DOI 10.2478/ama-2019-0022

DESIGN OF AN OPTIMAL FUZZY CONTROLLER OF AN UNDER-ACTUATED MANIPULATOR BASED ON TEACHING-LEARNING-BASED OPTIMIZATION

Mohammad Javad MAHMOODABADI,* Amineh YAZDIZADEH BAGHINI*

*Department of Mechanical Engineering, Sirjan University of Technology, Sirjan, P.O. Box: 78185-439, Iran

mahmoodabadi@sirjantech.ac.ir, amineh.yazdizadeh@yahoo.com

received 24 April 2019, revised 20 September 2019, accepted 24 September 2019

Abstract: In this paper, an optimal fuzzy controller based on the Teaching-Learning-Based Optimization (TLBO) algorithm has been presented for the stabilization of a two-link planar horizontal under-actuated manipulator with two revolute (2R) joints. For the considered fuzzy control method, a singleton fuzzifier, a centre average defuzzifier and a product inference engine have been used. The TLBO algorithm has been implemented for searching the optimum parameters of the fuzzy controller with consideration of time integral of the absolute error of the state variables as the objective function. The proposed control method has been utilized for the 2R under-actuated manipulator with the second passive joint wherein the model moves in the horizontal plane and friction forces have been considered. Simulation results of the offered control method have been illustrated for the stabilization of the considered robot system. Moreover, for different initial conditions, the effectiveness and the robustness of the mentioned strategy have been challenged.

Keywords: Optimal controller, Fuzzy control, Teaching-learning-based optimization, Under-actuated system, 2R planar horizontal manipulator

1. INTRODUCTION

In the last two decades, many researchers have shown interest in the control and stabilization of under-actuated manipulators and it has remained an open problem till now. A system with lower number of control inputs than degrees of freedom is said to be an under-actuated one. Utilizing the under-actuated systems is economical because of energy saving, reduced cost and weight, although they have the disadvantage that the control of these systems is more complicate. Furthermore, if one of actuators of a fully actuated system fails, it is convenient to have a controller for the under-actuated system in this situation.

A popular and powerful solution to attack this problem is utilizing control methods based on fuzzy logic. These techniques are implemented to design stabilizers according to the synthesis of fuzzy IF-THEN rules heuristically determined by the knowledge and experience of an expert. There are variant versions of the fuzzy control methods based on the kinds of fuzzifiers, the defuzzifiers and the inference engines applied by researchers. For instance, in the field of fuzzy control of under-actuated systems, Jianiang et al. (2001) presented a new fuzzy controller based on a sum-product inference engine system for stabilization of a ball and beam system. Ho et al. (2007) proposed a stable adaptive fuzzy-based tracking control for a robot manipulator. Karimi et al. (2011) introduced a new type-2 fuzzy logic controller to handle a nonlinear inverted pendulum system. Mahmoodabadi et al. (2016) implemented an optimal fuzzy technique to stabilize two nonlinear systems. Mahmoodabadi and Danesh (2017) applied a fuzzy approach based on the gravitational search algorithm to set the state variables of a ball and beam system. Caoyang et al. (2017) introduced nonlinear guidance and fuzzy control for three-dimensional path following of an autonomous underwater vehicle. Naghibi et al. (2017) presented a fuzzy controller with integrator for control of manipulators. Nguyen et al. (2018) designed and experimented fuzzy steering control for autonomous vehicles' saturation. Lin et al. (2018) exhibited the performance of the adaptive fuzzy output feedback stabilization control for the surface vessel. Zakeri et al. (2019) introduced an optimal interval type-2 fuzzy fractional order super twisting algorithm for stabilization of dynamical systems. Deng et al. (2019) displayed event-triggered robust fuzzy path following control for ships with input saturation. Wang et al. (2019) represented fuzzy unknown observer-based robust adaptive path following the control of underactuated surface vehicles subject to multiple unknowns. Vahidi-Moghaddam et al. (2019) organized disturbance-observer-based fuzzy terminal sliding mode control for uncertain nonlinear systems.

On the other hand, the synthesis of control policies has been presented as optimization problems of certain performance measures of the controlled systems. A very effective means of solving such optimum controller design problems is utilizing evolutionary algorithms such as Teaching-Learning-Based Optimization (TLBO). This algorithm as a new nature-inspired optimization algorithm works based on the influence of a teacher on its learners. It is also a population-based method and uses a population of solutions to find the optimum global solution. At first, Rao et al. (2011) showed the better performance of TLBO over other nature-inspired optimization methods for the constrained benchmark functions and mechanical design problems. Furthermore, Rao et al. (2012) proposed it to find a solution for continuous non-linear large scale optimization problems. Khoban (2014) used TLBO for optimum design of a feedback linearization controller to achieve the best trajectory tracking for non-holonomic wheeled mobile robots.

sciendo
DOI 10.2478/ama-2019-0022 acta mechanica et automatica, vol.13 no.3 (2019)

This research tries to present an optimal fuzzy controller based on a TLBO algorithm for stabilization of a two-link under-actuated manipulator. For this fuzzy control method, a singleton fuzzifier, a centre average defuzzifier and a product inference engine have been used. The TLBO algorithm has been designed for searching the optimum design variables of the fuzzy controller. The proposed control method has been utilized for the 2R under-actuated manipulator with a second passive joint wherein the model moves in the horizontal plane and the friction forces have been considered.

