Prof. Dr.Sc.,PhD.,Eng. Viktar Tur
Brest State Technical University, Republic of Belarus

Bialystok University of Technology, Poland

PhD., M.Sc.,Eng. Vitali Nadolski
Belarusian National Technical University, Republic of Belarus

Belarusian national annex to eurocode 3: basic variables formulation for the partial factors calibration

1. Introduction

The reliability concept for the building structures within the actual codes is based on Limit States Design approach. In accordance with 1.5.2.12 EN 1990[1] the limit state is defined: "*states beyond which the structure no longer fulfils the relevant design criteria*". The following methods are used for verification of the limit states: probabilistic method; semiprobabilistic method, i.e. so-called *partial factor method*; designing supported by testing [1]. Use of the probabilistic calculation methods in the designing is restrained due to complexity of their implementation. Therefore, the partial factor method (semiprobabilistic method) has been widely practised.

In the partial factor method the variability and uncertainty of the design models of the resistance, action effects and basic variables included in these models are taken into account by means of the partial factor system applicable to the characteristic values of the basic variables. The partial factor system is one of the tools for differentiation and assurance of the target structural reliability levels; therefore, the justification of their values with due account to the specific geographic, social and economic conditions is a top-priority objective for every state.

The most advanced method of determination of the partial factors values is calibration method with application of the probabilistic methods on the basis of the target (required) reliability level. The general recommendations for calibration have the status of statutory requirements in ISO 2394 [2] and EN 1990 [1] standards and reflected in the JCSS recommendations [3]. The procedure of calibration of the partial factors is described in the following publications [4-7].

In general case, the calibration of the partial factors values is based on formulation of the probabilistic performance (limit state) function $g(\boldsymbol{X})$ and subsequent calculation of the conditional probability of exceeding of the limit state by one of the reliability theory methods. To verify the ultimate limit states of the steel elements, the state function $g(\boldsymbol{X})$ characterising the safety factor of a structural element can be written in general terms:

$$g(\boldsymbol{X}) = R - E = K_R\, z\, f_y - K_E\,[G + C_{0,Q1}\, Q_1 + C_{0,Q2}\, Q_2]$$

where:

K_R – the basic variable describing the uncertainty of the resistance model ;

z – the basic variable describing the geometric parameters of the cross section of the element (area, section modulus);

f_y – the basic variable describing the steel yield strength;

K_E – the basic variable describing the uncertainty of the action effect model;

G – the basic variable describing the permanent load;

$C_{0,Qi}$ – the basic variable describing the uncertainty of the action model of the i-th action;

Q_i – the basic variable describing the i-th variable action.

Given the probabilistic models of the basic variables $\boldsymbol{X}$, the failure probability for the basic period of time can be determined by the reliability theory methods. The calculated values of the failure probability are compared with the target reliability level. Should the result be unsatisfactory, a new set of partial factors values shall be established and the calculation shall be repeated until the moment of achievement of the reliability level.

The present investigation is devoted to the probabilistic descriptions of the basic variables for the verification of Ultimate Limit States (ULS) of steel elements in accordance with Eurocode 3 for condition of the Republic of Belarus.

2. Probabilistic Models of the Basis Variables

The initial data for the probabilistic calculation is the information on the basic variables used in the performance (limit state) function, therefore the accuracy and adequacy of the probabilistic models of the variables exert predominating influence on the calculation results. This circumstance predetermines the necessity of the systemic investigations of the statistical parameters of the basic variables and formation of the unified principles of their assignment.

A special place is held by the matter of establishment of the probability distribution law for the basic variable. Usually, the distribution law is established on the basis of the statistical analysis of the available experimental data. In the construction industry, the availability of experimental data is limited making it impossible to obtain the statistically valid results. Therefore, theoretical preconditions are often used when choosing the distribution law. It should be noted that there is a general problem of use in the reliability theory of any of reliability random value distribution laws in the range of very low probability values, i.e. outside the range where the applicability of the law was experimentally justified and its parameters were determined. The general recommendations for assignment of the distribution laws to the basic variables have statutory form in the documents [1-3].

