

EBRAHIM GHASEMI\*<sup>1</sup>, MOHAMMAD ATAEE\*\* , KOUROSH SHAHRIAR\*\*\***IMPROVING THE METHOD OF ROOF FALL SUSCEPTIBILITY ASSESSMENT BASED ON FUZZY APPROACH****UDOSKONALENIE METODY OKREŚLANIA SKŁONNOŚCI STROPU DO ZAWAŁU W OPARCIU O ELEMENTY LOGIKI ROZMYTEJ**

Retreat mining is always accompanied by a great amount of accidents and most of them are due to roof fall. Therefore, development of methodologies to evaluate the roof fall susceptibility (RFS) seems essential. Ghasemi et al. (2012) proposed a systematic methodology to assess the roof fall risk during retreat mining based on risk assessment classic approach. The main defect of this method is ignorance of subjective uncertainties due to linguistic input value of some factors, low resolution, fixed weighting, sharp class boundaries, etc. To remove this deflection and improve the mentioned method, in this paper, a novel methodology is presented to assess the RFS using fuzzy approach. The application of fuzzy approach provides an effective tool to handle the subjective uncertainties. Furthermore, fuzzy analytical hierarchy process (AHP) is used to structure and prioritize various risk factors and sub-factors during development of this method. This methodology is applied to identify the susceptibility of roof fall occurrence in main panel of Tabas Central Mine (TCM), Iran. The results indicate that this methodology is effective and efficient in assessing RFS.

**Keywords:** Coal mining; Room and pillar; Retreat mining; Roof fall susceptibility (RFS); Analytical hierarchy process (AHP); Risk assessment fuzzy approach

Wybieraniu w kierunku od pola towarzyszy zazwyczaj większa ilość wypadków, większość z nich spowodowana jest zawalem stropu. Dlatego też opracowanie skutecznej metody oceny skłonności stropu do zawalu jest kwestią kluczową. Ghasemi et al. (2012) zaproponował metodologię określania ryzyka zawalu stropu w trakcie prowadzenia prac górniczych w kierunku od pola w oparciu o klasyczne metody oceny ryzyka. Główną wadą tej metody jest to, iż nie uwzględnia ona subiektywnych niepewności na

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poziomie językowym związanych z określaniem wartości wejściowych charakteryzujących czynniki ryzyka, inne niedociągnięcia to niska rozdzielczość metody, stałe przyporządkowania wag, przyjęcie ostrych granic pomiędzy kolejnymi klasami. Aby usunąć te niedociągnięcia i w ten sposób udoskonalić metodę, zaproponowano nowe podejście do określania stabilności stropu wykorzystujące elementy logiki rozmytej. Zastosowanie logiki rozmytej jest efektywnym narzędziem w przypadku niepewności na poziomie językowym. Ponadto podejście bazujące na określeniu hierarchii procesów i wykorzystujące elementy logiki rozmytej zastosować można do określania wagi poszczególnych czynników ryzyka oraz czynników cząstkowych. Opracowaną metodę zastosowano do oceny skłonności stropu do zawału w polu głównym wybierania w kopalni Tabas Central Mine, w Iranie. Uzyskane wyniki potwierdzają skuteczność metody prognozowania stabilności stropu.

**Słowa kluczowe:** górnictwo węgla, wybieranie filarowo-komorowe, wybieranie w kierunku od pola, podatność stropu na zawał, analityczne badanie hierarchii procesów, ocena ryzyka z wykorzystaniem elementów logiki rozmytej

## 1. Introduction

In underground coal mining, room and pillar is one of the oldest methods used for the extraction of flat and tabular coal seams (Peng, 2008). In this method, a series of rooms are driven in the solid coal using continuous miner and generally Shuttle cars and pillars are formed in the development panels. Pillars are left behind to support the roof and prevent collapse. To increase the utilization of coal resources, the pillars are removed in a later operation (known as retreat mining or pillar recovery). Retreat mining is one of the most hazardous activities because it creates an inherently unstable situation. The process of retreat mining removes the main support for overburden and allows the ground to cave. As a result, the pillar line is an extremely dynamic and highly stressed environment. In other words, the roof at the pillar line is subjected to severe stresses and deformations. Retreat mining accounting for about 10% of all US underground coal production, yet has historically been associated with more than 25% of all roof and rib fall fatalities between 1986 and 1996 (Mark et al., 2003). Furthermore, similar statistics are observed in coal mining of Australia and South Africa (Lind, 2005). During a 14 years period, 1995-2008 in US, there was a total of 112 ground fall (roof and rib) fatalities in bituminous underground coal mines that 21% of total fatalities have occurred during retreat mining (Mark et al., 2009). These statistics and reviews emphasize the need for continuing efforts to reduce roof fall fatalities and injuries. Unfortunately, there are not enough researches about roof fall during retreat mining. One of the most valuable studies in this field is that was performed by Mark et al. (2003). They introduced the risk factors associated with retreat mining for reducing the risk of roof falls. They provided a risk factor checklist which can evaluate the overall level of roof fall risk and possible ways to reduce the roof fall. Similar studies were carried out for reducing roof fall accidents during retreat mining by Mark et al. (2002), Mark and Zelanko (2005), Feddock and Ma (2006). Furthermore, extensive researches have been conducted to control and assess roof fall risk in coal mines but not during retreat mining. Some of these researches have been carried out by Molinda et al. (2000), van der Merve et al. (2001), Deb (2003), Molinda (2003), Duzgun and Einstein (2004), Duzgun (2005), Palei and Das (2008), Shahriar and Bakhtavar (2009), Maiti and Khanzode (2009), Palei and Das (2009), Ghasemi et al. (2013), Razani et al. (2013) and Farid et al. (2013).

Recently, Ghasemi et al. (2012) have carried out a detailed study on roof fall risk in room and pillar coal mines during retreat mining. At first, they identified the major effective parameters on roof fall and explained the role of each one. Then, they presented a systematic method for

roof fall risk assessment using classic approach of risk assessment. In this method a quantifiable value is assigned to roof fall risk based on which the roof fall can be prevented and the safety is improved. Ignorance of subjective uncertainties during the process of risk assessment is the most remarkable limitation of proposed method. These uncertainties originate from the linguistic input value of some parameters, low resolution, fixed weighting, sharp class boundaries, etc. Fuzzy set theory enables a soft approach to account for these uncertainties by allowing the expert to participate in this process. Therefore, in this study a risk assessment fuzzy approach is developed to improve the accuracy and efficiency of classic method. In other words, the main purpose of this paper is to develop a new methodology to assess the RFS during retreat mining in room and pillar coal mines.

