Dziula Przemysław
ORCID ID: 0000-0002-3930-1316

Kołowrocki Krzysztof
ORCID ID: 0000-0002-4836-4976
Gdynia Maritime University, Gdynia, Poland

Integrated impact model on global Baltic network of critical infrastructure networks safety related to its operation process

Keywords

Safety, Global Baltic Network, Critical Infrastructure Network, Network of Networks

Abstract

The purpose of the paper is to present the impact model for Global Baltic Network of Critical Infrastructure Networks (GBNCIN), related to its operation process. At first, basic issues regarding the GBNCIN operation at variable conditions have been described. Then, aspects regarding safety of the multistate GBNCIN at Variable operation conditions have been pointed. Finally, safety characteristics of multistate GBNCIN, consisting of exponential BCIN Networks at variable operation conditions are presented.

1. Introduction

Integrated impact model of Global Baltic Network of Critical Infrastructure Networks (GBNCIN) safety related to its operation process, is the continuation of works processed within the report [EU-CIRCLE Report D3.3-GMU11, 2016], that specified the GBNCIN operation process and safety model. The models were developed basing on outcomes of the report [EU-CIRCLE Report D1.2-GMU1, 2016], that analysed nature of some critical infrastructures operating within the Baltic Sea area, their interconnections and interdependencies, resulting with distinguishing certain critical infrastructure networks, defined as a set of interconnected and interdependent critical infrastructures, interacting directly and indirectly at various levels of their complexity and operating activity [EU-CIRCLE Report D1.1. EU-CIRCLE Taxonomy, 2015]. The networks have been abbreviated as the Baltic Critical Infrastructure Networks (BCIN). Consequently, distinguished networks, operating within the Baltic Sea area, interacting, and being also interconnected and interdependent, were classified as the Global Baltic Network of Critical Infrastructure Networks.

The GBNCIN operation process model was defined basing on the operation process of critical infrastructure, critical infrastructure network, and their parameters. The parameters of the GBNCIN operation process, specified in the report [EU-CIRCLE Report D3.3-GMU11, 2016], were: the vector of probabilities of the process staying at initial operation states, the matrix of probabilities of the process transitions between the operation states, and the matrices of conditional distribution and density functions of the process conditional sojourn times at the operation states.

The GBNCIN safety model was developed with the use of the multi-state approach [Amari, 1997], [Aven, 1985, 1999, 1993], [Barlow, Wu, 1978], [Brunelle, Kapur, 1999], [Hudson, Kapur, 1982, 1985], [Lisnianski, Levitin, 2003], [Natvig, 1982], [Ohio, Nishida, 1984], [Xue, 1985], [Xue, Yang, 1995a,b], [Yu et al, 1994], [Kołowrocki, Soszyńska-Budny, 2011], with the assumption that each particular network is composed of multi-state assets [EU-CIRCLE Report D3.1-GMU4], with safety states degrading in time [Guze, Kołowrocki, 2008], [Kołowrocki, 2004, 2014], [Kołowrocki, Soszyńska-Budny, 2011], [Xue, 1985], [Xue, Yang 1995 a, b], that gave the possibility to precise analysing of their safety and operational processes' effectiveness. This assumption allowed to distinguish a network safety critical state to exceed which is either dangerous for the environment or does not assure the necessary level of its operation process effectiveness. Then, an important network safety characteristic specified were: the time to the moment of exceeding its safety critical state and its distribution, which was called the network risk function. This distribution was strictly related to the safety function that are basic characteristics of the multi-state network. Then, the multistate asset and the multistate network main safety characteristics, i.e. their mean values of the lifetimes and in the safety state subsets and in the particular safety states and standard deviations and the moment when the network risk function exceeds a fixed permitted level, were determined.

The integrated impact model of Global Baltic Network of Critical Infrastructure Networks safety related to its operation process, is linking the GBNCIN safety and operation process models, taking into account its variable safety structure at different operation states, and particular BCIN safety parameters. The model introduces additional safety indices, typical for the critical infrastructure related to its varying in time safety structures and its components' safety parameters caused by its operation process, extending previous models with the set of safety indicators by the assets, BCIN and GBNCIN conditional intensities of ageing at particular operation states, and conditional and unconditional coefficients of the operation process impact on intensities of ageing.

2. The GBNCIN operation at variable conditions

We assume that the Global Baltic Network of Critical Infrastructure Networks (*GBNCIN*), during its operation process is taking $\iota = \nu^{(1)} \cdot \nu^{(2)} \cdot \ldots \cdot \nu^{(8)}$, different operation states $z_1, z_2, \ldots, z_\iota$, of the form $[z^{(1)}, z^{(2)}, \ldots, z^{(8)}]$, where

$$z^{(i)} \in \{z_1^{(i)}, z_2^{(i)}, \ldots, z_{\nu^{(i)}}^{(i)}\},\ i = 1,2,\ldots,8,$$

are particular operation states of particular networks $BCIN^{(i)}$, $i = 1,2,\ldots,8$, and $\nu^{(i)}$, $i = 1,2,\ldots,8$, are the numbers of operation states of those particular networks respectively.

Further, it was assumed in the above report that the *GBNCIN* operation process $Z_{GBNCIN}(t)$, $t \in \langle 0, +\infty)$, has operation states from the set $\{z_1, z_2, \ldots, z_\iota\}$.