The probabilistic models of the basic variables adopted in various investigations differ often from each other. The reliability investigations based on different probabilistic models can lead to different results and, as a consequence, to different values of the partial factors, combination factors and other parameters ensuring the achievement of the target reliability levels. It is important to take into account that the calibrated values of the reliability parameters belong to a specific set of probabilistic models of the basic variables included in the models of resistance and action effects. As noted in ISO 2394 standard [2] *“The use of calibrated values jointly with other models can cause the unintended high or low reliability levels*”.

A special kind of the basic variable represents the combined effect of several actions. In this article, these matters are not considered.

3. Basic Variable for the Resistance Models of Steel Structures

Strength characteristics of steel

The sources of variability of the statistical parameters of the strength and deformation properties of steel are variations in chemical composition, differences in the manufacturing technologies, quality control methods, testing procedure, sample size, etc. To perform the full-fledge and adequate refinement (correction) of the actual laws of distribution of the strength and deformation properties of steel, it is necessary to have the actual experimental data for different steel grades, different kinds and thickness of the rolled steel products. Such works require engagement of a wide range of scientific and production organisations. Carrying out of such investigations in the territory of the Republic of Belarus is complicated by the fact that the rolled steel products is mainly delivered by foreign manufacturers. In the circumstances concerned, it seems reasonable to assess the statistical parameters of the steel properties with due account for variability of the rolled steel products properties for different conditions of its delivery on the basis of the results of the today's investigations performed abroad. To solve this problem, the article [8] summarises the results of the investigation of the yield strength variability performed at different times in different countries. On the basis of these data, it is recommended to use the mean value of the ratio of the actual value of the yield strength to the characteristic one (μ_{fy}/f_y) equal to 1.10-1.20, and the coefficient of variation $V = 0.05\text{-}0.08$. To determine the partial factors, it is recommended to take the statistical parameters of distribution of the yield strength within the range of equally possible values.

For comparison, the statistical parameters of the yield strength adopted in different works devoted to the calibration of the partial factors are presented:

– without accounting variability of geometrical parameters: $\mu_{fy}/f_{\mathrm{y}} = 1.19, V = 0.08$ [9]; $\mu_{fy}/f_{\mathrm{y}} = 1.27$, $V = 0.057$ [10];

– with accounting the variability of geometrical parameters: $\mu_R/X_{\mathrm{k}} = 1.25$, $V = 0.1$ [11]; $\mu_R = R_{\mathrm{k}}\mathrm{e}^{2\,V_R}$, $V = 0.08$ [12]; $\mu_R/X_{\mathrm{k}} = 1.18$, $V = 0.15$ [4]; $\mu_R/X_{\mathrm{k}} = 1.18$, $V = 0.08$ [7]; $\mu_R = R_{\mathrm{k}} + 2\,\sigma$, $V = 0.08$ [13].

To describe the yield strength probabilistically, the Normal and Lognormal distribution law is in the most common use [9-13]. Choosing the Normal distribution law is usually justified by the fact that the steel properties depend on the cumulative influence of independent random values, none of which exerts prevailing influence. Then, according to the Central Limit Theorem of the probability theory, the probability distribution functions of the yield strength may be taken as Normal ones. It should be noted that this precondition is true for the initial population; however, due to the steel quality control procedure rejecting the off-grade steel, the deviation from the standard law takes place as a rule [14]. After the control procedure, the distributions become truncated from one side or bimodal.

Geometrical parameters of the cross section

The tolerances of the geometrical dimensions regulated in the standards for rolled steel products, manufacture and installation of steel structures serve as an underlying cause when creating the probabilistic model of their deviations. Here the actual statistical parameters of distribution of geometrical dimensions of the cross sections shall be determined by the direct measurements of the dimensions. Taking into account the circumstance that the rolled steel products is delivered mainly by foreign manufacturers, the mean value of the ratio of the actual value of the geometrical parameters of the most common cross section (I-shaped rolled steel products) to its characteristic value of 0.99…1.03 and the coefficient of variation of 0.01…0.03[8] is recommended as a first approximation.

