Problems regarding stress between offset printing rollers

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At present, companies show particular care about quality and profit which are inherent with costs. An in adequately set stress between the printing rollers may significantly worsen the quality and increase costs of printing. Until now, a contact between rollers used to be set by a printer, where it was based on his experience. A width of the surface contact was measured or a distance between the form plate or the rubber to the bearer rings (rings made of hardened metal) was calculated. This paper includes a presentation regarding an attempt to solve the contact-related problems by way of mathematical methods.

Keywords and phrases: contact problem, stress, offset, blanket cylinder.

Introduction

Nowadays, economic aspects are of particular importance in every area, the printing industry included. Clients would like to be provided with print-outs of the best quality at the best price. So, printer operators are interested in reducing the time of making the printing machines ready for printing, as well as in decreasing the amount of paper wasted in the printing process. In order to maintain a good quality of print-outs, it is advisable to obtain such printouts with the quality kept invariable as much as possible at every moment of the printing process.

Setting an inadequate stress between printing cylinders is reflected both in terms of economic and quality aspects. Too much stress between the plate cylinder and the blanket one results in the flow of ink outside boundaries of print dots, thus resulting in a too big dot gain (print dots growing in area) and in a change of the print-out contrast, as well as in the wear-down of the said print dots. This also results in faster wear and tear of plate and rubber, deformation of bearer rings (hardened metal rings located at the ends of the two cylinders), cylinders slide and displacement between bearer rings, where it can additionally bring about increasing vibrations in the printing machine as well as lead to printing errors.

Too much stress between the blanket cylinder and the impression one can cause too much extension of paper in one direction and sometimes even result in breaking the paper or in deforming the bearer rings and the cylinders slide.

Setting too little stress between the plate cylinder and the blanket one, as well as between the blanket cylinder and the impression one, may lead to negative consequences. It can cause too small print contrast in result of failing to absorb enough ink by rubber and paper. It also results in dosing bigger amounts of ink and its over-consumption. Too small stress may cause an inadequate contact between bearer rings, which in turn may make the plate cylinder and the blanket one to bounce from each other.

So far applied methods of measurement of contact zone and pressure

A width of contact zone between the neighbouring cylinders is very often referred to as the stress. It is determined by way of reciprocal pushing a cylinder covered with ink and a clean cylinder, which is next followed with measuring a width of ink stripe printed on either the clean cylinder or on a piece of paper, where the so printed stripe, in turn, is compared to the figures specified in the machinery manual.

A stress between the cylinders is checked with the use of paper strips (sometimes foil stripes) which are placed between the cylinders in various places at all the length of the cylinders. Then, the paper stripes are pulled out by hand out of the stressed cylinders. Single strips can be used. However, better results are obtained by using three strips where the central one of them is to be pulled out (the said strip being narrower and longer than the upper and the lower strips). This allows to reduce friction between paper and the cylinders. It is better yet to use two paper strips, especially if cylinders have been ink coated. One of the strips needs to be folded in half, with the narrower strip being inserted inside the folded strip, where the latter of them is next pulled out. This way, however, allows only to say whether the stress at all the length is equal, or not too small, or not too large. Due to being based on a subjective impression of a person responsible for pulling out the strips, the measurement has significant errors and it does not allow to determines the stress value too. A cylinder setting measurement gauge is also used, where its end point is inserted in between two paper strips located between the cylinders. The gauge shows maximum resistance which is generated while pulling out the gauge.

The stress between the plate cylinder and the blanket cylinder is computed with the involvement of bearer rings. If bearer rings are in contact, the stress is computed by adding the height of the plate over bearer rings of the plate cylinder to the height of rubber over bearer rings of the blanket cylinder. If bearer rings are not in contact (during the printing process), then a distance between the bearer rings is subtracted from the aforementioned figure. However, a stress computed in that way provides neither for friction nor for wear and tear of the rubber.

The simplest method used to compute the said stress consists in dividing a loading mass which affects pivots on the two ends of the cylinder through the contact zone area. The stress is different in each place of the contact zone, and the said method implies that the stress level is identical at the entire contact zone area.

History of contact problems in printing and in continuum mechanics

Problem of distribution of pressure between two contact spheres as well as between a sphere and a surface was developed by H. Hertz in the year 1881 [2]. Following the publication of 1951, M. Hannah [3] went into the problem of thickness of elastic layer coated over a metal cylinder and its impact on the relationship between the amount of loading and deformation of such a cylinder. She compared pressure distribution in terms of finite and infinite thickness of elastic layer. She arrived at a conclusion that thickness of elastic layer is crucial nearly just as a cylinder diameter and an elastic modulus. In case of a thinner layer, the loading needs to be bigger than in case of a bigger layer thickness to obtain an identical contact area. In that case, there are smaller bulges between two contact cylinders located near the contact zone. According to her findings, the elastic layer has a smaller vertical deflection, i.e. central parts of cylinders are in a bigger distance from each other, whereas the loading is identical for layers of both smaller and bigger thickness. She also proved that changing Poisson's ratio brought to a significant change in the loading value required to obtain an identical contact zone.

