Electrostatic microactuators for precise positioning and comparison of their parameters

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Comparison of several types of electrostatic microactuators is carried out, particularly with respect to their resultant force effects. The continuous mathematical model of such actuators is mostly described by the Laplace equation. In this paper, its numerical solution is performed by a fully adaptive higher-order finite element method, using a code developed by the authors. The methodology is illustrated with typical examples whose results are discussed.

KEYWORDS: electrostatic microactuators, force effects, fully adaptive higher-order finite element method, numerical analysis, efficiency.

1. Formulation of technical problem

Precise position control is required in numerous scientific disciplines such as optics, microscope techniques or microsurgery. This control may be realized on the basis of several physical principles (mechanical, hydraulic, pneumatic, thermoelastic, piezoelectric, electrostatic, etc.). In very small applications working with low energies and powers, the micro-actuators based on the electrostatic principles may represent an efficient and reliable solution. They are generally very fast and their operation requires only a relatively low voltage. Three basic types of these actuators (longitudinal actuator, transversal actuator and actuator with the dielectric armature [1-3]) are schematically depicted in Figs. 1–3.



Fig. 1. Basic arrangement of longitudinal micro-actuator

The first two of these micro-actuators work on the principle of the Coulomb force (acting between electrically charged bodies), the third one works with the V. Starman, J. Kacerovsky, J. Jansa, P. Karban, I. Dolezel / Electrostatic ...

Maxwell force (acting on dielectric material in electric field). The aim of the paper is to find and evaluate their principal operation parameters.



Fig. 2. Basic arrangement of transversal micro-actuator



Fig. 3. Basic arrangement of micro-actuator with dielectric armature

2. Continuous mathematical model

Electric field in the domain of the micro-actuator is described by the equation for the electric potential φ in the form

$$\operatorname{div}(\varepsilon \operatorname{grad} \varphi) = 0, \qquad (1)$$

where symbol ε stands for the dielectric permittivity. The boundary conditions are given by the known values of the electric potential on the electrodes and the Neumann condition along the artificial boundary placed at a sufficiently distance from the device.

The force acting on the actuator can be determined from the change of the total electric energy W_e in the definition area V of the system. This energy can easily be determined from the volumetric energy w_e . The relevant formulas are as follows

$$w_{\rm e} = \frac{1}{2} ED, \quad W_{\rm e} = \int_{V} w_{\rm e} \,\mathrm{d}V, \tag{2}$$

where E is the vector of electric field strength at a point and D denotes the vector of the corresponding dielectric flux density.

The electric force F_{e} acting on the movable part of the micro-actuator is generally given by the formula

$$F_{\rm e} = -\operatorname{grad} W_{\rm e}$$

but its component $F_{e\zeta}$ in the direction ξ (see Figs. [1–3]) can be expressed as

$$F_{e\zeta} = -\frac{\partial W_e}{\partial \xi} \,. \tag{3}$$

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3. Numerical solution

The continuous mathematical model given by the equations (1) and (2) is solved numerically. We used our own codes Agros2D [4] and Hermes2D [5]. While Hermes is a library of numerical algorithms for monolithic and fully adaptive solution of systems of generally nonlinear and non-stationary partial differential equations (PDEs) based on the finite element method of higher order of accuracy, Agros2D is a powerful GUI serving for pre-processing and post-processing of the problems solved. Both codes written in C++ are intended for the solution of complex coupled problems rooting in various domains of physics. They are freely distributable under the GNU General Public License v2. The most important (and in some cases quite unique) features of the codes follow:

- Fully automatic *hp*-adaptivity. In every iteration step the solution is compared with the reference solution (realized on an approximately twice finer mesh), and the distribution of error is then used for selection of candidates for adaptivity. Based on sophisticated and subtle algorithms the adaptivity is realized either by a subdivision of the candidate element or by its description by a polynomial of a higher order [6].
- Each physical field can be solved on quite a different mesh that best corresponds to its particulars. Special powerful higher-order techniques of mapping are then used to avoid any numerical errors in the process of assembly of the stiffness matrix.
- Easy treatment of the hanging nodes [7] appearing on the boundaries of subdomains whose elements have to be refined. Usually, the hanging nodes bring about a considerable increase of the number of the degrees of freedom (DOFs). The code contains higher-order algorithms for respecting these nodes without any need of an additional refinement of the external parts neighboring with the refined subdomain.
- Curved elements able to replace curvilinear parts of any boundary by a system of circular or elliptic arcs. These elements mostly allow reaching highly accurate results near the curvilinear boundaries with very low numbers of the DOFs.

4. Illustrative examples

The voltage between the electrodes of all three micro-actuators U = 100 V. The length of the electrode $l = 45 \,\mu\text{m}$, the relative permittivity of the dielectric material (see Fig. 3) $\varepsilon_r = 12$. The distance of the artificial boundary was placed (after evaluating several testing examples) 270 μ m from the investigated system.

For illustration, Fig. 4 shows the final discretization mesh used for covering of the definition area of the longitudinal actuator. This mesh was built using the *hp* adaptive process and the rectangles on its right-hand side contain the polynomial degrees of particular elements. For the same type of micro-actuator, Fig. 5 shows the convergence curve of solution (in other words, the dependence of the relative error of solution on the number of the degrees of freedom). The threshold was set $\eta = 3$ %.

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Fig. 4. Mesh and polynomial order after adaptive process (longitudinal micro-actuator)



Fig. 5. Dependence of relative error on degrees of freedom (longitudinal microactuator)

Figures 6–8 show the distribution of equipotentials in all three arrangements.



Fig. 6. Equipotential lines of electric potential in longitudinal micro-actuator



Fig. 7. Equipotential lines of electric potential in transversal micro-actuator

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Fig. 8. Equipotential lines of electric potential in micro-actuator with dielectric material

Finally, Figs. 9-11 depict the static characteristics of the particular microactuators. From the viewpoint of the force acting on the movable part of the device, the highest values are reached in the case of the transversal micro-actuator. The principal reason is a very short distance between both oppositely charged surfaces.





Fig. 11. Dependence of force on displacement (dielectric material)

20

 $\xi (\mu m)$

25

30

35

40

15

10

5

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5. Conclusion

Mathematical and computer modeling of the investigated actuators fully confirmed their expected operation properties. The results show that the electrostatic micro-actuators can be used in two ways. Either they can be used in devices whose operation requires large forces or in devices using small and precise setting of the position. The MEMS devices provide a high performance and low cost solution for satisfying servo requirements.

In next work, the authors will address the dynamics of these devices and creation of more precise models in the 3D arrangements.

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