Fuzzy approach can be used to represent subjective, vague, linguistic and imprecise data and information effectively. The fuzzy approach was first introduced by Zadeh (1965) and its details can be found in the literatures. Because of ambiguity and vagueness involved in risk analysis, the fuzzy approach has been extensively used in different fields such as software development (Lee, 1996), environmental risk assessment (Sadiq & Husain, 2005), assessment of soil slopes failure (Saboya Jr et al., 2006), bridge risk assessment (Wang & Elhag, 2007), safety management (Dagdeviren & Yuksel, 2008), pipelines safety (Markowski & Mannan, 2009), underground mining method selection (Mikaeil et al., 2009), construction safety (Nieto-Morrote & Ruz-Vila, 2011), offshore risk assessment (Miri Lavasani et al., 2011), and etc. Herein, we report the application of fuzzy approach in assessment of roof fall susceptibility (RFS) for the first time.

## 2. Methodology

The proposed fuzzy risk assessment approach to determine the roof fall susceptibility (RFS) in room and pillar coal mines during retreat mining is composed of the following steps:

- Step 1: Identification of the factors and sub-factors affecting the roof fall.
- Step 2: Developing the decision model using analytic hierarchy process (AHP) technique based on the factors and sub-factors identified at step 1.
- Step 3: Determination of the local weights of the factors and sub-factors using fuzzy AHP approach.
- Step 4: Calculating the global weights for the sub-factors.
- Step 5: Representing the sub-factors in fuzzy form that is the determination of the linguistic variables for each of sub-factors.
- Step 6: Calculating the RFS index for panel of retreat mining using the global sub-factor weights and linguistic values.
- Step 7: Assessing the RFS index.

Schematic diagram of the proposed fuzzy model for determining and assessing RFS is provided in Fig. 1.

### 2.1. Step 1: Identifying the factors and sub-factors

As mentioned before, Ghasemi et al. (2012) determined the factors and sub-factors affecting roof fall risk during retreat mining. Based on findings of the field investigation, literature review and collected assistant data, they found 15 factors relevant to roof instability. These 15 factors

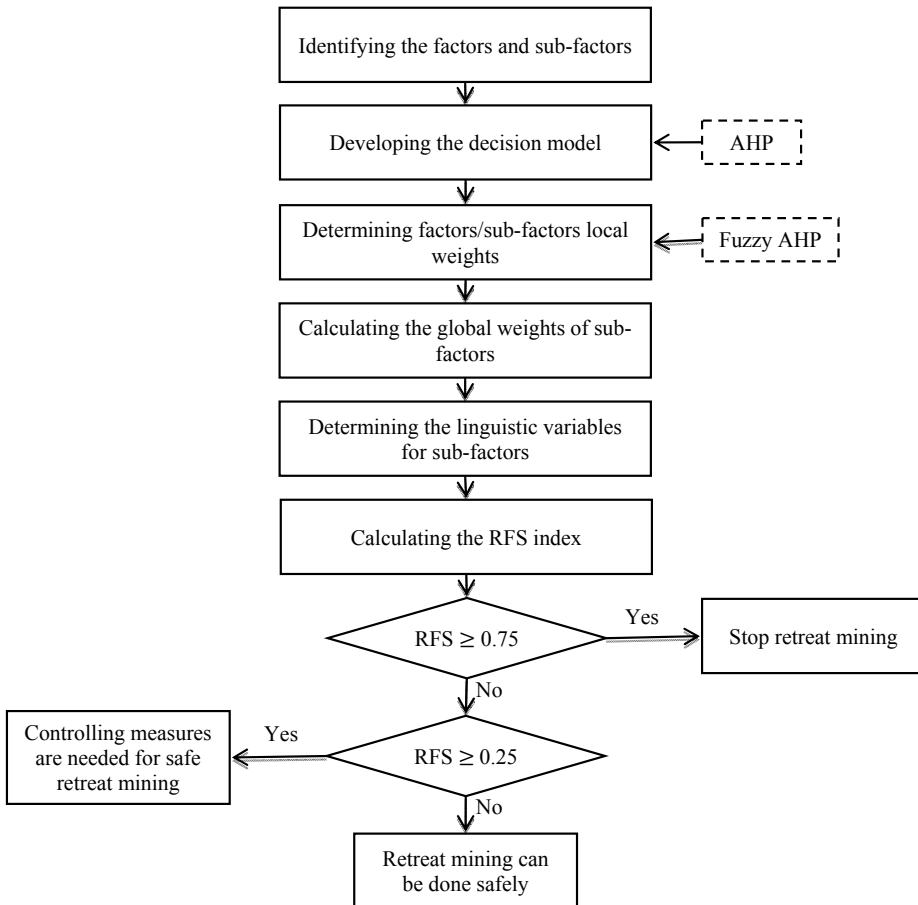


Fig. 1. Schematic diagram of the proposed fuzzy model for determining and assessing RFS

are classified into three groups as geological, design and operational factors. These groups are accepted as factors in this study and the factors belonging to these groups are accepted as sub-factors. The sub-factors that are classified as geological, design and operational are given below:

**Geological factors (A)**

- Depth of cover (A1)
- Roof rock quality (A2)
- Floor rock quality (A3)
- Groundwater (A4)
- Overlying massive strata (A5)
- Multiple-seam interaction (A6)

**Design factors (B)**

- Panel width (B1)
- Panel uniformity (B2)
- Entry width (B3)

- Pillar design (B4)
- Roof bolting (B5)

#### **Operational factors (C)**

- Panel age (C1)
- Supplemental support (C2)
- Cut sequence (C3)
- Final stump (C4)

The role of each sub-factor on roof fall can be found in Ghasemi et al. (2012).

