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A New Envelope Computational Method for Meshing Heliacal Gears Profiling Applied to Air Compressor Screw Pair

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ABSTRACT

The theory of enveloping, which determines the envelope to a family of surfaces, the meshing equations of kinematics pairs, including pinion and gear, cutting toll and gear, is widely used in Mechanical Engineering. This paper presents a new envelope computational method for profiling the meshing gear pair. It uses the normal projection of the instantaneous relative rotation axis of the kinematic pair onto the pinion surface to generate the contact line and then automatically computes geometric data of this contact line to create the gear tooth surface. As the profile complexity, the reverse engineering of compressor screw pair was a typical proper example to verify and clarify the proposed method. The method can generate the meshing surface pair with high accuracy: the 3D comparison average error of the surfaces generated by the proposed method and the Boolean method is 0.004 mm, and their RMS error is 0.01 mm. The novel idea of the proposed method is that the contact line, which is used to calculate gear surface, is created easier than solving complex meshing equations, which most previous work used. The proposed method in detail with an algorithm can be used as well for reverse engineering of air compressor screw pair as for parallel axixes heliacal gear pair.

Keywords: gear profile, compressor screw, contact line, meshing condition, envelope theory.

INTRODUCTION

The theory of enveloping, which determines the envelope to a family of surfaces, is widely used in mechanical engineering when dealing with the following problems:

- Given the tooth surface (or tooth profile) of the driving gear and the relative movement of the meshing gear pair, calculate the tooth surface (or tooth profile) of the driven gear;
- Given the gear tooth surface and the relative movement between the gear and its envelope cutting tool, calculate the primary peripheral surface of the cutting toll, such as a formed hobbing tool or disc tool;
- Calculate the contact path of the pinion and gear.

Livin [1] and Stosic [2] have published general theoretical studies on determining the envelope to a family of surfaces by giving the meshing equations of kinematics pairs, including pinion and gear, cutting toll and gear, and suggested algorithms for solving these equations. Many other specific research works in the field proposed new methods and algorithms in particular studies. These works should be regarded in the following categories:

Generating new gear profile and/or analyzing the contact path

Liu et al. [3] proposed a design method for generating tooth profiles, which control the relative curvature of meshing tooth profiles to enhance the contact strength of tooth surfaces and improve the lubrication of tooth surfaces. Wang et al. [4] suggested a method for designing gear tooth profiles using a parabolic curve instead of a straight line as a line of action. The method is more flexible in controlling the shape of the tooth profile by changing the parameters of the parabola to minimize the teeth number of the proposed gear without undercutting to lower contact and bending stress than those of the involute gear drive. Cao et al. [5] proposed a method for the reverse design of screw rotor profiles based on a B-spline curve. The meshing line is generated as the B-spline curve, and then female and male rotor profiles are specified reversely. The modified female and male rotor profiles have better gas compression performance than their original profiles. Hoang et al. [6] focus on gate screw profiling from the main screw profile and disc tool profiling for manufacturing the screw pair. The suggested method is based on the Boolean operation in 3D CAD, so it does not require representing the given data as mathematical equations and solving these complicated equations. Stosic et al. [7] described the synthesis of gear profiles using 3D CAD software. They called the procedure Direct Digital Simulation, or DDS. Cao et al. [8] proposed a new method to gear tooth contact analysis. The method overcomes the two disadvantages of the traditional algorithm: numerical instability due to solving a system of five nonlinear equations to search for each contact point and the complex computation coming from differences between tooth surface contact and edge contact. Petrescu and Petrescu [9] presented a method for determining gear transmission efficiency and analyzed the effect of parameters on gear drive efficiency as well as the contact ratio. Miltenović et al. [10, 11] focused on the wear load capacity and temperature distribution in crossed helical gears as well as in a worm gear transmission, which leads to the most common damage since their point contact difference from line contact in other gear types.