The rest of the paper is organized as follows. The mathematical model of the considered under actuated manipulator is explained in Section 2. The descriptions of the considered fuzzy system and the proposed fuzzy controller are stated in Sections 3 and 4, respectively. Moreover, Section 4 briefly introduces the TLBO algorithm, design variables and objective functions. The introduced fuzzy controller is implemented on the 2R planar under-actuated manipulator in Section 6. Furthermore, the optimization of the control parameters by the TLBO algorithm and the simulation results are included in this section. Finally, the conclusion is made in Section 7.

2. THE MATHEMATICAL MODEL OF THE UNDER-ACTUATED MANIPULATOR

The configuration of the considered manipulator is illustrated in Fig. 1, which has two degrees of freedom with revolute joints and two links that move on the frictional horizontal plane. The first revolute joint is active whereas the second one is considered as a passive pin.

If the generalized coordinates q_i, $i = 1,2$ are regarded as the joint angles, then the dynamic model of the system can be written as follow (Spong et al., 2005).

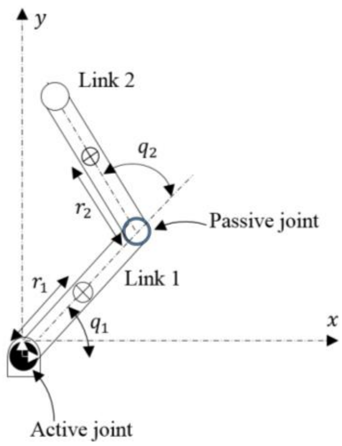

Fig 1. A schematic of the 2R planar under-actuated manipulator with the second passive joint

$$M(q)\ddot{q} + h(q,\dot{q}) = \tau \tag{1}$$

where, $\dot{q}$ is the vector of the generalized velocities, $\ddot{q}$ is the vector of the generalized accelerations, $M(q) \in R^{2\times2}$ denotes the inertia matrix and $h(\dot{q},\ddot{q})$ contains the centrifugal, Coriolis and possibly gravitational terms; although here, the gravitational terms have not been considered because the manipulator has been assumed to move on the horizontal plane.

In order to simplify the motion equations, the following constant parameters are defined.

$$z_1 = m_1 r_1^2 + m_2 l_1^2 + I_1 \tag{2}$$

$$z_2 = m_2 r_2^2 + I_2 \tag{3}$$

$$z_3 = m_2 l_1 r_2 \tag{4}$$

where, m_1 and m_2 illustrate the link masses, l_1 and l_2 show the link lengths, I_1 and I_2 are the moments of inertia, and r_1 and r_2 depict the centers of the masses. Hence, the elements of $M(q)$ and $h(q,\dot{q})$ are considered as follow:

$$m_{11} = z_1 + z_2 + 2z_3 \operatorname{Cos} q_2 \tag{5}$$

$$m_{12} = z_2 + z_3 \operatorname{Cos} q_2 \tag{6}$$

$$m_{21} = m_{12} \tag{7}$$

$$m_{22} = z_2 \tag{8}$$

$$h_1 = -z_3(2\dot{q}_1\dot{q}_2 + \dot{q}_2^2) \operatorname{Sin} q_2 \tag{9}$$

$$h_2 = z_3 \dot{q}_1^2 \operatorname{Sin} q_2 \tag{10}$$

It must be mentioned that both the frictions (Coulomb and viscous) are considered for the passive joint. The viscous friction F_V is defined as proportional to the velocity of the second joint.

$$F_V = b\dot{q} \tag{11}$$

where, b is a viscous friction constant. The Coulomb friction F_c is regarded as constant with a sign dependence on the joint velocity and is given by:

$$F_c = SGN(\dot{q})\, c \tag{12}$$

where, c is the Coulomb friction constant. In order to eliminate the chattering phenomena caused by the sign function, it will be replaced by the following saturation function.

$$sat(\dot{q}) = \begin{cases} 1 & \dot{q} > k \\ -1 & \dot{q} < -k \\ \dot{q} & -k < \dot{q} < k \end{cases} \tag{13}$$

where, k is a positive constant; and here, regarded as k = 0.01. Therefore, the equations of motion can be formulated as follows (Spong et al., 2005).

$$m_{11}\ddot{q}_1 + m_{12}\ddot{q}_2 + h_1 = \tau - sat(\dot{q}_1)c_1 - b_1\dot{q}_1 \tag{14}$$

$$m_{21}\ddot{q}_1 + m_{22}\ddot{q}_2 + h_2 = -sat(\dot{q}_2)c_2 - b_2\dot{q}_2 \tag{15}$$

