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Sources of Error in Satellite Navigation Positioning

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ABSTRACT: An uninterrupted information about the user's position can be obtained generally from satellite navigation system (SNS). At the time of this writing (January 2017) currently two global SNSs, GPS and GLONASS, are fully operational, two next, also global, Galileo and BeiDou are under construction. In each SNS the accuracy of the user's position is affected by the three main factors: accuracy of each satellite position, accuracy of pseudorange measurement and satellite geometry. The user's position error is a function of both the pseudorange error called UERE (User Equivalent Range Error) and user/satellite geometry expressed by right Dilution Of Precision (DOP) coefficient. This error is decomposed into two types of errors: the signal in space ranging error called URE (User Range Error) and the user equipment error UEE. The detailed analyses of URE, UEE, UERE and DOP coefficients, and the changes of DOP coefficients in different days are presented in this paper.

1 INTRODUCTION

An uninterrupted information about the user's position can be obtained generally from specialized electronic position-fixing system, in particular, satellite navigation system (SNS). At the time of this writing (January 2017) two global SNSs, the American GPS and the Russian GLONASS, are fully operational, two next, also global, Galileo in Europe and BeiDou in China are under construction (www.globaln-ews.ca, www.gpsworld.com, www.insidegnss.com).

In the case of each SNS the accuracy of the user's position determined using a single standalone receiver is affected by the three main factors:
– position's accuracy of each satellite used for calculation,
– accuracy of pseudorange measurements,
– actual satellite geometry.

The user's position error is a function of both the pseudorange error called UERE (User Equivalent Range Error) and user/satellite geometry expressed by Dilution Of Precision (DOP) coefficient. Assuming that the measurements errors for all satellites are identical and independent, the UERE may be defined as the root square of the various errors and biases. Multiplying the UERE by the appropriate DOP value produces the expected precision of the GPS positioning at the one sigma level (el Rabbany 2016, Misra & Enge 2006). The best guideline for position error analyses is pseudorange error budget.

2 USER EQUIVALENT RANGE ERROR

Overall UERE is the ranging error along the vector between the user's receiver and a particular satellite. This error is decomposed into two types of errors: the signal in space ranging error called URE (User Range Error) or SISRE (Signal In Space Ranging Error) and the user equipment error UEE.

$$UERE = \sqrt{URE^2 + UEE^2} \tag{1}$$

URE accounts for pseudorange error due to the space segment and terrestrial segment while UEE for effects of propagation and receiver processing. In the case of URE dominate three different error sources – clock error (estimation, prediction, curve fit and stability), ephemeris error (estimation, prediction and curve fit) and satellite group delay error. Additionally before May 2, 2000 URE of GPS (one only global fully operational SNS at that time) was dominated, very significantly, by Selective Availability (SA), the intentional distortion of the civil signal's clock correction and ephemeris parameters.

Ephemeris errors result when navigation message does not transmit the correct satellite location. It is typical that the radial component of this error is the smallest: the tangential and cross-track errors may be larger by an order of magnitude. Fortunately, the larger components do not affect ranging accuracy to the same degree. Fundamental to SNS is the one-way ranging measurement that ultimately depends on the clock predictability. These satellite clock errors affect both the C/A and P code users in the same way.

In the case of UEE dominate five distinct error sources – ionospheric, tropospheric, multipath, group delay and noise and interference [Beitz 2016]. To eliminate ionospheric error single frequency SNS receivers apply model of ionosphere with eight coefficients transmitted in navigation message by satellites. If the GPS receiver can work in differential mode the Pseudo Range Corrections (PRC) transmitted by reference stations can be used. The corrections to the clock, ephemeris and ionospheric information can be provided by geostationary satellites of SBAS (Satellite Based Augmentation Systems) also. That's why the number of integrated SNS/SBAS receivers increases since few years incessantly. Dual frequency SNS receivers can use the simultaneously measurements of pseudorange on two transmitted satellite frequencies (Januszewski 2012, Misra & Enge 2006).

Unlike the ionosphere, the troposphere is non-dispersive medium for all frequencies carrier of all SNS and SBAS. The refractive index n doesn't depend upon the frequency of the signal. As this index is larger than unity, the speed of propagation of SNS and SBAS signals is lower than in free space and, therefore, the apparent range to a satellite appears larger. The phase and group velocities on all satellites frequencies, the measurements of code and carrier of all these frequencies experience a common delay. The disadvantages is that an elimination of the tropospheric refraction by dual frequency methods, unlike ionospheric delay is not possible (Januszewski 2013, Misra & Enge 2006).