## **2.2. Step 2: Developing the decision model (constructing hierarchical structure of factors and sub-factors)**

The decision (AHP) model formed by the factors and sub-factors is shown in Fig. 2. Hierarchical structure is composed of three levels. The goal of model is located in the first level (determining sub-factor weights). The factors are located at the second level and the sub-factors related to them are located at the third level.

## **2.3. Step 3: Determining the local weights of factors and sub-factors**

Since the effects of different factors and sub-factors on the roof fall are not the same, it is necessary to give a weight to each factor and sub-factor. Each weight represents the importance of specified factor or sub-factor on roof fall occurrence. In this study, the fuzzy AHP approach is used for determining the weights of factors and sub-factors.

### **2.3.1. Fuzzy AHP**

The AHP, introduced by Saaty (1980), addresses how to determine the relative importance of a set of activities in a multi-criteria decision problem. The process makes it possible to incorporate judgments on intangible qualitative criteria alongside tangible quantitative criteria. When applying AHP, a hierarchical decision model is constructed by decomposing the decision problem into its decision criteria. The importance and preference of the decision criteria are compared in a pairwise comparison manner with regard to the criterion preceding them in the hierarchy. The use of such pairwise comparison to collect data from the decision maker offers significant advantages. It allows the decision maker to focus on the comparison of just two objects, which makes the observation as free as possible from extraneous influences.

Despite AHP popularity and simplicity in concept, this method is often criticized for its inability to adequately handle the inherent uncertainty and imprecision associated with the mapping of the decision maker's perception to crisp values. In the traditional formulation of the AHP, human's judgments are represented as crisp values. However, in many practical cases the human preference model is uncertain and decision makers might be reluctant or unable to assign crisp values to the comparison judgments. Having to use crisp values is one of the problematic points in the crisp evaluation process. One reason is that decision makers usually feel more confident to give interval judgments rather than expressing their judgments in the form of single numeric values. As some criteria are difficult to measure by crisp values, they are usually neglected during the evaluation. Another reason is mathematical models that are based on crisp values. Thus, these

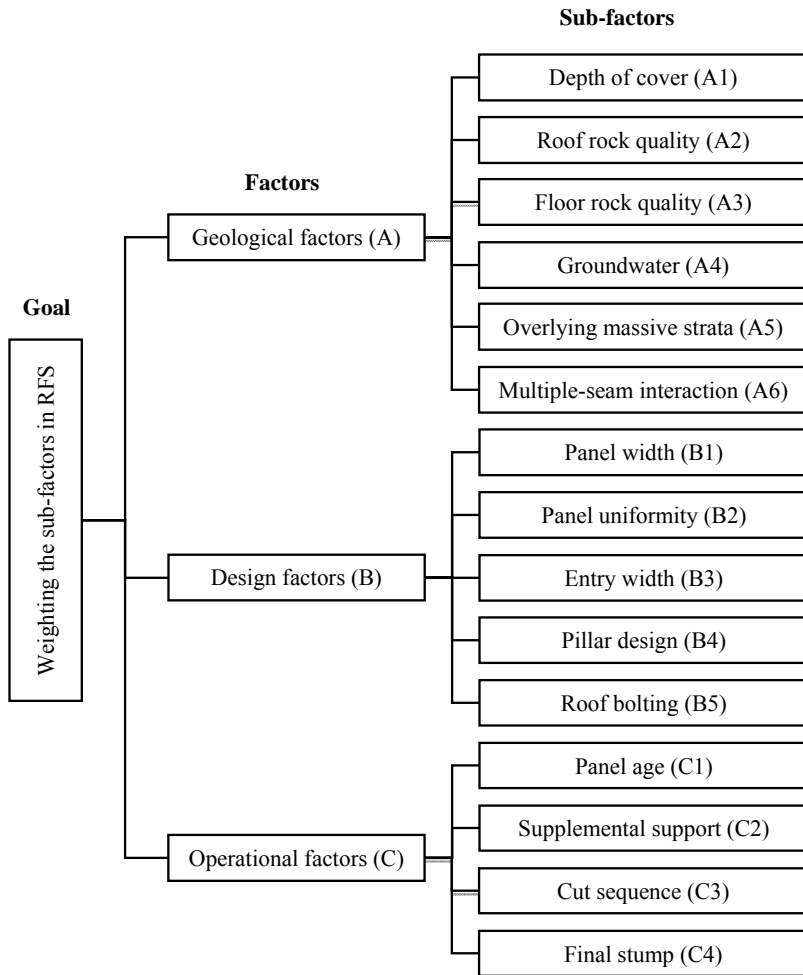


Fig. 2. Hierarchical structure of factors and sub-factors

models cannot deal with decision makers' ambiguities, uncertainties and vagueness which cannot be handled by crisp values. The use of fuzzy set theory allows the decision makers to incorporate unquantifiable information, incomplete information, non-obtainable information and partially ignorant facts into decision model (Zadeh, 1965). As a result, fuzzy AHP and its extensions are developed to solve alternative selection and justification problems. Although fuzzy AHP requires tedious computations, it is capable of capturing a human's appraisal of ambiguity when complex multi-criteria decision making problems are considered (Erensal et al., 2006).

### 2.3.2. Chang's extent analysis method

There are many fuzzy AHP methods proposed by various authors: Van Laarhoven and Pe-drycz (1983), Buckley (1985), Chang (1996), Cheng (1997), Deng (1999), Leung and Cao (2000),

and Mikhailov (2004). In this study, we use Chang's (1996) extent analysis method because the steps of this approach are easier than the other fuzzy AHP approaches. This method uses the triangular fuzzy numbers as a pairwise comparison scale for deriving the priorities of factors and sub-factors. The reason for using a triangular fuzzy number is that it is intuitively easy for the decision makers to use and calculate. In addition, modeling using triangular fuzzy numbers has proven to be an effective way for formulating decision problems where the information available is subjective and imprecise. The steps of Chang's (1996) extent analysis approach are as follows: Let  $X = \{x_1, x_2, \dots, x_n\}$  be an object set, and  $U = \{u_1, u_2, \dots, u_m\}$  be a goal set. According to the method of Chang's extent analysis, each object is taken and extent analysis for each goal,  $g_i$ , is performed, respectively. Therefore,  $m$  extent analysis values for each object can be obtained, with the following signs:

$$M_{gi}^1, M_{gi}^2, \dots, M_{gi}^m \quad i = 1, 2, \dots, n \quad (1)$$

where all the  $M_{gi}^j$  ( $j = 1, 2, \dots, m$ ) are triangular fuzzy numbers. A triangular fuzzy number is denoted simply as  $(l, m, u)$ . The parameters  $l$ ,  $m$  and  $u$ , respectively, denote the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event.