For the probabilistic calculations, the geometrical parameters are accepted as being determined or their variability in other basic variables is taken into account. In some works, the variability of the geometrical parameters was taken into account by introducing a separate basic variable:

$\mu_x / X_n = 1.025$, $V = 0.032$ – for the cross section of the I-shaped section IPE 140 [10];

$\mu_x / X_n = 1.0$, $V = 0.04$ – for the area, section modulus, moment of inertia of rolled profiles [6];

$\mu_x / X_n = 1.0$, $V = 0.03$ – for the moment of inertia [13].

The uncertainty of the Resistance model

In the general case, this uncertainty is conditioned by the inaccuracies occurring as a result of the assumptions and idealisations which are made when formulating the mathematical model of resistance.

The insufficient lighting of the problem of the probabilistic description of uncertainties of design models in the scientific literature should be noted. The analysis shows that the model uncertainties is ignored or uncertainties are taken into account rather arbitrarily when performing the probabilistic calculations. Due to this situation, the results of investigations for refinement of the resistance models are not reflected in standardization of the partial factors, and all the resistance models are placed on the same level from the standpoint of their accuracy.

The generalisation of the statistical characteristics of uncertainties of the resistance models is complicated due to continual improvement of the design models. However, in case of insignificant differences in the resistance models, the statistical parameters of the model uncertainty adopted in other normative documents may be used as an approximation. In this case, for the models of *Resistance of Cross Sections* such parameters may be used as a rule as a trustworthy estimation, and for the models of *Resistance of the Elements* to bending and axial compression when performing the stability verifications – as an approximate estimation. For other resistance models, as a rule it is necessary to investigate the uncertainties directly on the basis of the experimental data processing.

When performing the probabilistic calculations, the most frequent references are made to the JCSS recommendations [3], the values of statistical parameters are only presented for the resistance model to bending ($\mu = 1$, $V = 0.05$) and shear ($\mu = 1$, $V = 0.05$) and for the resistance model of welded ($\mu = 1.15$, $V = 0.15$) and bolted ($\mu = 1.25$, $V = 0.15$) connections only. The results of investigations of the uncertainties of the resistance models of the steel elements generalised in the work [15] are of interest.

4. Basic Variables Included in the Action Effects models

Permanent loads

The permanent loads include the self-weight of structures and stationary equipment, prestressing and indirect actions caused by rheological properties of materials and relative settlements of structural system elements. The self- weight variability is affected by uncertainties of the size, density, additional loads from connections of the structural elements, possible modifications in the process of reconstruction or repair, environment and level of quality control of the work performance. The effect of the uncertainty of the dimensions and density can be taken into account with the highest justification.

The dimensions of the elements and densities of the materials are random values and described in the most cases by the Normal, truncated Normal or Lognormal distribution laws. The statistical parameters of traditional material have been well studied.

The probabilistic models of the permanent load, what are presented below adopted when performing the calibrations of the partial factors of the Eurocodes: $\mu_G = 1.05\ G_\mathrm{k}$, $V_G = 0.1$ in the works [4, 5]; $\mu_G = 1.05\ G_\mathrm{k}$, $V_G = 0.07$[6]; $\mu_G = 1.0\ G_\mathrm{k}$, $V_G = 0.1$ [12, 16].

In the SAKO's technical report [17], the permanent load was divided into the dead weight (coefficient of variation for concrete and glued wood is adopted to be 0.06 and that for steel – to be 0.02) and other permanent loads (coefficient of variation is 0.1). This approach is more realistic and true in methodological aspect than application of one coefficient of variation.

Imposed loads

The probabilistic model of the imposed loads have no strict dependence on the territorial peculiarities of the construction region, therefore the generally accepted models can be used. In most works the statistical parameters published in JCSS recommendations [3] are used for probabilistic description of the model of imposed loads. It should be noted that these models are consistent with the results of investigations published by Riser, Bulychev et al [18, 19].