Moreover, the state-space equations are written as follows:

$$\begin{bmatrix}\dot{y}_1\\ \dot{y}_2\\ \dot{y}_3\\ \dot{y}_4\end{bmatrix} = \begin{bmatrix}0 & 1 & 0 & 0\\ 0 & -\frac{m_{12}}{m_{22}}b_1 & 0 & b_2\\ 0 & 0 & 0 & 1\\ 0 & \frac{m_{12}}{m_{11}}b_1 & 0 & -b_2\end{bmatrix}\begin{bmatrix}y_1\\ y_2\\ y_3\\ y_4\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{m_{11}-m_{12}^2/m_{22}}\\ 0\\ -\frac{m_{12}/m_{11}}{m_{22}-m_{12}^2/m_{11}}\end{bmatrix}\tau + \begin{bmatrix}0\\ \frac{c_1 sat(y_2)-h_1+m_{12}/m_{22}(c_2 sat(y_4)+h_2}{m_{11}-m_{12}^2/m_{22}}\\ 0\\ \frac{-c_2 sat(y_4)-h_2-m_{12}/m_{11}(-c_1 sat(y_2)-h_1)}{m_{22}-m_{12}^2/m_{11}}\end{bmatrix} \tag{16}$$

DOI 10.2478/ama-2019-0022

where, $[y_1, y_2, y_3, y_4]$ are the state variables corresponding to $[q_1, \dot{q}_1, q_2, \dot{q}_2]$, respectively. The considered control method proposes a suitable manipulated variable (U) to move the two links from the initial conditions to the desired positions. The relationship between U and torque τ is given as follows:

$$\tau = c_1\, sat(y_2) + b_1 y_2 + h_1 - \left(\frac{m_{12}}{m_{22}}\left(c_2\, sat(y_4) + b_2 y_4 + h_2\right)\right) + \left(m_{11} - \frac{m_{12}^2}{m_{22}}\right) U \quad (17)$$

3. THE CONSIDERED FUZZY SYSTEM AND PROPOSED FUZZY CONTROLLER

Three types of fuzzy systems are commonly implemented in the literature: (i) pure fuzzy systems, (ii) Takagi-Sugeno-Kang (TSK) fuzzy systems, and (iii) fuzzy systems with fuzzifier and defuzzifier. The main problem with the pure fuzzy system is that its inputs and outputs are fuzzy sets, whereas in engineering systems, the inputs and outputs are real-valued variables. To solve this problem, Takagi, Sugeno and Kang (Takagi and Sugeno, 1985; Sugeno and Kang, 1988) proposed another fuzzy system whose inputs and outputs are real-valued variables. The main problems with the Takagi-Sugeno-Kang fuzzy system are: (i) its THEN part is a mathematical formula, and therefore, may not provide a natural framework to represent human knowledge, and (ii) there is not much freedom left to apply different principles in fuzzy logic. To elucidate these drawbacks, the third type of fuzzy systems has been proposed with fuzzifier and defuzzifier (Wang, 1996).

In general, a third type fuzzy system consists of four parts; the fuzzifier, the fuzzy rule base, the inference engine and the defuzzifier (Wang, 1996). The used fuzzy system in this work implements the singleton fuzzifier, the product inference engine and the centre average defuzzifier as the following form:

$$\Psi_i = \frac{\sum_{l=1}^{m} \overline{\psi_l}\,(\prod_{i=1}^{n} \mu_{A_i^l}(x_i))}{\sum_{l=1}^{m} \prod_{i=1}^{n} \mu_{A_i^l}(x_i)} \quad (18)$$

where, $\mu_{A_i^l}(x_i)$ is the membership function of the input linguistic variable x_i for the rule L-th. Because the calculations with triangular memberships are easy, this kind of functions are used here. The variable x_i is the normalized form of the state variable y_i. $\overline{\psi_l}$ presents the center of the output membership function $\mu_{B_i^l}$ for the rule L-th.

The fuzzy rules are separately regulated for each input item as follow:

$$rule - l \quad \{R_i^l : IF \quad x_i = \mu_{A_i^l} \quad THEN \quad \Psi_i = \mu_{B_i^l}\}_{l=1}^{m} \quad (19)$$

that, R_i^l is the rule L-th. The normalized variable x_i is considered as an input item corresponding to the output variable Ψ_i.

In this paper, the following equation is proposed to calculate the manipulated variable U as the summation of the multiplication of the deviation and priority variables.

$$U = \sum_{i=1}^{n} D_i \times P_i \quad (20)$$

where, n is the number of the system states. D_i denotes the i-th deviation variable and P_i represents the priority variable.