The steps of Chang's extent analysis can be given as in the following:

**Step 1:** The value of fuzzy synthetic extent with respect to the  $i^{\text{th}}$  object is defined as:

$$S_i = \sum_{j=1}^m M_{gi}^j \otimes \left[ \sum_{i=1}^n \sum_{j=1}^m M_{gi}^j \right]^{-1} \quad (2)$$

where  $\otimes$  denotes the extended multiplication of two fuzzy numbers. In order to obtain  $\sum_{j=1}^m M_{gi}^j$ , perform the fuzzy addition of  $m$  extent analysis values for a particular matrix such that:

$$\sum_{j=1}^m M_{gi}^j = \left( \sum_{j=1}^m l_j, \sum_{j=1}^m m_j, \sum_{j=1}^m u_j \right) \quad (3)$$

and to obtain  $\left[ \sum_{i=1}^n \sum_{j=1}^m M_{gi}^j \right]^{-1}$ , perform the fuzzy addition operation of  $M_{gi}^j$  ( $j = 1, 2, \dots, m$ ) values such that:

$$\sum_{i=1}^n \sum_{j=1}^m M_{gi}^j = \left( \sum_{i=1}^n l_i, \sum_{i=1}^n m_i, \sum_{i=1}^n u_i \right) \quad (4)$$

and then compute the inverse of the vector in Eq. (4) such that:

$$\left[ \sum_{i=1}^n \sum_{j=1}^m M_{gi}^j \right]^{-1} = \left( \frac{1}{\sum_{i=1}^n u_i}, \frac{1}{\sum_{i=1}^n m_i}, \frac{1}{\sum_{i=1}^n l_i} \right) \quad (5)$$

**Step 2:** The degree of possibility of  $M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$  is defined as:

$$V(M_2 \geq M_1) = \sup [\min(\mu_{M_1}(x), \mu_{M_2}(y))] \tag{6}$$

and can be equivalently expressed as follows:

$$V(M_2 \geq M_1) = \text{hgt}(M_1 \cap M_2) = \mu_{M_2}(d) = \begin{cases} 1, & \text{if } m_2 \geq m_1 \\ 0, & \text{if } l_1 \geq u_2 \\ \frac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise} \end{cases} \tag{7}$$

where  $d$  is the ordinate of the highest intersection point  $D$  between  $\mu_{M_1}$  and  $\mu_{M_2}$  (see Fig. 3). To compare  $M_1$  and  $M_2$ , we need both the values of  $V(M_1 \geq M_2)$  and  $V(M_2 \geq M_1)$ .

**Step 3:** The degree possibility for a convex fuzzy number to be greater than  $k$  convex fuzzy numbers  $M_i (i=1, 2, \dots, k)$  can be defined by:

$$V(M \geq M_1, M_2, \dots, M_k) = \min V(M \geq M_i), \quad i = 1, 2, \dots, k \tag{8}$$

**Step 4:** Finally,  $W = (\min V(S_1 \geq S_k), \min V(S_2 \geq S_k), \dots, \min V(S_n \geq S_k))^T$  is the weight for  $k = 1, 2, \dots$ .

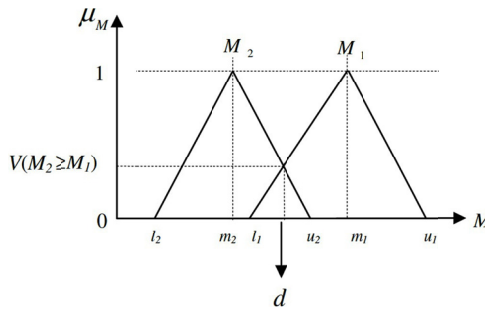


Fig. 3. The intersection between  $M_1$  and  $M_2$

### 2.3.3. Local weights of factors and sub-factors

To determine the local weights of factors and sub-factors, at first the pairwise comparison matrices should be constructed. The fuzzy scale that is used for pairwise comparison is given in Table 1 and Fig. 4. This scale is proposed by Kahraman et al. (2006) and used for solving fuzzy decision making problems in the literatures.

The pairwise comparison matrices are formed by the expert team (including mining engineers and ground control experts) based on the scale described above. The pairwise comparison matrix for the factors is presented in Table 2. Fuzzy evaluations are performed in the pairwise



TABLE 1

Linguistic scale for relative importance

Linguistic scale for importance	Triangular fuzzy scale	Triangular fuzzy reciprocal scale
Just equal	(1, 1, 1)	(1, 1, 1)
Equally important (EI)	(1/2, 1, 3/2)	(2/3, 1, 2)
Weakly more important (WMI)	(1, 3/2, 2)	(1/2, 2/3, 1)
Strongly more important (SMI)	(3/2, 2, 5/2)	(2/5, 1/2, 2/3)
Very strongly more important (VSMI)	(2, 5/2, 3)	(1/3, 2/5, 1/2)
Absolutely more important (AMI)	(5/2, 3, 7/2)	(2/7, 1/3, 2/5)

