – flow resistance; *c* [ms-1] – speed of sound in the *gas; r, l*[m] – dimensions of the inlet pipe.

By specifying the pulsatance ω_0 and damping ratio ξ as:

$$\omega_0=\frac{1}{\sqrt{L_pC_p}}=\sqrt{\frac{3\pi^2c^2}{4lV}} \quad (3a)$$

$$\xi=\frac{R_pC_p\omega_0}{2}=$$

$$=\frac{R_p}{2}\sqrt{\frac{C_p}{L_p}}=2\frac{\eta\sqrt{\frac{3lV}{\pi}}}{r\rho c}=2\sqrt{\frac{3\eta^2lV}{\pi r^2\rho^2c^2}} \quad (3b)$$

equation (1) finally assumes the form:

$$\frac{d^2p(t)}{dt^2}+2\xi\omega_0\frac{dp(t)}{dt}+\omega_{0^2}p(t)=\omega_{0^2}p_0(t) \quad (4)$$

Equation (4) is a mathematical description of the dynamics of the analyzed pneumatic system, using classical differential calculus (of integer orders). The impulse response of the analyzed pneumatic system is given by:

$$g(t)=\frac{\omega_0}{\sqrt{1-\xi^2}}e^{-\xi\omega_0t}\sin\omega_0\sqrt{1-\xi^2}\,t \quad (5)$$

The step response of the system is expressed by:

$$h(t)=1-\frac{e^{-\omega_0\xi t}}{\sqrt{1-\xi^2}}\sin\left(\omega_0\sqrt{1-\xi^2}\,t+\varphi\right) \quad (6)$$

where:

$$\varphi=arctg\frac{\sqrt{1-\xi^2}}{\xi} \quad (7)$$

Given that the derivatives of integer orders in the fractional calculus are a special case of fractional derivatives, equation (4) can be written as:

$${}_0^{RL}D_t^{2v}p(t)+2\xi\omega_0\,{}_0^{RL}D_t^{v}p(t)+\omega_{0^2}p(t)=\omega_{0^2}p_0(t) \quad (8)$$

where: $v>0$.

In order to determine the pressure in the transducer chamber, the definition of Riemann - Liouville differintegral of non-integer order was used, defined by a following formula [5, 12, 13]:

$${}_a^{RL}D_t^{\alpha}f(t)=\frac{1}{\Gamma(k-\alpha)}\frac{d^k}{dt^k}\int_a^t(t-\tau)^{k-\alpha-1}f(\tau)d\tau \quad (9)$$

where:

α – the order of integration within bounds *(a,t)* of the function *f(t)*, $k-1\le a\le k$ and: $\alpha\in R^+$, $\Gamma(x)=\int_0^\infty e^{-t}t^{x-1}dt$ - Euler's gamma function

The Laplace transform for a fractional derivative defined by Riemann - Liouville takes the form:

$$L\left[{}_0^{R-L}D_t^{\alpha}f(t)\right]=s^{\alpha}F(s)-\sum_{k=0}^{j-1}s^k\,{}_0^{R-L}D_t^{\alpha-k-1}f(0) \quad (10)$$

where: $j-1\le á\le j\in N$

The practical application of the formula determining the Laplace transform of a Riemann-Liouville fractional derivative faces some difficulties related to the lack of physical interpretation of the initial values of successive derivatives of fractional orders. Assuming zero initial conditions, the difficulties associated with their physical interpretation will be eliminated.

Using the Laplace transform to equation (8), for zero initial conditions, we obtain:

$$s^{2v}p(s)+2\xi\omega_0s^vp(s)+\omega_{0^2}p(s)=\omega_{0^2}p_0(s) \quad (11)$$

Thus the transfer function of non-integer order of the analyzed pneumatic system is obtained:

$$G^{(v)}(s)=\frac{\omega_{0^2}}{s^{2v}+2\xi\omega_0s^v+\omega_{0^2}} \quad (12)$$

Transfer function denominator of non-integer order has two complex roots as the system damping is $\xi<1$.

