archives
of thermodynamics
Vol. **34**(2013), No. 4, 161–173
DOI: 10.2478/aoter-2013-0035

Modeling of cooling of ceramic heat accumulator

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Abstract The analyzed heat accumulator is a key element in a hybrid heating system. In this paper, analytical and numerical models of the ceramic heat accumulator are presented.The accuracy of finite difference methods will be assessed by comparing the results with those obtained from the exact analytical solution.

Keywords: Electric resistance heating; Hybrid heating system; Heat accumulator with dynamic discharge; Central heating system

Nomenclature

A	–	heat exchange area, m^2
c_p	–	specific heat, kJ/(kgK)
L_x	–	accumulator length, m
m	–	mass, kg
$\dot{m}$	–	mass flow rate, kg/s
n	–	time step number
N	–	number of nodes in finite difference grid
N_1	–	air number of heat transfer units
t	–	time, s
t_{pr}	–	transit time of the fluid particle, s
T	–	temperature, °C

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x – cartesian coordinate
x^+ – dimensionless coordinate
w – velocity, m/s

Greek symbols

α – heat transfer coefficient, W/(m^{2}K)
Δt – time step size, s
Δx – grid size, m
ε – relative error, %
θ – excess temperature, °C
ρ – density, kg/m^3
τ – time constant, s

Subscripts

0 – initial
1 – air
c – corrector
in – inlet
p – predictor
w – accumulator bed

1 Introduction

The European Union energy management tends to reduce CO_2 emission and to increase the share of energy obtained from renewable energy sources in the total energy consumption. As a consequence of these actions there are plans for reduction and eventually elimination of natural gas, oil and coal use in building heating system. Electrically-heated ceramic heat accumulator is an alternative to this kind of installation.

The heat accumulator (Fig. 1) analyzed in the paper is a key element in a hybrid heating system. During the night the heat accumulator 1 (Fig. 2) is warmed up with the low-price electrical energy. During the day the heat accumulator is discharged by the flowing air. The hot air is cooled in the plate fin and tube heat exchanger 2 by the water flowing inside the tubes. The heat exchanger performs the function of a low-temperature water boiler. The cooled air stream flows through the reversing duct and is forced by a centrifugal fan through the accumulator. Water is heated up in the plate fin-and-tube heat exchanger and feeds to the central heating system.

Mathematical models of a cylindrical heat accumulator will be presented. An accumulator bed consists of cylindrical ceramic elements which are arranged in an orderly manner (Fig. 1). Small solid cylinders with a di-

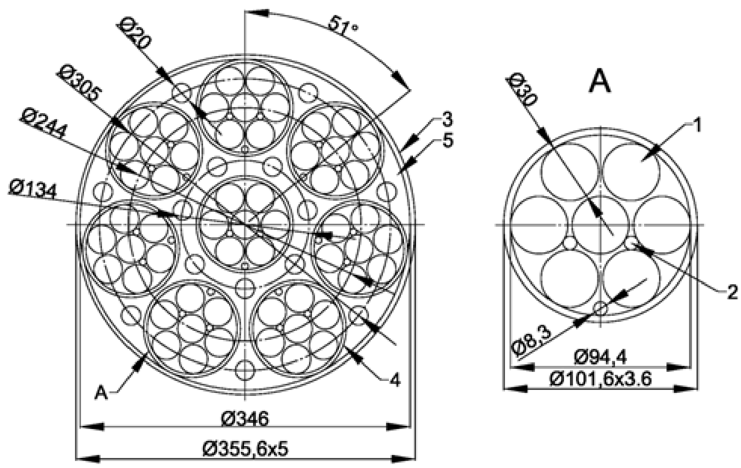

Figure 1. Cross-section of cylindrical heat accumulator: 1 – ceramic cylinder of finite length, 2 – sheath electrical heater, 3 – steel cylindrical casing , 4 – steel tube located inside the accumulator, 5 – bottom of the accumulator with holes through which flows air.