Multipath is mainly caused by reflecting surfaces near the user's receiver, a satellite emitted signal arrives at the receiver by more than one path. Secondary effects are reflections at the satellite during signal transmission. In code tracking the result can be tens or even hundreds of meters of error. To reduce or estimate the multipath effects, various methods were developed – antenna-based mitigation, improved receiver technology and signal and data processing (Beitz 2016, Hofmann-Wellenhof 2008).

Group delay errors occur in the user's receiver. In the case of CDMA (Code Division Multiple Access) receiver (currently GPS, in the future Galileo and BeiDou) these errors are very small (fractional nanosecond) when all measurements are made using a single signal type. In the case of FDMA (Frequency Division Multiple Access) receiver (currently GLONASS) the signals from different satellites have different center frequencies and consequently experience different group delays that can be significant.

Noise and interference errors produce jitter in the estimates of a signal's time of arrival (Beitz 2016).

Finally we can say that in the case of GPS system during last years URE decreased considerably, i.e. in 2001 – 1.6 m, in 2014 – 0.7 m. In this period Standard Positioning Service (SPS) Performance Standard (PS) changed also; before 2008 it was 6 m RMS, later it was considered that equivalent RMS decreased to 4 m only. Additionally in 2008 the worst of any healthy satellite it was 7.8 m (95%), currently it is about 3 m (95%) only (Munchen 2016).

3 DILUTION OF PRECISION COEFFICIENTS

Satellite navigation system (SNS) positioning accuracy is measured by the combined effect of the unmodeled measurement errors and the effect of the satellite geometry. The last effect can be measured by a single dimensionless coefficient called the dilution of precision (DOP). A low DOP coefficient represents a better positional precision due to the wider angular reparative between the satellites used to calculate a user's position. The lower value of this coefficient, the better the geometric strength, and vice versa. DOP value is computed based on the relative user's receiver – satellite geometry at any instance, that is, it requires the availability of both the receiver and the satellite coordinates. Due to the relative motion of the satellites and the user the value of the DOP will change over time (Forsell 2008).

The various DOP forms are used, depending on the user's need. GDOP (Geometric Dilution of Precision) coefficient determines the multiplication factor of the estimated distance measurement error for the estimation of total position and time errors. Its value is the root sum square of the variances along the coordinate axes, that's why the quantity can be regarded as the distance between two points in space and, consequently, it is independent of the selection of coordinate system (Prasad & Ruggieri 2005).

The GDOP represents the combined effect of two coefficients: PDOP (Position Dilution Of Precision) and TDOP (Time Dilution Of Precision). PDOP can be broken into two components: HDOP (Horizontal Dilution Of Precision) coefficient and VDOP (Vertical Dilution Of Precision) coefficient. PDOP represents the contribution of the satellite geometry to the 3D positioning accuracy and HDOP the contribution to the 2D positioning accuracy. Because the receiver can track only those satellites above horizon ($H_{min} > 0^O$), VDOP will always be larger than HDOP.

In order to know the distribution four DOP coefficients, VDOP, HDOP, PDOP and GDOP, values of all four global mentioned above SNSs author's simulating program was used. The calculations based on reference ellipsoid WGS–84 were made for the observer at latitudes 50 – 60^O for masking elevation angle H_{min} = 5^O (the most frequently used value in SNS receiver) and H_{min} = 25^O (representative for the positioning in restricted area where the visibility of satellites can be limited). The parameters of spatial segment, the number of MEO satellites and time interval of constellation repeatability of all four SNSs are presented in the table 1. The geographical longitude of ascending node and argument of latitude of all GPS and GLONASS satellites were taken from current constellations of these operational systems, in the case of Galileo and BeiDou it was nominal future constellation.

Table 1. Global satellite navigation systems, parameters of spatial segment and time interval of constellation repeatability (www.beidou.gov.cn; www.glonass-ianc.rsa.ru; www.gps.gov; European GNSS, 2016, www.gsc.europa.eu).