3. Impulse response to the membrane pressure transducer

By transforming equation (12), we obtain:

$$G^{(v)}(s)=\frac{\omega_{0^2}}{s^{2v}+2\xi\omega_0 s^v}\cdot\frac{1}{1+\frac{\omega_{0^2}}{s^{2v}+2\xi\omega_0 s^v}} \tag{13}$$

Using the properties of the geometric series:

$$G^{(v)}(s)=\frac{\omega_{0^2}}{s^{2v}+2\xi\omega_0 s^v}\cdot\sum_{k=0}^{\infty}\frac{\left(-\omega_{0^2}\right)^k}{\left(s^{2v}+2\xi\omega_0 s^v\right)^k} \tag{14}$$

Conducting elementary transformations, we obtain:

$$G^{(v)}(s)=\sum_{k=0}^{\infty}\frac{\left(-\omega_{0^2}\right)^k}{k!}\frac{k!\omega_0{}^2 s^{v-(2v+vk)}}{\left(s^v-\left(-2\xi\omega_0\right)\right)^{k+1}} \tag{15}$$

Using the formula:

$$L\left\{t^{\alpha m+\beta-1}E_{\alpha,\beta}^{(m)}\left(at^\alpha\right)\right\}=\frac{m!s^{\alpha-\beta}}{\left(s^\alpha-a\right)^{m+1}} \tag{16}$$

we obtain:

$$g^{(v)}(t)=L^{-1}\left(G^{(v)}(s)\right)=\sum_{k=0}^{\infty}\frac{\left(-\omega_{0^2}\right)^k\omega_{0^2}}{k!}t^{vk+2v+vk-1}E_{v,2v+vk}^{k}\left(-2\xi\omega_0 t^v\right) \tag{17}$$

where:

$$E_{\alpha,\beta}^{(k)}=\sum_{n=0}^{\infty}\frac{(n+k)!}{n!}\frac{t^n}{\Gamma\left(\alpha n+\alpha k+\beta\right)} \tag{18}$$

The visualization of the pressure impulse response in the transducer chamber required a program to be written in the MATLAB environment [14]. The program for the given parameters and derivative orders calculates the function values and draws out their impulse response. We present, for comparison, the graphs of the function obtained for the classic solution (*v=1*) and for several fractional orders.

In Fig. 2, the impulse response of the analyzed pneumatic transducer was determined by simulating equation (17) for the selected parameter values:

$$v\left(F_{0,5}-0{,}5\quad F_{0,7}-0{,}7\quad F_{0,9}-0{,}9\quad F_{1,0}-1\right).$$

The impulse response (characteristics C_2 in the above figure) was also presented, by simulating a computer equation (5) which is a mathematical model of the analyzed pneumatic system, with the use of a classical differential calculus (described by ordinary differential equation). It is worth noting that while reducing the row, it reduces the response time, which is desirable in measuring transducers.

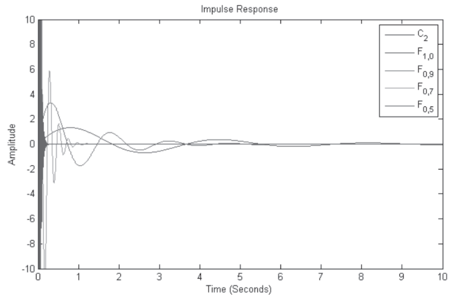

Fig. 2. Impulse response of a pneumatic transducer described with an integer and non integer order: $F_{0,5}$- for *v=0,5*, $F_{0,7}$ - for *v=0,7*, $F_{0,9}$- for *v=0,9*, $F_{1,0}$- for v=1,0, C_2- classic model (integer order) [7]

4. Step response of the membrane pressure transducer

The step response of the tested transducer is defined by the relationship:

$$H^{(v)}(s)=\frac{\omega_{0^2}}{s\left(s^{2v}+2\xi\omega_0 s^v+\omega_{0^2}\right)}=\frac{\omega_{0^2}s^{-1}}{s^{2v}+2\xi\omega_0 s^v}\cdot\frac{1}{1+\frac{\omega_{0^2}}{s^{2v}+2\xi\omega_0 s^v}} \tag{19}$$

Using the properties of geometric series, we obtain:

$$H^{(v)}(s)=\frac{\omega_{0^2}s^{-1}}{s^{2v}+2\xi\omega_0 s^v}\cdot\sum_{k=0}^{\infty}\frac{\left(-\omega_{0^2}\right)^k}{\left(s^{2v}+2\xi\omega_0 s^v\right)^k} \tag{20}$$

