JAISCR, 2012, Vol. 2, No. 4, pp. 343 – 356

ORTHONORMAL BASIS FUNCTION FUZZY SYSTEMS FOR BIOLOGICAL WASTEWATER TREATMENT PROCESSES MODELING

Ali Chaibakhsh[1], Naz Chaibakhsh[2], Mehdi Abbasi[3] and Ardalan Norouzi[4]

[1]*Department of Mechanical Engineering, University of Guilan, Rasht, 4199613769 Guilan, Iran*

[2]*Department of Chemistry, University of Guilan, Rasht, 4199613769 Guilan, Iran*

[3]*WASTE, University of Stuttgart, 70569 Stuttgart, Germany*

[4]*Department of Mechanical Engineering, Politecnico di Milano, 20156 Milan, Italy*

Abstract

In this paper, fuzzy models with orthonormal basis functions (OBF) framework are employed for modeling the nonlinear dynamics of biological treatment processes. These models are consisting of a linear part describing the system dynamics (Laguerre filters) followed by a non-linear static part (fuzzy system). The training procedure contains of two main steps: 1) obtaining the optimum time-scale and the order of truncated Laguerre network as the linear part and 2) defining membership functions, corresponding rules and adjusting the consequent parameters of fuzzy system as the nonlinear part. A comparison between the responses of the developed model and the original plant was performed in order to validate the accuracy of the developed model.

1 Introduction

Biological treatment processes are one of the most important parts of wastewater treatment systems, which are widely employed due to their advantages in terms of low capital investment, low operating costs and high flexibility to be used for different types of wastewater [1].

Enhancing the performances of wastewater treatment plants (WWTP) has been taken into consideration in recent years, in order to improve effluent water qualities. In this regard, employing more expert control strategies is highly in demand to improve the reliability and availability of treatment plants with higher operation performance. Better control quality is always required more information from the plant performances. In this case, gathering the adequate information about the system dynamics would be the main key [1].

Inherent nonlinearities of bioprocess systems, large variations in wastewater compositions, concentrations, and influent flow rates are the main reasons that make the modeling process of such systems very complicated [2].

In general, fundamental rules of biological and chemical reactions can be employed for developing the analytical plant model to describe the behavior of WWTP. However, these models could be very complicated in structure with many tunable parameters that have to be adjusted with respect to boundaries, inputs and outputs [3].

System identification techniques can be also employed to develop black-box models based on measured data obtained from real performance of the plant [4-5]. Many different methodologies have been developed and applied successfully for nonlinear system modeling.

A common class of nonlinear models frequently used for system identification is the NARX (Nonlinear AutoRegressive with eXogenous input) models. One of the most effective approaches to deal with the system nonlinearities is employing models with the block-oriented structures [6-7]. Wiener type model is a best-known member of the block-oriented model class, which consists of a linear dynamic part followed by a static nonlinear element [8]. This topology can be used for developing dynamic fuzzy system [9]. The TSK (Takagi-Sugeno-Kang) types of fuzzy models particularly come to consideration to deal with systems nonlinearities and uncertainties [10]. In dynamic fuzzy structure, the model's output at a given time instant is estimated based on a finite number of the past input and output signals. This structure is still suffering from the usual drawbacks of the NARX such as requirements for identifying the past terms of process variables and high computational efforts [8][11]. Employing orthonormal basis functions such as the Laguerre or Kautz functions as the linear dynamic part of Wiener models could be very helpful to cope with these problems [8]. For the systems with slow dynamics and non-oscillatory behaviors, the Laguerre network based models have a great performance [12]. Having much smaller number of tunable parameters and no requirement for identifying the past terms of the input variables are some advantages of Laguerre network based models. These could considerably reduce the computational efforts in identification process [8].

This work presents an application of Laguerre network based fuzzy modeling approach for describing the nonlinear behavior of aerobic WWTPs. The proposed model is constituted by a linear part describing the system dynamics (Laguerre filters) followed by a non-linear static part (fuzzy system), which are trained based on the plant performances at different operating conditions. The optimal time scales for linear parts were defined in order to minimize the modeling error.

The number of inputs for fuzzy part was selected based on step response of plant with respect to optimum pole parameters.

For training the nonlinear part of the models, fuzzy c-means (FCM) clustering technique was employed to define the structure of the fuzzy system, where the corresponding consequent parameters were adjusted by least-square methods. A comparison between the responses of the developed model and the original plant was performed in order to validate the accuracy of the developed model. In addition, in order to show the advantages of the modeling approach, a comparison between the performances of the proposed method and others recent modeling approach was performed.

In the next section, the structure of Laguerre-based fuzzy model is presented. It is followed by training procedure of the process model in section 3. A brief description of the aerobic activated sludge wastewater treatment process and its dynamic behavior are presented in section 4. A detailed discussion on simulation experiments and the obtained results comes in Section 5. The last section is the conclusion and suggestions.

2 Laguerre-based fuzzy system

A nonlinear system can be described by a NARX model as a discrete-time nonlinear mapping on some previous measured outputs and inputs as follows,

$$y(k) = f\big(u(k),\, u(k-1),\, \ldots,\, u(k-n_u),\\ y(k-1),\, \ldots,\, y(k-n_y)\big) \quad (1)$$

where n_u and n_y are the number of past terms for input u and output y, respectively, that represent the dynamic order of the system. The function *f()* is a nonlinear mapping function that can be considered as neural networks, fuzzy systems, polynomials and etc. A combination of Laguerre basis filters as the dynamic linear part and fuzzy logic systems as the nonlinear static part of a Wiener-type model is proposed as an appropriate method to nonlinear system identification. An improved representation of Laguerre network based fuzzy systems is proposed in

