

Power line induced electric field and electromotive force in a nearby overhead circuit

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The study presents methods of the calculation of electric field induced in vicinity of an overhead current carrying conductor. Exact method of the determination of the induced electric field bases on the Fourier transform technique, whereas a simplified method uses the concept of complex ground return plane.

Exact and simplified methods of the calculation of electromotive force (*emf*) induced in a loop located under an overhead power line conductor are also presented. The loop is treated as a rectangular loop (two-conductor closed loop) located near the power line horizontal to the earth surface. The exact method bases on the earth return circuit theory, whereas the simplified method allows one to calculate the induced *emf* under the assumption neglecting earth currents. The methods presented are illustrated by exemplary calculations.

KEYWORDS: overhead power line, earth return, induced electric field, loop conductor, electromotive force

1. Introduction

Transmission of the electric power is accompanied with generation of low – frequency electromagnetic fields. Nowadays of special concern is the possibility of detrimental environmental effects arising from the electrical and magnetic fields formed adjacent to the overhead transmission lines. These fields may affect both operation of near electric and electronic devices and appliances and also various living organisms, e.g. [1, 2].

Computation of the overhead power line electric field intensity can be sought through the well-known equation which combines scalar electric potential V and vector magnetic potential \vec{A} :

$$\vec{E} = -gradV - j\omega\vec{A} = \vec{E}_s + \vec{E}_i \quad (1)$$

where E_s and E_i mean the static and induced electric field intensity, respectively.

The knowledge of the induced electric field is necessary e.g. in calculation of the self and mutual impedances of earth return circuits and also in calculation of electromotive force induced in nearby circuits.

In the paper exact and approximate methods are presented for analyzing electric fields induced by current flowing in a straight overhead conductor with earth return. The exact method bases on the Fourier transform technique, whereas the simplified method uses the concept of complex ground return plane.

In the study exact and simplified methods of the determination of the electromotive force (*emf*) induces in a loop conductor hanging under a straight overhead current carrying conductor of infinite length are also presented. The loop can be treated as a rectangular loop (two-conductor closed mitigation loop) located near the power line horizontal to the earth surface.

The exact method bases on the earth return circuit theory taking into account the earth current induced in the earth, whereas the simplified method allows one to calculate the induced *emf* under the assumption neglecting earth currents.

2. Electric field of a straight overhead conductor

2.1. Exact method

An infinitely long conductor is placed at height h_k above the earth surface, Fig. 1, and carries the current I_k , which flows in direction of the x -axis. The current varies with the time as $\exp(j\omega t)$ where ω is the radian frequency. The x, y plane is considered to be the earth surface. It is assumed that the earth is an isotropic, homogeneous medium of finite conductivity γ . The magnetic permeability of the soil and of the air is μ_0 . The displacement currents in both regions: the air and the earth are neglected.

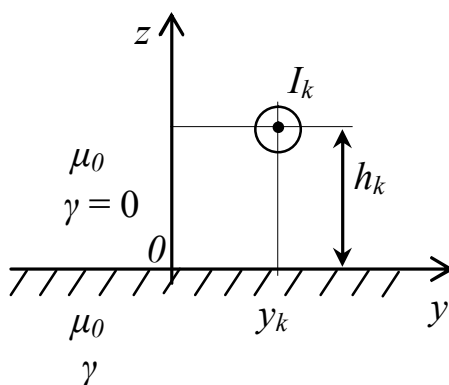


Fig. 1. An infinitely long current carrying conductor above the earth surface

The vector potential of the electromagnetic field has the x -component only denoted $A_x(y,z)$ which satisfies the following equations:

- the Poisson equation in the air:

$$\frac{\partial^2 A_x(y, z)}{\partial y^2} + \frac{\partial^2 A_x(y, z)}{\partial z^2} = -\mu_0 I_k \delta(y - y_k) \delta(z - h_k) \quad (2)$$

– the Helmholtz equation in the earth:

$$\frac{\partial^2 A_x(y, z)}{\partial y^2} + \frac{\partial^2 A_x(y, z)}{\partial z^2} = k^2 A_x(y, z) \quad (3)$$

where: δ – Dirac delta function, $k^2 = j\omega\mu_0\gamma$.

The vector potential \vec{A} in the air can be obtained if the Fourier transform is used and the boundary conditions in the system considered expressing the continuity of the normal component of the magnetic flux density and the tangential components of the magnetic as well as electric intensities are taken into account.