Conducting elementary transformations, we obtain:

$$H^{(v)}(s)=\sum_{k=0}^{\infty}\frac{\left(-\omega_{0^2}\right)^k}{k!}\frac{k!\omega_0{}^2 s^{v-(2v+vk+1)}}{\left(s^v-\left(-2\xi\omega_0\right)\right)^{k+1}}$$

Using the formula (16), we obtain:

$$h^{(v)}(t)=\sum_{k=0}^{\infty}\frac{\left(-\omega_{0^2}\right)^k\omega_{0^2}}{k!}t^{vk+2v+vk+1-1}E_{v,2v+vk+1}^{k}\left(-2\xi\omega_0 t^v\right)$$

in which $E_{\alpha,\beta}^{(k)}$ is the Mittag-Leffler function defined by the equation (18).

Running a simulation of a pneumatic transducer, a unit step signal was applied and the received step response is shown in Fig. 3.

The model described by equation (17) and (22) correctly reproduces the amplitude of the input signal as the classical model - the graphs coincide (graph $F_{1,0}$ - the parameter *v=1* coincides with C_2 –the classical model). This confirms the correctness of the method and that the differential calculus with derivatives of integer orders is a special case of fractional calculus. The step response (Fig. 3) shows that regardless of the differential order, the amplitude of the signal is constant. The smaller the order of the derivative, the faster the reaction of the system to the unit step.

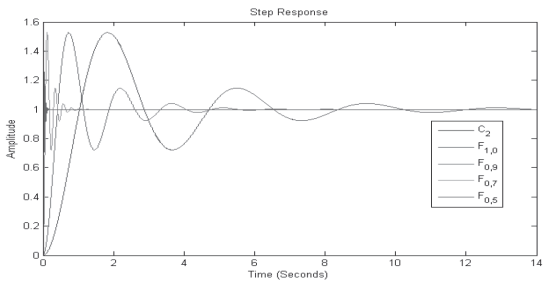

Fig. 3. The step response of the pneumatic system: $F_{0,5}$ - for *v=0,5*, $F_{0,7}$ - for *v=0,7*, $F_{0,9}$ - for *v=0,9*, $F_{1,0}$ - for v=1,0, C_2 - classic model (integer order) [7]

5. Frequency response of the membrane pressure transducer

In order to determine the relationships describing the frequency response, the spectral transfer function of the tested transducer was determined. Substituting:

$$s = j\omega = \omega e^{j\frac{\pi}{2}} = \omega\left[\cos\left(\frac{\pi}{2}\right) + j\sin\left(\frac{\pi}{2}\right)\right] \tag{23}$$

to the formula (12), the spectral transfer function of the transducer is obtained:

$$G^{(v)}(j\omega) = \frac{\omega_{0^2}}{(j\omega)^{2v} + 2\xi\omega_0 (j\omega)^v + \omega_{0^2}} \tag{24a}$$

$$G^{(v)}(j\omega) = \frac{\omega_{0^2}}{\omega^{2v}\left[\cos(v\pi) + j\sin(v\pi)\right] + 2\xi\omega_0\omega^v\left[\cos\left(v\frac{\pi}{2}\right) + j\sin\left(v\frac{\pi}{2}\right)\right] + \omega_{0^2}} \tag{24b}$$

By making elementary transformations, the real and imaginary part of the spectral transfer function was calculated:

$$G^{(v)}(j\omega) = P^{(v)}(\omega) + jQ^{(v)}(\omega) \tag{25}$$

where:

$$P^{(v)}(\omega) = \frac{\omega_{0^2}\omega^{2v}\cos(v\pi) + 2\xi\omega_{0^3}\omega^v\cos\left(\frac{v\pi}{2}\right) + \omega_{0^4}}{\left[\omega^{2v}\cos(v\pi) + 2\xi\omega_0\omega^v\cos\left(\frac{v\pi}{2}\right) + \omega_{0^2}\right]^2 + \left[\omega^{2v}\sin(v\pi) + 2\xi\omega^v\omega_0\sin\left(\frac{v\pi}{2}\right)\right]^2} \tag{26a}$$

$$Q^{(v)}(\omega) = -\frac{\omega_{0^2}\omega^{2v}\sin(v\pi) + 2\xi\omega_{0^3}\omega^v\sin\left(\frac{v\pi}{2}\right)}{\left[\omega^{2v}\cos(v\pi) + 2\xi\omega_0\omega^v\cos\left(\frac{v\pi}{2}\right) + \omega_{0^2}\right]^2 + \left[\omega^{2v}\sin(v\pi) + 2\xi\omega_0\omega^v\sin\left(\frac{v\pi}{2}\right)\right]^2} \tag{26b}$$

