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FATIGUE LIFE ASSESSMENT WITH THE USE OF THE SPECTRAL METHOD FOR NON-GAUSSIAN LOADING HISTORIES WITH THE USE OF THE ENERGY PARAMETER

Key words: spectral method, non-Gaussian loading, fatigue life, energy parameter.

Abstract: The objective of the paper is to calculate fatigue life with the use of the spectral method for 10HNAP steel under non-Gaussian loading conditions with the use energy parameter and non-Gaussian correction. The energy parameter has not yet been used in the frequency domain for fatigue calculations under non-Gaussian loading conditions. Some calculations regarding the energy parameter and spectral method have been performed by Banvilett et al. [1] for the case with non-Gaussian loading but with no non-Gaussian correction. In terms of non-Gaussian correction recently, a paper has been published by Niesłony et al. [20] where the authors are using a correction factor presented by Bracessi et al. [8], and it is used to correct the damage degree of the loading signal. The correction factor used in the calculations has been modified to address the specific description of the W_a-N_f curve. The results for the energy parameter have been compared with results for the stress calculation with spectral method with the use of the Bracessi correction factor.

Wyznaczanie trwałości zmęczeniowej za pomocą metody spektralnej dla historii obciążeń o charakterze nie-Gaussowskim z wykorzystaniem parametru energetycznego

Słowa kluczowe: metoda spektralna, nie-Gaussowskie obciążenie, trwałość zmęczeniowa, parametr energetyczny.

Streszczenie: Celem pracy jest obliczenie trwałości zmęczeniowej za pomocą metody spektralnej dla stali 10HNAP w warunkach obciążeń o charakterze nie-Gaussowskim z parametrem energetycznym i korektą z uwagi na nie-Gaussowość. Parametr energetyczny nie został jeszcze wykorzystany w dziedzinie częstotliwości do obliczeń zmęczeniowych w warunkach obciążenia o charakterze nie-Gaussowskim. Niektóre obliczenia dotyczące parametru energetycznego i metody spektralnej zostały wykonane przez Banviletta i innych [1], dla przypadku z obciążeniem o charakterze nie-Gaussowskim, ale bez korekcji z uwagi na nie-Gaussowość. Pod względem nie-Gaussowskiej korekty ostatnio opublikowano pracę Niesłonego i innych [20], w której autorzy stosują współczynnik korekcyjny przedstawiony przez Bracessiego i innych [8]. Służy on do korekcji stopnia uszkodzenia przebiegu obciążenia. Współczynnik korekcyjny zastosowany w obliczeniach został zmodyfikowany, aby uwzględnić konkretny opis krzywej W_a-N_f. Wyniki dla parametru energetycznego zostały porównane z wynikami obliczeń dla naprężeń za pomocą metody spektralnej z wykorzystaniem współczynnika korekcji Bracessiego i innych.

Introduction

The main idea behind the paper is to calculate fatigue life with the use of the spectral method for 10HNAP steel under non-Gaussian loading conditions with the use of strain energy density and non-Gaussian correction. The strain energy density (often referred to as energy parameter) has not yet been used in the frequency domain for fatigue calculations under non-Gaussian loading conditions. The frequency domain methods used to calculate fatigue life are often called by the name of spectral methods [7, 17]. These methods use the statistical information about the working loads that cause fatigue in the form of a power spectral density (PSD) [18, 19]. It seems to be an interesting case due to the fact that the strain energy density is used in some important areas of industry [25]. Some calculations regarding the energy parameter and spectral method have been performed by Banvilet et al. [1] for the case with non-Gaussian loading but with no non-Gaussian correction. While analysing the literature, we can find some papers that deal with non-Gaussian correction inter alia by Niesłony et al. [20], where the authors are using a correction factor presented by Bracessi et al. [8], and it is used to correct the damage degree of the loading signal. There are other solutions to the problem of non-Gaussianity like those presented by Benasciutti and Tovo [2–5] or by Wolfensteiner and Sedlmair [24]. Nevertheless, the problem of spectral method fatigue calculations has been discussed by many more, especially in papers by Slovak and Italian research groups [9, 10, 15, 21]. The correction factor used in the calculations in this paper has been specifically modified from the Bracessi et al. [8] formula to address the specific description of the W_a-N_f curve. The results for the energy parameter have been compared with results for the stress calculation with the spectral method with the use of the Bracessi correction factor. Specific errors for both stress and strain energy density have been calculated according to the Walat and Łagoda model [23].