R. Miller [4], in turn, was interested in the problem of contact of the rigid cylinder and the cylinder coated with rubber (or with another material). He measured cylinders' rotational speed differences, line pressure and indentation of the cylinder's coating layer at the time of contact these cylinders. Next, the so obtained experimental results were compared versus the theory regarding indentation resulted from a loading made to the coating layer. According to his findings, a rotational speed difference between the rubber coated cylinder and the rigid on depends on indentation of the coating layer, its thickness and kind of the coating material.

The paper by A. A. Elsharkawy and B. J. Hamrock [5] includes a description of the numerical solution as regards dry contact of two bodies. Centres of these bodies were subject to loadings. An analysis almost statistic slide was made with respect to two cases. The first of them comprises cylinders coated with a single hard layer. In the second case, the both cylinders are provided with two layers: the inner one which is hard and the outer one which is soft. As appears from the performed calculations, the tangential loading has significant impact on the pressure profile. Both as regards the cylinder coated with soft layer and that coated with the hard one, friction is of significant meaning for the location of the centre of the contact after the loading being made as well as on the pressure profile. Only as regards cylinders coated with soft layer, friction has significant impact on the contact zone and on the maximum contact pressure. The said paper includes computation of the subsurface stress and elastic surface deformation.

Johnson's book [7] includes problems related to contact between bodies for which the contact zone is significantly smaller than dimensions of the contacting bodies. Problems were approached which related to forces observed at a contact point, line loading and point loading, contact of elastic bodies and inelastic bodies, tangential loading and contact at sliding, rolling contact of elastic bodies and inelastic bodies.

Mathematical approach to solve problems as regards contact between offset cylinders

We assume that contact Hertz conditions [2] are kept. Considered issue was reduced to issue described two elastic half-space contact with assumption $R_1 >> a$, $R_2 >> a$. It allows to write in analytic form the contact pressure $P(x)$ and contact zone $2a$ [7].

More complicated issues lead to necessity of work out of contact problems solution methodics and methodics of measurement contact characteristic

The equilibrium equation in plain strain can be described as follows:

$$\begin{cases} (\lambda+2\mu)\frac{d^2U_x}{dx^2}+\mu\frac{d^2U_x}{dz^2}+(\lambda+\mu)\frac{d^2U_z}{dxdz}=0 \\ \mu\frac{d^2U_z}{dx}+(\lambda+2\mu)\frac{d^2U_z}{dz^2}+(\lambda+\mu)\frac{d^2U_x}{dxdz}=0 \end{cases} \tag{1}$$

where λ, μ – Lame parameters, $U_x(x,z)$, $U_z(x,z)$ – displacement in the direction x and z.

For purposes of further calculation, the Fourier transform applies:

$$F[U_x]=\widetilde{U_x}(\xi,z)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}U_x(x,z)e^{i\xi x}dx, \tag{2}$$

as well as inverse Fourier transform is used:

$$F^{-1}[\widetilde{U_x}]=U_x(x,z)= =\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\widetilde{U_x}(\xi,z)e^{-i\xi x}d\xi. \tag{3}$$

Having used the inverse Fourier transform, the equation (1) will take the form of two equations:

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}e^{-i\xi x}d\xi[(-i\xi)^2(\lambda+2\mu)\widetilde{U_x}+ +\mu\frac{d^2}{dz^2}\widetilde{U_x}+(\lambda+\mu)(-i\xi)\frac{d}{dz}\widetilde{U_z}]=0 \tag{4}$$

and

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}e^{-i\xi x}d\xi[\mu(-i\xi)^2\widetilde{U_z}+ +(\lambda+2\mu)\frac{d^2}{dz^2}\widetilde{U_z}+(\lambda+\mu)(-i\xi)\frac{d}{dz}\widetilde{U_x}]=0 \tag{5}$$

The equations (4) and (5) form the following system of equations:

$$\begin{cases} (-i\xi)^2(\lambda+2\mu)\widetilde{U_x}+\mu\frac{d^2}{dz^2}\widetilde{U_x}+(\lambda+\mu)(-i\xi)\frac{d}{dz}\widetilde{U_z}=0 \\ \mu(-i\xi)^2\widetilde{U_z}+(\lambda+2\mu)\frac{d^2}{dz^2}\widetilde{U_z}+(\lambda+\mu)(-i\xi)\frac{d}{dz}\widetilde{U_x}=0 \end{cases} \tag{6}$$