Deviation fuzzy variable shows the deviation of state i-th from the corresponding desired state. In order to calculate D_i, as the first fuzzy variable, based on the fuzzy system introduced in Equation (18), Table 1 and Fig. 2 are regarded. Table 1 mentions the fuzzy rule base for the input items corresponding to the deviation fuzzy variables. The related triangular membership functions have been determined in Fig. 2 as negative (N), zero (Z) and positive (P). Based on Table 1 and Fig. 2, if the joint variables are located near the desired values, the control torque will be zero. Moreover, if the joint variables are less than the desired values, then a negative torque will rotate the arm counter clockwise. Inversely, if the angles are more than the desired values, a positive torque will rotate the arm clockwise.

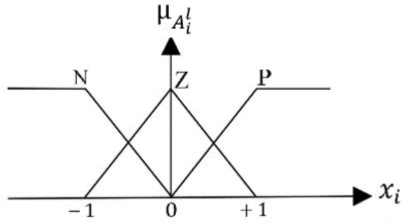

Fig. 2. Triangular membership functions for the input items corresponding to the deviation fuzzy variable D_i

Tab. 1. Fuzzy rule base corresponding to the deviation fuzzy variable D_i

x_i	$\overline{D_l}$
N	-1
Z	0
P	1

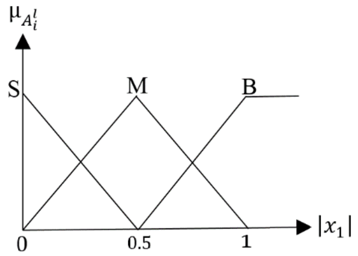

Fig. 3. Triangular membership functions for the input items corresponding to the second fuzzy variable S_i

Tab. 2. Fuzzy rule base corresponding to the second fuzzy variable S_i.

$\lvert x_1 \rvert$	$\bar{S}_l$
S	0
M	0.5
B	1

Priority variable represents the control priority of the considered joint. The priority variable P_i is proposed based on this fact that each state variable has its own effect on the control performance which may be different from others. For example, the stabilizing of the first joint angle and its angular velocity should be more important than those of the second joint when the first joint is not balanced. So, the first joint angular control takes more priority than the second joint. To express the role of each input item separately, the control priority P_i is defined as follows.

$$P_i = v_1^i + v_2^i \times S_i\ ;\ i = 1, 2, 3, 4. \tag{21}$$

where, v_1^i and v_2^i are the constant parameters. S_i, as the second fuzzy variable, is calculated based on the fuzzy system defined in Equation (18), Table 2 and Fig. 3. Table 2 shows the fuzzy rule base for the absolute of the normalized joint angles and the joint angular velocities. Based on Fig. 3 and Table 2, for example when the first angle is small, the control priority of this joint is zero but when this angle is big, it takes the maximum value.

Because the changes range of the inputs for the membership functions are $[-1; +1]$ or $[0; +1]$, the four state variables, i.e. the joint angles q_1 and q_2 and the joint angular velocities $\dot{q_1}$ and $\dot{q_2}$, have been normalized ($[x_1, x_2, x_3, x_4]$) with the scaling factors.

4. TEACHING-LEARNING-BASED OPTIMIZATION

For searching the optimum parameters of the fuzzy controller, the TLBO algorithm has been implemented. Teaching-learning-based optimization as a new nature-inspired optimization algorithm works based on the influence of a teacher on learners. It is also a population-based method and uses a population of solutions to find the global optimum solution. The wide application of TLBO in engineering fields is reflected in the improvement of the performance of distinguished systems, such as fatty acid methyl esters (Baghban et al., 2018), scheduling of projects (Kumar et al., 2018), plasma arc cutting (Patel et al., 2018) and power consumption optimization (Rao, 2019). The population is considered as a group of learners or a class of learners. The optimization process in TLBO is divided into two parts: the first part consists of the 'teacher phase' and the second part consists of the 'learner phase'. The first one means learning from the teacher and the second means learning from the interaction between learners (Rao et al., 2011 and 2012).

4.1. Teacher phase

In this phase, the teacher conveys information among the learners for the improvement of their mean result in the class. It is assumed that m and n are the number of subjects and students, respectively. Furthermore, Υ_m^{ji} denotes the mean result of the learners at subject $j = 1, 2, \ldots, m$ and teaching-learning sequence i. The supreme leaner of the entire population is assigned as the teacher because he/she is the most knowledgeable person (Υ_{best}).