1. Energy parameter

The energy parameter describes the energy of the combined stress-strain state of the material. This parameter allows one to take into account either the elastic or plastic state of the material in terms of calculations [12, 14]. Some forms of the parameter allow one to take the whole elastic-plastic state of the material. It is suitable for both the low cycle calculation range, where normally we calculate with the use of strain models, as well as for the high cycle range, where we calculate fatigue with the use of stress models. The parameter can be defined for the elastic state with a simple formula as follows [14]:

$$W_a = 0.5 \cdot \sigma_a \cdot \varepsilon_a \qquad (1)$$

where σ_a – stress amplitude, ε_a – strain amplitude.

It can be also presented in a relatable form:

$$W_a = \frac{(\sigma_f{}')^2}{2E}\left(2N_f\right)^{2b} \qquad (2)$$

where $\sigma_f{}'$ – is the fatigue strength coefficient, E – is the Young modulus, N_f – number of cycles till failure, b – fatigue strength exponent.

The energy parameter calculations with the spectral method seem promising while reading the literature, but his main drawback is the missing possibility to calculate fatigue life for non-Gaussian loading characteristics. The potential of use, especially in areas where stress or strain cannot directly describe the material damage, seems important to be solved.

2. Material and loading characteristic

The material used for the calculations is 10HNAP steel, which is quite popular, due to his good fatigue properties. The material has undergone random uniaxial tension-compression fatigue tests under non-Gaussian loading conditions. The tests have been performed by Lachowicz et al. [13]. The material properties are presented in Table 1. The power spectral density (PSD) of one of registered signals is presented in Fig.1. The PSD has been calculated with the Welch method [18, 19, 22]. The probability density function of energy parameter amplitudes is presented in Fig. 2 [16].

Table 1. Mechanical properties of 10HNAP steel [13]

R_e,MPa	R_m,MPa	A_{10}, %	Z, %	E,GPa	n
414	556	31	35	215	0.29

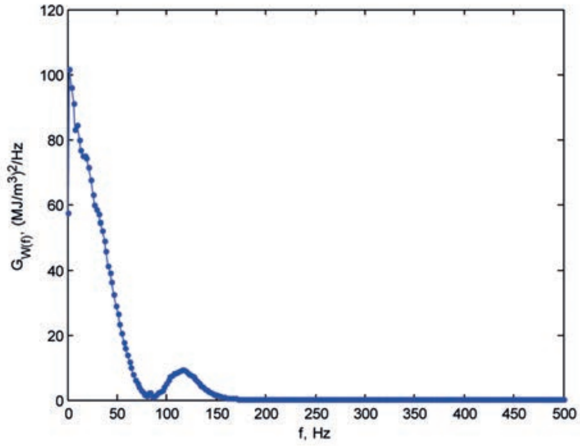

Fig. 1. Power spectral density of the energy parameter $G_{w(f)}$

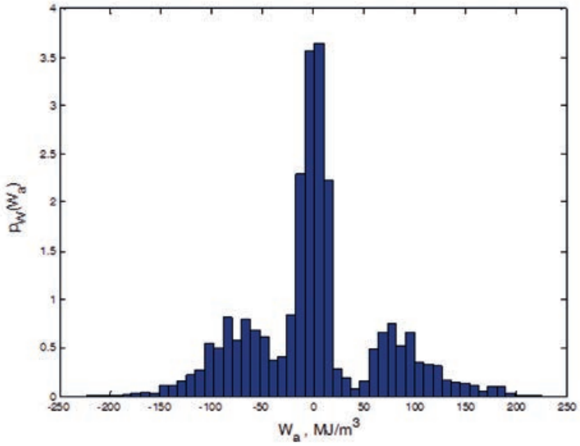

Fig. 2. Probability distribution function (PDF) of the non-Gaussian energy parameter signal