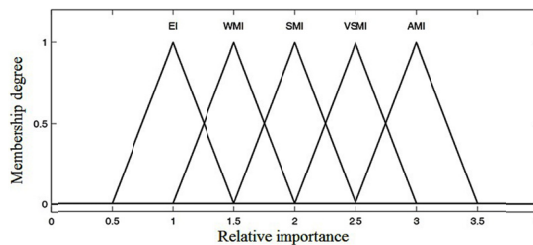


Fig. 4. Linguistic scale for relative importance

comparisons as follows: Geological factors and Operational factors are compared using the question “How important are Geological Factors (A) when it is compared with Operational Factors (C)?” and if the answer is “Strongly more important (SMI)”, for this linguistic scale the triangular fuzzy number placed in the relevant cell against it is (3/2, 2, 5/2). All the fuzzy evaluation matrices are produced in the same manner. Local weights of the factors are calculated using the fuzzy comparison values presented in Table 2 through Chang’s extent analysis method as follows:

$$S_A = (3.00, 4.00, 5.00) \otimes (1/13.17, 1/9.50, 1/7.23) \approx (0.23, 0.42, 0.69),$$

$$S_B = (2.17, 3.00, 4.50) \otimes (1/13.17, 1/9.50, 1/7.23) \approx (0.17, 0.32, 0.62),$$

$$S_C = (2.06, 2.50, 3.67) \otimes (1/13.17, 1/9.50, 1/7.23) \approx (0.16, 0.26, 0.51)$$

are obtained. Using these vectors:

$$V(S_A \geq S_B) = 1.00, V(S_A \geq S_C) = 1.00,$$

$$V(S_B \geq S_A) = 0.79, V(S_B \geq S_C) = 1.00,$$

$$V(S_C \geq S_A) = 0.64, V(S_C \geq S_B) = 0.87$$

are obtained. Thus the weight vector from Table 2 is calculated as  $W_{Factors} = (0.41, 0.33, 0.26)^T$ . The local weights for the sub-factors are calculated in a similar fashion to the fuzzy evaluation matrices, as shown above. Pairwise comparison matrices for sub-factors are given in Tables 3-5 together with the calculated local weights.

TABLE 2

Local weights and pairwise comparison matrix of factors

Factors	A	B	C	Local weight
A	(1, 1, 1)	(1/2, 1, 3/2)	(3/2, 2, 5/2)	0.41
B	(2/3, 1, 2)	(1, 1, 1)	(1/2, 1, 3/2)	0.33
C	(2/5, 1/2, 2/3)	(2/3, 1, 2)	(1, 1, 1)	0.26

TABLE 3

Local weights and pairwise comparison matrix of geological sub-factors

Geological sub-factors	A1	A2	A3	A4	A5	A6	Local weight
A1	(1, 1, 1)	(2/3, 1, 2)	(1, 3/2, 2)	(1, 3/2, 2)	(1, 3/2, 2)	(1/2, 1, 3/2)	0.19
A2	(1/2, 1, 3/2)	(1, 1, 1)	(3/2, 2, 5/2)	(3/2, 2, 5/2)	(1, 3/2, 2)	(1, 3/2, 2)	0.22
A3	(1/2, 2/3, 1)	(2/5, 1/2, 2/3)	(1, 1, 1)	(1/2, 1, 3/2)	(2/3, 1, 2)	(2/3, 1, 2)	0.14
A4	(1/2, 2/3, 1)	(2/5, 1/2, 2/3)	(2/3, 1, 2)	(1, 1, 1)	(1/2, 2/3, 1)	(1/2, 2/3, 1)	0.12
A5	(1/2, 2/3, 1)	(1/2, 2/3, 1)	(1/2, 1, 3/2)	(1, 3/2, 2)	(1, 1, 1)	(2/3, 1, 2)	0.16
A6	(2/3, 1, 2)	(1/2, 2/3, 1)	(1/2, 1, 3/2)	(1, 3/2, 2)	(1/2, 1, 3/2)	(1, 1, 1)	0.17

TABLE 4

Local weights and pairwise comparison matrix of design sub-factors

Design sub-factors	B1	B2	B3	B4	B5	Local weight
B1	(1, 1, 1)	(1/2, 1, 3/2)	(1/2, 2/3, 1)	(2/3, 1, 2)	(1/2, 2/3, 1)	0.17
B2	(2/3, 1, 2)	(1, 1, 1)	(1/3, 2/5, 1/2)	(1/2, 2/3, 1)	(2/5, 1/2, 2/3)	0.13
B3	(1, 3/2, 2)	(2, 5/2, 3)	(1, 1, 1)	(1/2, 1, 3/2)	(1/2, 1, 3/2)	0.25
B4	(1/2, 1, 3/2)	(1, 3/2, 2)	(2/3, 1, 2)	(1, 1, 1)	(2/3, 1, 2)	0.21
B5	(1, 3/2, 2)	(3/2, 2, 5/2)	(2/3, 1, 2)	(1/2, 1, 3/2)	(1, 1, 1)	0.24

TABLE 5

Local weights and pairwise comparison matrix of operational sub-factors

Operational sub-factors	C1	C2	C3	C4	Local weight
C1	(1, 1, 1)	(1/2, 2/3, 1)	(1/2, 2/3, 1)	(2/5, 1/2, 2/3)	0.16
C2	(1, 3/2, 2)	(1, 1, 1)	(1/2, 1, 3/2)	(2/3, 1, 2)	0.27
C3	(1, 3/2, 2)	(2/3, 1, 2)	(1, 1, 1)	(2/3, 1, 2)	0.28
C4	(3/2, 2, 5/2)	(1/2, 1, 3/2)	(1/2, 1, 3/2)	(1, 1, 1)	0.29

## 2.4. Step 4: Calculating the global weights of sub-factors

Using local weights of the factors and sub-factors, global weights for the sub-factors are calculated in this step. Global sub-factor weights are computed by multiplying local weight of the sub-factor with the local weight of the factor in which it belongs. Computed global weights for sub-factors are shown in Table 6. According to the global sub-factor weights, shown in Table 6,

the five most important sub-factors which can cause roof fall are roof rock quality (A2), entry width (B3), roof bolting (B5), depth of cover (A1), and final stump (C4).