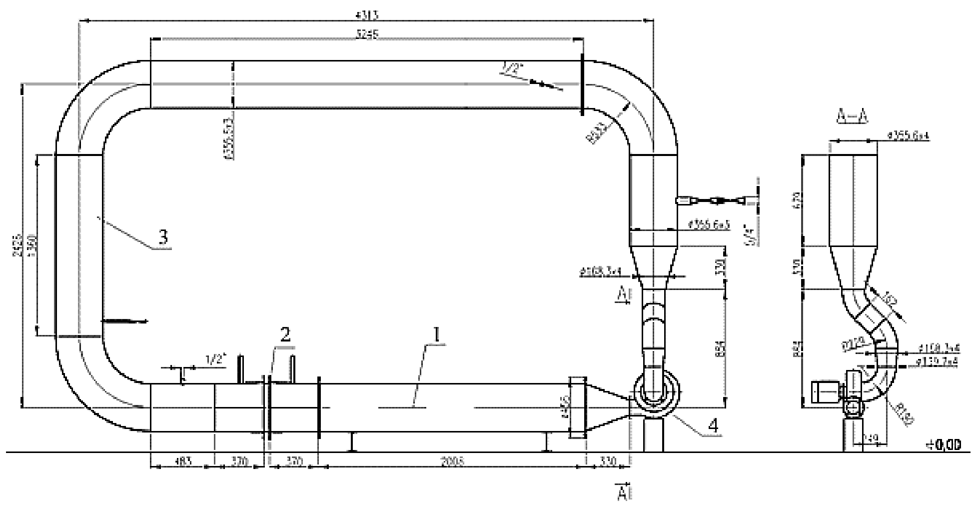

Figure 2. Hybrid heating system with the heat accumulator: 1 – ceramic accumulator heated electrically, 2 – plate fin and tube heat exchanger connected to central heating system, 3 – reversing duct, 4 – ventilator.

ameter of 30 mm and finite height of 30 mm are placed inside steel tubes with an inner diameter of 94.4 mm. The air flows through free spaces between ceramic cylinders and steel tubes. Heat accumulator is the main part of a test facility shown in Fig. 2.

In this paper, analytical and numerical models of the regenerator are presented.The accuracy of finite difference method will be assessed by comparing the results with those obtained from the exact analytical solutions. The temperature of air flowing through the heat accumulator is a function of time and position. The accumulator bed is modeled as an object with the lumped capacity, i.e., the bed temperature is uniform and is a function of time only. Mathematical modeling of the accumulator operation will be conducted for a step-wise increase in air temperature at the inlet of the heat accumulator (Fig. 3). For such case an exact analytical solution can be found in [1].

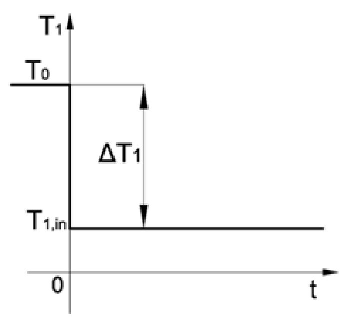

Figure 3. Air temperature changes at the inlet of the accumulator.

2 Mathematical formulation of the problem

The differential equations describing the transient temperature distribution in the air and bed were derived under following simplifying assumptions:

- outer accumulator surface is perfectly insulated;
- heat is only transferred from the bed to the air, but not to the surroundings:
  - air temperature is a function of a coordinate, x, and time, t; at a given cross-section the air temperature is uniform,
  - bed temperature is a function of time only,
  - physical properties of air and ceramic bed are constant.

Temperature changes of the air inside the accumulator are described by the energy conservation equation

$$\tau_1 \frac{\partial T_1}{\partial t} + \frac{1}{N_1}\frac{\partial T_1}{\partial x^+} = -(T_1 - T_w) \ . \tag{1}$$

The number of transfer units N_1 and the time constant τ_1 are defined as follows:

$$N_1 = \frac{\alpha_1 A_w}{\dot{m}_1 c_{p1}} = \frac{\alpha_1 A_w}{\rho_1 w_1 A_1 c_{p1}} \ , \tag{2}$$

$$\tau_1 = \frac{m_1 c_{p1}}{\alpha_1 A_w} \ . \tag{3}$$

The energy conservation equation for the bed is

$$\tau_w \frac{\partial T_w}{\partial t} = T_1 - T_w \ , \tag{4}$$

where the symbol τ_w denotes the time constant of the bed, defined as

$$\tau_w = \frac{m_w c_{pw}}{\alpha_1 A_w} \ . \tag{5}$$

Equation (1) is subject to the following boundary condition at the accumulator inlet

$$T_1 \left.\right|_{x=0} = T_{1,in} \ , \tag{6}$$

where $T_{1,in}$ is the known air temperature at the accumulator inlet.