Enveloping cutting tools

Veliko, Genycho [12], and Mohan [13, 14] presented a 3D CAD-based envelope method using the Boolean operator to design rotation tools machining helical surfaces. These methods are universal for every complex profile of the helical surfaces. Stoic [15] used the gearing envelope theory in the case of crossed helical gears to calculate the stock distribution to gain the uniform wear of a disc tool when grinding screw compressor rotors. By the same method, Stoic [16] quantifies the tool setting errors, including the mismatch of the angle formed by the tool and rotor shafts, the centre distance between the rotor and their tool, and the axial tool position, to compensate for tool deformation and wear. Berbinschi et al. [17] proposed an envelope method based on the CATIA graphical platform for profiling rotation tools generating constant pitch cylindrical helical surfaces.

The meshing equations of kinematics pairs, including pinion and gear or cutting toll and generated surface, can be solved by purely analytical methods [1, 2], numerically analytical methods [3, 4, 5, 8, 15, 16], 3D CAD Boolean operator method [6, 12, 13, 14]. Each method has specific advantages and disadvantages and is suitable for a particular purpose. The purely analytical methods are the fundament of the other one and give precise solutions. Still, they are complex and more difficult in the case of undercutting or singular points. Besides, the meshing equations can only be solved accurately in some exceptional cases and require a surface representation through a system of equations, which would be inconvenient with reverse engineering. The numerically analytical methods come out only from the purely analytical methods, which solve approximately a system of five nonlinear meshing equations, so it has some disadvantages similar to the original analysis method. The Boolean method comes from a novel idea, which simulates the actual envelope process like the material cutting process, so the surface is not required to be regular and without any singular points. Still, the method takes a long time to perform and cannot be directly used for reverse engineering because the object is not solid.

Based on the state of the previous envelope method mentioned above, avoiding the very complex meshing equations, improving accuracy, and being suitable for reverse engineering of the air compressor screw pair, this work aims to create a new computational method for the envelope problem: given the tooth surface of the driving helical gear and the relative movement of the meshing helical gear pair, calculate the tooth surface of the driven gear. Then, the proposed method is applied to the air compressor screw pair to verify the correctness of the proposed method as well as to clarify the application of the method.

TRADITIONAL ENVELOPE METHOD AS A BASIS FOR MESHING GEAR PROFILING

The meshing condition of a kinematic pair, including pinion and gear pair and gear and cutting tool pair, is derived from the envelope concept to a family of surfaces. Litvin [1] generally denotes Σ_1 and Σ_2 for the generating and generated surfaces, respectively. The coordinate systems S_1 and S_2 are attached to Σ_1 and Σ_2, respectively. Σ_1 is represented in vector function form $r_1(u, \theta)$. The surfaces family Σ_1 is represented in S_2 in the vector function form $r_2(u, \theta, \tau)$, where τ is the motion parameter. The Σ_1 and Σ_2 must have a common tangent plane that leads to the meshing Equation below

$$\left(\frac{\partial r_2}{\partial u}\times\frac{\partial r_2}{\partial \theta}\right)\cdot\frac{\partial r_2}{\partial \tau}=0 \tag{1}$$

The meshing equation of the helical gear pair (Fig. 1) is now analyzed. The coordinate systems $x_1y_1z_1$ and $x_2y_2z_2$ are fixed to the bearings of gear 1 and gear 2, respectively. Σ is the angle formed by the two bearing axes denoted as z_1 and z_2. The issue starts with a given generating surface $r_1(u,\theta)$ of gear 1, from which the generated surface r_2 of gear 2 is determined later. Some Eq. below is used for specifying the tangent plane at a point on the surface $r_1(u, \theta)$ [2]:

$$r_1 = r_1(u,\theta) = [x_1, y_1, z_1]$$

$$r_1 = [x_{o1}\cos\theta - y_{o1}\sin\theta, x_{o1}\sin\theta + y_{o1}\cos\theta,\ p_1\theta] \tag{2}$$

$$\frac{\partial r_1}{\partial u} = \left[\frac{\partial x_1}{\partial u}, \frac{\partial y_1}{\partial u}, 0\right] =$$
$$= \left[\frac{\partial x_{01}}{\partial u}\cos\theta - \frac{\partial y_{01}}{\partial u}\sin\theta, \frac{\partial x_{01}}{\partial u}\sin\theta + \frac{\partial y_{01}}{\partial u}\cos\theta, 0\right] \tag{3}$$