Knowing the real and imaginary part of the spectral transfer function of the transducer, one can determine the equation describing the logarithmic amplitude step:

$$L^{(v)}(\omega) = 20\log\sqrt{\left[P^{(v)}(\omega)\right]^2 + \left[Q^{(v)}(\omega)\right]^2} \tag{27}$$

and the equation describing the logarithmic phase step:

$$\varphi^{(v)}(\omega) = arctg\left[\frac{Q^{(v)}(\omega)}{P^{(v)}(\omega)}\right] = -arctg\left[\frac{\omega^{2v}\sin(v\pi) + 2\xi\omega_0\omega^v\sin\left(\frac{v\pi}{2}\right)}{\omega^{2v}\cos(v\pi) + 2\xi\omega_0\omega^v\cos\left(\frac{v\pi}{2}\right) + \omega_{0^2}}\right] \tag{28}$$

In order to verify the relationships describing logarithmic steps of amplitude (27) and phase (28) of the tested transducer, the pneumatic pressure transducer was modeled in the MATLAB environment, described by means of ordinary differential equation and differential equation with derivatives of non-integer order. Describing the transducer with the use of a differential equation of non-integer orders, the parameter *v=1* was assumed and the obtained logarithmic amplitude and phase steps were compared to the logarithmic amplitude and phase steps obtained from the transducer description by means of the ordinary differential equation.

The simulations assumed:

- pulsatance $\omega_0 = 500\left[rad/s\right]$,
- damping ratio $\xi = 2$

The transfer function of the pneumatic pressure transducer, calculated from the ordinary differential equation, has the form:

$$G(s) = \frac{p(s)}{p_0(s)} = \frac{\omega_{0^2}}{s^2 + 2\xi\omega_0 s + \omega_{0^2}} \tag{29}$$

By performing the simulation of equation (29) which presents the dynamics of the phenomena occurring in the analyzed pneumatic system, in the MATLAB programming environment, the frequency response presented in Fig. 4 was obtained:

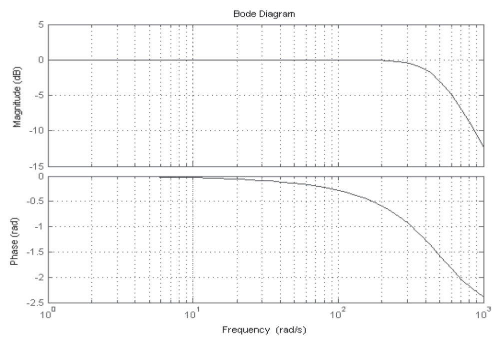

Fig. 4. Logarithmic frequency response of the transducer described by the ordinary differential equation [6]

When simulating equations (27) and (28) in the MATLAB environment which describe a pneumatic pressure transmitter by means of a differential equation of non-integer order, assuming a coefficient *v=1* for damping *ξ=0,8*, the response shown in Fig. 5 was obtained:

Logarithmic frequency response of amplitude and phase presented by the simulation of ordinary differential equation (Fig. 4), coincide with frequency response obtained by the simulation of the equations describing logarithmic response of amplitude (27) and phase (28), obtained from the equation of the transducer described with the help of non-integer orders (Fig. 5) for the parameter *v=1*.