Uncertainties of the imposed load model. As it was shown in [1], the equivalent imposed load was determined on the basis of the theoretical coefficients of the load area and area-averaged load. For the analysis of uncertainties of the imposed load model, the work by P.V. Avramenko [20] presenting not only the statistical parameters of the area-equivalent load, but also those for the load being equivalent as to forces in the elements and support reactions is of practical significance. The mean value and coefficients of variation for the load equivalence coefficient (uncertainties of the imposed load model) as to moment were 0.94 and 0.11, and as to transverse force – 1.1 and 0.06 [20, table 1]. As a rule, the uncertainties of the imposed load model is not considered when performing the analysis of reliability of the building structures or adopted according to the JCSS recommendations [3] with the following parameters: $\mu = 1$, $V = 0.1$.

Snow load

The probabilistic model of the sequence of the annual maxima of snow load is the most common model for the snow load at the ground and provides sufficient accuracy. This precondition allow passing from the probabilistic description of a random process to the description of the random value.

The statistical parameters of the snow load at the ground are variable in their nature that requires the systematic and purposeful investigations for refining them. As a rule, empirical series data of the snow load are rather limited and include 40-60 values that introduces some uncertainties into the estimation result.

The actual values of the statistical parameters of the snow load specified for the territory of the Republic of Belarus are presented in the works [7, 21].

For approximation of the annual maxima of the snow load, the Gumbel first limit distribution, Lognormal distribution and Weibull distribution are in the most common use. It should be noted that the three types of distribution, namely the Gumbel, Weibull and Frechet ones are used for estimation the characteristic values of the snow load (to estimate the so-called " hard tail" part of the distribution) for the territory of the Republic of Belarus. The use of these distributions has allowed the more justified characteristic values of the snow load to be obtained. However, the use of the Gumbel law is more preferable for analysis of the structural reliability characterised by very low probability values that is consistent with the common practice and today's tendencies of the probabilistic description of the snow load within the reliability concept adopted in the Eurocodes [1]. It should be noted that the standardization of the characteristic values of the snow load on the ground is performed in the new edition of the Belarusian National Annex to the EN 1991-1-3 using the Order Statistic Method. On the one hand, this approach has made it possible to determine the position of the fractile estimator under the limited sample conditions more accurately, but, on the other hand, it has deprived the specialists in the field of the theory of reliability of structures of the information both on the distribution type and on the values of the statistical parameters (within this method, no revealing of the distribution law is required).

Statistical parameters of the snow load at the ground for the assessment period of 50 years as adopted in the works for calibrations of the partial factors of the Eurocodes can be established as: $\mu_S = S_k$, $V_S = 0.22$ [12]; $\mu_S = 0.7$ kPa, $V_S = 0.3$ [6]; $\mu_S = 1.11\ S_k$, $V_S = 0.27$ [9]. In all these publications, the Gumbel distribution is adopted.

Uncertainties of the snow load model. As a rule, the uncertainties of the snow load model is determined by the variability of the coefficients of “transition” (shape coefficient, exposure coefficient, thermal coefficient) from the load at the ground to the snow load on the roof. Here the statistical parameters of the coefficients of “transition” are studied insufficiently. In most works, the probabilistic models of the coefficients of “transition” are adopted according to the JCSS recommendations [3].

Wind action

To describe the probabilistic model of the wind model, it is necessary to have the statistical parameters of the basic wind speed, basic wind pressure; coefficients of “transition” from the basic wind speed to the wind profile; coefficients of “transition” from the from the wind speed to the wind action (pressure, forces) on the structure and uncertainties of the models of determination of the wind action effects (statistical and dynamical reaction of the structures).

The values of the statistical parameters of the basic wind speed for the territory of the Republic of Belarus are adopted on the basis of the works [7, 22]. Uncertainties of the wind model are adopted according to the JCSS recommendations [3].