Hence the x -component of the vector potential in the air can be written in the form:

$$A_x(y, z) = \frac{\mu_0 I_k}{2\pi} \int_0^\infty \left[\frac{e^{-u|z-h_k|}}{u} + \frac{(u-\alpha)e^{-u(z+h_k)}}{u(u+\alpha)} \right] \cos[(y-y_k)u] du \quad (4)$$

where:

$$\alpha = \sqrt{u^2 + k^2} \quad (5)$$

It should be noted that the formula (4) is the same that obtained by Carson [3].

The induced electric field intensity in the air can be obtained from the relation:

$$\vec{E}_i = -j\omega\vec{A} \quad (6)$$

The x -component of the induced electric field intensity can be now written in the form:

$$E_{ix}(y, z) = -\frac{j\omega\mu_0 I_k}{\pi} \left\{ \frac{1}{2} \int_0^\infty \frac{e^{-u|z-h_k|} - e^{-u(z+h_k)}}{u} \cos[(y-y_k)u] du + \int_0^\infty \frac{e^{-u(z+h_k)}}{u + \sqrt{u^2 + k^2}} \cos[(y-y_k)u] du \right\} \quad (7)$$

Taking into account the formula [4]:

$$\int_0^\infty \frac{e^{-au} - e^{-\beta u}}{u} \cos(au) du = \frac{1}{2} \ln \frac{a^2 + \beta^2}{a^2 + \alpha^2} \quad (8)$$

where: $\alpha > 0$, $\text{Re}(\alpha) \geq 0$, $\text{Re}(\beta) > 0$ we get:

$$E_{ix}(y, z) = -\frac{j\omega\mu_0 I_k}{\pi} \left\{ \frac{1}{4} \ln \frac{(z+h_k)^2 + (y-y_k)^2}{(z-h_k)^2 + (y-y_k)^2} + \int_0^\infty \frac{e^{-u(z+h_k)}}{u + \sqrt{u^2 + k^2}} \cos[(y-y_k)u] du \right\} \quad (9)$$

Denoting:

$$u = |k|n \quad (10)$$

$$\sqrt{n^2 + j} = a + jb \quad (11)$$

where:

$$a = \sqrt{\frac{\sqrt{n^4 + 1} + n^2}{2}} \quad (12)$$

$$b = \sqrt{\frac{\sqrt{n^4 + 1} - n^2}{2}} \quad (13)$$

the electric field intensity becomes:

$$E_{ix}(y, z) = -\frac{j\omega\mu_0 I_k}{\pi} \left\{ \frac{1}{4} \ln \frac{(z+h_k)^2 + (y-y_k)^2}{(z-h_k)^2 + (y-y_k)^2} + j \int_0^\infty (n - a - jb) e^{-(z+h_k)|k|n} \cos[(y-y_k)|k|n] dn \right\} \quad (14)$$

2.2. Complex ground return plane approach

As an alternative for the accurate method of the evaluation of the influence of currents induced in the earth on total electric field intensity produced by an overhead power line the concept of the complex ground return plane can be applied. The concept, initially proposed by Dubanton at Electricité de France and published by Gary [5], has been developed for simple and sufficiently accurate calculations for line impedances, valid for the whole range of frequencies. The formal proof of the complex ground return plane approach for calculation of transmission line impedances has been presented in [6, 7]. According to [6, 7] a return current flow in homogeneous earth can be modelled by a perfect conducting plane which is located at a complex depth p below the earth surface, as shown in Fig. 2.

The complex plane appears as a mirroring surface, so that conductor images can be used to derive simple formulas for impedances of earth return circuits. Its depth is defined by:

$$p = \frac{1}{\sqrt{j\omega\mu_0\gamma}} \quad (15)$$

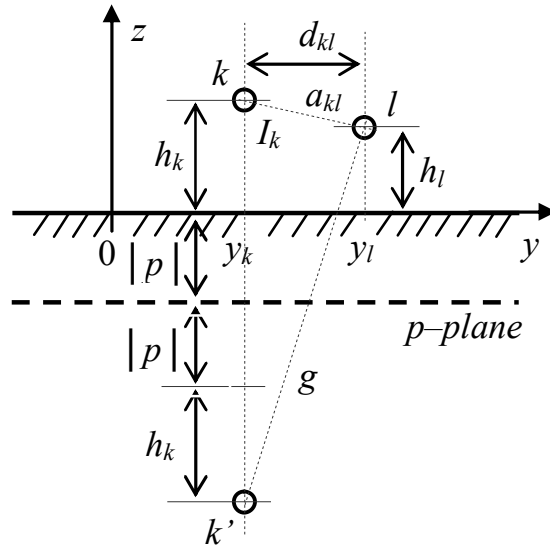


Fig. 2. To the calculation with the use of the complex ground return plane approach