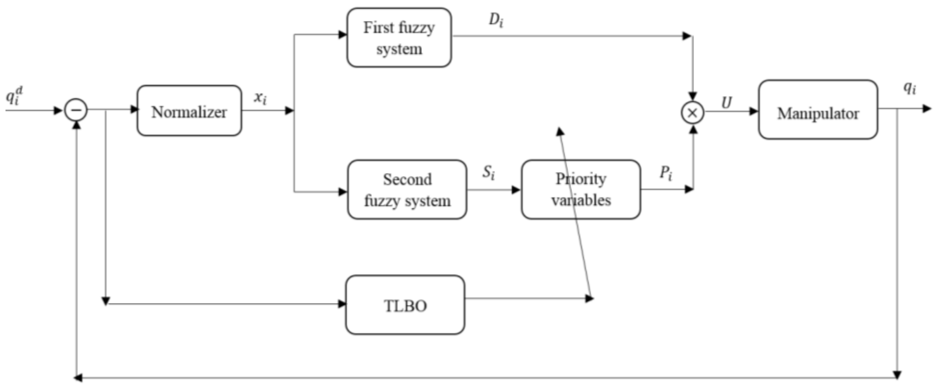

Fig. 4. Block diagram of the proposed optimal fuzzy controller of the under-actuated manipulator based on TLBO

A random weighted differential vector is formed in subject j for student k at sequence i as follows.

$$\Upsilon_{Dif}^{jki} = rand(\Upsilon_{best}^{jki} - \alpha\Upsilon_m^{ji}) \tag{22}$$

where, rand is a random number in range [0,1]. Moreover, α is the teaching factor and random takes the values of 1 or 2. A new learner is determined using the following equation.

$$\Upsilon_{new}^{jki} = \Upsilon_{old}^{jki} - \Upsilon_{Dif}^{jki} \tag{23}$$

where, Υ_{new}^{jki} represents the corresponding result of the improved learner and Υ_{old}^{jki} shows the grade achieved by each student in that class.

4.2. Learner phase

In this phase, the students might modify their knowledge via the mutual interaction. At first, two different student are selected, i.e. Υ_1^j and Υ_2^j. Then, based on the following conditions, the new situations would be obtained.

$$\text{IF } \Upsilon_1^j < \Upsilon_2^j \text{ then } \Upsilon_{1_{new}}^j = \Upsilon_1^j + rand(\Upsilon_2^j - \Upsilon_1^j) \tag{24}$$

$$\text{IF } \Upsilon_2^j < \Upsilon_1^j \text{ then } \Upsilon_{1_{new}}^j = \Upsilon_1^j + rand(\Upsilon_1^j - \Upsilon_2^j) \tag{25}$$

Moreover, $\Upsilon_{1_{new}}^j$ would be accepted in the condition that it has a more effective function value.

5. OPTIMAL FUZZY CONTROL OF THE UNDER-ACTUATED MANIPULATOR

Fig. 4 provides a graphical representation of the proposed strategy as a block diagram. This figure illustrates that the state variables (the joint angles and the joint angular velocities) relevant to the manipulator are compared with their desired values. Then, those are inputted into the normalizer block for normalization by the associated scaling factors. The fuzzy systems are utilizes to

DOI 10.2478/ama-2019-0022

produce deviation and priority variables. The summation of the production of these variables related to the joint angles and the joint angular velocities create the control effort. Finally, the TLBO algorithm described in the previous section is implemented for optimization of the constant parameters of the priority variables. The eight control parameters, i.e. $v_1^1, v_2^1, v_1^2, v_2^2, v_1^3, v_2^3, v_1^4, v_2^4$ have been regarded as the design variables and the summation of the integral of the absolute errors of the first and second joint angles has been considered as the cost function F as follows:

$$F = \int |q_1 - q_1^d|\, dt + \int |q_2 - q_2^d| dt \tag{26}$$

where, q_1^d and q_2^d respectively denote the desired values for the first and second joint angles.

6. RESULTS AND DISCUSSIONS

The manipulator parameters considered in this paper are as follows (Mahindrakar et al., 2006):
$z_1 = 0.725, \quad z_2 = 0.3179, z_3 = 0.3147, c_1 = 0.26,$
$c_2 = 0.116, b_1 = 0.6236, \ b_2 = 0.1223.$

Here, two cases with different initial values are considered; that for both, the desired values for the first and second joints are regarded as zero.

With considering an initial population 50 and maximum iterations 100, it is tried to optimize the cost function (26) and find the best parameters of the controller so that the system moves to the desired positions at a minimum possible time. Two different initial values:
$\left([q_1(0), \dot{q}_1(0), q_2(0), \dot{q}_2(0)] = \left[\frac{\pi}{5}, 0, \frac{\pi}{4}, 0\right] \ and \ \left[\frac{\pi}{2}, 0, 0, 0\right]\right)$
have been regarded, and their associate optimum values are listed Tables 4 and 5, respectively. The optimization graph of the first initial conditions illustrated in Fig. 5 shows that the population size and maximum iterations are well organized because the best cost has fixed after 42 iterations. The value of the best cost function in the first repetition equals 2.217 and in the last repetition, equals 2.061. The numerical values of the scaling factors obtained by a try and error process are as $[25\,°, 50\,°/s, 25\,°, 50\,°/s]$ for $[y_1, y_2, y_3, y_4] = [q_1, \dot{q_1}, q_2, \dot{q_2}]$, respectively.

