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COMPUTER USE IN THE DISCOVERY OF PROPERTIES OF FUNCTIONS AND THEIR PROOFS

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Abstract. In this paper we present the results of research which was conducted in years 2006/2007 and 2007/2008 on groups of students of Pedagogical University of Cracow. The results are related with the role of the computer in the process of solving the tasks on the functions with parameter, discovering some properties of functions, formulating statements describing observed regularity and finding the method of proof.

1. Introduction

The second half of the XX century and the beginning of the XXI century are the period of rapid development of computer technology. Its importance in various fields of human life constantly increases so it is no wonder that it also gains entrance to education. Throughout the world, many people carry out research on the effective use of computer technology in teaching and learning mathematics. One of the issues is an attempt to answer the questions: How to use effectively computer technology to shape the image of mathematical concepts? and: How to help learners to develop creative activity? The work in this field was also carried out at the Pedagogical University of Cracow. The results of this research can be found in the papers [2], [3], [4], [6] and [8], among others.

2. Methodological notes

Our results presented in this paper were obtained during attempts to use the computer in the formation of some mathematical concepts in the process of solving certain tasks. In these studies, we used the computer to help students:

- to find the solutions to the problem concerning the properties of functions with the parameter,
- to justify the correctness of the obtained solutions,
- to discover certain properties of functions and formulate them in mathematical language and discover methods of proof.

We were interested in finding answers to the following research questions:

- How can a computer with appropriate software help understanding the content of the mathematical problem?
- Can a computer be useful in discovering the solution to a mathematical problem?
- How much can a computer help to deepen understanding the mathematical concepts?
- How much can a computer be useful in discovering methods of proof of observed facts?

Some results concerning these issues have already been signaled in our paper in 2005. In this paper we present results of our research which was conducted in years 2006–2008 among three groups of postgraduate students (Group A – a group of 14 people in the academic year 2006/2007, Group B – a group of 18 people in the academic year 2006/2007, Group C – a group of 17 people in the academic year 2007/2008). The research involved 49 students. Most of them were teachers who widened their mathematical knowledge to get the right to teach this subject. Their study lasted three semesters. During their studies they had to learn many new mathematical contents. We expect that the use of computer can be a positive motivation for them to try to solve more difficult tasks.

As the research tools we used the sets of tasks with the parameter which relate to the properties of elementary functions and questionnaire which contains twelve questions on the previous experience of using a computer and an opinion on the role of the computer in the class of mathematical analysis and future plans related to the use of the computer. We use the following working

methods: observation of students' work, discussion with students, analysis of homework solutions and questionnaires. Research were conducted according to the following plan:

1. Students work on solutions sets of tasks in pairs without the aid of a computer.

2. Common discussion about the obtained solutions with use of a computer during the discussion.

3. Students working alone – solving homework.

4. Students complete the questionnaire.

5. Analysis of the collected material and putting the research hypotheses.

Research started with work on tasks without a computer. The idea was to check whether students will feel the need to use a computer. In our research we used three tasks. The first of them concerned a linear function $f_m(x) = (2m-3)x+3$, where m is a real parameter. The idea was to investigate whether students perceive that a change of the location of plots of these functions depends on a change of the value of the parameter m. We will not analyze this task in current paper, because it is already described in other papers. In the task 2 the goal was to answer the question: what can be said – depending on the parameter m – about the domain and the set of values for the following functions: $g_m(x) = \frac{1}{f_m(x)}$ and $h_m(x) = \sqrt{f_m(x)}$, where f_m is defined as in Task 1. We chose this task to research how the respondents understand the concept of composite functions and if they are able to indicate those functions' domains and sets of values. In addition, we were interested to see whether they can designate the limit cases. Task 3 deals with the description of the position of a plot of the function $f(x) = ax^2 + bx + c$ depending on the real values of the parameters a, b, c. The solution of this task can help you to analyze graphs of functions drawn on a computer. In our research we used programs Derive 5.0 and Wykresy 3 (Graphs 3). We wanted to check our assumption that the location of the graph when you change the coefficient b will be difficult to describe for students. The role of homework and use of computers in solving problems will be discussed in another paper.

Wykresy 3 is a simple program useful for drawing graphs of functions. This program allows us to enter formulas and draw graphs of functions with a parameter. Using it one can observe how the position of the graph depends on the parameters. The program is available at
http://www.up.krakow.pl/mat/komputery/prokomp.html.

The questionnaire was anonymous and consisted of 12 questions divided into three groups. The first group includes general questions about computer use by students during independent problem solving or during classes in school. The second group consists of questions concerning the role of a computer during the exercise and in the process of solving problems. Students have to indicate one from five situations which have occurred:

a) a student independently discovered the solution and the computer did not help in any way,

b) a student independently discovered the solution and a computer confirmed the obtained result,

c) a student independently discovered a way to solve the problem and computer simulations helped him to find an error in reasoning,

d) a student has not discovered the solution and understand how to solve them after the analysis using computer,

e) a student has not discovered the solution and a computer did not help him in any way.

In this group there was also a question which presentation helped them better to discover and formulate the observed hypothesis. The last group of questions concerned the respondents subjective feelings about computer-aided teaching. They were asked about whether:

- more topics should be implemented in a similar way?
- the participant is satisfied that the classes were conducted using a computer?
- the respondent would like to use a computer in the future in the classroom?