The following statistical parameters of the wind action for the assignment period of 50 years ware adopted in the calculation for calibrations of the partial factors of the Eurocodes: $\mu_W = 0.7\ W_\mathrm{k}$, $V_W = 0.35$ [5]; $\mu_W = 0.9\ W_\mathrm{k}$, $V_W = 0.34$ [4]; $\mu_W = 1$ kPa, $V_W = 0.3$ [6]; $\mu_W = 0.7\ W_\mathrm{k}$, $V_W = 0.33$ [9]

Uncertainties of the action effect model

Uncertainties of the action effect model takes into account the inaccuracies in determining the action effect (internal forces) appearing due to idealisation of the geometry, supporting conditions (boundary conditions), simplifications adopted when determining the forces, etc. This uncertainty is described by the random variable K_R, the statistical parameters of which are adopted from JCSS recommendations [3].

Conclusion

The probabilistic descriptions of the basic variables have been justified with due account for the territorial peculiarities (conditions) of the Republic of Belarus. Table 1 presents the summary of the probabilistic models of the basic variables included in the design models of resistances and action effects when designing the steel structures, on the basis of which the investigations of the target values of the reliability index were performed and the partial factors values were obtained.

Table 1. Probabilistic models of the basic variables for conditions of the Republic of Belarus

Basic variables	Distribution	μ / X_k	V
Steel element resistance	Lognormal	1.1 – 1.2	0.05 – 0.08
Uncertainties of the resistance model	Lognormal	1.0 – 1.15	0.05 – 0.10
Self-weight	Normal	1.0	0.03 – 0.06
Permanent load	Normal	1.0 – 1.05	0.07 – 0.10
Imposed load	Gumbel distribution	0.45 – 0.6	0.35 – 0.40
Uncertainties of the imposed load model	Normal	1.0	0.10
Snow load	Gumbel distribution	0.9 – 1.1	0.19 – 0.23
Uncertainties of the snow load model	Normal	1.0	0.15
Wind action	Gumbel distribution	1.0 – 1.1	0.17 – 0.20
Uncertainties of the wind action model	Normal	0.8	0.30
Uncertainties of the action effect model	Lognormal	1.0	0.10

μ is the mean value; V is the coefficient of variation, and X_k is the characteristic value.

The necessity of carrying out further systematic investigations of the variability of the basic variables and formation of the unified principles of assigning thereof has been confirmed.

Summary

Studies of reliability based on different probability models can lead to uncomparable results. Therefore, an important task is to develop common approaches to define probabilistic models of basic variables. The modern approach to probabilistic modelling of action with an emphasis on European trends is shown in the article. Probabilistic models snow and wind loads taking into account the territorial conditions of the Republic of Belarus are clarified. A review of probabilistic models adopted in the calibration of partial coefficients for the Eurocodes was performed. The probabilistic model of action are recommended based on the investigations. The problem of probabilistic description of the resistance of steel elements is considered. Generalized statistics of the strength characteristics of steel, geometric parameters of sections and uncertainty of resistance models of steel elements are presented based on the analysis of modern research.

Basic variables formulations (descriptions), which are presented in Table 1, was used for calibration of the partial factors for the Belarusian National Annex to Eurocode 3

References

1. EN 1990 Eurocode: Basis of structural design – Brussells: European Committee for Standardization, 2002.

2. ISO 2394:2015. General principles on reliability for structures.

3. JCSS Probabilistic Model Code //Joint Committee of Structural Safety [Electronic resource]. – 2001. –Mode of access: http://www.jcss.ethz.ch. –Date of access: 15.01.2012.

4. Beck, A.T. A firts attempt towards reliability- based calibration of Brazilian Structural Design codes / A.T. Beck, Jr.Souza // Journal of the Brazilian Society of Mechanical Sciences and Engineering. –2010. –№.2(XXXII). –P. 119-127

5. Holicky, M. Reliability assessment of alternative Eurocode and South African load combination schemes for structural design / M. Holicky, J. Retief // Journal of the South African Institution of Civil Engineering. – 2005. – Vol. 47, № 1. – P. 15–20.

6. Vrouwenvelder, A.C.W.M. Probabilistic calibration procedure for the derivation of partial safety factors for the Netherlands building codes / A.C.W.M. Vrouwenvelder, A.J.M. Siemes // Delft University of Technology. – 1987. – Vol. 32 (4). – P.9-29.