The heuristic Dubanton method gives equations for the unit-length self impedance of the conductor k and the unit-length mutual impedances between conductors k and l . According to Fig.2, the unit-length mutual impedance can be obtained from the relation [6, 7]:

$$Z_m = j\omega \frac{\mu_0}{2\pi} \ln \frac{g}{a_{kl}} \quad (16)$$

On the other hand the mutual impedance between two earth return circuits k and l can be defined as the negative ratio of the electromotive force induced in the second conductor to the current in the first conductor [8]:

$$Z_m = -\frac{1}{I_k} \int_c \vec{E}_i \cdot d\vec{l} \quad (17)$$

whereas the integration path c lies on the axis of the second conductor.

Thus, one can deduce that the induced electric field intensity generated in an observation point (y, z) by an overhead conductor with the current I_k can be expressed in the form:

$$E_{ix\ approx}(y, z) = -j\omega \frac{\mu_0 I_k}{2\pi} \ln \frac{\sqrt{(z + h_k + 2p)^2 + (y - y_k)^2}}{\sqrt{(z - h_k)^2 + (y - y_k)^2}} \quad (18)$$

3. Electromotive force induced in a loop

3.1. Exact solution

Consider the electromotive force (*emf*) induced in the horizontal, rectangular loop located underneath an infinitely long overhead conductor with a current, as in Fig. 3.

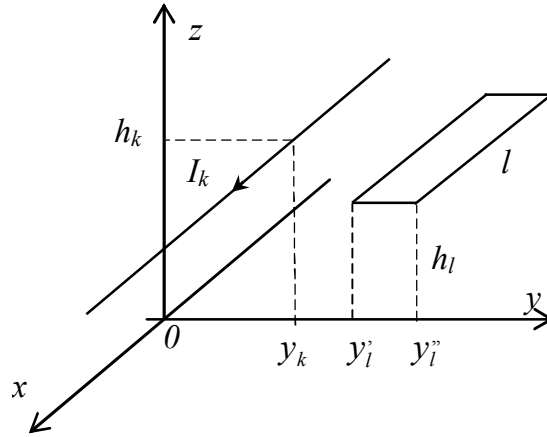


Fig. 3. Current carrying conductor and a loop

Electromotive force (*emf*) induced in the loop is obtained by applying the general equation:

$$emf = \oint_l \vec{E}_{ix} \cdot d\vec{l} \quad (19)$$

Taking into account that the induced electric field is given by eqn.(14), it is easy to show that the electromotive force (*emf*) induced in the loop takes the form:

$$emf = -\frac{j\omega\mu_0 I_k l}{\pi} \left\{ \frac{1}{4} \ln \left[\frac{(h_k - h_l)^2 + (y_l'' - y_k)^2}{(h_k - h_l)^2 + (y_l' - y_k)^2} \right] \frac{(h_k + h_l)^2 + (y_l' - y_k)^2}{(h_k + h_l)^2 + (y_l'' - y_k)^2} \right\} + \int_0^\infty b e^{-(h_k + h_l)|k|n} [\cos[(y_l' - y_k)|k|n] - \cos[(y_l'' - y_k)|k|n]] dn + \left. + j \int_0^\infty (n - a) e^{-(h_k + h_l)|k|n} [\cos[(y_l' - y_k)|k|n] - \cos[(y_l'' - y_k)|k|n]] dn \right\} \quad (20)$$

where l denotes the length of the longitudinal loop-conductor.

3.2. Approximate solution

Earth return circuit theory

The possibility to derive the approximate formula for the induced *emf* consists in the calculation of the *emf* with the application of mutual impedances defined in the earth return circuit theory [3, 8]. Consider the equivalent circuit of a loop as shown in Fig. 4.