3. Analysis of results

Students considering the family of functions $f_m(x) = (2m-3)x + 3$ should observed that the point $(0, 3)$ belongs to the graphs of all functions from this family. For $m > 1.5$ these functions are increasing, for $m < 1.5$ these functions are decreasing, and for $m = 1.5$ the function f_m is constant. Research has shown that the greatest difficulties for students were caused by constant function. Most respondents generally did not include it in the solutions. Only

after using a computer one noticed that it meets the conditions of tasks 1 and 2. This is probably because the respondents do not have yet well formed concept image of linear function (see [7]). They did not see that in the family under consideration there is a constant function which has slightly different properties from other functions of this family. Therefore, in a situation complicated by the use of a parameter, students did not reflect on the limiting case which corresponds to the function $f(x) = 3$ ([1]). Regarding to the task 2, it should be noticed that the domain of the function g_m is the set of real numbers without the root of the function f_m for $m \neq 1.5$ or the set $\mathbb{R}$ for $m = 1.5$. However, the domain of the function h_m is the set of real numbers for which the function f_m takes the non-negative values, and the whole $\mathbb{R}$, when $m = 3$. The answers to general questions show that almost 45% of respondents use a computer while solving tasks independently, and 67% use it at their lessons. This fact can arouse optimism, but let us notice here that the most of the teachers were not learners of mathematics. The most frequently reported methods used were:

- Drawing graphs of functions;
- Search the web definitions, theorems and examples of the solutions;
- Perform calculations and check the results of various activities, in particular, for operations on matrices, calculating determinants, solving systems of equations or the tasks from statistics and economics.

Analysis of responses to the question about the role of a computer during solving the tasks shows that 6% of the students solved the task without the help of a computer and 30% sought confirmation of the correctness of their solution by computer. Approximately 17% of respondents knew the way to solve the task, but using a computer has detected errors in their solution. The largest group (about 47%) were people who could not solve the task and understood the solution after the analysis of drawings on the computer screen viewing. A group of students, representing 11% of respondents, were not able to solve a task and usage of a computer did not help them.

From these data we can conclude that the appropriate use of a computer can contribute to understanding the solution of the problem. It is still an open issue whether a student can independently use a computer to find a solution.

Observation of students' work while solving the task 3 showed that they do not really know how to graph the function changes when the parameter b is changing and the numbers a and c are constant. The first way to use a computer in the class was to show students an image made using the program Derive 5.0 (see Figure 1).

Analysis of this Figure does not lead the students closer to the discovery of a complete solution, although there are some hypotheses. Illustration using Wykresy 3 which allows observation of changes in variable values has led the students to the formulation of appropriate hypotheses. In this case, the plot moves along a parabola with the equation $y = -ax^2 + c$, as shown in Figure 2. Analysis of the vertex coordinates led one of the respondents to justify the correctness of an appropriate solution.

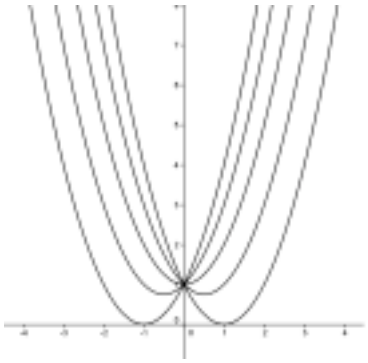

Figure 1.

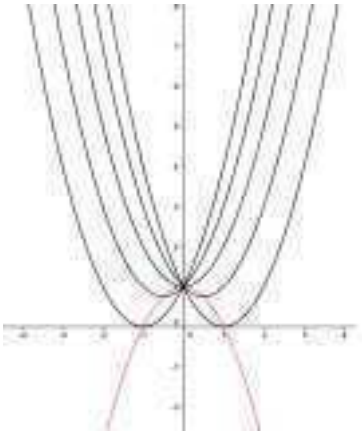

Figure 2.

The fact that this result was confirmed by a mathematical operations surprised 55% of respondents. At the same time, almost 80% of study participants were in favor of the use of dynamic presentations that really helped them to understand the solution. Moreover 98% of the final workshop participants expressed their satisfaction with this form of work and 90% would like to participate in similar activities in other subjects. There occurred some surprising answers to questions about use of a computer in classes conducted by respondents. Only 33% of respondents expressed the willingness to use a computer in their lessons. Comparing this response with answers to general questions, we can see a percentage decrease of the number of people who need to support their teaching with computers.

4. Conclusions and research hypotheses

The conducted observations have revealed a certain behavior of students during solving tasks with a computer. Most respondents sought for a solution by empirical observations, i.e. by examining the correlation between the graph and the specific parameter value. Thereby, obtaining a correct solution is not the intuitive feel of proving the correctness of their observations. Such

a behavior is already known and also occurred in other studies of this type (cf. [3]). Answering research questions, it should be noticed that a computer with appropriate software can be helpful in finding a solution, but requires that the user knowledge and skills matches to software suitable for the problem under consideration. The knowledge of mathematical concepts related with the problem which occurs and some experience of students in the creative action are also necessary.

During the research students do not choose the software by themselves, but they use the software suggested by the teacher. Moreover, in the case of difficulty one could ask the teacher for the help. It was also observed in some tasks that there occurred difficulties in interpreting data visualization on the screen. Possible causes of this phenomenon consist in incomprehension terms in the task and inexperience in the use of parameter expressions. In independent work on a task one may also do not receive computer application skills as a tool for exploration of creative solutions.

We note that nearly half of study participants said that they understood the task solution by working with computer. In our view, it is an argument for the use of this tool in the teaching process.

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