The initial temperature, T_0, of the air and accumulator bed are assumed to be constant, i.e., the initial conditions have the form

$$T_1 \left.\right|_{t=0} = T_0 \ , \tag{7}$$

$$T_w \left.\right|_{t=0} = T_0 \ . \tag{8}$$

The initial-boundary value problem (1)–(8) will be solved at first using the finite difference method and analytically for a time variation in the inlet temperature of the fluid in the form of a unit jump.

3 Finite difference method application

In the numerical model of the regenerator, the governing system of differential equations (1) and (4) with a boundary condition (6) and initial conditions (7) and (8) will be solved by a finite difference method. Three methods will be used: explicit and implicit finite difference methods of the first-order accuracy and the MacCormac's predictor-corrector method of second-order accuracy [3–5].

3.1 Explicit finite difference method

The explicit finite difference method was used to solve the problem (1)–(8). The arrangement of nodes in the finite difference grid is depicted in Fig. 4. Equations (1) and (4) are approximated by the finite difference equations:

$$\tau_1 \frac{T^{n+1}_{1,i+1} - T^{n}_{1,i+1}}{\Delta t} = -\frac{1}{N_1}\frac{T^{n}_{1,i+1} - T^{n}_{1,i}}{\Delta x^+} - \left(\frac{T^{n}_{1,i} + T^{n}_{1,i+1}}{2} - T^{n}_{w,i}\right), \qquad i = 1,\ldots,N; \quad n = 0,1,\ldots, \tag{9}$$

$$\tau_w \frac{T^{n+1}_{w,i} - T^{n}_{w,i}}{\Delta t} = \tau_w \frac{T^{n+1}_{w,i} - T^{n}_{w,i}}{\Delta t} = \frac{T^{n}_{1,i} + T^{n}_{1,i+1}}{2} - T^{n}_{w,i} \,. \tag{10}$$

The coordinates of the difference grid are:

$x_i = (i-1)\,\Delta x,$ $\quad i = 1,\ldots,N+1$ $\quad$ for air,
$x_i = (i-1)\,\Delta x + \frac{1}{2}\,\Delta x,$ $\quad i = 1,\ldots,N+1$ $\quad$ for accumulator bed,
$t_n = n\Delta t,$ $\quad n = 0,1,\ldots$ $\quad$ for accumulator bed and air.

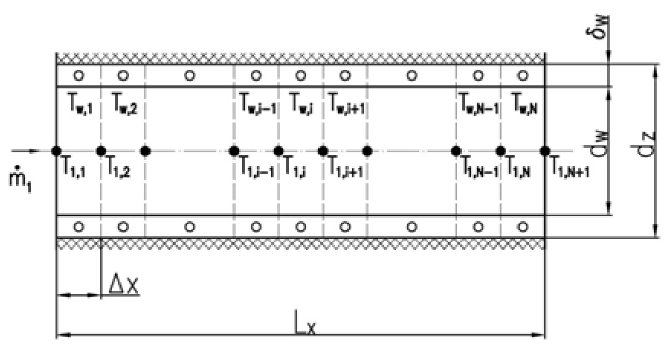

Figure 4. Finite difference grid for accumulator bed and air: • – bed, o – air.

The dimensionless grid size is

$$\Delta x^+ = \Delta x / L_x = 1/N \,,$$

where L_x is the length of the accumulator.