Nomenclature	
a	order of truncated Laguerre network
$A_{i,j}$	membership function
b_i	Laguerre coefficient
c	number of the clusters
D	distance
$DI(t)$	dilution rate (flow/basin volume) (h^{-1})
DO_{in}	inlet dissolved oxygen cons (*mg/l*)
DO_{max}	maximum dissolved oxygen concentration(*mg/l*)
DO_r	dissolved oxygen concentration in the recycle flow (*mg/l*)
e	error
$f()$	mapping function
$H(z)$	discrete-time transfer function
J	Laguerre time-scale performance index
J_m	clustering objective function
K_0	model constant
K_d	endogenous decay constant (*mg/l*)
K_{DO}	half oxygen saturation constant (*mg/l*)
K_s	half-velocity constant (*mg/l*)
$L(z)$	discrete-time Laguerre function
l_k	Laguerre signal
q	cluster centers
$\tilde{q}_i$	optimum positions of cluster centers
Q_{XB}	cluster validity function
r	ratio of recycled to influent flow rate
R_i	rule numbers
S_{in}	inlet soluble substrate concentration (*mg/l*)
$u(k)$	system input
V/V_r	ratio between aeration basin and settling basin volume
W	ratio of air flow rate to basin volume (h^{-1})
x	unlabeled data set for clustering
Y	growth rate
$y(k)$	system output

Abbreviation	
AAD	average absolute deviation
DO	dissolved oxygen
FCM	fuzzy c-means clustering
FIR	finite input response
FIS	fuzzy interference system
LNBF	Laguerre network based fuzzy system
LSE	least-squares estimation
MAE	mean absolute error
MISO	multi input single output system
NARX	Nonlinear AutoRegressive with eXogenous input
SSC	soluble substrate concentration
TSK	Takagi-Sugeno-Kang fuzzy type system
WWTP	wastewater treatment processes

Greek letters	
γ	oxygen transfer rate
α	pole parameter
Θ	consequent parameter matrix
Γ	weighted input dataset matrix
ψ	output target matrix
Λ	the vector of membership degree
σ	the center of Gaussian membership function
ζ	the spread of Gaussian membership function
$\lambda_{i,j}$	membership degree
$\tilde{\lambda}_{i,j}$	optimum membership degree
β	ratio of waste to influent flow rate
μ_{max}	maximum specific growth rate (h^{-1})
μ_s	specific growth rate (h^{-1})

reference [13], where the delay shift operators was replaced by Laguerre basis filter as follows,

$$y(k) = f(l_0(k) * u(k), l_1(k) * u(k), \ldots \\ \ldots, l_a(k) * u(k), y(k-1)) \quad (2)$$

where $l_i(k)$ are Laguerre basis filters where the nonlinear mapping function $f()$ is a TSK type fuzzy system, which is illustrated in Figure 1. This model is able to approximate the nonlinear systems by performing an interpolation of local models via a fuzzy inference mechanism. The antecedents are describing fuzzy regions in the input space, where the consequents describe the local linear models in corresponding fuzzy subspaces [14][9].

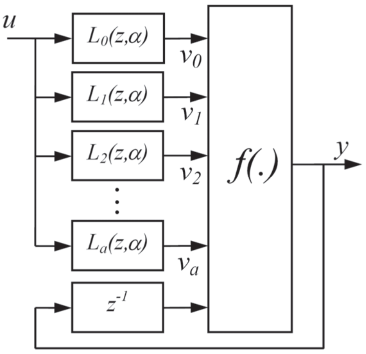

Figure 1. Laguerre network based fuzzy model

2.1 Laguerre filter

Discrete-time Laguerre functions as complete orthonormal set in z-domain can be represented as follows,

$$L_i(z,a) = \frac{\sqrt{1-\alpha^2}}{1-\alpha z^{-1}} \left(\frac{-\alpha + z^{-1}}{1-\alpha z^{-1}} \right)^i \quad i = 0,1,2,\ldots \quad (3)$$

where $\alpha \in \{\Re : |\alpha| < 1\}$ is the pole dominant parameter that determines the rate of exponential decay for Laguerre functions responses. This parameter can be obtained through optimization or be chosen by experiments [15].

For any linear discrete-time system, the transfer function $H(z)$ can be approximated by the limited order of Laguerre polynomial as follows,

$$H(z) = \frac{Y(z)}{U(z)} = \sum_{i=0}^{a} b_i(\alpha) L_i(z,\alpha) \quad (4)$$

in which $U(z)$ and $Y(z)$ are input and output of the system and $b_i\ (\alpha)$ are the Laguerre coefficients [15]. As a result, we have,

$$Y(z) = \sum_{i=0}^{a} b_i(\alpha) V_i(z) \quad (5)$$

and,

$$V_i(z) = L_i(z,\alpha) U(z) \quad i = 0,1,2,\ldots,a \quad (6)$$

The model is a Laguerre network, which consists of a first-order low-pass factor and $(i\text{-}1)^{th}$-order identical all-pass filters. The Laguerre filters $L_i(z,\alpha)$ will turn to regular delay operators as $\alpha=0$ [15].

For each region at input space, a local linear model can be developed by using the polynomial presented in Eq. (5). The local linear models can be interpolated by means of fuzzy inference mechanism that is presented in next section

2.2 Neuro-fuzzy architecture

The rule-base of the model's fuzzy part is represented by a set of if-then rules as follows [16],

$$R_i: \quad if\ v_0\ is\ A_{i,1}\ and\ \ldots\ v_a\ is\ A_{i,a+1}\ and\ y(k-1)\ is\ A_{i,a+2}$$
$$then\ y_i(k) = \textstyle\sum_{j=0}^{a} b_{i,j}\, v_j(k) + b_{i(a+1)}\, y(k-1) + b_{i(a+2)}$$

where $A_{i,j}$ is the membership function associated with input variable v_j. In this system, a linear combination of the input variables are considered as the conclusion functions of fuzzy rules. The fulfillment degrees of the fuzzy rules are estimated through the five layers of fuzzy system [16]. According to rules R_i, the weighted sum average is obtained as follows,

$$y = \sum_{i=1}^{c} y_i\, \bar{w}_i \tag{7}$$

where,

$$\bar{w}_i = \prod_{j=1}^{N} A_{i,j}(.) \Bigg/ \sum_{i=1}^{c} \left[\prod_{j=1}^{N} A_{i,j}(.)\right] \tag{8}$$

and N is the number of inputs to the fuzzy system that is equal to a+2. Also, c is the number of fuzzy rules. The membership function $A_{i,j}$ is considered to be Gaussian specified by the center σ and the spread ζ,

$$A_{i,j}(x_r) = \exp(-((x_r - \zeta_{i,j})/\sigma_{i,j})^2) \tag{9}$$

In the next section, the procedure for adjusting the model parameters is presented.

3 Model parameters adjustment

The parameters and the structure of the model have to be defined with respect to the collected data from the performances of the plant. For this aim, three following main steps are considered.