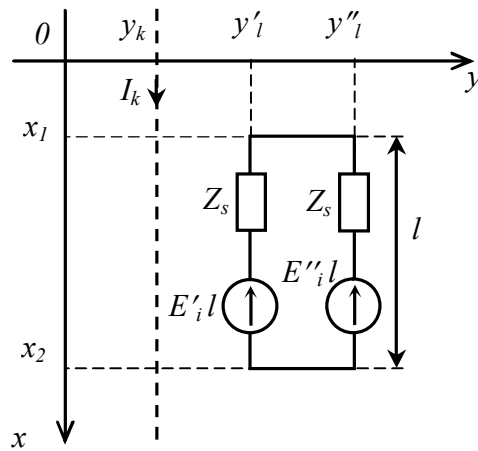


Fig. 4. Equivalent circuit of a loop subjected to inductive effects of a current carrying overhead conductor

The active elements of the circuit represent voltage sources with the source voltages:

$$E'_{ix}l = -I_k Z'_{kl}l, \quad E''_{ix}l = -I_k Z''_{kl}l \quad (21)$$

where E'_{ix} and E''_{ix} denote the induced electric field intensities along the longitudinal conductors of the loop, I_k is the current in the overhead conductor, l is the length of the longitudinal loop conductor, and Z'_{kl} and Z''_{kl} are the unit-length mutual impedances between the overhead conductor and the longitudinal loop-conductors (as in Fig. 5) given by the general relation:

$$Z'_{km} = \frac{\mu_0 I}{2\pi} \int_0^\infty \left[\frac{e^{-u|h_m-h_k|}}{u} + \frac{(u-\alpha)e^{-u(h_m+h_k)}}{u(u+\alpha)} \right] \cos(a_{km}u) du \quad (22)$$

where:

$$\alpha = \sqrt{u^2 + k^2} \quad (23)$$

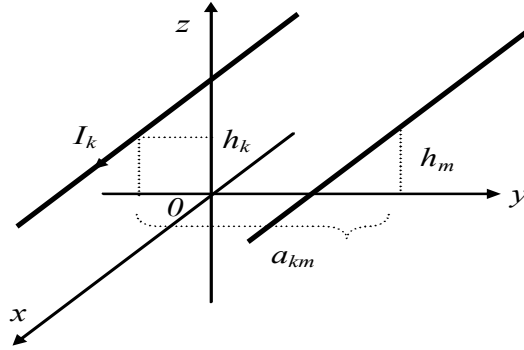


Fig. 5. Two parallel overhead conductors with earth return

The improper integral appearing in (22) has no a closed-form solution. If the distance between conductors $s_{km} < 0,3\Delta$, where:

$$s_{km} = \sqrt{(h_k - h_m)^2 + a_{km}^2} \quad (24)$$

and

$$\Delta = \frac{1,85}{\sqrt{\omega\mu_0\gamma}} \quad (25)$$

the unit-length mutual impedance can be approximately calculated from the relationship [9]:

$$Z'_{km} = \frac{\omega\mu_0}{8} + j \frac{\omega\mu_0}{2\pi} \ln \frac{\Delta}{s_{km}} \quad (26)$$

The passive element Z_s of the circuit shown in Fig.4 is the self impedance of the longitudinal loop-conductor. The equation for the unit-length self impedance of the conductor k with the radius r_k is [6, 7]:

$$Z'_s = j\omega \frac{\mu_0}{2\pi} \ln \frac{2(h_k + p)}{r_k} \quad (27)$$

The resultant driving emf in the system shown in Fig. 4 takes the form:

$$emf_{approx} = I_k l (Z'_{kl'} - Z'_{kl}) = -I_k \frac{j\omega\mu_0 l}{2\pi} \ln \frac{s_{kl}''}{s_{kl}'} \quad (28)$$

Complex ground return plane approach

Taking into account that according to the complex ground return approach, the induced electric field intensity generated in an observation point (y,z) by an overhead conductor with the current I_k is given by eqn. (18), from eqn.(19) we get:

$$emf_{approx} = -\frac{j\omega\mu_0 I_k l}{4\pi} \ln \frac{[(h_l + h_k + 2p)^2 + (y_l' - y_k)^2][(h_l - h_k)^2 + (y_l'' - y_k)^2]}{[(h_l + h_k + 2p)^2 + (y_l'' - y_k)^2][(h_l - h_k)^2 + (y_l' - y_k)^2]} \quad (29)$$