$$\frac{\partial r_1}{\partial \theta} = \left[\frac{\partial x_1}{\partial \theta}, \frac{\partial y_1}{\partial \theta}, p_1\right] = [-y_1, x_1, p_1] \tag{4}$$

where: p_1 is the helical surface r_1 lead given for the unit rotation angle; u and θ are the surface parameters; x_{01} and y_{01} are the coordinates of the points on the gear end cross-section. It is similar to analyzing the generated surface $r_2(\rho, \tau)$ as follows.

$$r_2 = r_2(\rho,\tau) = [x_2, y_2, z_2] =$$
$$= [x_1 - C, y_1\cos\Sigma - z_1\sin\Sigma, y_1\sin\Sigma + z_1\cos\Sigma] = \tag{5}$$
$$= [x_{02}\cos\tau - y_{02}\sin\tau, x_{02}\sin\tau + y_{02}\cos\tau,\ p_2\tau]$$

$$\frac{\partial r_2}{\partial \tau} = \left[\frac{\partial x_2}{\partial \tau}, \frac{\partial y_2}{\partial \tau}, p_2\right] = [-y_2, x_2, p_2] =$$
$$= [p_1\theta\sin\Sigma - y_1\cos\Sigma + C, p_2\sin\Sigma + \tag{6}$$
$$(x_1 - C)\cos\Sigma, p_2\cos\Sigma + (x_1 - C)\sin\Sigma]$$

The surfaces r_2 and r_1 must have a common tangent plane at the common point, which leads to Equation 7 below:

$$\left(\frac{\partial r_{\ddot{u}}}{\partial u}\times\frac{\partial r}{\partial \theta}\right).\frac{\partial r}{\partial \tau}=0 \tag{7}$$

Insert Equation 3, 4, and 6 into Equation 7 leads to Equation 8 below:

$$\left[C - x_1 + (p_1 - p_2)cot\,\Sigma\right]\left(x_1\frac{\partial x_1}{\partial u} + y_1\frac{\partial y_1}{\partial u}\right) +$$
$$+ p_1\left[p_1\theta\frac{\partial y_1}{\partial u} + (p_2 - C\cos\Sigma)\frac{\partial x_1}{\partial u}\right] = 0 \tag{8}$$

The meshing condition of the compressor screw pair is exceptional in cases of helical gears [2]. The problem of specifying air compressor screw profiles is solved by placing $\Sigma = 0$ into Equation 8. The resulting meshing equation, in this case, becomes Equation 9 below [2]:

$$\frac{\partial y_{01}}{\partial x_{01}}\left(ky_{01} - \frac{C}{i}\sin\theta\right) + kx_{01} + \frac{C}{i}\cos\theta = 0 \tag{9}$$

where: $i = p_2/p_1$; $k = 1\text{-}1/i$.

At the start of the rotor profiling process, the profile point coordinates in the transverse plane of one rotor, x_{01}, $y_{01,}$ and their first derivatives must be known. Equation 8 and 9 can be solved only numerically by the algorithms suggested in [2].

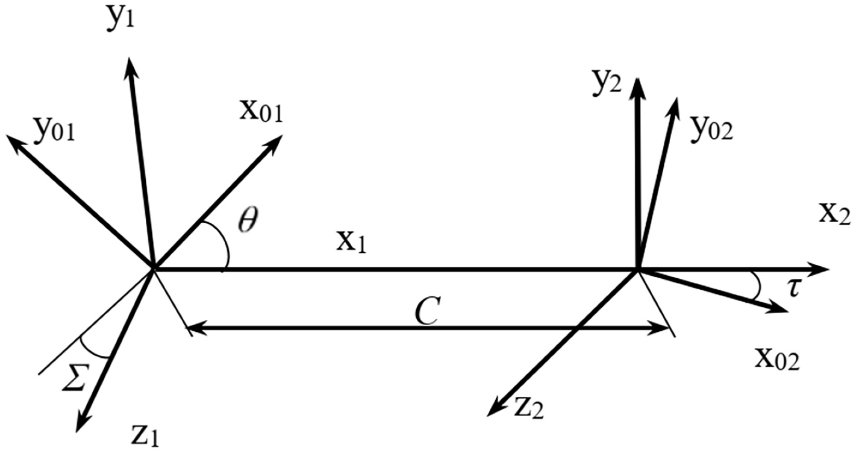

Figure 1. Coordinate systems of helical gears [2]