System	Orbit altitude [km]	Orbit inclination [°]	Number of orbital periods / time interval [min]	Number of MEO satellites
BeiDou	21,500	55	13 / 10,091.48	27
Galileo	23,222	56	17 / 14,360.75	24
GLONASS	19,100	64.8	17 / 11,488.44	24
GPS	20,183	55	2 / 1435.94	31

For each system and for each angle (H_{min}) one thousand (1000) geographic–time coordinates of the observer were generated by random–number generator with uniform distribution:

– latitude interval 0 – 600 minutes (10O),
– longitude interval 0 – 21600 minutes (360O),
– time interval in minutes equal time of constellation repeatability (table 1).

For each geographic–time coordinates the values of all four DOP coefficients were calculated. This value (v) was divided for H_{min} = 5^O into 5 intervals (1≤v<1.5, 1.5≤v<2, 2≤v<3, 3≤v<4, 4≤v<8) and for H_{min} = 25^O into 8 intervals (v<2, 2≤v<3, 3≤v<4, 4≤v<5, 5≤v<6, 6≤v<8, 8≤v<20, v≥20).

Distributions of Dilution Of Precision (DOP) coefficients values at latitudes 50 – 60^O for all four global SNS for masking elevation angle H_{min} = 5^O and 25^O are presented in the table 2 and 3 respectively. We can say that in the first case:

– DOP coefficient value is for all SNSs greater than 1 and less than 3 except for BeiDou (less than 4),
– the percentage of the lowest values of HDOP (less than 1.5) is the greatest for GPS system,
– coefficient HDOP value is for all SNSs greater than 1 and less than 3,
– coefficients PDOP and GDOP are for all SNSs greater than 2 and less than 8,
– and in the second case:
– as for two SNSs, Galileo and GPS, the number of satellite visible above Hmin = 25O can be less than 4 (3D position cannot be determined) No Fix (in percentage) greater than 0, is equal 0.3 and 1.4, respectively,
– for all four SNSs HDOP coefficient can be less than 2, coefficient VDOP is greater than 2 and PDOP and GDOP are greater than 3, the value of all DOP coefficients can be for all SNSs greater than 20 but for GLONASS system this percentage is the greatest,
– the percentage of HDOP coefficient value less than 2 is for GPS system greater than for other three SNSs, considerably,
– the percentage of HDOP coefficient less than 3 and PDOP and GDOP coefficients less than 4 is the lowest for GLONASS system.

The number of satellites (ls) used in GPS or DGPS position determination and the changes of HDOP and VDOP coefficient values in different days in Gdynia are presented in the table 4. All data were registered each minute. For all series of measurements the number ls is for GPS system greater than for DGPS system, in the case of DOP coefficients the both external values of interval are for VDOP greater than for HDOP. Additionally we can say that if ls is greater both coefficients can be and in the most cases are less and vice versa if ls is lower both coefficients can be and are greater (table 5).

Table 2. Distribution of Dilution Of Precision (DOP) coefficient values (v) for satellite navigation systems, BeiDou (BeiD), Galileo (GAL), GLONASS (GLO) and GPS, elevation mask 5^O, latitude 50 – 60^O (own study).

DOP	System	Coefficient value – v				
		1<v≤1.5	1.5<v≤2	2<v≤3	3<v≤4	4<v≤8
HDOP	BeiD	3.2	87.4	9.4	–	–
	GAL	4.3	75.6	20.1	–	–
	GLO	4.1	71.1	24.8	–	–
	GPS	6.9	89.9	3.2	–	–
VDOP	BeiD	28.1	64.4	6.9	0.6	–
	GAL	23.8	67.8	8.4	–	–
	GLO	16.9	68.9	14.2	–	–
	GPS	41.6	48.6	9.8	–	–
PDOP	BeiD	–	–	88.7	11.2	0.1
	GAL	–	–	83.2	16.8	–
	GLO	–	–	78.8	21.2	–
	GPS	–	–	91.2	8.6	0.2
GDOP	BeiD	–	–	70.3	27.8	1.9
	GAL	–	–	68.7	30.8	0.5
	GLO	–	–	61.3	38.7	–
	GPS	–	–	79.5	19.7	0.8

The values of three DOP coefficients indicated by five different stationary GPS receivers (Furuno GP 33, Leica MX 420, Magnavox MX 200, Saab R5 Su-preme Nav, Simrad MX512) located in Gdynia for different masking elevation angles H_{min} and different numbers of satellites visible and used for position determination are presented in the table 6. In the case of Furuno 33 receiver the lowest value of H_{min} is 5^O.