Thus, we denote particular operation states of the *GBNCIN* operation process $Z_{GBNCIN}(t)$, $t \in \langle 0, +\infty)$, by

$$z_{j_1 j_2 \ldots j_8},\ j_i \in \{1,2,\ldots,\nu^{(i)}\},\ i = 1,2,\ldots,8,$$

and understand them according to the following relationship

$$(Z_{GBNCIN}(t) = z_{j_1 j_2 \ldots j_8}) \Leftrightarrow$$
$$(Z^{(1)}(t) = z_{j_1}^{(1)} \wedge Z^{(2)}(t) = z_{j_2}^{(2)} \wedge \ldots$$
$$\wedge Z^{(8)}(t) = z_{j_8}^{(8)}),\ j_i \in \{1,2,\ldots,\nu^{(i)}\},\ i = 1,2,\ldots,8,$$

where $z_{j_1}^{(1)}, z_{j_2}^{(2)}, \ldots, z_{j_8}^{(8)}$, $j_i \in \{1,2,\ldots,\nu^{(i)}\}$, $i = 1,2,\ldots,8$, are particular operation states of single operation processes

$$Z^{(1)}(t), Z^{(2)}(t), \ldots, Z^{(8)}(t),\ t \in \langle 0, +\infty),$$

of particular *BCIN* networks.

Moreover, it was assumed, that the *GBNCIN* Network operation process $Z_{GBNCIN}(t)$ is a semi-Markov process [Grabski, 2014], [Habibullah et al, 2009], [Kołowrocki, Soszyńska-Budny, 2011] with the conditional sojourn times $\theta_{j_1 j_2 \ldots j_8\ k_1 k_2 \ldots k_8}^{GBNCIN}$, $j_i \in \{1,2,\ldots,\nu^{(i)}\}$, $i = 1,2,\ldots,8$, at the operation states $z_{j_1 j_2 \ldots j_8}$, when its next operation state is $z_{k_1 k_2 \ldots k_8}$, $j_i, k_i \in \{1,2,\ldots,\nu^{(i)}\}$, $i = 1,2,\ldots,8$, $j_1 j_2 \ldots j_8 \neq k_1 k_2 \ldots k_8$.

Under these assumptions, the *GBNCIN* operation process may be described by:

– the vector

$$[p_{j_1 j_2 \ldots j_8}^{GBNCIN}(0)]_{1x\iota},\quad \iota = \nu^{(1)} \cdot \nu^{(2)} \cdot \ldots \cdot \nu^{(8)},$$

of the initial probabilities

$$p_{j_1 j_2 \ldots j_8}^{GBNCIN}(0) = P(Z_{GBNCIN}(0) = z_{j_1 j_2 \ldots j_8}),$$
$$j_i \in \{1,2,\ldots,\nu^{(i)}\},\ i = 1,2,\ldots,8,$$

of the *GBNCIN* operation process $Z_{GBNCIN}(t)$ staying at particular operation states at the moment $t=0$;

– the matrix

$$[p^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8}]_{\iota\times\iota},\ \iota=\nu^{(1)}\cdot\nu^{(2)}\cdot\ldots\cdot\nu^{(8)},$$

of probabilities

$$p^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8},\ j_i,k_i\in\{1,2,...,\nu^{(i)}\},\ i=1,2,...,8,$$
$$j_1j_2...j_8\neq k_1k_2..k_8,$$

of the *GBNCIN* Network operation process $Z_{GBNCIN}(t)$ transitions between the operation states $z_{j_1j_2..j_8}$, and $z_{k_1k_2..k_8}$;

– the matrix

$$[H^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8}(t)]_{\iota\times\iota},\ \iota=\nu^{(1)}\cdot\nu^{(2)}\cdot\ldots\cdot\nu^{(8)},$$

of conditional distribution functions

$$H^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8}(t)=P(\theta^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8}<t),\ t\geq 0,$$
$$j_i,k_i\in\{1,2,...,\nu^{(i)}\},\ i=1,2,...,8,\ j_1j_2...j_8\neq k_1k_2..k_8,$$

of the *GBNCIN* operation process $Z_{GBNCIN}(t)$ conditional sojourn times $\theta^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8}$ at the operation states.

It is well known that the mean values $E[\theta^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8}]$ of the conditional sojourn times $\theta^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8}$ are given by

$$M^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8}=E[\theta^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8}]$$
$$=\int_0^\infty t dH^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8}(t),\ j_ik_i\in\{1,2,...,\nu^{(i)}\},$$
$$i=1,2,...,8,\ j_1j_2...j_8\neq k_1k_2..k_8. \quad (1)$$

Then, from the formula for total probability, it follows that the unconditional distribution functions of the sojourn times $\theta^{GBNCIN}_{j_1j_2..j_8}$, $j_i\in\{1,2,...,\nu^{(i)}\}$, $i=1,2,...,8$, of the *GBNCIN* operation process $Z_{GBNCIN}(t)$ at the operation states $z_{j_1j_2..j_8}$, are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$H^{GBNCIN}_{j_1j_2..j_8}(t)=\sum_{k_1k_2..k_8=1}^{\iota}p^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8}H^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8}(t),$$
$$t\geq 0,\ j_i\in\{1,2,...,\nu^{(i)}\},\ i=1,2,...,8, \quad (2)$$

Hence and from (1), the mean values $E[\theta^{GBNCIN}_{j_1j_2..j_8}]$ of the *GBNCIN* operation process $Z_{GBNCIN}(t)$ unconditional sojourn times $\theta^{GBNCIN}_{j_1j_2..j_8}$, $j_i\in\{1,2,...,\nu^{(i)}\}$, $i=1,2,...,8$, at the operation states are given by

$$M^{GBNCIN}_{j_1j_2..j_8}=E[\theta^{GBNCIN}_{j_1j_2..j_8}]=$$
$$\sum_{k_1k_2..k_8=1}^{\iota}p^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8}M^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8},$$
$$j_i\in\{1,2,...,\nu^{(i)}\},\ i=1,2,...,8, \quad (3)$$

where $M^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8}$ are defined by the formula (1).