Solving Eq. (9) for $T^{n+1}_{1,i+1}$ and Eq. (10) for $T^{n+1}_{w,i}$ gives, respectively

$$T^{n+1}_{1,i+1} = T^{n}_{1,i+1} - \frac{\Delta t}{\tau_1}\left[\frac{1}{N_1}\frac{T^{n}_{1,i+1} - T^{n}_{1,i}}{\Delta x^+} + \left(\frac{T^{n}_{1,i} + T^{n}_{1,i+1}}{2} - T^{n}_{w,i}\right)\right], \qquad i = 1,\ldots,N, \quad n = 0,1,\ldots \tag{11}$$

$$T_{w,i}^{n+1} = T_{w,i}^{n} + \frac{\Delta t}{\tau_w}\left(\frac{T_{1,i}^{n} + T_{1,i+1}^{n}}{2} - T_{w,i}^{n}\right). \tag{12}$$

The boundary condition can be written as

$$T_{1,1}^{n} = T_0, \quad n = 0, 1, \ldots \tag{13}$$

The initial conditions (7) and (8) take the following form

$$T_{1,i}^{0} = T_0, \quad i = 1, \ldots, N+1 \quad , \tag{14}$$

$$T_{w,i}^{0} = T_0, \quad i = 1, \ldots, N\ . \tag{15}$$

Formulas (11) and (12), after taking into account the boundary condition (13) and the initial conditions (14)–(15), allow to determine the temperature of the air and the bed as a function of position and time. The test calculations were performed for the step size $\Delta t = 0.01$ s.

3.2 Implicit finite difference method

Discretizing Eqs. (1) and (4) using the implicit finite difference method gives:

$$T_{1,i+1}^{n+1} = T_{1,i+1}^{n} + \left[T_{w,i}^{n} - \frac{T_{1,i}^{n+1} + T_{1,i+1}^{n+1}}{2} - \frac{1}{N_1}\frac{T_{1,i+1}^{n+1} - T_{1,i}^{n+1}}{\Delta x^{+}}\right]\frac{\Delta t}{\tau_1}, \tag{16}$$

$$T_{w,i}^{n+1} = T_{w,i}^{n} + \left(\frac{T_{1,i}^{n+1} + T_{1,i+1}^{n+1}}{2} - T_{w,i}^{n+1}\right)\frac{\Delta t}{\tau_w}\ . \tag{17}$$

The system of algebraic equations (16) and (17) which are subject to the boundary conditions (13) and initial conditions (14)–(15) was solved using the Gauss-Seidel method [2]. The step size Δt was also equal to 0.01 s.

3.3 The MacCormac predictor-corrector method

MacCormac's method is an explicit finite difference technique which is second-order accurate in both space and time [3–5].

Predictor step for the air and bed temperature were derived by discretizing Eqs. (1) and (4) using two-stage approximation:

- predictor step

$$Tp_{1,i+1}^{n+1} = T_{1,i+1}^{n} + \left[T_{w,i}^{n} - \frac{T_{1,i}^{n} + T_{1,i+1}^{n}}{2} - \frac{1}{N_1} \frac{T_{1,i+1}^{n} - T_{1,i}^{n}}{\Delta x^{+}} \right] \frac{\Delta t}{\tau_1} , \qquad (18)$$

$$Tp_{w,i}^{n+1} = T_{w,i}^{n} + \left(\frac{T_{1,i}^{n} + T_{1,i+1}^{n}}{2} - T_{w,i}^{n} \right) \frac{\Delta t}{\tau_w} , \qquad (19)$$

- corrector step.

Based on the temperatures $Tp_{1,i+1}^{n+1}$ and $Tp_{w,i}^{n+1}$, calculated in the predictor step, the following finite difference equations for the corrector step are as follows:

$$Tc_{1,i+1}^{n+1} = T_{1,i+1}^{n} + \left[Tp_{w,i}^{n+1} - \frac{Tp_{1,i}^{n+1} + Tp_{1,i+1}^{n+1}}{2} - \frac{1}{N_1} \frac{Tp_{1,i+1}^{n+1} - Tp_{1,i}^{n+1}}{\Delta x^{+}} \right] \frac{\Delta t}{\tau_1} , \qquad (20)$$

$$Tc_{w,i}^{n+1} = T_{w,i}^{n} + \left(\frac{Tp_{1,i}^{n} + Tp_{1,i+1}^{n}}{2} - Tp_{w,i}^{n} \right) \frac{\Delta t}{\tau_w} . \qquad (21)$$