7. Марковский Д.М. Калибровка значений параметров безопасности железобетонных конструкций с учётом заданных показателей надёжности: автореф. дис. … канд. техн. наук: 05.23.01 / Д.М. Марковский; Брестский государственный технический университет. – Брест, 2009. – 24с.

8. Мартынов, Ю.С. Статистические параметры базисных переменных, входящих в модели сопротивления стального элемента / Ю.С. Мартынов, В.В. Надольский // Архитектура и строительные науки. – 2014. – № 1, 2(18, 19). – С. 39-41

9. Sýkora, M. Reliability Analysis of a Steel Frame /M. Sýkora // In: ActaPolytechnica, Vydavatelství ČVUT, Prague, Czech Republic. – 2002. –№ 4(42). – P. 27-34

10. Kala, Z. Influence of partial safety factors on design reliability of steel structures – probability and fuzzy probability assessments /Z. Kala// Journal of civil engineering and management. –2007. –№ 4(XIII). –P.291–296

11. Holicky, M. Safety design of lightweight roofs exposed to snow loads / M.Holicky // Engineering Sciences. –2007. –№58. –P. 51–57

12. Sýkora, M. Reliability-based design of roofs exposed to a snow load /M. Sýkora, M.Holicky // In Li, J. - Zhao, Y.-G. - Chen, J. (eds.) Reliability Engineering - Proceedings of the International Workshop on Reliability Engineering and Risk Management IWRERM 2008, Shanghai, 21 - 23 August 2008. Shanghai: Tongji University Press. –2009. – P. 183-188.

13. Honfi, D. Reliability of beams according to Eurocodes in serviceability limit state/ D. Honfi, A.Mårtensson, S. Thelandersson// Engineering Structures. – 2012. –Vol.35. –P.48-54

14. Melcher, J. Design characteristics of structural steels based on statistical analysis of metallurgical products / J.Melcher, Z.Kala, M.Holicky, M.Fajkus, L.Rozlivka// Journal of Constructional Steel Research. –2004. Vol. 60, № 3–5,– P.795-808.

15. Nadolski, V. Uncertainty in Resistance Models for Steel Members / V. Nadolski, M. Sýkora // In: Transactions of the VSB - Technical University of Ostrava. Construction Series. –2014. – Vol.14 –P. 26–37.

16. Holicky, M. Calibration of Reliability Elements for a Column / M. Holicky, J. Markova // Workshop on Reliability Based Code Calibration : Press Release, Zurich, March 21-22, 2002 [Electronic resource] / Swiss Federal Institute of Technology (ETH Zurich). – 2002. –Mode of access : http://www.jcss.ethz.ch/events/WS_2002-03/WS_2002-03.html. – Date of access : 08.07.2011.

17. SAKO. Joint Committee of NKB and INSTA-B. NKB Report: 1999:01 E, Basis of Design of Structures. Proposals for modification of Partial Safety Factors in Eurocodes

18. Райзер, В.Д. Методы теории надежности в задачах нормирования расчетных параметров строительных конструкций. – М.: Стройиздат, 1986.-192с.

19. Булычев А.П., Десятник Е.И., Семченков А.С. Временные нагрузки на несущие конструкции зданий торговли // Строит. механика и расчет сооружений. 1989. - № 3. - С. 57-59.

20. Авраменко, П.В. Временные нагрузки на перекрытия многоэтажных административных зданий/ П.В. Авраменко// Строит. механика и расчет сооружений.- 1980. №1. - С. 67-71.

21. Тур, В.В. Нормирование снеговых нагрузок для территории Республики Беларусь / В.В.Тур, В.Е. Валуев, С.С. Дереченник, О.П. Мешик, И.С. Воскобойников // Строительная наука и техника. – 2008. – № 2. – С. 27–45.

22. Черноиван, А.В. Нормирование ветровой нагрузки на здания и сооружения для климатических условий Республики Беларусь: автореф. дис. … канд. техн. наук: 05.23.01 / А.В. Черноиван; Брестский государственный технический университет. – Брест, 2012. – 24с.