It is obvious from Tables 4 and 5 that the summation of the two parameters for the first joint angle and the first joint angular velocity is larger than of those for the second joint. It is resulted that the first joint control takes more priority than the second joint control. The time responses of the first and second joint angles, joint angular velocities and control torque for the two initial values are depicted in Figs. 6–11. It is observable from these figures that all the state variables have converged to zero, and the complete stabilization have occurred.

Tab. 4. The optimum control parameters obtained by TLBO for the initial values $[q_1(0), \dot{q}_1(0), q_2(0), \dot{q}_2(0)] = \left[\frac{\pi}{5}, 0, \frac{\pi}{4}, 0\right]$

Input item	v_1^i	v_2^i
$i = 1$	0.2500	4.1000
$i = 2$	1.4000	3.0000
$i = 3$	0.5000	1.5044
$i = 4$	1.4000	0.9600

Tab. 5. The optimum control parameters obtained by TLBO for the initial values $[q_1(0), \dot{q}_1(0), q_2(0), \dot{q}_2(0)] = \left[\frac{\pi}{2}, 0,0,0\right]$

Input item	v_2^i	v_1^i
$i = 1$	0.10050	3.1002
$i = 2$	1.72820	1.9097
$i = 3$	0.19978	0.1299
$i = 4$	1.59450	0.9895

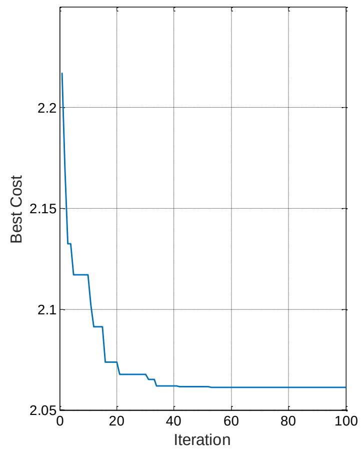

Fig. 5. The optimization trajectory for the fuzzy control of the under-actuated manipulator

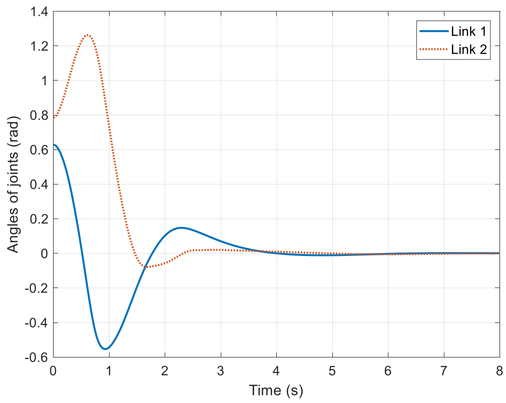

Fig. 6. Time trajectories of the first and second joint angles for initial values $[q_1(0), \dot{q}_1(0), q_2(0), \dot{q}_2(0)] = \left[\frac{\pi}{5}, 0, \frac{\pi}{4}, 0\right]$

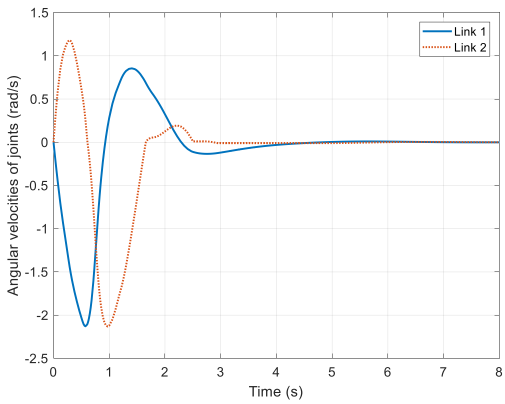

Fig. 7. Time trajectories of the first and second joint angular velocities for initial values $[q_1(0), \dot{q}_1(0), q_2(0), \dot{q}_2(0)] = \left[\frac{\pi}{5}, 0, \frac{\pi}{4}, 0\right]$

DOI 10.2478/ama-2019-0022

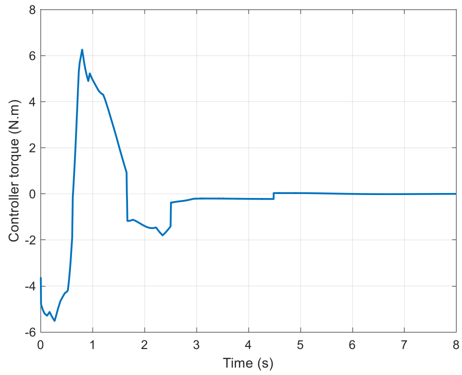

Fig. 8. Time trajectory of the actuator torque for initial values $[q_1(0), \dot{q}_1(0), q_2(0), \dot{q}_2(0)] = \left[\frac{\pi}{5}, 0, \frac{\pi}{4}, 0\right]$