TABLE 6

Computed global weights for sub-factors

Factor and local weight	Sub-factor	Local weight	Global weight
Geological factors (A) (0.41)	Depth of cover (A1)	0.19	0.08
	Roof rock quality (A2)	0.22	0.09
	Floor rock quality (A3)	0.14	0.06
	Groundwater (A4)	0.12	0.05
	Overlying massive strata (A5)	0.16	0.06
	Multiple-seam interaction (A6)	0.17	0.07
Design factors (B) (0.33)	Panel width (B1)	0.17	0.06
	Panel uniformity (B2)	0.13	0.04
	Entry width (B3)	0.25	0.08
	Pillar design (B4)	0.21	0.07
	Roof bolting (B5)	0.24	0.08
Operational factors (C) (0.26)	Panel age (C1)	0.16	0.04
	Supplemental support (C2)	0.27	0.07
	Cut sequence (C3)	0.28	0.07
	Final stump (C4)	0.29	0.08

**2.5. Step 5: Determination the linguistic variables for sub-factors**

Sub-factors based on their natures can be divided into two categories: continuous and discrete. In order to determine the linguistic variables for continuous sub-factors, the fuzzy approach is applied. To achieve this, the trapezoidal and triangular membership functions are used because of simplicity and computational efficiency. Furthermore, the discrete sub-factors are described in linguistic form using classic approach. Depth of cover, roof rock quality, floor rock quality, entry width, pillar design, roof bolting, and panel age are continuous sub-factors and their linguistic variables are indicated in Figs. 5-11, respectively. Furthermore, Table 7 shows the linguistic variables, their linguistic values and associated parameters for each continuous sub-factor. Groundwater, overlying massive strata, multiple-seam interaction, panel width, panel uniformity, supplemental support, cut sequence, and final stump are discrete parameters, which

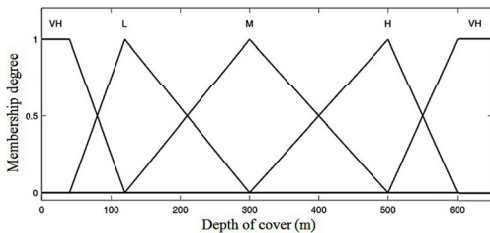


Fig. 5. Representation of linguistic variables for depth of cover

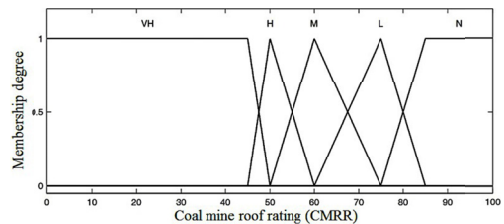


Fig. 6. Representation of linguistic variables for roof rock quality

their linguistic variables are shown in Tables 8-15. As can be seen in this stage, five linguistic variables (negligible (N), low (L), medium (M), high (H) and very high (VH)) are used and the mean of fuzzy number (FN) related with these variables are shown in Table 16.

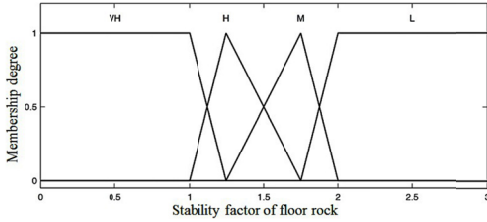


Fig. 7. Representation of linguistic variables for floor rock quality

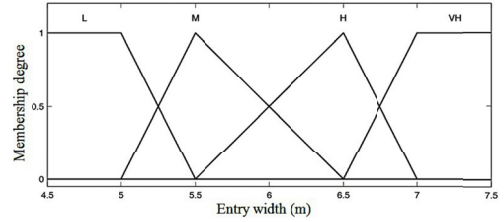


Fig. 8. Representation of linguistic variables for entry width

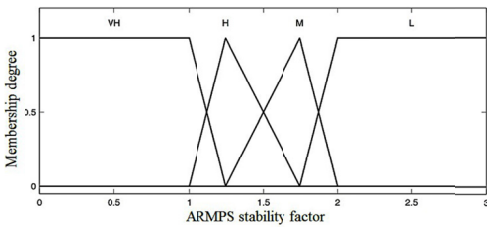


Fig. 9. Representation of linguistic variables for pillar design

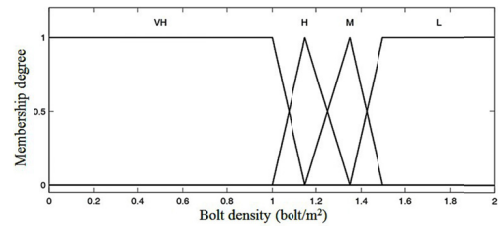


Fig. 10. Representation of linguistic variables for roof bolting

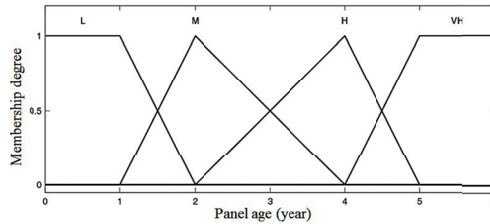


Fig. 11. Representation of linguistic variables for panel age

TABLE 7

Representation of linguistic variables and their parameters for continuous sub-factors

Sub-factor	Linguistic variable	Type of membership function	Parameters
1	2	3	4
Depth of cover	Very high	Trapezoidal	[0 0 40 120]
	Low	Triangular	[40 120 300]
	Medium	Triangular	[120 300 500]
	High	Triangular	[300 500 600]
	Very high	Trapezoidal	[500 600 650 650]

1	2	3	4
Roof rock quality	Very high	Trapezoidal	[0 0 45 50]
	High	Triangular	[45 50 60]
	Medium	Triangular	[50 60 75]
	Low	Triangular	[60 75 85]
	Negligible	Trapezoidal	[75 85 100 100]
Floor rock quality	Very high	Trapezoidal	[0 0 1 1.25]
	High	Triangular	[1 1.25 1.75]
	Medium	Triangular	[1.25 1.75 2]
	Low	Trapezoidal	[1.75 2 3 3]
Entry width	Low	Trapezoidal	[4.5 4.5 5 5.5]
	Medium	Triangular	[5 5.5 6.5]
	High	Triangular	[5.5 6.5 7]
	Very high	Trapezoidal	[6.5 7 7.5 7.5]
Pillar design	Very high	Trapezoidal	[0 0 1 1.25]
	High	Triangular	[1 1.25 1.75]
	Medium	Triangular	[1.25 1.75 2]
	Low	Trapezoidal	[1.75 2 3 3]
Roof bolting	Very high	Trapezoidal	[0 0 1 1.15]
	High	Triangular	[1 1.15 1.35]
	Medium	Triangular	[1.15 1.35 1.5]
	Low	Trapezoidal	[1.35 1.5 2 2]
Panel age	Low	Trapezoidal	[0 0 1 2]
	Medium	Triangular	[1 2 4]
	High	Triangular	[2 4 5]
	Very high	Trapezoidal	[4 5 6 6]