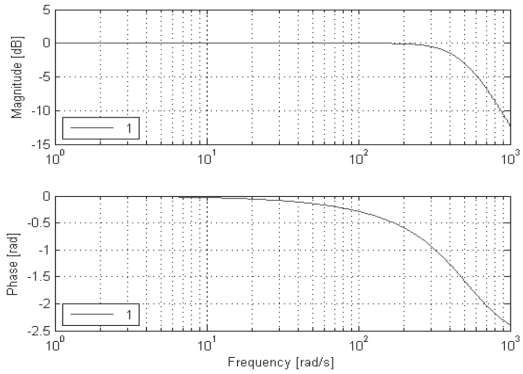

Fig. 5. Logarithmic frequency response of a pneumatic transducer described by means of a differential equation with non-integer order for *v=1* (equation 27 and 28) [6]

In order to obtain a Bode plot, the equations (27) and (28) were simulated by writing an appropriate program in the MATLAB environment. Written in the MATLAB environment, the program allows analyzing the transducer for different orders of derivatives, with any step, because the order was given as a parameter. The simulation results for the selected values of parameter v are shown in Fig. 6 and Fig. 7.

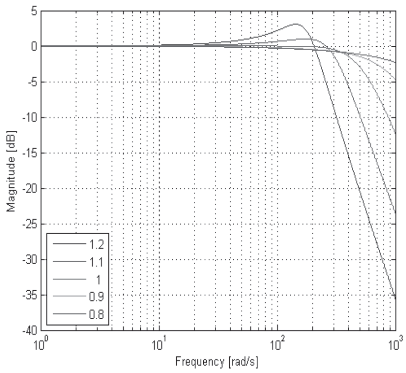

Fig. 6. Logarithmic amplitude response of a pneumatic transducer described by means of differential equation with fractional derivatives of non-integer orders in the range (0.8-1.2) [7]

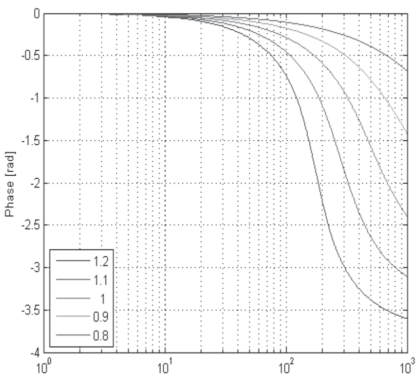

Fig. 7. Logarithmic phase response of a pneumatic transducer described by means of differential equation with fractional derivatives of non-integer orders in the range (0.8-1.2) [7]

The analysis of the responses shows that for the parameter *v<1*, the logarithmic amplitude responses (Fig. 6) are monotonically decreasing functions. For the parameter *v>1*, the logarithmic amplitude responses have a maximum depending on the order of the differential. The maximum is achieved with resonant frequency $\omega_0 = 110\frac{rad}{s}$.

Increasing the order of derivative, the frequency responses acquire the character of a second-order oscillatory element, and while decreasing the order of the derivative, the responses acquire the character of the first order inertial element. Decreasing the order of the derivative causes the transducer to become more linear, which allows the scope of work to be increased. Increasing the parameter *v* above one results in resonance, although it should not be visible in the response, because the simulation was carried out for the damping *v=0,8*. The model then does not reflect the actual layout.

6. Selected laboratory tests of the transducer

In order to identify the dynamics of the pressure transducer, the measuring system was constructed as it is shown in Fig. 8. The AVL single-cylinder automatic ignition engine was used for testing. It is an internal combustion engine with a capacity of 511cm^3, cylinder diameter 85.01mm and a piston stroke of 90mm. The measurements were made in the inlet air system to the engine. The air supply was provided by an additional system consisting of a rotary screw compressor. Thanks to this system it was possible to regulate the air pressure in the intake system [8].

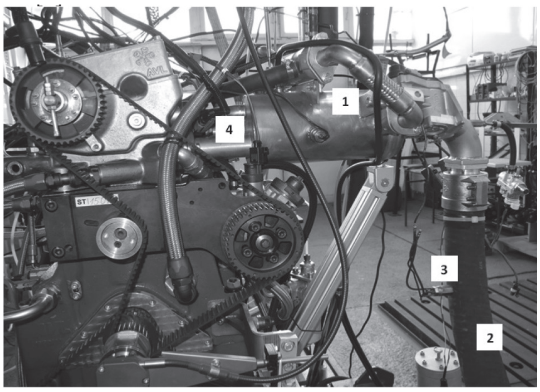

Fig. 8. View of measuring station: 1 - measuring chamber, 2 - intake manifold, 3-input pressure transducer, 4 - output pressure transducer [8]

In the air intake system leading into the measuring chamber, the first pressure transducer was installed. The second transducer was installed inside the measuring chamber, at the outlet of the air into the combustion chamber. Kulite pressure transducers, type ETL-189-190M-10 BARA were used. Two identical pressure transducers were used in the system.