A NEW PROPOSED ENVELOPE COMPUTATIONAL METHOD FOR MESHING GEAR PROFILING

Now we analyze Equation 1, in which $\partial r_1/\partial u$ and $\partial r_1/\partial \theta$ represent two tangent lines of the common tangent plane; $\partial r_2/\partial \tau$ represents relative velocity at a contact point, which leads to another form of the meshing condition below.

$$N.V = 0 \qquad (10)$$

where: $N = (\partial r_1/\partial u \times \partial r_1/\partial \theta)$ represents a normal vector to surface r_1, and $V = \partial r_1/\partial \tau$ represents a relative velocity vector of the points on the surfaces r_1 with respect to the surfaces r_2.

In the case of two parallel axises gear pair, such as compressor screw pair, an instantaneous axis of relative rotation is determined, so the contact line containing contact points is the normal projection of the instantaneous rotation axis onto the surface r_1. Such contact lines can be created in the CATIA platform by projection command (see Fig. 3). Import the generating surface with the contact line into AutoCAD and run a subroutine written in Autolisp following the algorithm shown in Fig. 2. In which the geometry data structure of the contact line is a list following the DXF code, which can be accessed by the AutoLISP commands, as shown in the text below.

((-1. < Entity name: 22b75950af0 >) (0. “SPLINE”) (330. < Entity name: 22b75951f00 >) (5. “117”) (100. “AcDbEntity”) (67. 0) (410. “Model”) (8. “10000”) (62. 0) (100. “AcDbSpline”) (70. 0) (71. 3) (72. 8) (73. 4) (74. 0) (42. 1.0e-09) (43. 1.0e-10) (40. 0.0) (40. 0.0) (40. 0.0) (40. 0.0) (40. 12.5563) (40. 12.5563) (40. 12.5563) (40. 12.5563) (10 -40.7184 19.1584 30.5951) (10 -41.8126 17.8388 31.9067) (10 -43.1055 16.6727 33.2498) (10 -44.563 15.6946 34.6461))…

The subroutine following the algorithm shown in Fig. 2 picks up every point on the contact line to calculate the x_{02} and y_{02} coordinates, based on Equation 5, to create the cross-end section profile of the surface r_2.

EXPERIMENTAL VALIDATION RESULT AND DISCUSSION

As the profile complexity, the compressor screw pair was a typical proper example to verify and clarify the proposed method.

The air compressor Airman PDS50 main rotor 3D model generated by reverse engineering using the Nikon MMDx100 handheld laser scanner with an accuracy of 0.01 mm was imported into the CATIA. The instantaneous axis of the screw pair’s relative rotation, which is determined by the transmission parameter of the screw pair and their axis distance, is projected onto the main rotor surface by the normal projection command to create the contact line, as shown in Fig. 3.

In AutoCAD, the subroutine following the algorithm picks up every point on the contact line to calculate the x_{02} and y_{02} coordinates to create the cross-end section profile of the gate rotor surface, as shown in Fig. 4.

The gate rotor surface generated by the proposed method is compared with the one generated by the Boolean method [6] to verify the reliability of the proposed method. The two 3D surfaces are imported into the Geomagic Control X, as shown in Fig. 5.

After aligning them in the best fit, the 3D surface comparison deviation is shown in Fig 6. The average deviation is -0.0042, and the root-mean-square deviation is 0.01 mm.