3.1 Laguerre pole placement

Optimum choice of pole parameter makes the Laguerre filter capable to provide better approximations of system. This can be captured through stable discrete systems approximating based on the impulse responses, in both time and frequency domains. In this regard, an analytical method is proposed by Fu and Dumont to evaluate the optimum time scale [17]. With respect to Eq. (4), for given impulse response of a discrete system, $h(n)$, we have,

$$h(n) = \sum_{k=1}^{\infty} g_k\, l_k(n) \tag{10}$$

where $l_k(n)$ is the Laguerre signal in time domain taken by the inverse z-transform of $L_k(z,\alpha)$ and g_k is the corresponding Laguerre coefficient. The following performance index was considered to determine the optimal Laguerre parameter,

$$J = \sum_{k=1}^{\infty} k\, g_k^2 \tag{11}$$

In this performance index, the weight of each additional Laguerre coefficient is linearly increased that leads to a fast convergence rate. In [17], by minimizing the performance index (11), an analytical expression for the optimal solution was obtained. By defining,

$$M_1 = \frac{1}{\|h\|^2} \sum_{n=0}^{\infty} n\, h^2(n) \tag{12}$$

and

$$M_2 = \frac{1}{\|h\|^2} \sum_{n=0}^{\infty} n\, [\Delta h(n)]^2 \tag{13}$$

in which,

$$\|h\|^2 = \sum_{n=0}^{\infty} h^2(n) = \sum_{j=1}^{\infty} g_k^2 \tag{14}$$

The optimal Laguerre parameter can be captured in terms of M_1 and M_2 as follows,

$$\alpha_{opt} = \frac{2M_1 - M_2 - 1}{2M_1 - 1 + \sqrt{4M_1M_2 - M_2^2 - 2M_2}} \tag{15}$$

In cases that exciting the system by an impulse input is difficult, we can use alternative approaches that employ arbitrary input signals to estimate the optimal pole position [18]. However, no considerable differences in obtained results can be observed. The order of Laguerre network can be chosen by a simple step response test. More explanations are presented in the next sections.

3.2 Fuzzy system configuration

In this section, fuzzy c-means (FCM) clustering algorithm is employed to define the structure of the fuzzy model. By using clustering techniques, the complex nonlinear regions can be divided into simpler subspaces. Integration of fuzzy system and clustering methodologies that allow for subspaces overlapping and accordingly smoother transitions between operating regions [19].

The FCM algorithm partitions the data set into c predefined subsets through optimizing an objective function, which indicates the desirability of each c-partition. The data partitioning into clusters depends on similarity/dissimilarity of each cluster members, which is generally defined by the distance of data points from cluster centers [20].

The distance between q^i and p^j is defined by$D(q^i, x^j)$, where $\{q^i\} \subset R^s$and $\{x^j\} \subset R^s$are the vector of cluster centers and unlabeled data set, respectively. In order to find the best possible solution, the following objective function has to be minimized [20].

$$Minimize: \quad J_m(\Lambda, Q) = \sum_{j=1}^{n}\sum_{i=1}^{c}(\lambda_{i,j})^m (D_{ij})^2 \quad (16)$$

where $\lambda_{i,j}$ is the membership of the j^{th} data point in the i^{th} cluster. In Eq. (16), the fuzziness degree of each cluster is controlled by the weighting exponent m ($1 \leq m < \infty$). Minimization of J_m is performed by considering the following constraints on the membership values that leads to the optimal partition.

$$\begin{array}{l} \forall j = 1...n, \forall i = 1...c, \\ \sum_{i=1}^{c}\lambda_{i,j} = 1 \quad and \quad 0 \leq \lambda_{i,j} \leq 1 \end{array} \quad (17)$$

The optimal positions of cluster centers and the corresponding membership functions can be captured using Eq. (18) and Eq. (19) via an iterative procedure [20].

$$\tilde{q}_i = \frac{\sum_{j=1}^{n}(\lambda_{i,j})^m x_j}{\sum_{j=1}^{n}(\lambda_{i,j})^m}, \qquad 1 \leq i \leq c \quad (18)$$

and

$$\tilde{\lambda}_{i,j} = \left[\sum_{k=1}^{c}\left(\frac{D_{ij}}{D_{jk}}\right)^{2/(m-1)}\right]^{-1}, \quad 1 \leq i \leq c, \quad 1 \leq j \leq n \quad (19)$$

While no further improvement is observed in $J_m(\Lambda, Q)$, the iteration will be stopped. Raising the number of the clusters may increase the accuracy of the model; however it could be led to models overfitting problems and the excessive computational costs. To deal with this problem, it is suggested that the number of the clusters be chosen through optimization [21]. In order to determine the optimal number of clusters in the data set, a cluster validity index can be employed. Many different validity indexes are available that can be applied [22]. Here, a validity function proposed by Xie and Beni [23] was employed. We have,

$$Q_{XB} = \sum_{i=1}^{c}\sum_{j=1}^{n}(\lambda_{i,j})^m (D_{ij})^2 \Big/ n \min_{i \neq j}(D_{ij})^2 \quad (20)$$

The optimum number of cluster centers can be obtained through an iterative algorithm [23]. In order to increase the performance of the proposed algorithm, some modifications have been considered in the procedure, which are presented in Appendix A

3.3 Consequent parameters tuning

By defining the fuzzy membership functions and corresponding fuzzy rules, the main requirement is that the parameters of fuzzy rules be adjusted. Here, the least-squares estimation (LSE) technique is employed for adjusting the parameters of consequent based on the plant data. For each input-output pattern the Eq. 7 can be written as,

$$\psi^{(i)} = \Gamma^{(i)}.\Theta^{(i)} \quad (21)$$

where,

$$\Theta^{(i)} = \begin{bmatrix} b_{1,0} & b_{1,1} & \dots & b_{1,N} & \dots & b_{c,0} & b_{c,1} & \dots & b_{c,N} \end{bmatrix}^T$$

$$\Gamma^{(i)} = \big[\ \bar{w}_1 v_1^{(i)} \ \dots \ \bar{w}_1 v_{N-2}^{(i)} \ \ \bar{w}_1 y^{(i)} \ \ \bar{w}_1 \ \dots \ \dots \ \bar{w}_c v_c^{(i)} \ \dots \ \bar{w}_c v_{N-2}^{(i)} \ \ \bar{w}_c y^{(i)} \ \ \bar{w}_c \ \big] \quad (22)$$

and ψ is the output y at present time. In this case, all input-output patterns can be defined as below,

$$\Psi_{M\times1} = \Gamma_{M\times(c+1)N}.\Theta_{(c+1)N\times1} \quad (23)$$

In this case, the parameters of consequent can be obtained by minimizing the squared error as follows,

$$\Theta = (\Gamma^T\Gamma)^{-1}\Gamma\Psi \quad (24)$$

In order to increase the performance of training process, the dataset are normalized by dividing each variable by its maximum value to distribute data over a range of [0, 1] or as the deviation from mean values in the range of [-1,1]. This ensures none of variables are dominant over the others during the training phase.