The tests were performed with Concerto and Puma software, whose interfaces are shown in Fig. 9. The presented measuring system allows studying the dynamic properties of the pressure transducer. The studies refer to the time and frequency analysis of the investigated pressure transducer described with integer and non-integer order.

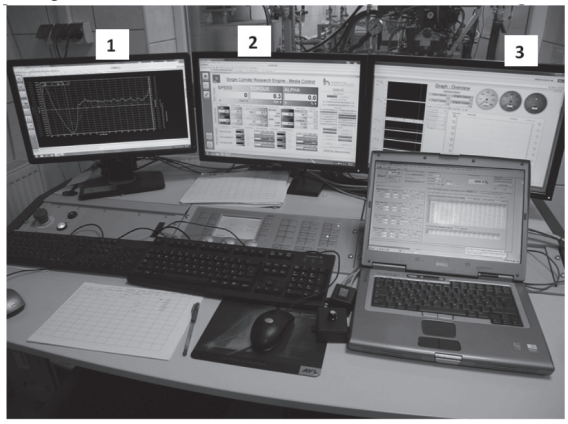

Fig. 9. Interfaces of the programs for motor control and recording fast-changing parameters: 1 - Concerto program window for fast variable parameters recording (monitor 1), 2 - Puma program window for controlling and archiving engine parameters (monitor 2), 3 - Puma program window for controlling and archiving engine parameters (monitor 3) [8]

In the measuring system, computers with Concerto and Puma software were used. The Concerto program allows recording fast-changing system parameters and recording them in time and numeric format. The Puma program was used to control the engine.

The air supplied into the measuring system is provided by a rotary screw compressor. The engine draws air into the combustion chamber from the measuring system by opening the valve located at the exit of the measuring chamber. The valve opens and closes every two turns of the engine crankshaft. Cyclical opening of the valve caused the same effect as supplying the system with a pneumatic rectangular signal generator, which allowed experimental evaluation of the step response of the measuring system. The step response of the system was determined using the Concerto program, and its graphic representation is shown in Fig. 10.

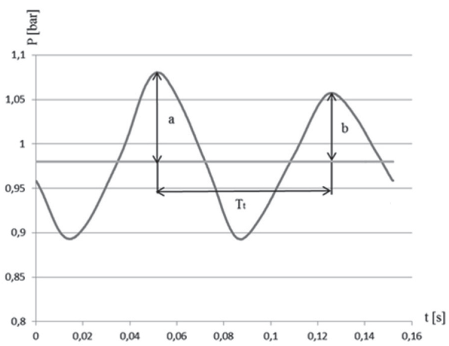

Fig. 10. The step response of the measuring system in the measuring chamber and inlet tube obtained experimentally [8]

The obtained graph is a step response of a typical oscillating element with pulsatance ω0 and damping ratio ξ. In order to identify the dynamic properties of the tested pneumatic system it is convenient to determine its transfer function.

The equation describing the logarithmic amplitude response can be determined from the equation (27):

$$L^{(v)}(\omega) = 20\log\sqrt{\left[P^{(v)}(\omega)\right]^2 + \left[Q^{(v)}(\omega)\right]^2} \qquad (30)$$

After simulating equation (37), a logarithmic amplitude response of the non-integer order of the tested measurement system was obtained for different values of the parameter *v*, which is shown in Fig. 12.Against the response obtained by means of simulation of the mathematical model in which the actual parameters of the tested transducer were applied, the transducer response obtained experimentally was presented.