The stationary receivers destined for maritime users can indicate one (Furuno and Magnavox), two (Leica and Simrad) or three (Saab) DOP coefficients. From among five mentioned above receivers HDOP coefficient is indicated by all except for Furuno, VDOP by Leica, Saab, Simrad, PDOP by Furuno and Saab. GDOP and TDOP are not indicated.

All receivers except for Magnavox have 12 channels (MX 200 – 6 only) for GPS pseudorange mea-surements but the number of satellites visible (lv) and used (lu) for position determination is different and these numbers depend on angle H_{min}. Numbers lv and lu are the greatest (12) for Saab if H_{min} = 0° and 5°. Additionally we can recapitulate that:

– HDOP value is the greatest for Magnavox for each H_{min}. This receiver has 6 channels only and therefore the number of satellites used for position fix and for calculate DOP coefficients cannot be greater than 6,
– VDOP coefficient value is for each H_{min} greater than HDOP value in the case of all three receivers (Leica, Saab, Simrad) indicating VDOP and the difference between these values is the greatest for H_{min} = 25° for two last receivers,
– some receivers, e.g. Furuno, indicate one DOP coefficient value only. As in the case of Furuno GP 33 all positions were calculated in mode 3D, it was considered that this coefficient means PDOP,
– if $H_{min} \leq 10°$ HDOP value is the lowest for Saab, if H_{min} = 15° and 20° for Saab and Leica, if H_{min} = 25° for Leica,

if $H_{min} \leq 5°$ VDOP value is the lowest for Saab, if $H_{min} > 5°$ for Leica.

Table 5. The number of satellites ls used in DGPS position determination and DOP coefficients values October 4, 2016 in Gdynia (54°31,0849'N, 018°33,2738'E), Leica MX 420 Navigation System receiver, H_{min} = 5° (own study).

Time [UTC]	ls	HDOP	VDOP
1133	7	1.5	2.4
1134	8	1.0	1.4
1116	9	0.9	1.1
1117	9	1.7	2.4

All values of DOP coefficients presented in the tables 2 and 3 for given geographic-time coordinates were calculated by author for this constellation of four satellites for which GDOP was the lowest. However in some receivers, e.g. Magnavox MX200, the user has the possibility to choose satellite selection criterion. The most frequently used and recommended was the best (the lowest) DOP.

Currently the receivers, particularly professional models, use own and at the same time unknown algorithm for the DOP coefficients calculation. That's why in different receivers DOP values for the same satellite constellation are in given moment and for the same geographic coordinates different (table 6), e.g. if H_{min} = 15° the number of satellites visible by two receivers, Saab and Simrad, was the same (12) but the number of satellites used for position fix was different, 9 and 7, respectively. Because of this difference HDOP and VDOP coefficients values were 1.0 & 2.0 and 1.2 & 2.3, respectively.

Table 3. Distribution of Dilution Of Precision (DOP) coefficients values (v) for satellite navigation systems, BeiDou, Galileo, GLONASS and GPS, and No Fix in percent, elevation mask 25°, latitude 50 – 60° (own study).

DOP	System	No Fix	Coefficient value – v							
		[%]	v≤2	2<v≤3	3<v≤4	4<v≤5	5<v≤6	6<v≤8	8<v≤20	v>20
VDOP	BeiDou	–	–	43.2	32.9	0.7	0.4	5.6	12.0	5.2
	Galileo	0.3	–	54.3	10.1	2.0	1.5	2.9	14.9	14.0
	GLONASS	–	–	42.4	24.3	3.6	2.5	2.7	3.7	20.8
	GPS	1.4	–	36.5	32.2	15.1	5.9	3.0	4.0	1.9
HDOP	BeiDou	–	6.5	69.2	11.8	5.3	1.9	0.9	2.4	3.0
	Galileo	0.3	2.5	62.0	6.1	5.0	5.1	5.0	7.4	6.6
	GLONASS	–	2.5	60.9	11.7	1.9	0.9	1.3	6.8	14.0
	GPS	1.4	17.2	56.1	15.0	3.5	1.7	1.8	2.0	1.3
PDOP	BeiDou	–	–	–	42.1	33.2	–	2.4	14.9	7.4
	Galileo	0.3	–	–	50.3	14.0	1.8	3.3	15.1	15.2
	GLONASS	–	–	–	32.4	33.8	3.7	4.1	4.7	21.3
	GPS	1.4	–	–	37.2	29.6	13.7	9.2	5.8	2.3
GDOP	BeiDou	–	–	–	13.6	39.4	22.3	0.1	14.7	9.9
	Galileo	0.3	–	–	15.3	43.3	5.7	2.7	12.7	20.0
	GLONASS	–	–	–	4.4	47.5	15.4	4.4	6.2	22.1
	GPS	1.4	–	–	18.4	31.0	20.1	16.6	9.4	3.1