3. Results and discussion

The calculation of fatigue life has been performed for both stress and energy parameter signal characteristic. For the case of the probability density of energy parameter amplitudes calculations, the Dirlik formula [11] has been used by Niesłony et al. [18, 19]:

$$p(\mathrm{W}_a) = \frac{1}{2\sqrt{\xi_0}} \cdot \left[\frac{K_1}{K_4} \cdot e^{\frac{-Z}{K_4}} + \frac{K_2 \cdot Z}{R^2} \cdot e^{\frac{-Z^2}{2 \cdot R^2}} + K_3 \cdot Z e^{\frac{-Z^2}{2}} \right] \quad (3)$$

where Z, K_1, K_2, K_3, K_4, R – factors which are functions of the first five moments of the transformed PSD.

The obtained PDF is presented in Fig. 3 in comparison with some popular PDF calculation models like Bendats [6] or Benasciutti [2–5]. It is clear that the Dirlik formula covers most of the rainflow amplitude range.

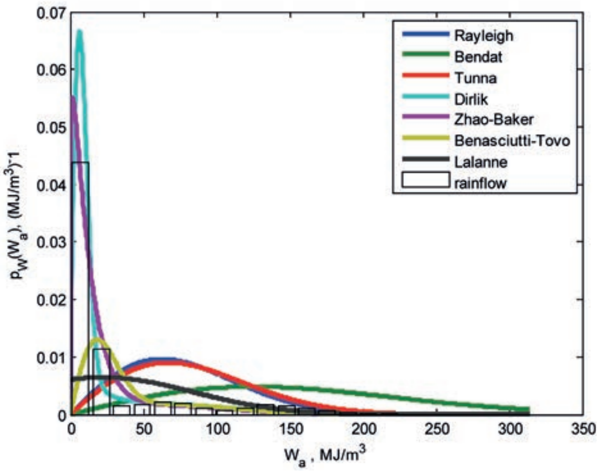

Fig. 3. **Chosen probability distribution function shapes for random data in comparison with rainflow amplitude distribution**

The fatigue life calculations have been performed with the use of the Palmgren-Miner hypothesis assumptions. We obtain the given formula [20]:

$$T_{cal} = \frac{1}{E[P] \int_0^\infty \frac{p(\mathrm{W}_a)}{N(\mathrm{W}_a)} d\mathrm{W}_a} \cdot \frac{1}{\lambda} \quad (4)$$

where $E[P]$ is the expected number of peaks, and non-Gaussianity is corrected with a corrective factor according to Braccesi et al. [8]:

$$\lambda' = \exp\left(\frac{(m')^{\frac{2}{3}}}{\pi} \left(\frac{K-3}{5} - \frac{S^2}{4} \right) \right) \quad (5)$$

where: m'– slope of the energy fatigue curve, K – kurtosis and S – skewness.

The root mean square error of calculation results were calculated with the use of the following formula presented by Walat and Łagoda [23]:

$$E_{RMS} = \sqrt{\frac{\sum_{i=1}^{n} \log^2 \frac{T_{\exp i}}{T_{cal i}}}{n}} \quad (6)$$

where $T_{\exp}$ and T_{cal} are the experimental and calculated fatigue life.