The 3D point comparison is shown in Fig. 7. The gap distance normal to the surface between the nearest point on two surfaces is shown for each selected point. The 3D point comparison is also detailed in Tab. 1, which contains the coordinates of the reference points and measured points and their distance. The deviation above is dependent not only on the accuracy of the proposed method but also mainly on the accuracy of the Boolean method, which is not absolutely accurate naturally, while the proposed method is based on the meshing equations, so it is absolutely accurate.

In principle, the air compressor’s gate rotor surface can also be used as the input to determine the main rotor surface; however, due to its concave surface, its scan data are less accurate than the scan data of the main rotor. Using the Nikon MMDx100 handheld laser scanner, the accuracy of the main rotor surface is 0,01 mm. Still, the meshing condition of the main and gate rotor generated by the proposed method is met nearly perfectly (the average error is 0.004 mm, as shown in Fig. 6). The novel idea of the proposed method is that the contact line, which is used to calculate gear surface, is created easier than solving complex meshing equations which most previous work used. The limitation of the proposed method is that it uses not only one CAD package (CATIA, AutoCAD with Autolisp programming).

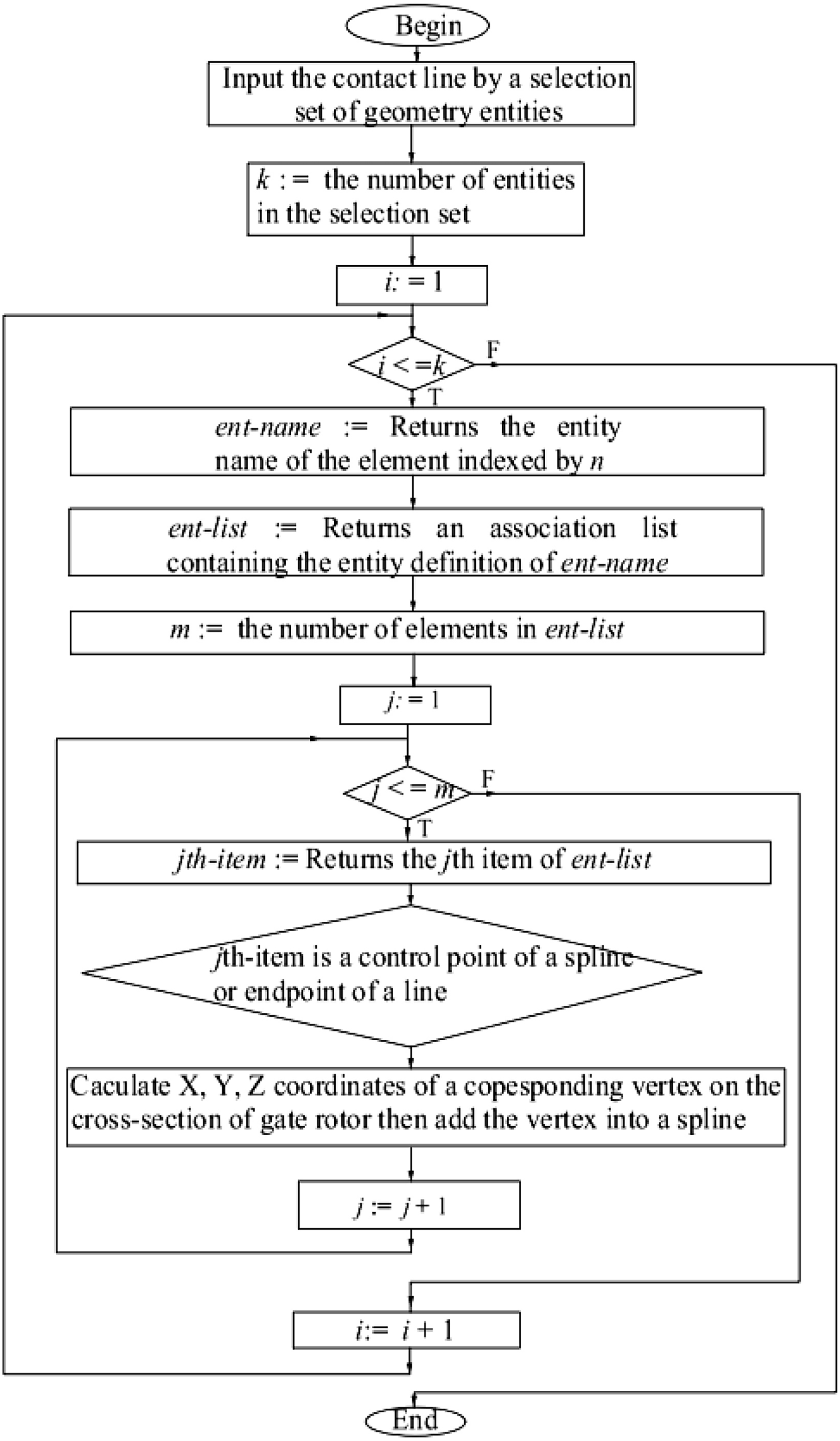

Figure 2. Algorithm of the Autolisp subroutine creating cross-section profile of the generated gear

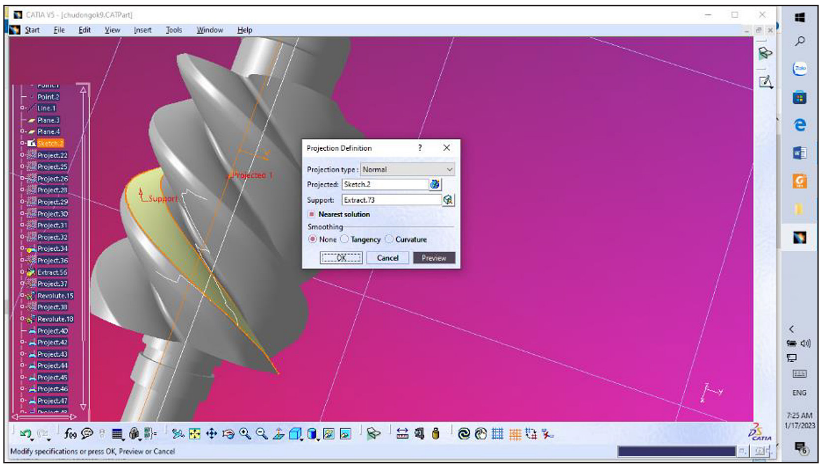

Figure 3. The contact line is created by the projection command in the CATIA

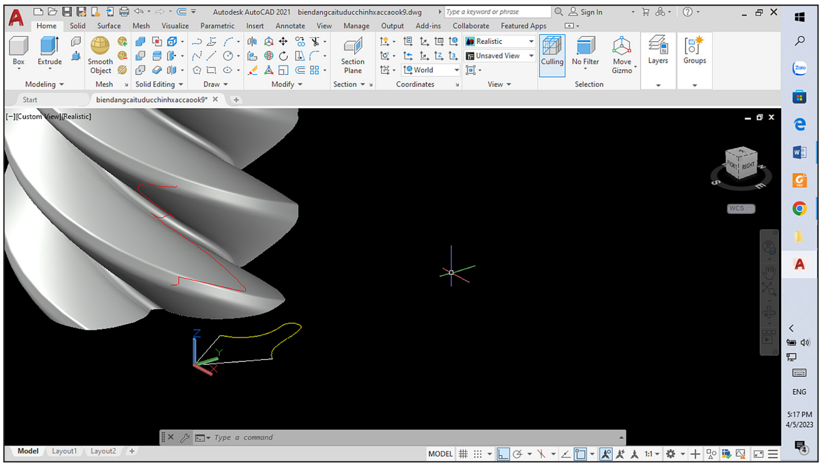

Figure 4. The gate rotor profile was created automatically

Table 1. 3D point comparison of two gate rotor surfaces generated by the two methods (units in mm)