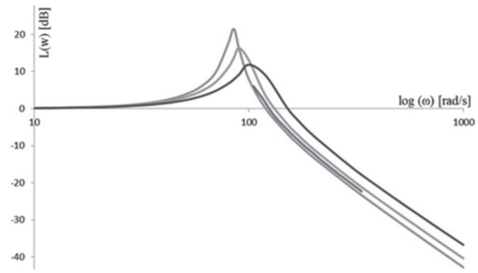

Fig. 11. Logarithmic amplitude response determined experimentally and theoretically for different values of the parameter *v* [8]

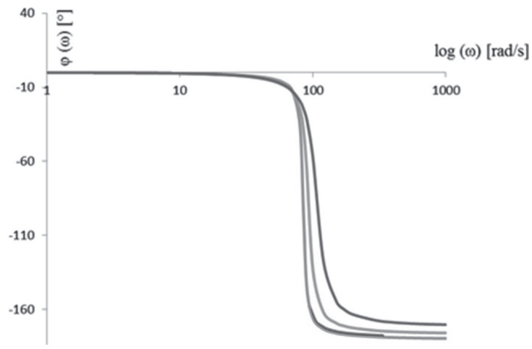

Fig. 12. Logarithmic phase response determined experimentally and for different values of the parameter *v* [8]

Logarithmic phase response was determined by measuring the phase shift between the output and input signal for each set motor shaft speed.

The minimum rotation speed of the internal combustion engine used in the tests is 1000[*obr/min*]. This limitation made it impossible to obtain experimentally the full frequency response shown in Fig. 11 and 12. The obtained responses were made from pulsation *ω=104,717[rad/s]* do *ω=335,093[rad/s]*.

By comparing the obtained responses with the responses of non-integer order for the parameter *v=1*, it can be stated that the tested transducer is of a slightly smaller order than the second order oscillating element. Experientially determined frequency responses are included between the simulated frequency response for parameter *v=0,98* and *v=1*. This means that the transducer should be modeled with the equations of non-integer order. It can therefore be concluded that the description with the classical method would be inaccurate.

7. Conclusion

The pneumatic system, analyzed in the article in the part on simulation, represents the second order damping oscillator with damping ratio $\xi<1$, which means that the characteristic equation of the model does not have real solutions. Therefore, the authors were required to develop the original method for determining the relationships describing the time and frequency responses for dynamic systems described with fractional calculus. In the construction of the mathematical model of the analyzed dynamic system, the definition of Riemann - Liouville differintegral (of fractional order) was used.

The paper presents the results of the laboratory tests of the pressure transducer, which was described with a mathematical model. The analysis of the dynamic properties of the model in terms of time and frequency was conducted. The parameters of the tested pressure transducer were determined experimentally - the damping ratio ξ and the pulsatance ω0, which were used to determine the transfer function of integer and non-integer order . Having the knowledge of transfer function, the step response was determined as well as logarithmic amplitude response and phase response of integer and non-integer order. It was found out that the time response obtained experimentally, coincides with the response determined by the developed model of non-integer order of the transducer for the parameter $v=1$. This confirms the correctness of the designated model.

Frequency responses obtained experimentally differ slightly from the responses obtained in computer simulation. The logarithmic amplitude response and phase response obtained experimentally are of the order of the oscillating element for the parameter v of the interval $0,98<v<1$.

In the real system shown in the article, the pressure in the transducer chamber was measured at the outlet of the air into the combustion chamber, where, at the moment of aspiration of air through the engine, the air reaches the speed of sound. Such conditions may account for a slight decrease in the order of the oscillating element under testing.

The analysis of the logarithmic amplitude response of the models presented in the article shows that the local maximum present in these characteristics is dependent on the order of the derivative and the bigger the amplitude, the higher the order of the derivative. For the parameter $v=1$ (classical model) the amplitude reaches the maximum at the resonant frequency for damping $\xi<1$. With the decreasing order of the derivative, the increase in the resonant frequency of the circuit can be observed.

Fractional calculus is particularly useful in building dynamic models of mathematical systems working in conditions that cannot be described with differential equations of integer orders. This can be deduced by analyzing systems such as the long electric line of infinitely large length or the supercapacitor of a few thousand Farads, which are now also described with fractional calculus.

The presented simulation studies were performed in the MATLAB development environment (manufacturer: The MathWorks) and Microsoft Office Excel (manufacturer: Microsoft). The authors of the paper declare that, using the above mentioned trademarks, they did so only with reference to this publication and with such an intention that it would be for the benefit of the trademark holders but without the intention of infringing the trademarks.

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