The limit values of the *GBNCIN* operation process $Z_{GBNCIN}(t)$ transient probabilities at the particular operation states

$$p^{GBNCIN}_{j_1j_2..j_8}(t)=P\big(Z_{GBNCIN}(t)=z_{j_1j_2..j_8}\big),\ t\in<0,+\infty),$$
$$j_i\in\{1,2,...,\nu^{(i)}\},\ i=1,2,...,8,$$

are given by [Kołowrocki, Soszyńska-Budny, 2011]

$$p^{GBNCIN}_{j_1j_2..j_8}=\lim_{t\to\infty}p^{GBNCIN}_{j_1j_2..j_8}(t)=\frac{\pi^{GBNCIN}_{j_1j_2..j_8}M^{GBNCIN}_{j_1j_2..j_8}}{\sum_{k_1k_2..k_8=1}^{\iota}\pi^{GBNCIN}_{k_1k_2..k_8}M^{GBNCIN}_{k_1k_2..k_8}},$$
$$j_i\in\{1,2,...,\nu^{(i)}\},\ i=1,2,...,8, \quad (4)$$

where $M^{GBNCIN}_{j_1j_2..j_8}$ are given by (3), while the steady probabilities $\pi^{GBNCIN}_{j_1j_2..j_8}$ of the vector $[\pi^{GBNCIN}_{j_1j_2..j_8}]_{1x\iota}$ satisfy the system of equations

$$\begin{cases}[\pi^{GBNCIN}_{j_1j_2..j_8}]=[\pi^{GBNCIN}_{j_1j_2..j_8}][p^{GBNCIN}_{j_1j_2..j_8\ \ k_1k_2..k_8}]\\ \sum_{k_1k_2..k_8=1}^{\iota}\pi^{GBNCIN}_{k_1k_2..k_8}=1\end{cases} \quad (5)$$

In the case of a periodic *GBNCIN* operation process, the limit transient probabilities $p^{GBNCIN}_{j_1j_2..j_8}$, $j_i\in\{1,2,...,\nu^{(i)}\}$, $i=1,2,...,8$, at the operation states given by (4), are the long term proportions of the *GBNCIN* operation process $Z_{GBNCIN}(t)$ sojourn times at the particular operation states $z_{j_1j_2..j_8}$, $j_i\in\{1,2,...,\nu^{(i)}\}$, $i=1,2,...,8$.

Other interesting characteristics of the *GBNCIN* operation process $Z_{GBNCIN}(t)$ possible to obtain are its total sojourn times $\hat{\theta}^{GBNCIN}_{j_1 j_2 .. j_8}$ at the particular operation states $z_{j_1 j_2 .. j_8}$, $j_i \in \{1,2,...,\nu^{(i)}\}$, $i = 1,2,...,8$, during the fixed *GBNCIN* operation time. It is well known [Kołowrocki, Soszyńska-Budny, 2011] that the *GBNCIN* operation process total sojourn times $\hat{\theta}^{GBNCIN}_{j_1 j_2 .. j_8}$ at the particular operation states $z_{j_1 j_2 .. j_8}$, $j_i \in \{1,2,...,\nu^{(i)}\}$, $i = 1,2,...,8$, for sufficiently large operation time θ have approximately normal distributions with the expected value given by

$$\hat{M}^{GBNCIN}_{j_1 j_2 .. j_8} = E[\hat{\theta}^{GBNCIN}_{j_1 j_2 .. j_8}] = p^{GBNCIN}_{j_1 j_2 .. j_8} \theta,$$
$$j_i \in \{1,2,...,\nu^{(i)}\}, \; i = 1,2,...,8, \quad (6)$$

where $p^{GBNCIN}_{j_1 j_2 .. j_8}$ are given by (4).

3. Safety of multistate GBNCIN at variable operation conditions

It is assumed that the changes of the operation states of the *GBNCIN* operation process $Z_{GBNCIN}(t)$ have an influence on particular *BCIN* networks E_i^{GBNCIN}, $i = 1,2,...,8$, safety and the *GBNCIN* safety structure as well. Consequently, we denote the *BCIN* network E_i^{GBNCIN}, $i = 1,2,...,8$, conditional lifetime in the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$, $u = 1,2,...,z^{BBNCIN}$, while the *GBNCIN* is at the operation state $z_{j_1 j_2 .. j_8}$, $j_i \in \{1,2,...,\nu^{(i)}\}$, $i = 1,2,...,8$, by

$$[T_i^{GBNCIN}]^{(j_1 j_2 .. j_8)}(u), \; u = 1,2,...,z^{BBNCIN},$$
$$j_i \in \{1,2,...,\nu^{(i)}\}, \; i = 1,2,...,8,$$

and its conditional safety function by the vector

$$[S_i^{GBNCIN}(t,\cdot)]^{(j_1 j_2 .. j_8)} = = [1, [S_i^{GBNCIN}(t,1)]^{(j_1 j_2 .. j_8)}, \ldots,$$
$$[S_i^{GBNCIN}(t, z^{GBNCIN})]^{(j_1 j_2 .. j_8)}], \; t \in <0, \infty),$$
$$j_i \in \{1,2,...,\nu^{(i)}\}, \; i = 1,2,...,8, \quad (7)$$

with the coordinates defined by

$$[S_i^{GBNCIN}(t,u)]^{(j_1 j_2 .. j_8)} =$$
$$= P([T_i^{GBNCIN}]^{(j_1 j_2 .. j_8)}(u) > t \,|\, Z^{GBNCIN}(t) = z_{j_1 j_2 .. j_8}), \quad (8)$$

for $t \in <0, \infty)$, $u = 1,2,...,z^{GBNCIN}$, $j_i \in \{1,2,...,\nu^{(i)}\}$, $i = 1,2,...,8$,

The safety function $[S_i^{GBNCIN}(t,u)]^{(j_1 j_2 .. j_8)}$ is the conditional probability that the *BCIN* network E_i^{GBNCIN} lifetime $[T_i^{GBNCIN}]^{(j_1 j_2 .. j_8)}(u)$ in the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$, is greater than t, while the *GBNCIN* operation process $Z_{GBNCIN}(t)$ is at the operation state $z_{j_1 j_2 .. j_8}$.