Name	Reference pos.			Measured pos.			Gap dist.	Tolerance
	X	Y	Z	X	Y	Z		
CMP1: 1	3.3009	31.6207	19.9998	3.291	31.6159	19.9909	-0.0142	±0.3
CMP1: 2	11.961	32.5647	44.9999	11.9518	32.564	44.9912	-0.0127	±0.3
CMP1: 3	19.9997	42.8024	37.2985	20.003	42.8045	37.3014	0.0048	±0.3
CMP1: 4	-4.9993	37.3232	60.0008	-5.001	37.3223	59.9988	-0.0027	±0.3
CMP1: 5	32.0875	27.0656	59.9998	32.0863	27.0661	59.9983	-0.002	±0.3
CMP1: 6	28.2042	20.1988	59.9998	28.1962	20.2033	59.9911	-0.0127	±0.3
CMP1: 7	27.4483	10.3119	54.9998	27.4329	10.3207	54.9879	-0.0213	±0.3
CMP1: 8	26.0202	4.8441	54.9997	26.0115	4.8452	54.9976	-0.0089	±0.3
CMP1: 9	26.0571	-0.2008	55	26.0522	-0.2008	55.0001	-0.0049	±0.3
CMP1: 10	26.1657	-9.7445	50.0008	26.1607	-9.7462	50.0036	-0.006	±0.3
CMP1: 11	29.051	-2.9885	60.0006	29.0482	-2.9915	60.0035	-0.0051	±0.3
CMP1: 12	34.8758	-6.464	60	34.8735	-6.472	60.0088	-0.0121	±0.3
CMP1: 13	44.5971	-7.5006	60.0001	44.5972	-7.5004	59.9998	0.0004	±0.3
CMP1: 14	30	35.8661	63.8705	30.0029	35.8659	63.8738	0.0045	±0.3
Min.	-4.9993	-9.7445	19.9998	-5.001	-9.7462	19.9909	-0.0213	
Max.	44.5971	42.8024	63.8705	44.5972	42.8045	63.8738	0.0048	

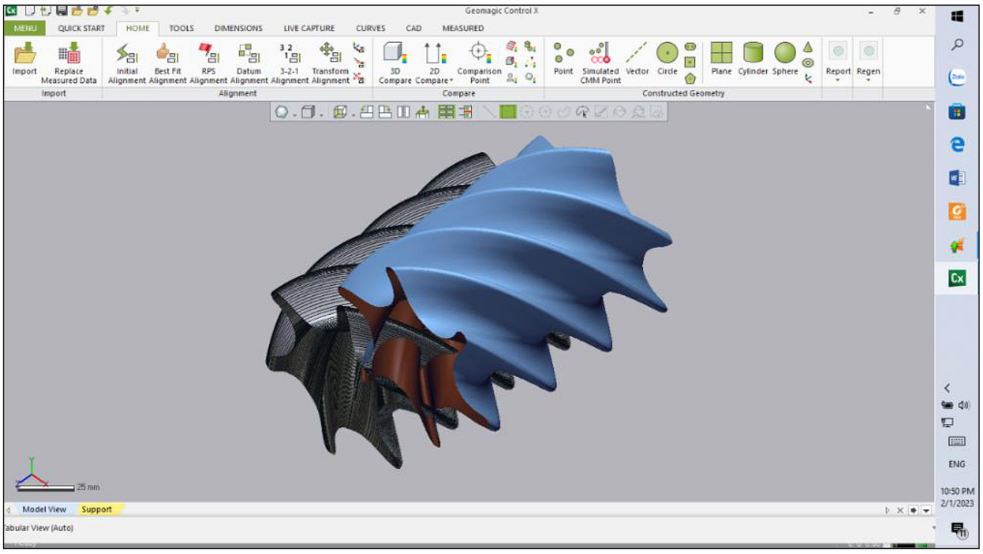

Figure 5. Two surfaces generated by the two methods are imported into the Geomagic.