General solution of the homogeneous system of equations (6) is as follows:

$$\begin{cases} \widetilde{U}_x^1=\left(-\frac{i(B(\lambda+3\mu)+A(\lambda+\mu)|\xi|)}{(\lambda+\mu)\xi}-\frac{iB|\xi|z}{\xi}\right)e^{|\xi|z}+\left(-\frac{i(D(\lambda+3\mu)-C(\lambda+\mu)|\xi|)}{(\lambda+\mu)\xi}-\frac{iC|\xi|z}{\xi}\right)e^{-|\xi|z} \\ \widetilde{U}_z^1=(A+zB)e^{|\xi|z}+(C+zD)e^{|\xi|z} \end{cases} \tag{7}$$

The following relationships are used:

$$\lambda=\frac{\nu E}{(1+\nu)(1-2\nu)} \tag{8}$$

and

$$\mu=\frac{E}{2(1+\nu)} \tag{9}$$

where ν – Poisson's ratio, E – Young's modulus.

The so obtained solution (7) makes it possible to solve various problems with respect to layers and half-space – e. g., the problem of distribution of stresses on the half-space surface was analysed [7].

The elastic half-space $z > 0$ was considered, where its boundary was loaded with normal loading $P(x)$ and tangential loading $Q(x)$ at a length $(-a, a)$. The half-space boundary laying outside the aforementioned length is free of loadings.

The displacement in half-space $U_x(x, z)$ and $U_z(x, z)$ described by differential equations (1).

Boundary conditions for the considered issues are as follows:

$$\begin{matrix} \sigma_{zz}(x,z)|_{z=+0}=-P(x) & \sigma_{xz}(x,0)|_{z=+0}=-Q(x) \\ \sigma_{zz}(x,z)|_{z\to\infty}=0 & \sigma_{xz}(x,0)|_{z\to\infty}=0 \end{matrix} \tag{10}$$

The general solution (7) includes 4 constants, which we get from 4 boundary conditions (10). The final solution for the issue being analysed is as follows:

$$\begin{cases} U_x(x,z)=-\frac{1+\nu}{E\sqrt{2\pi}}\left(-\frac{2xz}{x^2+z^2}+(1-2\nu)2arctg\frac{x}{z}\right)*P(x)- \\ \frac{2(1-\nu^2)}{E\sqrt{2\pi}}\left(2C_x+\frac{z^2}{(1-\nu)(x^2+z^2)}+\ln(x^2+z^2)\right)*Q(x) \\ U_z(x,z)=-\frac{2(1-\nu^2)}{E\sqrt{2\pi}}\left(2C_z-\frac{z^2}{(1-\nu)(x^2+z^2)}+\ln(x^2+z^2)\right)*P(x)- \\ \frac{1+\nu}{E\sqrt{2\pi}}\left(-\frac{2xz}{x^2+z^2}-(1-2\nu)2arctg\frac{x}{z}\right)*Q(x) \end{cases} \tag{11}$$

where "*" – the convolution of function $f(x)*\varphi(x)=\int f(x-s)\varphi(s)ds$.

The aforementioned solution is not included in Johnson's book [7].

While making calculations according to formulas (10) for displacement in half-space boundary ($z\to+0$) the following formulas, as known from the literature [7], are obtain:

$$\begin{cases} U_x(x,+0)=-\frac{(1+\nu)(1-2\nu)\pi}{E\sqrt{2\pi}}sign(x)*P(x)-\frac{2(1-\nu^2)}{E\sqrt{2\pi}}(2C+\ln|x|)*Q(x) \\ U_z(x,+0)=\frac{(1+\nu)(1-2\nu)\pi}{E\sqrt{2\pi}}sign(x)*Q(x)-\frac{2(1-\nu^2)}{E\sqrt{2\pi}}(2C+\ln|x|)*P(x) \end{cases} \tag{12}$$

The so obtained displacement make it possible to compute stress in half-space which result from loadings as referred to in (10). It constitutes correction to the formulas specified in book [9].

Experimental approach of stress in contact zone between offset cylinders

The mathematical formulas, as derived above, and any and all future ones will be verified experimentally. For the experimental purposes, the Roller Nip Control device will be used. The said device allows to measure both a contact zone between cylinders and the cylinders pressure. The contact zone is measured when the contacting cylinders are static (the measuring unit is in mm or inch). The stress is measured when the cylinders are subject to slow rotation (the measuring unit is in $\frac{N}{cm^2}$. The device displays the maximum stress for a given moment. Either static or dynamic sensor should be used accordingly.

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