The final solution is an arithmetic average of the predictor and corrector step values:

$$T_{1,i+1}^{n+1} = \frac{Tp_{1,i+1}^{n+1} + Tc_{1,i+1}^{n+1}}{2} , \qquad (22)$$

$$T_{w,i}^{n+1} = \frac{Tp_{w,i}^{n+1} + Tc_{w,i}^{n+1}}{2} . \qquad (23)$$

Similar as in previous methods the boundary conditions (13) and initial conditions (14)–(15) must be accounted for. The test calculations were performed for step size $\Delta t = 0.02$ s.

4 Exact analytical solution

To assess the accuracy of finite difference approximations an analytical exact solution [1] will be applied.

The excess air temperature, θ_1, and bed temperature, θ_w, over the initial temperature, T_0, are defined as the difference between the real and the initial temperatures, i.e.,

$$\theta_1 = T_1 - T_0 , \qquad (24)$$

$$\theta_w = T_w - T_0 . \qquad (25)$$

After an initial jump at $t = 0$, the air inlet temperature to the accumulator remains constant (Fig. 5)

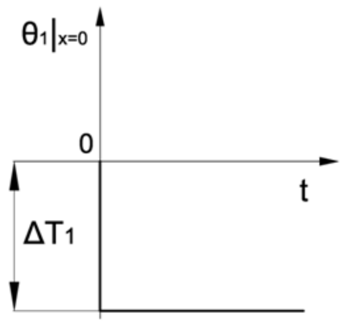

Figure 5. Transformed boundary condition for the air; $\theta|_{x=0} = T_1\,|_{x=0} - T_0$.

$$\theta_1\,|_{x=0}\; = -\Delta T_1\ . \tag{26}$$

The initial conditions are:

$$\theta_1\,|_{t=0}\; = 0\ , \tag{27}$$

$$\theta_w\,|_{t=0}\; = 0\ . \tag{28}$$

The exact analytical solution of Eqs. (1) and (4) subject to the boundary condition (26) and initial conditions (27)–(28) is:

$$\frac{T_1 - T_0}{\Delta T_1} = -U(\xi,\eta)\ \exp[-(\xi+\eta)], \quad t \geq t_{pr}\ , \tag{29}$$

$$\frac{\theta_1 - T_0}{\Delta T_1} = -\big[U(\xi,\eta) - I_o(2\sqrt{\xi\eta})\big]\exp\big[-(\xi+\eta)\big], \quad t \geq t_{pr}\ , \tag{30}$$

where

$$\xi = \frac{x\,N_1}{L_x}\ , \quad \eta = \frac{t - t_{pr}}{\tau_w}\ , \quad t_{pr} = x^{+}N_1\,\tau_1\ . \tag{31}$$

The symbols $I_0\,(x)$, $I_1\,(x)$ and $I_n\,(x)$ denote the modified Bessel functions of the first kind, of order zero, one, and n, respectively. The symbol t_{pr} designates the transit time of the fluid particle from the inlet ($x = 0$) to the x position given by $t_{pr} = x/w_1$.

Calculations were performed for the following data: $N_1 = 1.275$, $\tau_1 = 0.357$ s, $\tau_w = 1013.63$ s, $T_0 = 400$ °C, $\Delta T_1 = 380$ °C. These parameters are calculated based on actual accumulator data, which was installed in the experimental facility. The dimensionless spatial and time steps in the finite difference method were: $\Delta x^{+} = 1/25$, $\Delta t = 0.01$ s, both for explicit and implicit method. For the predictor-corrector method the time step was larger and equal to: $\Delta t = 0.02$ s. The adopted time step should satisfy the Courant condition [3–5], when the explicit finite difference method is used

$$\frac{\Delta x}{w_1\,\Delta t} = \frac{\Delta t}{N_1\,\tau_1\,\Delta x^+} \leq 1\ .$$

Substituting the data gives

$$\frac{\Delta t}{N_1\,\tau_1\,\Delta x^+} = \frac{0.01}{1.275 \cdot 0.357 \cdot 0.04} = 0.55\ ,$$

it means that the Currant condition is satisfied.