4 Process description

A most common configuration for WWTP is an aerobic biological treatment process, which generally consists of an aeration tank followed by a settling tank. In Figure 2, the flow diagram for the conventional activated sludge process is presented. In this process, in the presence of oxygen, the activated sludge and organic matter (influent) are biodegraded [24]. The simplified reaction for the process is as follows,

$$\text{organics} + O_2 \xrightarrow{\text{aerobic microorganisms}} CO_2 + H_2O + \text{more aerobic microorganism} \tag{25}$$

in which the organics are consumed by microorganisms for preservation and are oxidized to CO_2 and H_2O. The mixture of treated wastewater and biological flocs are directed to a settling tank, where biological solids are separated by gravity. The activated sludge flocs settle at the bottom of the settling tank and the treated wastewater (effluent) leaves the basin from top of the settling tank. Most part of settled sludge is returned to aeration tank and a small part is wasted [25].

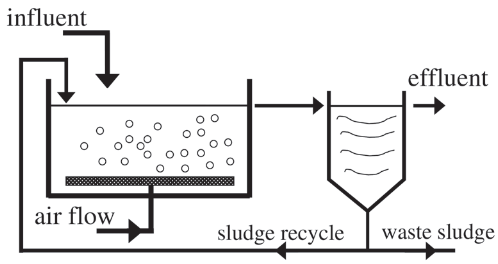

Figure 2. Schematic representation of the aeration unit

The performance of microorganisms can be governed by controlling the dissolved oxygen concentration in aeration tank.

Many different nonlinear models have been presented for the process of transporting oxygen from aeration system to the cells of microorganisms and their metabolism. Here, we have employed a first principle model given by [26] in order to describe the unit dynamic behavior. The mass balance for the concentration of biomass *X(t)* is expressed by the following equation,

$$\frac{dX(t)}{dt} = \mu_s(t)X(t) - DI(t)(1+r)X(t) + rDI(t)X_r(t)$$

The mass balance for the concentration of substrate *S(t)* is shown by Eq. (26).

$$\frac{dS(t)}{dt} = -\frac{\mu_s(t)}{Y}X(t) - DI(t)(1+r)S(t) + DI(t)S_{in}(t) + rDI(t)S_r(t) \tag{26}$$

The mass balance for the concentration of dissolved oxygen *DO(t)* can be captured as,

$$\frac{dDO(t)}{dt} = -\frac{K_0\mu_s(t)}{Y}X(t) - DI(t)(1+r)DO(t) + DI(t)DO_{in} + \gamma W(DO_{\max} - DO(t)) + rDI(t)DO_r(t)$$

By assuming a constant volume for the settling tank, the mass balance equation for the recycled biomass $X_r(t)$ is obtained as,

$$\frac{dX_r(t)}{dt} = \frac{V}{V_r}[DI(t)(1+r)(X(t) - DI(t)(\beta+r)X_x(t)]$$

According to the Monod's relationship, the specific bacterial growth rate μ_s can be estimated as a function of soluble substrate and biomass concentration as follows,

$$\mu_s(t) = \mu_{\max}\frac{S(t)}{K_s + S(t)}\frac{DO(t)}{K_{DO} + DO(t)} - K_d \tag{27}$$

The nominal values of Stoichiometric and kinetic parameters of the model and the initial conditions to run the simulations are presented in Appendix B. Here, the air flow rate is considered as the manipulated input, the inlet soluble substrate concentration and the dilution rate are the measureable disturbance. Also, the other inputs are considered as unmeasured disturbances. The concentration of dissolved oxygen is considered as the main plant output, which required to be controlled. Due to very slow dynamics of the plant, measurement noise was neglected. The ranges of variations for the plant inputs are presented in Appendix B. The input-output data obtained from the plant model running in the full-scale are employed for system identification in the next section

4.1 Simulation Experiments and Results

As it has been mentioned previously, the implementation of proposed identification algorithm is based on two main steps: 1) identifying the linear part that consists evaluating the optimum time scale and the order of truncated Laguerre network; 2) identifying the nonlinear part that consists defining the fuzzy membership functions, corresponding fuzzy rules and adjusting the consequent parameters.

The required input-output information for identification process was captured by simulating the dynamic equations presented in the previous section. Simulation experiments were performed in *MATLAB Simulink* environment. One input variable (air flow rate) and two measurable disturbances (dilution flow rate and inlet soluble substrate concentration) were selected as the main inputs for the model.

In practice, a physical system can be approximated by the limited order Laguerre series. In this case, in order to minimize the truncation error, the time scale should be optimized for a given number of filters. The optimum pole parameter can be estimated from a finite sequence of the impulse responses data. If *h(n)* is the impulse response of the plant, the value of $||h||^2$ and respectively M_1 and M_2 can be captured for N sequences of impulse response by using Eqs. (12) to (14). In Table 1, the optimum time scales (α_{opt}) and corresponding parameters M_1 and M_2 are estimated for the inputs. Here, the total number of samples are considered to be N = 40 while the sampling interval is 0.1. In Figure 3, the responses of Laguerre basis with respect to the impulse changes in air flow rate (α_{opt} = 0.2601) are presented.

Employing the step responses of the plant can be helpful for defining the order of Laguerre network. In Figure 4, the step responses of the plant and corresponding Laguerre filters responses are presented. As it is observed, for the higher stages of Laguerre network, the coefficients are very small. Furthermore, by increasing the number of Laguerre bases filters the computational efforts for the nonlinear part identification will increase. Therefore, with respect to Figure 4, considering a=3 would be an appropriate selection for describing the dynamics of the plant.