In the case, when the *BCIN* networks E_i^{GBNCIN}, $i = 1,2,...,8$, at the *GBNCIN* operation process $Z_{GBNCIN}(t)$ states $z_{j_1 j_2 .. j_8}$ have the exponential safety functions, the coordinates of the vector (7) are given by

$$[S_i^{GBNCIN}(t,u)]^{(j_1 j_2 .. j_8)} =$$
$$= P([T_i^{GBNCIN}]^{(j_1 j_2 .. j_8)}(u) > t \,|\, Z^{GBNCIN}(t) = z_{j_1 j_2 .. j_8}) =$$
$$= \exp[-[\lambda_i^{GBNCIN}(u)]^{(j_1 j_2 .. j_8)} t], \; t \in <0, \infty),$$
$$j_i \in \{1,2,...,\nu^{(i)}\}, \; i = 1,2,...,8, \quad (9)$$

The intensities of ageing/degradation of the *BCIN* networks E_i^{GBNCIN}, $i = 1,2,...,8$, given by (9) - the intensities of the *BCIN* networks E_i^{GBNCIN}, $i = 1,2,...,8$, departure from the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$, at the *GBNCIN* operation states $z_{j_1 j_2 .. j_8}$, i.e. the coordinates of the vector

$$[\lambda_i^{GBNCIN}(\cdot)]^{(j_1 j_2 .. j_8)} =$$
$$= [0, [\lambda_i^{GBNCIN}(1)]^{(j_1 j_2 .. j_8)}, ..., [\lambda_i^{GBNCIN}(z^{GBNCIN})]^{(j_1 j_2 .. j_8)}],$$
$$t \in <0, +\infty), \; j_i \in \{1,2,...,\nu^{(i)}\}, \; i = 1,2,...,8, \quad (10)$$

are given by

$$[\lambda_i^{GBNCIN}(u)]^{(j_1 j_2 .. j_8)} =$$
$$= [\rho_i^{GBNCIN}]^{(j_1 j_2 .. j_8)}(u) \cdot \lambda_i^{GBNCIN}(u),$$
$$u = 1,2,..., z^{GBNCIN}, \; j_i \in \{1,2,...,\nu^{(i)}\},$$
$$i = 1,2,...,8, \quad (11)$$

where $\lambda_i^{GBNCIN}(u)$ are the intensities of ageing of the *BCIN* networks E_i^{GBNCIN}, $i = 1,2,...,8$, (the intensities of the *BCIN* networks E_i^{GBNCIN}, $i = 1,2,...,8$, departure from the safety state subset

$\{u, u+1, ..., z^{GBNCIN}\}$), without operation process impact, i.e. the coordinate of the vector

$$\lambda_i^{GBNCIN}(\cdot) = [0, \lambda_i^{GBNCIN}(1), ..., \lambda_i^{GBNCIN}(z^{GBNCIN})],$$
$$i = 1,2,...,8, \tag{12}$$

and

$$[\rho_i^{GBNCIN}(u)]^{(j_1 j_2 ... j_8)}, \ u = 1,2,...,z^{GBNCIN},$$
$$j_i \in \{1,2,...,\nu^{(i)}\}, \ i = 1,2,...,8, \tag{13}$$

are the coefficients of operation impact on the *BCIN* networks E_i^{GBNCIN}, $i = 1,2,...,8$, intensities of ageing (the coefficients of operation impact on *BCIN* networks E_i^{GBNCIN}, $i = 1,2,...,8$, intensities of departure from the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$), at the *GBNCIN* operation states $z_{j_1 j_2 ... j_8}$, i.e. the coordinate of the vector

$$[\rho_i^{GBNCIN}(\cdot)]^{(j_1 j_2 ... j_8)} =$$
$$[0, [\rho_i^{GBNCIN}(1)]^{(j_1 j_2 ... j_8)}, ..., [\rho_i^{GBNCIN}(z^{GBNCIN})]^{(j_1 j_2 ... j_8)}],$$
$$j_i \in \{1,2,...,\nu^{(i)}\}, \ i = 1,2,...,8, \tag{14}$$

The *BCIN* network safety function (7), the *BCIN* network intensities of ageing (10) and the coefficients of the operation impact on the *BCIN* network intensities of ageing (14), are main *BCIN* network safety indices.

Similarly, we denote the *GBNCIN* conditional lifetime in the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$, while the *GBNCIN* is at the operation state $z_{j_1 j_2 ... j_8}$, $j_i \in \{1,2,...,\nu^{(i)}\}$, $i = 1,2,...,8$, by $T_{GBNCIN}^{(j_1 j_2 ... j_8)}(u)$, and the conditional safety function of the *GBNCIN* Network by the vector

$$[S^{GBNCIN}(t,\cdot)]^{(j_1 j_2 ... j_8)} = [1, [S^{GBNCIN}(t,1)]^{(j_1 j_2 ... j_8)}, ...,$$
$$[S^{GBNCIN}(t, z^{GBNCIN})]^{(j_1 j_2 ... j_8)}], \ j_i \in \{1,2,...,\nu^{(i)}\},$$
$$i = 1,2,...,8, \tag{15}$$

with the coordinates defined by

$$[S^{GBNCIN}(t,u)]^{(j_1 j_2 ... j_8)} =$$
$$= P(T_{GBNCIN}^{(j_1 j_2 ... j_8)}(u) > t \mid Z^{GBNCIN}(t) = z_{j_1 j_2 ... j_8}) \tag{16}$$

for $t \in <0,\infty)$, $u = 1,2,...,z^{GBNCIN}$, $j_i \in \{1,2,...,\nu^{(i)}\}$, $i = 1,2,...,8$,

The safety function $[S^{GBNCIN}(t,u)]^{(j_1 j_2 ... j_8)}$ is the conditional probability that the *GBNCIN* lifetime $T_{GBNCIN}^{(j_1 j_2 ... j_8)}(u)$ in the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$, is greater than t, while the *GBNCIN* operation process $Z_{GBNCIN}(t)$ is at the operation state $z_{j_1 j_2 ... j_8}$.