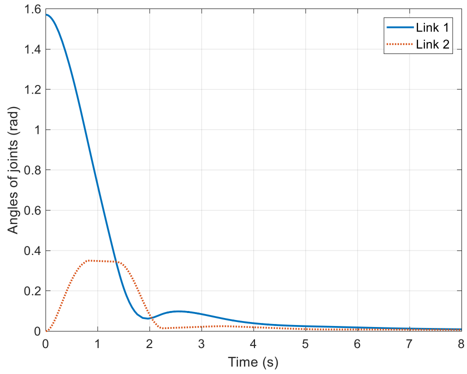

Fig. 9. Time trajectories of the first and second joint angles for initial values $[q_1(0), \dot{q}_1(0), q_2(0), \dot{q}_2(0)] = \left[\frac{\pi}{2}, 0,0,0\right]$

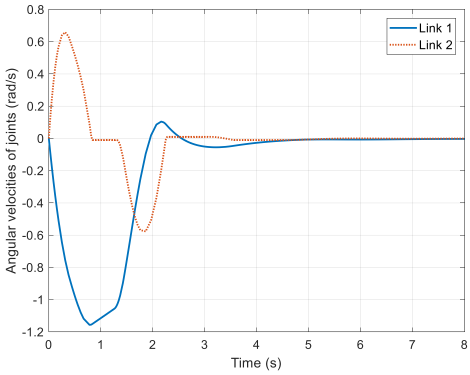

Fig. 10. Time trajectories of the first and second joint angular velocities for initial values $[q_1(0), \dot{q}_1(0), q_2(0), \dot{q}_2(0)] = \left[\frac{\pi}{2}, 0,0,0\right]$

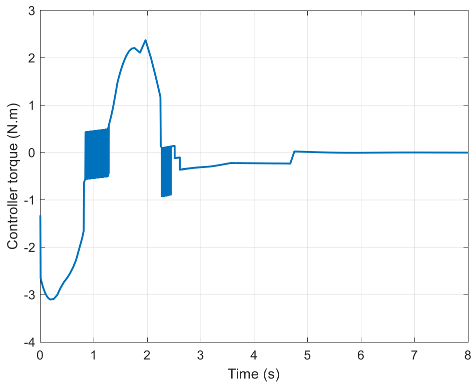

Fig. 11. Time trajectory of the actuator torque for initial values $[q_1(0), \dot{q}_1(0), q_2(0), \dot{q}_2(0)] = \left[\frac{\pi}{2}, 0,0,0\right]$

7. CONCLUSION

In the present research, an optimal controller based on fuzzy rules was proposed as a general scheme to control a class of nonlinear underactuated systems. Teaching-Learning-Based Optimization (TLBO) was utilized to ascertain the optimal parameters of the proposed controller with regard to the design criteria. By utilizing the optimal design variables in the proposed controller, the performance of the controller was evaluated considering a two-link manipulator system. The results and analysis demonstrate the proper performance of the proposed controller in the aspects of stability and minimum tracking error. As a potential future study, making the controller online through using approaches, such as neural networks and moving least squares interpolation; ascertaining better optimal solutions for the parameters of the controller via other smart optimization algorithms; and considering different objective functions due to different design criteria can be regarded to extend this study in order to establish general approaches in the case of designing optimal fuzzy controllers.

REFERENCES

1. **Baghban A., Kardani M.N., Mohammadi A.H.** (2018), Improved estimation of Cetane number of fatty acid methyl esters (FAMEs) based biodiesels using TLBO-NN and PSO-NN models, *Fuel*, 232, 620–631.
2. **Craig J.J.** (1989), *Introduction to robotics. Mechanics and Control, Boston*, MA, USA, Addison-Wesley Longman Publishing Co, Inc.
3. **Deng Y., Zhang X., Im N., Zhang G., Zhang Q.** (2019), Event-triggered robust fuzzy path following control for underactuated ships with input saturation, *Ocean Engineering*, 186(1), 106122.
4. **He G.P., Wang Z.L., Zhang J., Geng Z.Y.** (2014), *Characteristics analysis and stabilization of a planar 2R underactuated manipulator*, Robotica, Cambridge University Press, 1–17.
5. **Ho H.F., Wong Y.K., Rad A.B.** (2007), Robust fuzzy tracking control for robotic manipulator, *IFSA World Congress and 20th NAFIPS International Conference*, 801–816.
6. **Karimi M., Safarinejadian B.** (2011), On-line control of the inverted pendulum with type-2 fuzzy logic controller, *2nd International Conference on Control, Instrumentation and Automation (ICCIA)*, 429–433.
7. **Khooban M.H.** (2014), Design an intelligent proportional-derivative (PD) feedback linearization control for nonholonomic-wheeled mobile robot, *Journal of Intelligent & Fuzzy Systems*, 26, 1833–1843.
8. **Kumar M., Mittal M.L., Soni G., Joshi D.** (2018), A hybrid TLBO-TS algorithm for integrated selection and scheduling of projects, *Computers & Industrial Engineering*, 119, 121–130.
9. **Lim C.M., Hiyama T.** (1991), Application of fuzzy logic control to a manipulator, *IEEE Transactions on Robotics and Automation* I, 5, 688–691.
10. **Lin X., Nie J., Jiao Y., Liang K., Li H.** (2018), Adaptive fuzzy output feedback stabilization control for the underactuated surface vessel, *Applied Ocean Research*, 74(1), 40–48.
11. **Ma X.J., Sun Z.Q., He Y.Y.** (1998), Analysis and design of fuzzy controller and fuzzy observer, *IEEE Transactions on Fuzzy Systems*, 6(1), 41–50.
12. **Mahindrakar A.D., Rao S., Banavar R.N.** (2006), Point-to-point control of a 2R planar horizontal underactuated manipulator, *Mechanism and Machine Theory*, 41, 838–844.
13. **Mahmoodabadi M.J., Danesh N.** (2017), Gravitational search algorithm based fuzzy control for a nonlinear ball and beam system, *Journal of Control and Decision*, 5(3), 229–240.
14. **Mahmoodabadi M.J., Mottaghi M.B.S., Mahmodinejad A.** (2016), Optimum design of fuzzy controllers for nonlinear systems using