Table 1. Pole estimation for truncated Laguerre networks

input	M_1	M_2	α_{opt}
air flow rate	21.5557	21.6137	0.2601
dilution rate	17.1954	17.5952	0.2515
inlet SSC	22.8093	22.7924	0.2613

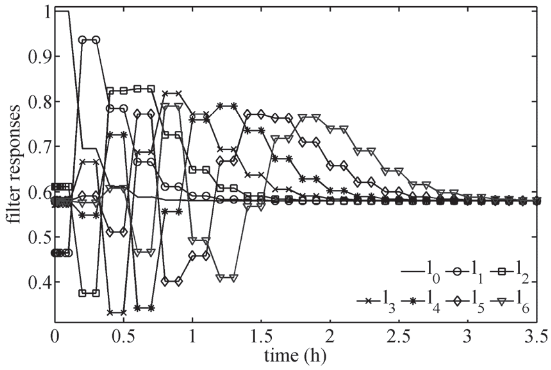

Figure 3. Normalized impulse responses of the Laguerre filters

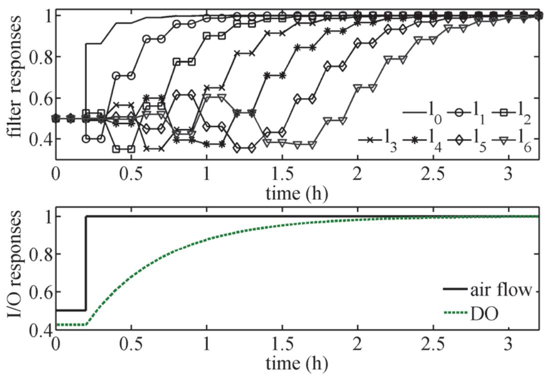

Figure 4. The step responses of Laguerre bases filters and plant

Defining the order of linear part and the optimal pole position, the nonlinear part of the model can be identified with respect to collected data. For this propose, the plant was excited with uniform random inputs. The air flow rate was varied every 10 hours in the range of 50 to 100 (h^{-1}), where the changes in the dilution rate were produced every 20 hours in the range of 0.07 to 0.08 (h^{-1}). In addition, the inlet SSC is considered to be changed in the range of 200 to 220 (*mg/l*) in a period of 4 hours. The input-output data used for nonlinear part identification including the transient and steady state conditions is presented in Figure 5. The system performance was recorded for duration of 500 hours with time interval 0.1. The first 3000 data points were employed in training stage, while a set of 2000 data points were used for validation purpose. In testing stage, real-time simulations were employed for the developed model assessment.

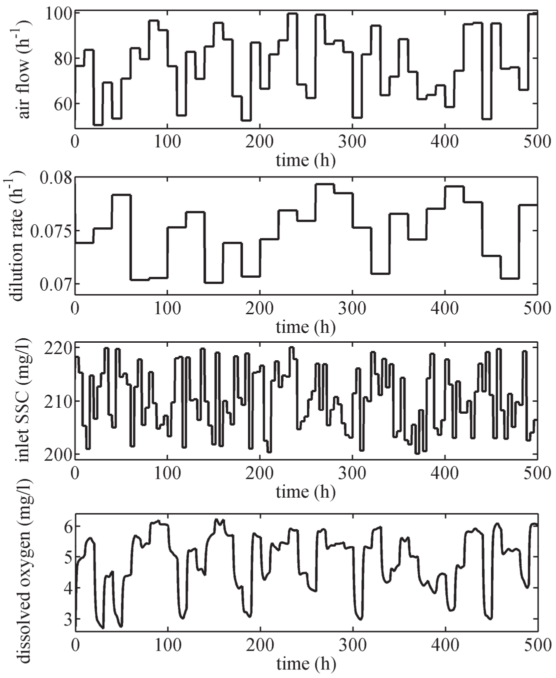

Figure 5. Input–output data for identifying the non-linear (fuzzy) part

The data generated through excitation of the plant are utilized for adjusting the nonlinear part of the model. The FCM algorithm is employed to define the optimal positions of cluster centers and corresponding membership functions through Eqs. (18) and (19). The optimal number of the clusters is obtained on the normalized input-output data set with respect to the cluster validity index presented in Eq. (20) and its corresponding algorithm. The cluster centers and corresponding region of each cluster are shown in Figure 6. The optimal parameters for membership functions are presented in Appendix C.

The number of fuzzy rules is determined by the number of clusters. In this regard, considering seven fuzzy rules may be appropriate to cover the entire range of operation. As a result, the nonlinear part of the model would be in the form of a MISO FIS with five inputs and one output with seven fuzzy rules, where for each FIS there are 42 parameters that have to be tuned. With respect to recorded data, the parameters of consequent can be adjusted by using the least square method presented in Eq. (24). The characteristics of the fuzzy part of the LNBF models are presented in Appendix C.

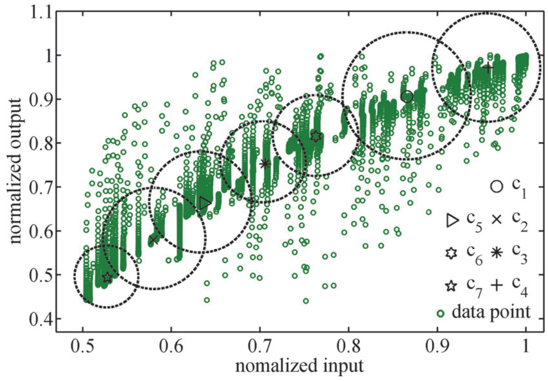

Figure 6. Cluster centers and corresponding operating regions

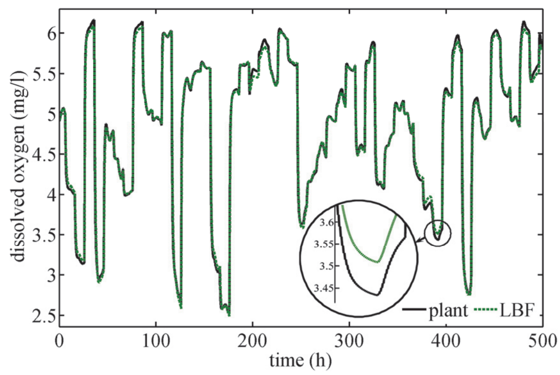

Figure 7. Response of the developed model for dissolved oxygen concentration

In order to validate the prediction accuracy of the developed model, the responses of the proposed model is compared with the responses of the plant over a wide range of operation. Real-time simulations are carried out while the plant is exposed to measured and unmeasured disturbances. In Figure 7, the predicted values of dissolved oxygen (DO) are presented for a period of 500 hours in both transient and steady state conditions. Simulation results show that the responses of the proposed model are