Finally, we obtain the mean deviation from the expected time to failure value with the use of the following formula:

$$T_{RMS} = 10^{E_{RMS}} \quad (7)$$

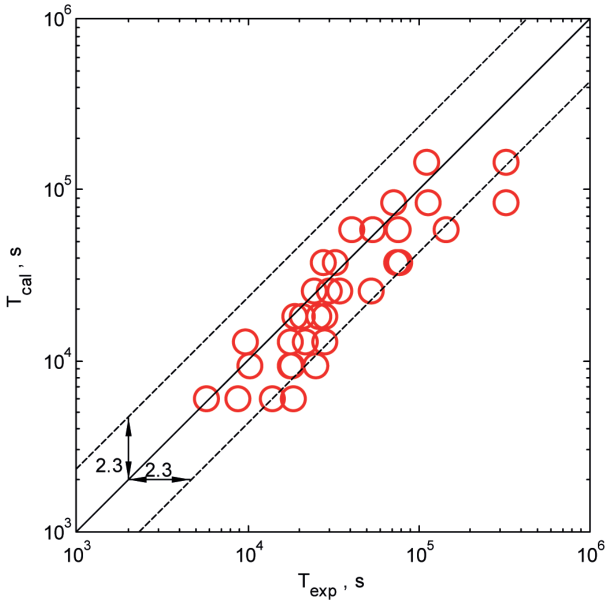

Fig. 4. **Comparison of experimental results with results for stress calculation**

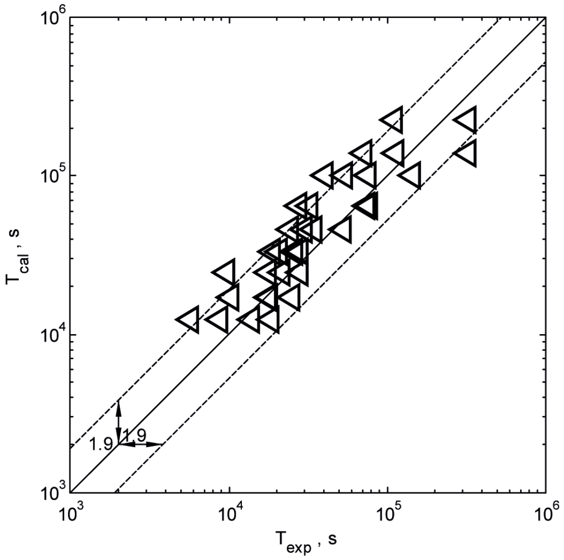

Fig. 5. **Comparison of experimental results with results for energy parameter calculation**

The obtained calculation results have been compared with experimental results and presented in Fig. 4 for individual scatter error calculated for stress, Fig. 5 for individual scatter error calculated for strain energy density, and a comparison of both results with the standard scatter error 3 in Fig. 6.

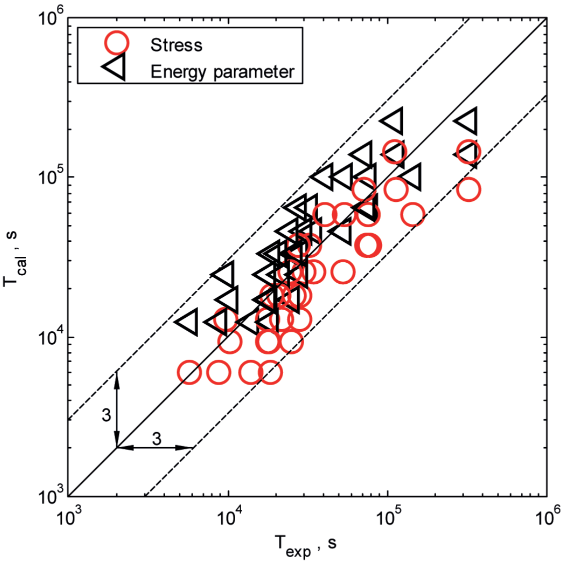

Fig. 6. Comparison of experimental results with results for stress and energy parameter calculations

4. Conclusions and observations

The calculations with the use of the energy parameter and with the use of the non-Gaussianity correction λ' have given satisfying results. All comparison results are within the scatter band of factor 3. While analysing the results we can formulate certain conclusions and observations:

The specific root mean square error of scatter area for individual methods has been calculated:

- For the calculations with the use of stress defined PSD, we have obtained a $T_{RMS\_N}=2.32$.
- For the calculations with the use of energy defined PSD, we have obtained a $T_{RMS\_E}=1.92$.

The energy parameter based calculations gave the best results of comparison with the experimental results.

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