DOI 10.2478/ama-2019-0022

multi-objective particle swarm optimization, *Journal of Vibration and Control*, 22(3), 769–783.

15. **Naghibi S.R., Pirmohamadi A.A., Moosavian S.A.A.** (2017), Fuzzy MTEJ controller with integrator for control of underactuated manipulators, *Robotics and Computer-Integrated Manufacturing*, 48(1), 93–101.
16. **Nguyen A.T., Sentouh C., Popieul J.C.** (2018), Fuzzy steering control for autonomous vehicles under actuator saturation: Design and experiments, *Journal of the Franklin Institute*, 355(18), 9374–9395.
17. **Patel P., Nakum B., Abhishek K., Kumar V.R., Kumar A.** (2018), Optimization of Surface Roughness in Plasma Arc Cutting of AISID2 Steel Using TLBO, *Materials Today: Proceedings*, 5 (9), 18927–18932.
18. **Rao K.V.** (2019), Power consumption optimization strategy in micro ball-end milling of D2 steel via TLBO coupled with 3D FEM simulation, *Measurement*, 132, 68–78.
19. **Rao R.V., Savsani V.J., Vakharia D.P.** (2011), Teaching-learning-based optimization: a novel method for constrained mechanical design optimization problems, *Computer-Aided Design*, 43, 303–315.
20. **Rao R.V., Savsani V.J., Vakharia D.P.** (2012), Teaching-learning-based optimization: an optimization method for continuous non-linear large scale problems, *Information Sciences*, 183(1), 1–12.
21. **Spong M.W., Hutchinson S., Vidyasagar M.** (2005), *Robot modeling and control*, John Wiley & Sons, Inc.
22. **Sugeno M., Kang G.T.** (1988), Structure identification of fuzzy model, *Fuzzy Sets and Systems*, 28(1), 15–33.
23. **Takagi T., Sugeno M.** (1985), Fuzzy identification of systems and its applications to modeling and control, *IEEE Transactions on Systems, Man and Cybernetics*, 15 (1), 116–132.
24. **Vahidi-Moghaddam A., Rajaei A., Ayati M.** (2019), Disturbance-observer-based fuzzy terminal sliding mode control for MIMO uncertain nonlinear systems, *Applied Mathematical Modelling*, 70(1), 109–127.
25. **Wang L.X.** (1996), *A Course in Fuzzy Systems and Control*, Upper Saddle River, United States Prentice-Hall International, Inc.
26. **Wang N., Sun Z., Yin J., Zou Z., Sud S.F.** (2019), Fuzzy unknown observer-based robust adaptive path following control of underactuated surface vehicles subject to multiple unknowns, *Ocean Engineering*, 176(1), 57–64.
27. **Yi J., Yubazaki N., Hirota K.** (2001), Stabilization control of ball and beam systems, *IFSA World Congress and 20th NAFIPS International Conference*, 2229–2234.
28. **Yoo B.K., Ham W.C.** (2000), Adaptive control of robot manipulator using fuzzy compensator, *IEEE Transactions on Fuzzy Systems*, 8(2), 186–199.
29. **Yu C., Xiang X., Lapierre L., Zhang Q.** (2017), Nonlinear guidance and fuzzy control for three-dimensional path following of an underactuated autonomous underwater vehicle, *Ocean Engineering*, 146(1), 457–467.
30. **Zakeri E., Moezi S.A., Eghtesad M.** (2019), Optimal interval type-2 fuzzy fractional order super twisting algorithm: A second order sliding mode controller for fully-actuated and under-actuated nonlinear systems, *ISA Transactions*, 85(1), 13–32.