TABLE 8

## Representation of linguistic variables for groundwater condition

Groundwater condition	Linguistic variable
Completely dry roof	N
Damp	L
Wet	M
Dripping	H
Flowing	VH

TABLE 9

## Representation of linguistic variables for overlying massive strata

Overlying massive strata/D	Linguistic variable
Not present	N
Present/Less than 20 m	L
Present/More than 20 m	H

D – Distance from the coal seam

TABLE 10

Representation of linguistic variables for multiple-seam interaction

<b>Multiple-seam interaction/Interburden thickness</b>	<b>Linguistic variable</b>
Not present	N
Present/Less than 10 h	VH
Present/Between 10h and 24 h	H
Present/Between 24h and 60 h	M
Present/More than 60 h	L
h – Thickness of the coal seam	

TABLE 11

Representation of linguistic variables for panel type

<b>Panel type</b>	<b>Linguistic variable</b>
Sub-critical	L
Critical	M
Super-critical	H

TABLE 12

Representation of linguistic variables for panel uniformity

<b>Panel uniformity</b>	<b>Linguistic variable</b>
Uniform	L
Partly uniform	M
Non-uniform	H

TABLE 13

Representation of linguistic variables for supplemental support

<b>Supplemental support</b>	<b>Linguistic variable</b>
Mobile roof support	L
Timber post	VH

TABLE 14

Representation of linguistic variables for cut sequence

<b>Cut sequence</b>	<b>Linguistic variable</b>
Outside lift	L
Left-right	M
Other sequence	H

TABLE 15

Representation of linguistic variables for final stump

<b>Final stump</b>	<b>Linguistic variable</b>
Proper	L
Improper	VH

TABLE 16

Linguistic variables and mean of fuzzy numbers

Linguistic variable	The mean of fuzzy number (FN)
Negligible (N)	0
Low (L)	0.25
Medium (M)	0.5
High (H)	0.75
Very high (VH)	1

## 2.6. Step 6: Calculating the RFS index

The RFS index can be calculated using sub-factor global weights and linguistic values. To achieve this purpose, the following equations are applied. Based on Eq. (9), the RFS index can be calculated for each individual sub-factor, whereas the Eq. (10) calculates the RFS index based on all sub-factors.

$$RFS_i = GW_i \times \sum_{j=1}^2 (MD_j \times FN_j) \quad (9)$$

$$RFS = \sum_{i=1}^{15} RFS_i \quad (10)$$

where  $RFS_i$  and  $GW_i$  are the RFS index and global weight for the  $i^{\text{th}}$  sub-factor, respectively.  $MD$  is the membership degree (membership degree is an indication of certainty with which a sub-factor belongs to a certain linguistic variable),  $FN$  is the mean of fuzzy number and is determined based on Table 16. The parameter  $j$  can be 1 or 2, and this number shows that each sub-factor belongs to one or two linguistic variables.

In the following, two examples are presented to explain how to use these equations.

*Example 1 (continuous variable):* suppose the depth of cover is 85 meter. Based on Fig. 4, this depth of cover belongs to low linguistic variable with the membership degree of 0.56 and it belongs to very high linguistic variable with the membership degree of 0.44. Now, using the following equation, the RFS index can be calculated for the depth of cover as a sub-factor.

$$RFS_{A1} = 0.08 \times [(0.56 \times 0.25) + (0.44 \times 1)] = 0.05 \quad (11)$$

*Example 2 (discrete variable):* suppose the roof of panel is wet. Based on Table 8, this groundwater condition belongs to medium linguistic variable with the membership degree of 1. Now, using the following equation, the RFS index can be calculated for the groundwater sub-factor.

$$RFS_{A4} = 0.05 \times (1 \times 0.5) = 0.03 \quad (12)$$

## 2.7. Step 7: Assessing the RFS index

The value of RFS index is between 0 and 1. When this value approaches 0, the roof fall risk is negligible and when the RFS value approaches 1, the roof fall susceptibility increases. In order to assess the RFS index more accurately, an upper limit (UL) and a lower limit (LM) are determined for the RFS index according to the structure of proposed model. The upper limit and lower limit are identified as 0.75 and 0.25, respectively. Computed RFS index from previous step is compared to the upper and lower limits. Depending on the comparison results, the following decisions are made:

- If  $RFS \geq 0.75$ , then the retreat mining should be stopped, because roof fall risk is high.
- If  $0.25 \leq RFS < 0.75$ , the controlling measures are needed to ensure safe retreat mining.

It should be noted that amongst roof fall susceptibility factors, geological factors cannot be changed and are uncontrollable. Design parameters are controllable but these parameters should be considered in design stage of mine. Operational parameters are also controllable and good selection of these parameters prior to retreat mining results in reduction of roof fall risk. Thus, the most practical measures to reduce the RFS index are proper selection of design factors (in designing stage of mine) and operational factors (prior to retreat mining).

- If  $RFS < 0.25$ , then the retreat mining can be done safely.

## 3. A practical application of proposed model

The proposed model of evaluating RFS is put into practice in the main panel of Tabas Central Mine (TCM). TCM is the only room and pillar coal mine in Iran which is located in Parvadeh 1 region in Tabas coalfield. This mine is placed in a desert area approximately 85 km south of Tabas town in Yazd province in the mid-eastern part of Iran. TCM is the first mechanized room and pillar mine in Iran whose reserves are 6 million tons of coking coal. The detailed information about TCM is available in the literatures (Ghasemi et al., 2010, 2012), but a summary of essential data for RFS calculation in main panel of TCM is shown in Table 17. As can be seen, three sub-factors that is supplemental support, cut sequence and final stump are unknown because the retreat mining has not been done, yet. According to Tables 13-15, each of these sub-factors has 2, 3, 2 subcategories, respectively. As a result, there are 12 various scenarios for implementation of retreat mining in this panel. The RFS value for each scenario is calculated using Eqs. (9) and (10) and is presented in Table 18. For example, for the scenario number 1 and 12, the RFS is computed on the basis of Eqs. (13) and (14):