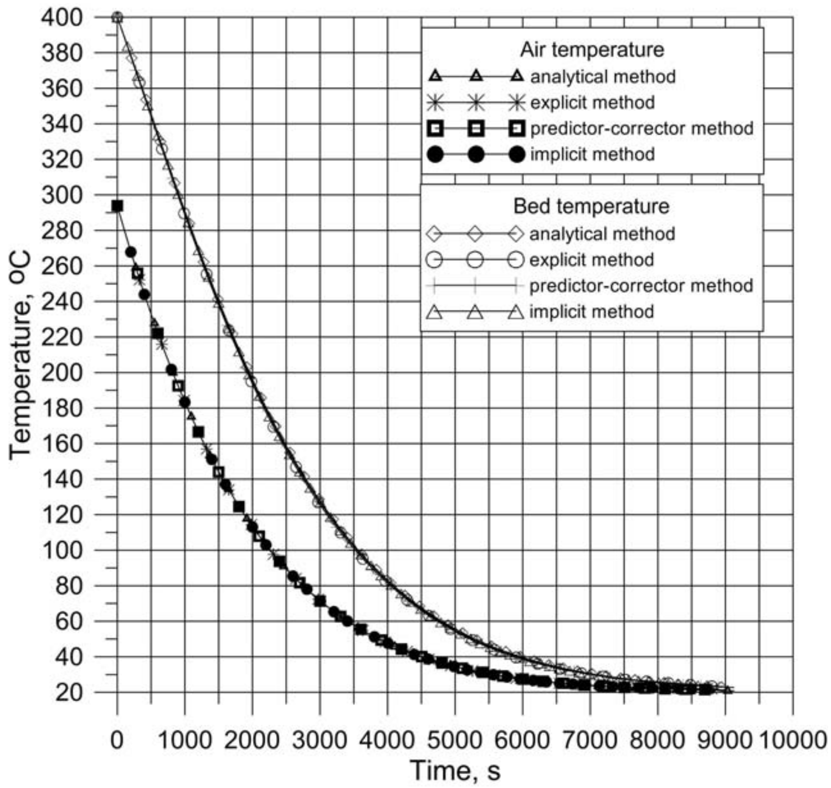

Figure 6. Comparison of the air and bed temperature at the accumulator outlet ($x^+ = 1$) – logarithmic scale.

The comparison of results obtained using the explicit (11)–(12), implicit (16)–(17), predictor-corrector (18)–(19) finite difference methods and