Table 2. Error functions for the LNBF Model

Max($\|e\|$)	Min($\|e\|$)	Mean	AAD	R^2
0.6167	4.188e-8	-0.0069	0.0393	0.9986

Table 3. Error functions for comparing the modeling approaches

	Max$\|(e)\|$	Min$\|(e)\|$	Mean	AAD	R^2
DynamicFuzzy	0.4813	6.223e-5	0.0682	0.1403	0.9835
WienerWavelet	1.0223	2.684e-5	-0.0381	0.1533	0.9736
Wiener Poly	0.9514	2.977e-5	-0.0316	0.1669	0.9710
Wienersigmoid	1.0329	5.831e-5	-0.0314	0.1734	0.9680

very close to the original output. Furthermore, The error functions of upper bound error *Max(*$|e|$*)*, lower bound error *Min(*$|e|$*)*, mean absolute error *MAE*, average absolute deviation *AAD(e)* and correlation coefficient $R^2(e)$ are calculated, where the error is defined as the difference between the response of the plant and the response of the developed model. The error functions for the developed model are presented in Table 2. In addition, in Fig. 8, a comparison between the correlation coefficients of the predicted values and the original output is performed. Obtained results indicate the accuracy of the developed model in terms of less deviation between the predicted and target values.

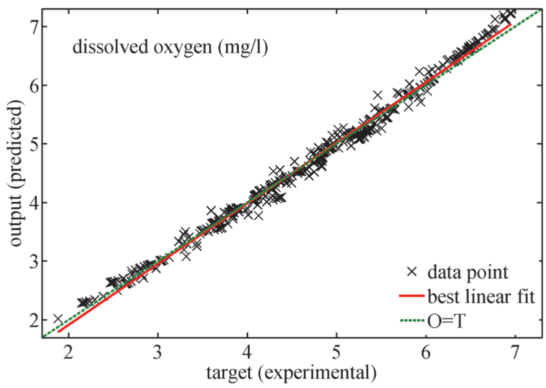

Figure 8. Correlation between the predicted and target values

4.2 Comparing with other models

Here, some comparisons between the performances of the LNBF modeling technique and other recent modeling approaches have been carried out in order to confirm the advantages of the proposed approach. For this aim, different models such as dynamic fuzzy system and Wiener-type models were developed. A conventional adaptive neuro-fuzzy inference system (ANFIS) was employed to develop dynamic fuzzy model. Four and one past terms of the plant inputs and output are considered as the inputs of a TSK model, respectively. The parameters of membership functions and fuzzy rules are adjusted using LSE and back-propagation approaches.

In addition, three types of Wiener-type models, with the nonlinearities of polynomial, wavelet neural network and sigmoid neural network are employed. Four and one past terms of the plant inputs and output are considered for the linear part of the models, respectively. To develop the nonlinear models, we have used the MATLAB§ identification toolbox. The processes of model training were carried out to obtain the best possible responses, where the wavelet neural network consists of 68 units, polynomial nonlinearity was a seven-order single-variable polynomial and sigmoid network consisted of 10 units [27].

The error functions are calculated for the developed models, which are presented in Table 3. The results indicate the accuracy and feasibility of the proposed approach.

5 Conclusion

This work presents an application of fuzzy models with orthonormal basis functions framework for modeling the nonlinear dynamics of biological treatment processes. The proposed model was a Wiener-type model, which consist of Laguerre basis network as the linear dynamic pat cascaded by a TSK-type fuzzy system as the nonlinear static part of the model. Evaluating the optimum time scale and the order of truncated Laguerre network for the linear part has been done in first step. In the second step, membership functions, corresponding fuzzy rules and the parameters of consequent for nonlinear part are adjusted. The developed model was employed for the prediction of DO concentration with respect to the changes in air flow rate as the main input and dilution flow rate and inlet SSC as the plant disturbances.

The performance and accuracy of the developed model were validated by performing a comparison between the responses of the developed model and the original plant. The performance of the developed model was also compared with the performances of other recent models to confirm the feasibility and accuracy of the proposed modeling approach. Obtained results indicate that the LNBF models are able to describe accurately the dynamics and nonlinearities of the system.

Further improvement may be achieved by considering the effects of other plant disturbances. Furthermore, using feedforward structure may improve the robustness of developed models to unmeasured disturbances and model prediction errors.

References

[1] Y.J. Chan, M.F. Chong, C.L. Law, D.G. Hassell, A review on anaerobic–aerobic treatment of industrial and municipal wastewater, Chemical Engineering Journal, 155, 2009, 1-18.

[2] K.V. Gernaey, M.C.M. van Loosdrecht, M. Henze, M. Lind, S.B. Jrgensen, Activated sludge wastewater treatment plant modelling and simulation: state of the art, Environmental Modelling & Software, 19, 2004, 763-783.

[3] I. Iacopozzi, V. Innocenti, S. Marsili-Libelli, E. Giusti, A modified Activated Sludge Model No. 3 (ASM3) with two-step nitrification – denitrification, Environmental Modelling & Software, 22, 2007, 847-861.

[4] L. Ljung, Perspectives on system identification, Annual Reviews in Control, 34, 2010, 1-12.

[5] E.K. Juuso, Integration of intelligent systems in development of smart adaptive systems, International Journal of Approximate Reasoning, 35, 2004, 307-337.

[6] Y.L. Hsu, J.S. Wang, A Wiener-type recurrent neural network and its control strategy for nonlinear dynamic applications, Journal of Process Control, 19, 2009, 942-953.

[7] U. Mehta, S. Majhi, Identification of a class of Wiener and Hammerstein-type nonlinear processes with monotonic static gains, ISA Transactions, 49, 2010, 501-509.

[8] R.J.G.B. Campello, F.J. Von Zuben, W.C. Amaral, L.A.C. Meleiro, R. Maciel Filho, Hierarchical fuzzy models within the framework of orthonormal basis functions and their application to bioprocess control, Chemical Engineering Science, 58, 2003, 4259- 4270.