$$\begin{aligned}
 RFS_{S1} = & 0.08 \times [(0.56 \times 0.25) + (0.44 \times 1)] + 0.09 \times (1 \times 1) + \\
 & + 0.06 \times [(0.84 \times 0.75) + (0.16 \times 1)] + 0.05 \times (1 \times 0.5) + 0.06 \times (1 \times 0) + \\
 & + 0.07 \times (1 \times 0) + 0.06 \times (1 \times 0.75) + 0.04 \times (1 \times 0.25) + 0.08 \times (1 \times 0.25) + \\
 & + 0.07 \times (1 \times 0.25) + 0.08 \times [(0.73 \times 0.75) + (0.27 \times 1)] + 0.04 \times (1 \times 1) + \\
 & + 0.07 \times (1 \times 0.25) + 0.07 \times (1 \times 0.25) + 0.08 \times (1 \times 0.25) = 0.46
 \end{aligned} \tag{13}$$



$$\begin{aligned}
RFS_{S12} = & 0.08 \times [(0.56 \times 0.25) + (0.44 \times 1)] + 0.09 \times (1 \times 1) + \\
& + 0.06 \times [(0.84 \times 0.75) + (0.16 \times 1)] + 0.05 \times (1 \times 0.5) + 0.06 \times (1 \times 0) + \\
& + 0.07 \times (1 \times 0) + 0.06 \times (1 \times 0.75) + 0.04 \times (1 \times 0.25) + 0.08 \times (1 \times 0.25) + \\
& + 0.07 \times (1 \times 0.25) + 0.08 \times [(0.73 \times 0.75) + (0.27 \times 1)] + 0.04 \times (1 \times 1) + \\
& + 0.07 \times (1 \times 1) + 0.07 \times (1 \times 0.75) + 0.08 \times (1 \times 1) = 0.61
\end{aligned}
\tag{14}$$

TABLE 17

Essential data for RFS calculation in main panel of TCM

Sub-factor	Value
Depth of cover	85 m
Coal mine roof rating (CMRR)	37
Stability factor of floor rock	1.21
Groundwater condition	Wet roof
Overlying massive strata	Not present
Multiple-seam interaction	Not present
Panel type	Super-critical
Panel uniformity	Uniform
Entry width	4.5 m
ARMPS stability factor	3.15
Bolt density	1.11 bolt/m <sup>2</sup>
Panel age	More than 5 years
Supplemental support	Unknown
Cut sequence	Unknown
Final stump	Unknown

TABLE 18

RFS index for various scenarios of retreat mining in main panel of TCM

Scenario No.	Supplemental support	Cut sequence	Final stump	RFS
S1	Mobile roof support	Outside lift	Proper	0.46
S2	Mobile roof support	Left-right	Proper	0.48
S3	Mobile roof support	Other sequence	Proper	0.50
S4	Mobile roof support	Outside lift	Improper	0.52
S5	Mobile roof support	Left-right	Improper	0.54
S6	Mobile roof support	Other sequence	Improper	0.56
S7	Timber post	Outside lift	Proper	0.51
S8	Timber post	Left-right	Proper	0.53
S9	Timber post	Other sequence	Proper	0.55
S10	Timber post	Outside lift	Improper	0.57
S11	Timber post	Left-right	Improper	0.59
S12	Timber post	Other sequence	Improper	0.61

As can be seen, the RFS value for all scenarios is between upper limit (UL) and lower limit (LL), so controlling measures are needed to ensure safe retreat mining. Based on panel condi-

tions and investigating sub-factors, the most important controlling measures that can be done are as follows:

1. Installation of new roof bolts prior to retreat mining especially in intersections because the old age of panel reduces the performance of roof and installed roof bolts.
2. Leaving final stump with proper size because of poor roof quality and super-critical width of panel. To find out the proper size of final stump, there are guidelines in the literatures based on detailed rock mechanic analysis of retreat mining experience (Mark & Zelanko, 2001).
3. Using mobile roof support (MRS) as supplemental support during retreat mining. Nowadays, using MRS is recommended strongly because using timber posts as pillar line supports has many disadvantages and the most important is that timber posts are passive supports and roof convergence would be small (Chase et al., 1997). Statistics in US coal mines showed that a miner on a timber panel is exposed to fatality 1.7 times more than a miner protected by MRSs (Mark & Zelanko, 2005). Furthermore, field observation revealed that the MRS reduces the roof-floor convergence (Maleki, 2008).
4. Pillar extraction using left-right method. In general, outside lift is used when the width of pillars is 10 m or less, and left-right methods are used when the pillars are too wide to be extracted completely from one side. As the width of pillars in main panel of TCM is 15.5 m, the left-right cut sequence is recommended.
5. Assessing the moisture sensitivity of roof rocks because the roof of main panel of TCM in the worst condition is wet. Moisture sensitivity of roof rocks can cause high numbers of roof falls in coal mines. If moisture sensitivity is detected, there are several engineering controls which can aid in the safe recovery of the coal. These controls include screen, sealants, increased support density, leaving top coal, removing moisture sensitive roof rock, narrower entries, shorter panel life, rib meshing, and conditioning the air (Klemetti & Molinda, 2009).

TCM managers and engineers can apply these precautions measures to reduce the susceptibility of roof fall occurrence during retreat mining and improve the safety of this operation.

## 4. Conclusions

The application of risk assessment classic approach may not give satisfactory results where high level of subjective uncertainty exists in the risk assessment process. It is therefore essential to develop new risk assessment methods where classic methods cannot be efficiently applied. To handle these uncertainties successfully, the fuzzy approach is a versatile and efficient tool. Therefore, this paper presents a novel model to evaluate the roof fall susceptibility during retreat mining using risk assessment fuzzy approach. The model provides a simple and effective mechanism for modeling risk assessment problems involving subjective uncertainties. This model is based on determining the most important factors and sub-factors that may cause roof fall. In this study, fuzzy AHP method was used to determine the importance degree of factors and sub-factors in the model. Chang's extent analysis method used in this paper has proved to be simpler, less time consuming and having less computational expense compared to other existing fuzzy AHP methods. To illustrate how the approach works, a problem on roof fall susceptibility assessment in main panel of TCM has been presented. The developed methodology is applicable to the general fuzzy risk assessment problem where a ranking of risks is required.

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