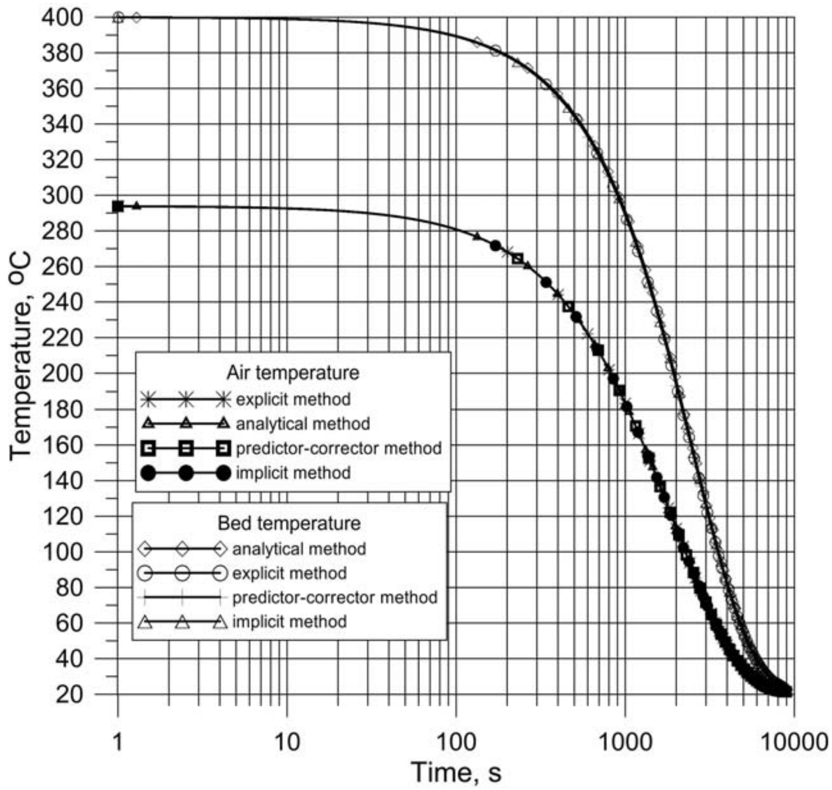

Figure 7. Comparison of the air and bed temperature at the accumulator outlet ($x^+ = 1$).

the exact analytical solution are presented in Figs. 6. and 7. The consistency between the approximate and exact solution is very satisfactory. The differences between the exact analytical solution and the finite difference solutions are almost invisible.

The relative error of the finite difference solutions with respect to the exact solution is given by

$$\varepsilon = \frac{T_m - T_{an}}{T_{an}} 100\% \ ,$$

where T_m and T_{an} are the numerical and analytical results, respectively. As shown in Fig. 8 the predictor-corrector, as a second order method, is characterized by the lowest relative error comparing to the analytical solution. Explicit ad implicit finite difference method relative errors are almost equal. It can be concluded that all the method can be used for the

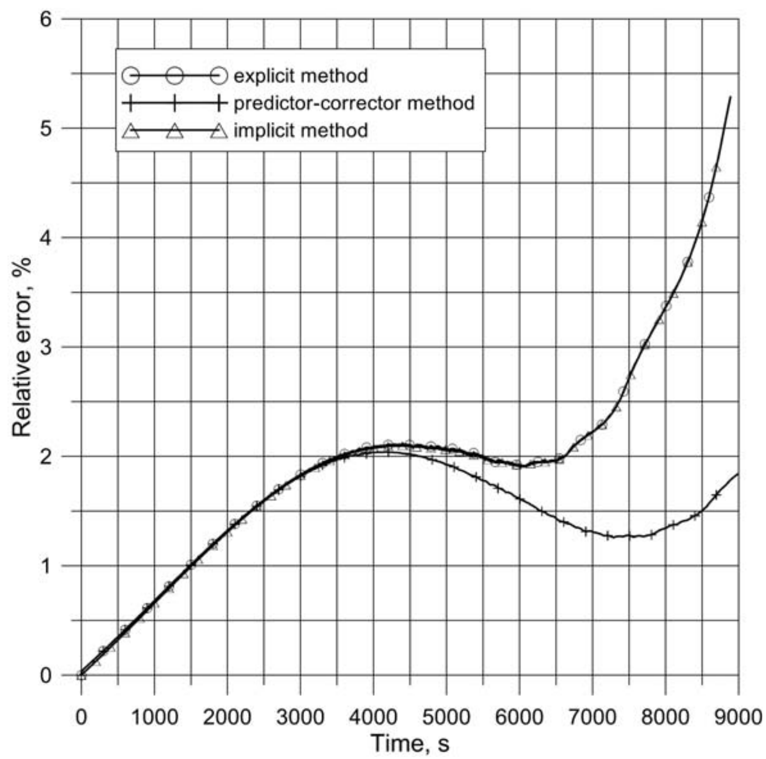

Figure 8. Relative error of the finite difference solutions with respect to the exact solution.

simulation of heat accumulators. After validation of the numerical models, they can be applied to the modeling of regenerator operation using actual boundary and initial conditions.

5 Conclusion

Consistency of the results obtained using the exact analytical method and finite difference method is very good. It was demonstrated that the accuracy of the explicit and implicit finite difference methods are comparable with the more accurate MacCormac method. The finite difference method can also be used to model the heat accumulators with time dependent inlet air velocity or temperature.

Received 14 October 2013

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