[9] I. Łkrjanca, S. Blai, O. Agamennoni, Identification of dynamical systems with a robust interval fuzzy model, Automatica, 41, 2005, 327-332.

[10] H. He, Fuzzy modeling and fuzzy control (book review), IEEE Computational Intelligence Magazine, 3, 2008, 8-10.

[11] R. Liutkeviius, Fuzzy Hammerstein Model of Nonlinear Plant, Nonlinear Analysis: Modelling and Control, 13, 2008, 201-212.

[12] B. Wahlberg, P.M. Makila, Approximation of stable linear dynamical systems using Laguerre and Kautz functions, Automatica, 32, 1996, 693-708.

[13] M. Alci, M.H. Asyali, Nonlinear system identification via Laguerre network based fuzzy systems, Fuzzy Sets and Systems, 160, 2009, 3518-3529.

[14] R. Babuka, H. Verbruggen, Neuro-fuzzy methods for nonlinear system identification, Annual Reviews in Control, 27, 2003, 73-85.

[15] M.A. Masnadi-Shirazi, M. Aleshams, Laguerre discrete-time filter design, Computers and Electrical Engineering, 29, 2003, 173-192.

[16] T. Takagi, M. Sugeno, fuzzy identification of systems and its applications to modeling and control," IEEE Transactions on Systems, Man and Cybernetics, 15, 1985, 116-132.

[17] Y. Fu, G.A. Dumont, An optimum time scale for discrete Laguerre network, IEEE Transaction on Automatic Control, 38, 1993, 934-938.

[18] T.O. E. Silva, On the determination of the optimal pole position of Laguerre filters, IEEE Transaction of Signal Processing, 43, 1999, 2079-2087.

[19] A.N. Venkat, P. Vijaysai, R.D. Gudi, Identification of complex nonlinear processes based on fuzzy decomposition of the steady state space, Journal of Process Control, 13, 2003, 473-488.

[20] G. Beliakov, M. King, Density based fuzzy c-means clustering of non-convex patterns, European Journal of Operational Research, 173, 2006, 717-728.

[21] B. Feil, J. Abonyi, F. Szeifert, Model order selection of nonlinear input-output models - a clustering based approach, Journal of Process Control, 14, 2004, 593-602.

[22] Y. Zhang, W. Wang, X. Zhang, Y. Li, A cluster validity index for fuzzy clustering, Information Sciences, 178, 2008, 1205-1218.

[23] X.L. Xie, G. Beni, A validity measure for fuzzy clustering, IEEE Transaction on Pattern Analyzing Machine Intelligence, 13, 1991, 841-847.

[24] G. Olsson, J.F. Andrews, The dissolved oxygen profile - a valuable tool for control of the activated sludge process, Water Research, 12, 1978, 985-1004.

[25] L.K. Wang, N.K. Shammas, Y.T. Hung, Advanced Biological Treatment Processes, Volume 9, in: the Handbook of Environmental Engineering series, Humana Press, 2009.

[26] F. Nejjari, A. Benhammou, B. Dahhou, G. Roux, Non-linear multivariable adaptive control of an activated sludge wastewater treatment process, International Journal of Adaptive Control and Signal Process, 13, 1999, 347-365.

[27] A. Janczak, Identification of Nonlinear Systems Using Neural Networks and Polynomial Models: A Block-Oriented Approach, Springer-Verlag, Berlin, 2004.

Appendix A. Cluster Validity Algorithm

There is a problem in implementing the proposed algorithm by Xie and Beni, whereby increasing the number of clusters Q_{XB} will have a tendency to decrease. To deal with this problem, it is proposed that the number of clusters to be limited. Here, the maximum number of clusters (c_{max}) is selected to be equal to ($n^{0.5}$) (in place of c_{max} = n/3), where n is the number of data points. Please see [23].

1. Initialize $c \leftarrow 2$, $Q_{XB}{*} \leftarrow 10^6$, $c_{max} \leftarrow \sqrt{n}$
2. Initialize fuzzy membership λ_{ij}
3. Find optimal positions of the cluster centers and the corresponding membership functions using Eq. (18) and (19) via an iterative procedure
4. Stop the iteration as $||\Lambda^k - \Lambda^{k-1}|| < \varepsilon$
5. Compute function Q_{XB} using Eq. (20)
6. IF $Q_{XB} < Q_{XB}{*}$ THEN $Q_{XB}{*} \leftarrow Q_{XB}$ ELSE goto 2
7. IF $c < c_{max}$ THEN c←c+1 AND goto 2
8. $c_{opt} = c|_{\min(Q_{xB})}\ 2 \le c \le c_{max}$

Appendix B. Parameters and Initial Condition for Analytical Model

Table 4. B.1. The parameters for analytical WWTP model

Parameter	unit	value
DO_{in}	*mg/l*	0.5
DO_{max}	*mg/l*	10
K_0	-	0.5
K_d	*mg/l*	0.1
K_{DO}	*mg/l*	2
K_s	*mg/l*	100
R	-	0.6
V/V_r	-	2
Y	-	0.65
γ	-	0.018
B	-	0.2
μ_{max}	h^{-1}	0.15

Table 5. B.2. Initial condition for analytical WWTP model

parameter	unit	initial value
DO	*mg/l*	3.2
S	*mg/l*	43.5
X	*mg/l*	423.9
X_r	*mg/l*	212

Table 6. B.3. The range of variations for WWTP inputs

inputs	unit	range
DI	h^{-1}	0.07˜ 0.08
DO_{in}	*mg/l*	0.5˜1
DO_r	*mg/l*	0˜0.2
S_{in}	*mg/l*	200˜ 220
S_r	*mg/l*	20˜30
W	h^{-1}	50˜100