Table 4. The number of satellites ls used in position determination and DOP coefficients values in different days in Gdynia (54°31,0849'N, 018°33,2738'E) for GPS and DGPS systems, Leica MX 420 Navigation System receiver, H_{min} = 5° (own study).

System	Day	UTC	Ls	HDOP	VDOP
DGPS	October 4, 2017	0824 – 1224	7 – 9	0.9 – 1.7	1.1 – 2.4
	October 5, 2017	0709 – 1108	8 – 9	0.9 – 1.2	1.2 – 1.8
	October 6, 2017	0710 – 0910	7 – 9	0.9 – 1.2	1.0 – 2.1
GPS	October 6, 2016	0911 – 1011	11 – 12	0.8 – 0.9	1.1 – 1.6

Table 5. The number of GPS satellites and DOP coefficients for different receivers and different masking elevation angle H_{min}, Gdynia (54°31,0849′N, 018°33,2738′E), October 17, 2016 (own study).

H_{min} [°]	Receiver	Number of satellites visible	Number of satellites used	Coefficient HDOP	Coefficient VDOP	Coefficient PDOP
0	Leica MX 420	12	9	1.0	1.5	–
	Magnavox MX 200	11	6	1.3	–	–
	SaaB R5 Supreme Nav	12	12	0.7	1.2	1.4
	Simrad MX512	12	8	1.0	2.0	–
5	Furuno GP−33	12	10	–	–	1.76
	Leica MX 420	12	9	1.0	1.4	–
	Magnavox MX 200	11	6	1.3	–	–
	SaaB R5 Supreme Nav	12	12	0.7	1.2	1.4
	Simrad MX512	12	8	1.0	2.0	–
10	Furuno GP−33	12	9	–	–	1.90
	Leica MX 420	11	8	1.0	1.5	–
	Magnavox MX 200	10	6	1.3	–	–
	SaaB R5 Supreme Nav	12	10	0.9	1.7	1.9
	Simrad MX512	12	8	1.0	2.0	–
15	Furuno GP−33	11	8	–	–	2.35
	Leica MX 420	11	8	1.0	1.5	–
	Magnavox MX 200	10	6	1.3	–	–
	SaaB R5 Supreme Nav	12	9	1.0	2.0	2.2
	Simrad MX512	12	7	1.2	2.3	–
20	Furuno GP−33	11	8	–	–	2.36
	Leica MX 420	11	7	1.0	1.6	–
	Magnavox MX 200	8	5	1.5	–	–
	SaaB R5 Supreme Nav	12	9	1.0	2.0	2.2
	Simrad MX512	12	7	1.2	2.3	–
25	Furuno GP−33	11	7	–	–	3.41
	Leica MX 420	11	7	1.0	1.6	–
	Magnavox MX 200	6	5	1.5	–	–
	SaaB R5 Supreme Nav	12	7	1.3	3.5	3.7
	Simrad MX512	12	6	1.4	3.5	–

4 CONCLUSIONS

– before May 2, 2000 the greatest and at the same time the most important source of the positioning error was Selective Availability (SA) after this day the ionosphere, currently (January 2017) it is user segment, receiver performance parameters,
– in the case of stand-alone GPS receiver the greatest influence has its "professionality", the number of channels in particular,
– in restricted area the multipath error must be taken into account by the user in the case of each SNS receiver,
– as currently the User Range Error (URE) is less than 1 meter the horizontal position's accuracy depends mainly on the user/satellite geometry expressed by DOP coefficient, ionospheric error and in restricted area multipath effects,
– DOP coefficient value depends on the number of satellites used for position fix, this number depends on the receiver, its performance parameters as the number of channels, number of signals and frequencies tracked, year of the production and manufacturer, in particular,
– the tropospheric propagation delay is critical for precise position and baseline determination, can significantly degrade the SNS accuracy, in particular in the height component.

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