Further, we denote the *GBNCIN* unconditional lifetime in the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$, by $T_{GBNCIN}(u)$ and the unconditional safety function of the *GBNCIN* by the vector

$$[S^{GBNCIN}(t,\cdot)]$$
$$= [1, S^{GBNCIN}(t,1), ..., S^{GBNCIN}(t, z^{GBNCIN})], \tag{17}$$

with the coordinates defined by

$$S^{GBNCIN}(t,u) = P(T_{GBNCIN}(u) > t) \tag{18}$$

for $t \in <0,\infty)$, $u = 1,2,...,z^{GBNCIN}$.

In the case when the *GBNCIN* operation time θ^{GBNCIN} is large enough, the coordinates (18) of the unconditional safety function of the *GBNCIN* defined by (17) are given by

$$S^{GBNCIN}(t,u)$$
$$\cong \sum_{j_1=1}^{\nu^{(1)}} \sum_{j_2=1}^{\nu^{(2)}} ... \sum_{j_8=1}^{\nu^{(8)}} p_{j_1 j_2 ... j_8} [S^{GBNCIN}(t,u)]^{(j_1 j_2 ... j_8)} \tag{19}$$

for $t \ge 0$, $u = 1,2,...,z^{GBNCIN}$, $j_i \in \{1,2,...,\nu^{(i)}\}$, $i = 1,2,...,8$,

where $[S^{GBNCIN}(t,u)]^{(j_1 j_2 ... j_8)}$, $u = 1,2,...,z^{GBNCIN}$, $j_i \in \{1,2,...,\nu^{(i)}\}$, $i = 1,2,...,8$, are the coordinates of the *GBNCIN* conditional safety functions defined by (15)-(16) and $p_{j_1 j_2 ... j_8}^{GBNCIN}$, $j_i \in \{1,2,...,\nu^{(i)}\}$, $i = 1,2,...,8$, are the *GBNCIN* operation process limit transient probabilities given by (4).

The mean value of the *GBNCIN* Network unconditional lifetime $T_{GBNCIN}(u)$ in the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$, is given by [Kołowrocki, Soszyńska-Budny, 2011]

$$\mu^{GBNCIN}(u) \cong \sum_{j_1 j_2 \ldots j_8=1}^{t} p^{GBNCIN}_{j_1 j_2 \ldots j_8} \mu^{GBNCIN}_{j_1 j_2 \ldots j_8}(u),$$
$$u = 1,2,\ldots,z^{GBNCIN},\ j_i \in \{1,2,\ldots,\nu^{(i)}\},\ i = 1,2,\ldots,8, \quad (20)$$

where $\mu^{GBNCIN}_{j_1 j_2 \ldots j_8}(u)$ are the mean values of the *GBNCIN* conditional lifetimes $T^{j_1 j_2 \ldots j_8}_{GBNCIN}(u)$ in the safety state subset $\{u, u+1, \ldots, z^{GBNCIN}\}$, at the operation state $z_{j_1 j_2 \ldots j_8}$, given by

$$\mu^{GBNCIN}_{j_1 j_2 \ldots j_8}(u) = \int_0^\infty [S^{GBNCIN}(t,u)]^{(j_1 j_2 \ldots j_8)} dt,$$
$$u = 1,2,\ldots,z^{GBNCIN}, \quad (21)$$

$[S^{GBNCIN}(t,u)]^{(j_1 j_2 \ldots j_8)}$, $u = 1,2,\ldots,z^{GBNCIN}$, $j_i \in \{1,2,\ldots,\nu^{(i)}\}$, $i = 1,2,\ldots,8$, are defined by (15)-(16) and $p^{GBNCIN}_{j_1 j_2 \ldots j_8}$ are given by (4). Whereas, the variance of the *GBNCIN* unconditional lifetime $T_{GBNCIN}(u)$ is given by

$$\sigma^2_{GBNCIN}(u) = 2\int_0^\infty t S^{GBNCIN}(t,u) dt - [\mu^{GBNCIN}(u)]^2,$$
$$u = 1,2,\ldots,z^{GBNCIN}, \quad (22)$$

where $S^{GBNCIN}(t,u)$, $u = 1,2,\ldots,z^{GBNCIN}$, are given by (17)-(18) and $\mu^{GBNCIN}(u)$, $u = 0,1,\ldots,z^{GBNCIN}$, are given by (20).

According to [Kołowrocki, Soszyńska-Budny, 2011], we get the following formulae for the mean values of the unconditional lifetimes of the *GBNCIN* in particular safety states

$$\bar{\mu}^{GBNCIN}(u) = \mu^{GBNCIN}(u) - \mu^{GBNCIN}(u+1),$$
$$u = 0,1,\ldots,z^{GBNCIN} - 1,$$
$$\bar{\mu}^{GBNCIN}(z^{GBNCIN}) = \mu^{GBNCIN}(z^{GBNCIN}), \quad (23)$$

where $\mu^{GBNCIN}(u)$, $u = 0,1,\ldots,z^{GBNCIN}$, are given by (20).