Appendix C. Cluster Centers and MF Parameters

Table 7. C.1. Cluster centers and MF parameters for air flow rate

	clstr1	*clstr2*	*clstr3*	*clstr4*	*clstr5*	*clstr6*	*clstr7*
$\sigma_{1,i}$	0.0211	0.0581	0.0645	0.0469	0.0491	0.0523	0.0723
$\zeta_{1,i}$	0.5164	0.5896	0.6371	0.7018	0.7665	0.8562	0.9622
$\sigma_{2,i}$	0.0675	0.0622	0.0606	0.0465	0.0487	0.0517	0.0724
$\zeta_{2,i}$	0.5296	0.5758	0.6348	0.7010	0.7661	0.8565	0.9636
$\sigma_{3,i}$	0.0700	0.0722	0.0526	0.0475	0.0479	0.0527	0.0709
$\zeta_{3,i}$	0.5309	0.5746	0.6345	0.6998	0.7654	0.8556	0.9636
$\sigma_{4,i}$	0.0614	0.0751	0.0688	0.0450	0.0463	0.0532	0.0736
$\zeta_{4,i}$	0.5321	0.5791	0.6358	0.6984	0.7639	0.8557	0.9612
$\sigma_{5,i}$	0.0917	0.0801	0.0605	0.0505	0.0483	0.0480	0.0656
$\zeta_{5,i}$	0.4991	0.5802	0.6623	0.7449	0.8143	0.9014	0.9756

Table 8. C.2. Cluster centers and MF parameters for dilution rate

	clstr1	*clstrr2*	*clstrr3*	*clstrr4*	*clstrr5*	*clstrr6*	*clstrr7*
$\sigma_{1,i}$	0.0807	0.0478	0.0577	0.0431	0.0409	0.0367	0.0692
$\zeta_{1,i}$	0.5333	0.6364	0.6868	0.7651	0.8287	0.8798	0.9629
$\sigma_{2,i}$	0.0768	0.0485	0.0553	0.0475	0.0207	0.0447	0.0687
$\zeta_{2,i}$	0.5297	0.6370	0.6863	0.7650	0.8261	0.8819	0.9648
$\sigma_{3,i}$	0.5297	0.6370	0.6863	0.7650	0.8261	.8819	0.9648
$\zeta_{3,i}$	0.5276	0.6358	0.6873	0.7648	0.8296	0.8808	0.9657
$\sigma_{4,i}$	0.0750	0.0567	0.0567	0.0473	0.0377	0.0475	0.0709
$\zeta_{4,i}$	0.5284	0.6345	0.6844	0.7675	0.8311	0.8789	0.9662
$\sigma_{5,i}$	0.2219	0.1811	0.1424	0.1499	0.1344	0.2059	0.2909
$\zeta_{5,i}$	-0.711	-0.461	-0.276	-0.045	0.2012	0.4483	0.8824

Table 9. C.3. Cluster centers and MF parameters for inlet SSC

	clstr1	*clstr2*	*clstr3*	*clstr4*	*clstr5*	*clstr6*	*clstr7*
$\sigma_{1,i}$	0.2094	0.1646	0.1427	0.1409	0.1281	0.1565	0.1875
$\zeta_{1,i}$	-0.527	-0.375	-0.124	-0.047	0.2104	0.2652	0.4900
$\sigma_{2,i}$	0.2045	0.1747	0.1438	0.1362	0.1253	0.1546	0.1888
$\zeta_{2,i}$	-0.539	-0.380	-0.134	-0.044	0.2077	0.2759	0.5011
$\sigma_{3,i}$	0.2027	0.1765	0.1470	0.1306	0.1326	0.1510	0.1881
$\zeta_{3,i}$	-0.539	-0.376	-0.143	-0.040	0.1977	0.2821	0.5014
$\sigma_{4,i}$	0.2094	0.1678	0.1437	0.1273	0.1375	0.1555	0.1914
$\zeta_{4,i}$	-0.529	-0.368	-0.149	-0.032	0.1760	0.2829	0.4927
$\sigma_{5,i}$	0.1470	0.1254	0.1106	0.1090	0.0708	0.1124	0.1214
$\zeta_{5,i}$	-0.085	-0.059	0.1489	0.2700	0.3540	0.3733	0.5898

Table 10. C.4. Consequent parameters for air flow rate

R_i	b_{i1}	b_{i2}	b_{i3}	b_{i4}	b_{i5}	b_{i6}
1	0.1528	0.1648	-0.1836	-0.0267	0.8541	0.0396
2	0.1918	0.2439	-0.1800	-0.0399	0.8027	-0.0037
3	0.1958	0.2557	-0.1550	0.0293	0.7433	-0.0386
4	0.2619	0.3103	-0.2006	0.0926	0.7715	-0.1399
5	1.0415	-1.1269	0.7382	-0.3855	0.9065	-0.0945
6	0.1815	0.2311	-0.1390	0.0221	0.7261	-0.0025
7	0.1414	0.1824	-0.1727	-0.0035	0.7720	0.0811

Table 11. C.5. Consequent parameters for dilution rate

R_i	b_{i1}	b_{i2}	b_{i3}	b_{i4}	b_{i5}	b_{i6}
1	-0.0896	-0.1582	-0.0154	-0.0385	0.8963	0.2164
2	-0.1240	-0.2388	-0.0251	-0.1228	0.8948	0.3757
3	-0.1277	-0.2490	0.0088	-0.1026	0.9060	0.3345
4	-0.1408	-0.2930	-0.0648	-0.1118	0.9150	0.4243
5	-0.1052	-0.1987	0.0126	-0.0897	0.9028	0.2843
6	-0.0415	-0.1644	0.0360	-0.0771	0.8874	0.1648
7	0.0085	-0.0383	-0.1658	0.2302	0.9088	-0.0552

Table 12. C.6. Consequent parameters for inlet SSC

R_i	b_{i1}	b_{i2}	b_{i3}	b_{i4}	b_{i5}	b_{i6}
1	-0.0109	-0.0389	-0.0295	-0.0155	0.9174	0.0192
2	-0.0131	-0.0492	-0.0073	-0.0333	0.9072	0.0153
3	-0.0131	-0.0362	-0.0234	0.0042	0.9375	0.0166
4	-0.0103	-0.0403	-0.0278	-0.0149	0.9274	0.0192
5	-0.0157	-0.0445	-0.0427	-0.0066	0.9475	0.0117
6	-0.0159	-0.0399	-0.0411	-0.0168	0.9182	0.0118
7	-0.0131	-0.0387	-0.0252	-0.0216	0.9113	0.0168

Table 13. C.7. Characteristics of fuzzy part of LNBF models

Type	Sugeno
inputs/outputs	5/1
number of clusters (fuzzy ruccles)	7
and method	product
or method	probabilistic or
implication method	product
aggregation	sum
defuzzification	weighted average