Simplified approach neglecting earth currents

If the effects of the currents induced in the earth on the total magnetic field produced under the overhead conductor are negligible as compared with the effects due to the current flowing in the overhead conductor only, the magnetic flux passing the loop can be determined from the relation (30):

$$\Phi_{approx} = l \int_{y_l'}^{y_l''} B_{z\ approx}(y, z = h_l) dy \quad (30)$$

In the relation (30) $B_{z\ approx}(y, z)$ denotes the z – component of the magnetic flux density produced by current carrying infinitely long conductor in homogeneous medium (air) [10]:

$$B_{z\ approx}(y, z) = \frac{\mu_0 I}{2\pi} \frac{y - y_k}{(z - h_k)^2 + (y - y_k)^2} \quad (31)$$

Thus

$$\Phi_{approx} = \frac{\mu_0 I_k l}{4\pi} \ln \frac{(h_l - h_k)^2 + (y_l'' - y_k)^2}{(h_l - h_k)^2 + (y_l' - y_k)^2} \quad (32)$$

The electromotive force can be next obtained from (32) in the simplified form:

$$emf_{approx} = -\frac{j\omega\mu_0 I_k l}{4\pi} \ln \frac{(h_l - h_k)^2 + (y_l'' - y_k)^2}{(h_l - h_k)^2 + (y_l' - y_k)^2} \quad (33)$$

It should be noted that the approximate formula (33) obtained for the resultant electromotive force in the loop is identical with the relation (28).

4. Exemplary calculations

4.1 Induced electric field

This numerical example concerns an overhead power line conductor, as shown in Fig.1. The infinitely long conductor with the unit-current $I_k = 1$ A is hanging over the earth surface ($y_k = 0$ m, $h_k = 10$ m); the earth conductivity $\gamma = 0,01$ S/m.

Computation of the induced electric field intensity is carried out at observation points along y -directed profile, positioned at 2 m above the earth surface ($z = 2$ m) and on the earth surface ($z = 0$ m), respectively.

Fig. 6 presents computed values of the electric field intensity, along the observation profile. The maximum values of the electric field intensity calculated in the observation point ($y = 0, z = 0$) equal 0,294 mV/m and 0,290 mV/m, whereas in the observation point ($y = 0, z = 2$ m) 0,308 mV/m and 0,304 mV/m for the approximate and exact method, respectively. The results obtained by the use of the exact method are in good agreement with those obtained by the approximate method.

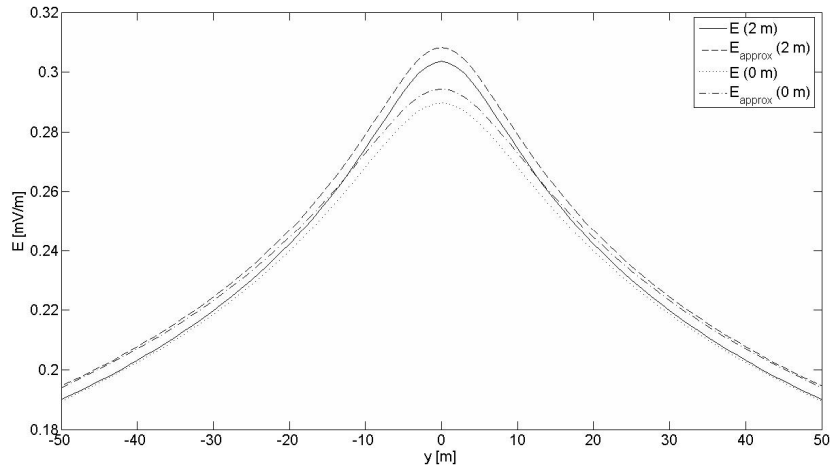


Fig. 6. Profiles of the induced electric field intensity under the overhead conductor with the unit-current

Next the maximum values of the E_i obtained from relationships (14) and (18) have been calculated for the system shown in Fig.1 for the unit current $I = 1$ A ($y_k = 0$ m) in the observation point $y=0$ m, $z=2$ m. Different values of the earth conductivity: $\gamma = 10^{-1}; 10^{-2}; 10^{-3}$ S/m and heights of the conductor: $h_k = 10$ m; 20 m have been considered. The results of the calculations are shown in the Table1.