Moreover, according to [Kołowrocki, Soszyńska-Budny, 2011], if r_{GBNCIN} is the *GBNCIN* critical safety state, then the *GBNCIN* risk function

$$r_{GBNCIN}(t) = P(S^{GBNCIN}(t) < r_{GBNCIN} \mid S^{GBNCIN}(0) = z^{GBNCIN})$$
$$= P(T_{GBNCIN}(r_{GBNCIN}) \le t),\ t \in <0,\infty), \quad (24)$$

defined as a probability that the *GBNCIN* is in the subset of safety states worse than the critical safety state r_{GBNCIN}, $r_{GBNCIN} \in \{1,2,\ldots,z^{GBNCIN}\}$, while it was in the safety state z^{GBNCIN} at the moment $t = 0$ [Kołowrocki 2004, 2014], [Kołowrocki, Soszyńska-Budny, 2011] is given by

$$r_{GBNCIN}(t) = 1 - S^{GBNCIN}(t, r_{GBNCIN}),\ t \in <0,\infty), \quad (25)$$

where $S^{GBNCIN}(t, r_{GBNCIN})$ is the coordinate of the *GBNCIN* unconditional safety function given by (19) for $u = r_{GBNCIN}$.

The *GBNCIN* safety function, the *GBNCIN* risk function and the *GBNCIN* fragility curve are main *GBNCIN* safety factors. Other practically useful *GBNCIN* safety factors are:

– the mean value of the unconditional *GBNCIN* lifetime $T_{GBNCIN}(r_{GBNCIN})$ up to the exceeding the critical safety state r_{GBNCIN} given by

$$\mu^{GBNCIN}(r_{GBNCIN}) \cong \sum_{j_1 j_2 \ldots j_8=1}^{t} p^{GBNCIN}_{j_1 j_2 \ldots j_8} \mu^{GBNCIN}_{j_1 j_2 \ldots j_8}(r_{GBNCIN}), \quad (26)$$

where $\mu^{GBNCIN}_{j_1 j_2 \ldots j_8}(r_{GBNCIN})$ are the mean values of the *GBNCIN* conditional lifetimes $T^{j_1 j_2 \ldots j_8}_{GBNCIN}(r_{GBNCIN})$ in the safety state subset $\{r_{GBNCIN}, r_{GBNCIN}+1, \ldots, z^{GBNCIN}\}$ at the operation state $z_{j_1 j_2 \ldots j_8}$, given by

$$\mu^{GBNCIN}_{j_1 j_2 \ldots j_8}(r_{GBNCIN}) = \int_0^\infty [S^{GBNCIN}(t, r_{GBNCIN})]^{(j_1 j_2 \ldots j_8)} dt,$$
$$j_i \in \{1,2,\ldots,\nu^{(i)}\},\ i = 1,2,\ldots,8, \quad (27)$$

$[S^{GBNCIN}(t, r_{GBNCIN})]^{(j_1 j_2 \ldots j_8)}$, $j_i \in \{1,2,\ldots,\nu^{(i)}\}$, $i = 1,2,\ldots,8$, are defined by (3.9)-(3.10) and $p^{GBNCIN}_{j_1 j_2 \ldots j_8}$ are given by (4);

– the standard deviation of the *GBNCIN* lifetime $T_{GBNCIN}(r_{GBNCIN})$ up to the exceeding the critical safety state r_{GBNCIN} given by

$$\sigma_{GBNCIN}(r_{GBNCIN})$$
$$= \sqrt{n(r_{GBNCIN}) - [\mu^{GBNCIN}(r_{GBNCIN})]^2} \quad (28)$$

where

$$n(r_{GBNCIN}) = 2\int_0^\infty t S^{GBNCIN}(t, r_{GBNCIN}) dt, \quad (29)$$

and $S^{GBNCIN}(t, r_{GBNCIN})$ is given by (19) and $\mu^{GBNCIN}(r_{GBNCIN})$ is given by (20) for $u = r_{GBNCIN}$.

– the moment τ_{GBNCIN} the *GBNCIN* risk function exceeds a permitted level δ_{GBNCIN} given by

$$\tau_{GBNCIN} = r^{-1}_{GBNCIN}(\delta_{GBNCIN}), \quad (30)$$

where $r^{-1}_{GBNCIN}(t)$, if exists, is the inverse function of the risk function $r_{GBNCIN}(t)$, given by (25).

Other *GBNCIN* safety indices are:

– the intensities of ageing/degradation of the *GBNCIN* (the intensities of *GBNCIN* departure from the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$) related to the operation process impact, i.e. the coordinates of the vector

$$\lambda^{GBNCIN}(t,\cdot) = [0, \lambda^{GBNCIN}(t,1), ..., \lambda^{GBNCIN}(t, z^{GBNCIN})],$$
$$t \in <0, +\infty), \quad (31)$$

where

$$\lambda^{GBNCIN}(t,u) = \frac{-\frac{dS^{GBNCIN}(t,u)}{dt}}{S^{GBNCIN}(t,u)}, \; t \in <0, +\infty),$$
$$u = 1, 2, ..., z^{GBNCIN}, \quad (32)$$

– the coefficients of operation process impact on the *GBNCIN* intensities of ageing (the coefficients of operation process impact on *GBNCIN* intensities of departure from the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$), i.e. the coordinates of the vector

$$\rho^{GBNCIN}(t,\cdot) = [0, \rho^{GBNCIN}(t,1), ..., \rho^{GBNCIN}(t, z^{GBNCIN})],$$
$$t \in <0, +\infty), \quad (33)$$

where

$$\lambda^{GBNCIN}(t,u) = \rho^{GBNCIN}(t,u) \cdot \lambda^{GBNCIN}(t,u),$$
$$t \in <0, +\infty), \; u = 1, 2, ..., z^{GBNCIN}, \quad (34)$$

and $\lambda^{GBNCIN}(t,u)$ are the intensities of ageing of the *GBNCIN* (the intensities of the *GBNCIN* departure from the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$) without climate-weather impact, i.e. the coordinate of the vector

$$\lambda^{GBNCIN}(t,\cdot) = [0, \lambda^{GBNCIN}(t,1), ..., \lambda^{GBNCIN}(t, z^{GBNCIN})],$$
$$t \in <0, +\infty), \quad (35)$$