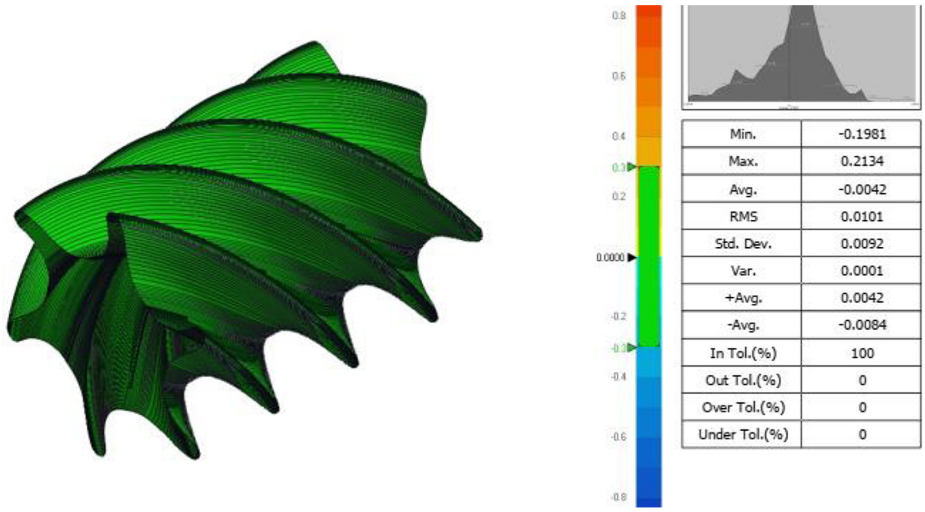

Figure 6. 3D surface comparison of two gate rotor surfaces generated by the two methods

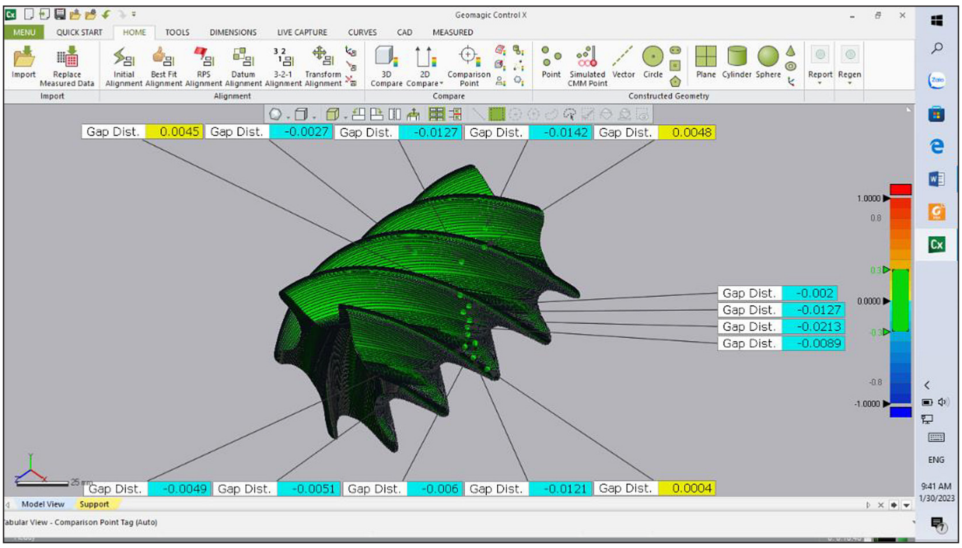

Figure 7. 3D point comparison of two gate rotor surfaces generated by the two methods

CONCLUSIONS

This paper has presented a new envelope computational method based on the traditional envelope theory for profiling the meshing gear pair. It uses the normal projection of the instantaneous relative rotation axis of the kinematic pair onto the pinion surface to generate the contact line and then automatically computes geometric data of this contact line to create the gear surface. The novel idea of the proposed method is that the contact line, which is used to calculate the gear surface automatically, is created easier than solving complex meshing equations, which most previous work has used. The advantages of the proposed method are as follows:

- It does not require generating surface representation through equations,
- It is suitable for the given generating surface created by the 3D scanning process,
- It is not difficult to implement and can quickly generate the meshing surface pair with high accuracy. The meshing condition of the main and gate rotor generated by the proposed method is met nearly perfectly (the average error is 0.004 mm).

The method is universal, so it could be used well for the meshing helical gear pairs as well as for other kinematic pairs, such as the cutting tool and gear. The method can also be used for creating new profiles of meshing gear pair or compressor screw pair, which we will address in the future.

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