Table 1. Maximum values of the induced electric field intensity and the ratio $E_{ixapprox}(0, 2) / E_{ix}(0, 2)$ 2 m above the ground

γ	h_k	E_{imax}	$E_{imaxapprox}$	$E_{imaxapprox} / E_{imax}$
[S/m]	[m]	[mV/m]	[mV/m]	[%]
0,1	10	0,23	0,24	101,85
	20	0,18	0,19	102,15
0,01	10	0,30	0,31	101,53
	20	0,25	0,26	101,78
0,001	10	0,37	0,38	101,27
	20	0,32	0,33	101,45

It follows from the results shown in the Table1, that in practical cases the induced electric field intensity can be calculated using the approximate method basing on the complex ground return plane approach.

4.2. Induced electromotive force

Exemplary calculations of the emf induced in the rectangular loop hanging under a power line conductor as shown in Fig. 3 have been carried out assuming that the conductor with the unit current $I_k = 1$ A is located horizontal to the earth at height $h_k = 10$ m; $y_k = 0$ m and $\gamma = 10^{-2}$ S/m. The results of calculations with the use of the exact method and the approximate methods are shown in the Table 2. The results show an excellent agreement.

Table 2. Electromotive force induced in the rectangular loop

Loop parameters	Induced emf [mV]		
	eqn. (20)	eqn. (29)	eqn. (33)
$L = 300$ m; $h_l = 8$ m $y_l' = 2$ m; $y_l'' = 10$ m	24,17	24,17	24,17
$L = 300$ m; $h_l = 8$ m $y_l' = 2$ m; $y_l'' = 30$ m	44,55	44,55	44,55
$L = 300$ m; $h_l = 8$ m $y_l' = 2$ m; $y_l'' = 50$ m	54,14	54,16	54,16

5. Conclusions

The design of installation generating low-frequency magnetic and electric fields requires access to effective analytical and computational tools. The paper presents procedures of determining the intensities of the induced electric field under power line conductor as well as the electromotive force produced in a current loop under power line conductor.

Exact and approximate methods are described for analyzing electric fields. It is shown, that the use of the approximate method of the induced electric field intensity calculations gives the results with considerable accuracy.

Similarly exact and approximate methods are developed for analyzing electromotive force in a loop located in the vicinity of an overhead conductor with the current having prescribed value. The loop is treated as a rectangular loop (two-conductor closed loop) located horizontal to the earth surface.

The exact methods are based on the theory of earth return circuits and take into account the earth currents whereas a simplified approach assumes that effects of earth currents on the emf are negligible.

It is shown that the use of the approximate method of the calculation of the induced *emf* can be successfully used in the practice.

The results derived can be used as the foundation for almost every study on the power line fields for conditions that are almost always satisfied for power engineering applications.

References

- [1] Olsen RG., Wong PS. Characteristics of low frequency electric and magnetic fields in the vicinity of electric power lines. *IEEE Transactions on Power Delivery* 1992; **7**: 2046-2053.
- [2] Nicolaou C.P., Papadakis A.P., Razis P.A., Kyriacou G.A., Sahalos J.N. Measurements and predictions of electric and magnetic fields from power lines. *Electric Power Systems Research* 2011, **81**: 1107 – 1116.
- [3] Carson JR. Wave propagation in overhead wires with ground return. *Bell System Technical Journal* 1926, **5**: 539-554.
- [4] Gradshteyn I.S., Ryzhik I.M. *Table of Integrals, Series and Products*. Academic Press: New York, 1965.
- [5] Gary C. Approche complète de la propagation multifilaire en haute fréquence par utilisation des matrices complexes. *EDF Bulletin de la Direction des Études et Recherches-Série B* 1976 ; No. 3/4: 5-20.
- [6] Deri A., Tevan G., Semlyen A., Castanheira A. The complex ground return plane a simplified model for homogeneous and multi-layer earth return. *IEEE Transactions on Power Apparatus and Systems* 1981; **100**: 3686-3693.
- [7] Olsen RG., Pankaskie TA. On the exact, Carson and image theories for wires at or above the earth's interface. *IEEE Transactions on Power Apparatus and Systems* 1983; **102**: 769-778.
- [8] Sunde ED., *Earth conduction effects in transmission systems*. New York, Dover 1968.
- [9] HÜTTE. *Taschenbücher der Technik, Elektrische Energietechnik*, Band 3, Elektrische Netze, Springer: Verlag, 1988.
- [10] Budnik K., Machczyński W.: Contribution to studies on calculation of the magnetic field under power lines. *European Transactions on Electrical Power ETPE* 2006: 345 – 364.