In the case, when the *GBNCIN* have the exponential safety functions, i.e.

$$S^{GBNCIN}(t,\cdot) = [0, S^{GBNCIN}(t,1), ..., S^{GBNCIN}(t, z^{GBNCIN})],$$
$$t \in <0, +\infty), \quad (36)$$

where

$$S^{GBNCIN}(t,u) = \exp[-\lambda^{GBNCIN}(u)t], \; t \in <0, +\infty),$$
$$u = 1, 2, ..., z^{GBNCIN}, \quad (37)$$

the *GBNCIN* safety indices defined by (31)-(35) take forms:

– the intensities of ageing of the *GBNCIN* (the intensities of *GBNCIN* departure from the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$) related to the operation impact, i.e. the coordinates of the vector

$$\lambda^{GBNCIN}(\cdot) = [0, \lambda^{GBNCIN}(1), ..., \lambda^{GBNCIN}(z^{GBNCIN})], \quad (38)$$

– the coefficients of the operation impact on the *GBNCIN* intensities of ageing (the coefficients of the climate-weather impact on *GBNCIN* intensities of departure from the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$), i.e. the coordinate of the vector

$$\rho^{GBNCIN}(\cdot) = [0, \rho^{GBNCIN}(1), ..., \rho^{GBNCIN}(z^{GBNCIN})], \quad (39)$$

where

$$\lambda^{GBNCIN}(u) = \rho^{GBNCIN}(u) \cdot \lambda^{GBNCIN}(u),$$
$$u = 1, 2, ..., z^{GBNCIN}, \quad (40)$$

and $\lambda^{GBNCIN}(u)$ are the intensities of ageing of the *GBNCIN* (the intensities of the *GBNCIN* departure from the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$) without operation impact, i.e. the coordinate of the vector

$$\lambda^{GBNCIN}(\cdot) = [0, \lambda^{GBNCIN}(1), ..., \lambda^{GBNCIN}(z^{GBNCIN})]. \quad (41)$$

4. Safety of multistate GBNCIN consisting of exponential BCIN networks at variable operation conditions

We assume that the $BCIN$ networks E_i^{GBNCIN}, $i = 1,2,...,8$, at the $GBNCIN$ operation states have the exponential safety functions.

This assumption and the results given in [Kołowrocki, Soszyńska-Budny, 2011] yield the following results.

If the $BCIN$ networks E_i^{GBNCIN} of the $GBNCIN$ at the operation states $z_{j_1 j_2 .. j_8}$, $j_i \in \{1,2,...,\nu^{(i)}\}$, $i = 1,2,...,8$, have the exponential safety functions given by

$$[S_i^{GBNCIN}(t,\cdot)]^{(j_1 j_2 .. j_8)} = [1, [S_i^{GBNCIN}(t,1)]^{(j_1 j_2 .. j_8)}, \ldots, [S_i^{GBNCIN}(t, z^{GBNCIN})]^{(j_1 j_2 .. j_8)}], \ t \in <0, \infty),$$
$$j_i \in \{1,2,...,\nu^{(i)}\}, \ i = 1,2,...,8, \quad (42)$$

with the coordinates

$$[S_i^{GBNCIN}(t,u)]^{(j_1 j_2 .. j_8)} = P([T_i^{GBNCIN}]^{(j_1 j_2 .. j_8)}(u) > t \mid Z^{GBNCIN}(t) = z_{j_1 j_2 .. j_8})$$
$$= \exp[-[\lambda_i^{GBNCIN}(u)]^{(j_1 j_2 .. j_8)} t], \ t \in <0, \infty),$$
$$j_i \in \{1,2,...,\nu^{(i)}\}, \ i = 1,2,...,8, \quad (43)$$

and the intensities of ageing of the $BCIN$ networks E_i^{GBNCIN} (the intensities of the $BCIN$ networks E_i^{GBNCIN} departure from the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$) related to operation impact, existing in (4.2), are given by

$$[\lambda_i^{GBNCIN}(u)]^{(j_1 j_2 .. j_8)} = [\rho_i^{GBNCIN}]^{(j_1 j_2 .. j_8)}(u) \cdot \lambda_i^{GBNCIN}(u),$$
$$u = 1,2,...,z^{GBNCIN}, \ j_i \in \{1,2,...,\nu^{(i)}\},$$
$$i = 1,2,...,8, \quad (44)$$

where $\lambda_i^{GBNCIN}(u)$ are the intensities of ageing of the $BCIN$ networks E_i^{GBNCIN} (the intensities of the $BCIN$ networks E_i^{GBNCIN} departure from the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$) without operation impact and

$$[\rho_i^{GBNCIN}(u)]^{(j_1 j_2 .. j_8)}, \ u = 1,2,...,z^{GBNCIN},$$
$$j_i \in \{1,2,...,\nu^{(i)}\}, \ i = 1,2,...,8, \quad (45)$$

are the coefficients of operation impact on the $BCIN$ networks E_i^{GBNCIN} intensities of ageing (the coefficients of operation impact on $BCIN$ networks E_i^{GBNCIN} intensities of departure from the safety state subset $\{u, u+1, ..., z^{GBNCIN}\}$), without operation impact, then in the case of series structure, the GBNCIN unconditional safety function is given by the vector:

$$\overline{S}_n^{GBNCIN}(t,\cdot) = [1, \overline{S}_n^{GBNCIN}(t,1), \ldots, \overline{S}_n^{GBNCIN}(t, z^{GBNCIN})],$$
$$\text{for } t \geq 0, \quad (46)$$

where

$$\overline{S}_n^{GBNCIN}(t,u) \cong$$
$$\cong \sum_{j_1 j_2 .. j_8 = 1}^{t} p_{j_1 j_2 .. j_8}^{GBNCIN} \exp[-\sum_{i=1}^{n} [\lambda_i^{GBNCIN}(u)]^{(j_1 j_2 .. j_8)} t],$$
$$\text{for } t \geq 0, \ u = 1,2,...,z^{GBNCIN}. \quad (47)$$

5. Conclusions

Integrated Impact Model of Global Baltic Network of Critical Infrastructure Networks Safety Related to Its Operation Process, proposed in this paper, is basic background for considerations in further Tasks of the EU-CIRCLE Project. The model, together with the probabilistic model of the network of critical infrastructure networks operation process, related to the Global Baltic Network of Critical Infrastructure Networks, and the Global Baltic Network of Critical Infrastructure Networks safety model, will be the base to work on climate-weather change influence on critical infrastructures, by evolving them further to include Operating Environment Threats (OET), and Extreme Weather Hazards (EWE) impact. The impact of OET will base on analysis of GBNCIN and BCIN networks intensities of degradation and the coefficients of operation process including OET influence on the GBNCIN and BCIN intensities of degradation. Next, a general safety analytical model of the GBNCIN safety related to the climate-weather change process in its operating area will be developed. The integrated model of GBNCIN safety, linking its multistate safety model and the model of the climate-weather change process at its operating area, considering variable at the different climate-weather states and impacted by them BCIN networks safety parameters. Finally, conditional safety functions at the climate-weather particular states, the unconditional safety function and the risk function of the GBNCIN at changing in time climate-weather conditions will be defined.

Acknowledgements

The paper presents the results developed in the scope of the EU-CIRCLE project titled "A pan – European framework for strengthening Critical Infrastructure resilience to climate change" that has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 653824. http://www.eu-circle.eu/

References

Amari S.V., Misra R.B., Comment on: Dynamic reliability analysis of coherent multistate systems. IEEE Transactions on Reliability, 46, 460-461, 1997

Aven T., Reliability evaluation of multistate systems with multistate components. IEEE Transactions on Reliability, 34, 473-479, 1985

Aven T., Jensen U., Stochastic Models in Reliability. Springer-Verlag, New York, 1999

Aven T., On performance measures for multistate monotone systems. Reliability Engineering and System Safety, 41, 259-266, 1993

Barlow R.E., Wu A.S., Coherent systems with multi-state components. Mathematics of Operations Research, 4, 275-281, 1978

Brunelle R.D., Kapur K.C., Review and classification of reliability measures for multistate and continuum models. IEEE Transactions,31, 1117-1180, 1999

EU-CIRCLE Report D1.1, EU-CIRCLE Taxonomy, 2015

EU-CIRCLE Report D3.3-GMU11, Global Baltic Network of Critical Infrastructure Networks Operation Process, 2016

EU-CIRCLE Report D1.2-GMU1, Identification of existing critical infrastructures at the Baltic Sea area and its seaside, their scopes, parameters and accidents in terms of climate change impacts, 2016

EU-CIRCLE Report D3.1-GMU4, Baltic Sea area critical infrastructures global network assets and interconnections ("network of networks" approach), 2016

EU-CIRCLE Report D3.3-GMU11, The Baltic Sea area critical infrastructures global network ("network of networks") safety modelling, identification and prediction (without climate-weather change influence), 2016

Grabski F., Semi-Markov Processes: Application in System Reliability and Maintenance, Amsterdam, Boston, Heidelberd, London, New York, Oxford, Paris, San Diego, San Francisco, Singapore, Sidney, Tokyo, Elsevier, 2014

Guze S., Kołowrocki K., Reliability analysis of multi-state ageing consecutive „k out of n: F" systems. International Journal of Materials and Structural Reliability, Vol. 6, No 1, 47-60, 2008

Habibullah M. S., Lumanpauw E., Kolowrocki K., Soszynska J., Ming N. G., A computational tool for general model of operation processes in industrial systems operation processes. Electronic Journal Reliability & Risk Analysis: Theory & Applications, Vol. 2, No 4, 181-191, 2009

Hudson J.C., Kapur K.C., Reliability theory for multistate systems with multistate components. Microelectronics Reliability, 22, 1-7, 1982

Hudson J., Kapur K., Reliability bounds for multistate systems with multistate components. Operations Research, 33, 735-744, 1985

Kołowrocki K., Reliability of Large and Complex Systems, Amsterdam, Boston, Heidelberd, London, New York, Oxford, Paris, San Diego, San Francisco, Singapore, Sidney, Tokyo, Elsevier, 2014b

Kołowrocki K., Soszyńska-Budny J., Reliability and Safety of Complex Technical Systems and Processes: Modeling - Identification - Prediction - Optimization, London, Dordrecht, Heildeberg, New York, Springer, 2011

Lisnianski A., Levitin G., Multi-State System Reliability. Assessment, Optimisation and Applications. World Scientific Publishing Co. Pte. Ltd., New Jersey, London, Singapore, Hong Kong, 2003

Natvig B., Two suggestions of how to define a multi-state coherent system. Advancesin Applied Probability, 14, 434-455, 1982

Ohio F., Nishida T., (1984) On multistate coherent systems. IEEE Transactions on Reliability, 33, 284-287, 1984

Xue J., On multi-state system analysis.IEEE Transactions on Reliability, 34, 329-337, 1985

Xue J., Yang K., Dynamic reliability analysis of coherent multi-state systems, *IEEE Trans on Reliab.* 4(44), 683-688, 1995a

Xue J., Yang K., Symmetric relations in multi-state systems, *IEEE Trans on Reliab* 4(44), 689-693, 1995b

Yu K., Koren I., Guo Y., Generalised multistate monotone coherent systems.IEEE Transactions on Reliability 43, 242-250, 1994