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ORIGINAL ARTICLE

Determining the area corrections affecting the map areas in GIS applications

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Abstract

Nowadays, there are many area-based Geographic Information Systems (GIS) applications such as real estate valuation, land tax, farming support and cost–benefit analysis. Areas used in such applications are calculated by means of two-dimensional plane geometry. However, the computed area value is not the exact area value in the terrain. In order to calculate the exact area value of a parcel, area corrections due to various factors must be taken into account. These factors are selection of projection, slope of the terrain, elevation of the terrain and scale of the map. Selection of projection and slope of terrain are available; elevation of the terrain and scale of map are not available in all GIS software. In this study, the effect of area corrections on the area value calculated from the map is examined with sample applications and the results are presented to the GIS users. According to the results, GIS users should select the equal area projection. In addition, scale of map, elevation and slope of terrain should be taken into account in the area calculation where land measurements are not possible.

Key words: Area-based GIS applications, area correction, exact area value, equal area projection, GIS software

1 Introduction

Today, area data is a basic parameter derived from analyses required for a large number of decision-making processes. The area needs to be calculated correctly because the values obtained as a result of calculations have an effect on cost. The values obtained as a result of area calculation are used in many fields such as engineering applications, geodetic studies, cadastral studies, zoning and expropriation applications, land valuation activities, forestry, agriculture and taxation. In such activities, Geographic Information Systems (GIS) are frequently used to obtain information such as cost–benefit analysis. Users obtain the area data of a parcel from the land register, land measurements or by digitisation from a scaled map. In any case, the area value is calculated from two-dimensional maps, in other words, the plane geometry. Maps are produced based on reference surfaces such as three-dimensional ellipsoid or sphere closest to the geoid using projection or map methods. In order to calculate the exact area value of a parcel on the map, area corrections must be added to the area value calculated on the map (Kundu and Pradhan, 2003; Zhang et al., 2011).

Large-scale maps (1:1000–1:10,000) are used in the registration process of the parcels. Area corrections in small parcels are neglected by most GIS users, as they have a very low value. However, in parcel types with large area values (forested land, pasture area, farming area and public land), area corrections are quite high. In this context, the larger the area of a parcel, the higher is the correction in the calculated area value for the area. In GIS applications, area corrections that need to be added to large parcels are not taken into account by users. Therefore, the values calculated by using the parcel areas do not reflect the exact value. GIS experts assume that better data leads to better decisions. An analysis that determines the effects of data quality on the quality of decisions should always be preferred. In GIS applications that area information of the parcels are used, the effect of the scale (Frank, 2008; Hejmanowska and Woźniak, 2009; Sindhuber et al., 2004), the effect of the slope (Kundu and Pradhan, 2003; Zhang et al., 2011), and the effect of the projection (Yildirim and Kaya, 2008) have been investigated. However, factors such as reference surface and elevation of terrain have not been taken into consideration in such studies. In this study, the effect of the errors on the calculated area in GIS applications will be examined. In order to calculate the exact area value of a parcel, the

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area correction for that area is required. In this context, in order to calculate the area corrections, the area of the parcel on the reference surface should be determined.

2 Materials and Methods

2.1 Area Calculation on the Ellipsoid Reference Surface

Maps are designed and produced based on reference surfaces using projection and other mapping methodologies. Large-scale maps, which are generally used in GIS applications, were obtained by mapping methods from the ellipsoid reference surface. In order to calculate the area corrections for a parcel, the parcel area (f) must first be calculated on the map plane. Then, the area of this parcel (F) on the ellipsoid reference surface is calculated based on the length of the edges as geodetic line. Thus, the area correction ($F - f$) of the parcel is calculated. As the area of the parcel grows, the difference ($F - f$) exceeds negligible levels in accordance with the mapping conditions. Area calculation on ellipsoid is more difficult due to its geometric structure compared to the sphere and plane. This difficulty can be easily eliminated considering today's software techniques. The shape of a parcel on the ellipsoid reference surface may be as parcels bounded by certain latitude and meridians, or parcels of concave shapes. Area calculation methods of a concave parcel with corners defined by geographical coordinates on the ellipsoid reference surface are frequently used in the literature. There exist a lot of remarkable methods in the literature proposed by Kimerling (1984), Danielsen (1989), Gillissen (1993), Sjöberg (2007), Freire and Vasconcellos (2010), Karney (2011, 2013) and Tseng et al. (2015) for calculation of area of a polygon using ellipsoidal geographical coordinates. Lumban-Gaol et al. (2019) examined the scale correction affecting the area and detected the best projection system in Indonesia in this study. The area has been computed using 72 projection systems with different scales employing MATLAB program. The minimum area distortion on the 1:5000 scale maps is shown by equal-area conic Albers standard projection system for the calculated study area (0.018 m^2). In addition, although Universal Transverse Mercator (UTM) projections have been used optimally for each map sheet so far, it has been determined that they are not sufficient to calculate the whole of Indonesia. Setiawan and Sediyono (2020) aim to have a line of sight for computation of the area of Indonesia depending on the limits of sub-district/village, district and regency/city in this study. In this paper, they have reported how to employ the circle method to detect the territory area of Indonesia depending on the limits of regency/city or district or village/sub-district from The Database of Global Administrative Areas (GADM database). The best outcomes gained are when the limit of the district is 1,965,443.51 km^2, which is 2.53% more than the reference area. Berk and Ferlan (2018) investigated the methodological difficulties of correct area calculation in the cadastre. The artıcle compared the uncertain legal explanation of the parcel boundary and parcel area in relation to the technically well-defined geodetic parcel boundary and the geodetic parcel area on the reference ellipsoid. Different approximate methods for area determination which can be employed in the cadastre are investigated. A highly correct method for parcel area calculation has been suggested, depending on an equal-area projection.

Since the Kimerling method is a spherical solution, the borders are not the geodesic curve, but the great circle. In the Danielsen and Sjöberg solutions, the parcel borders are taken as a geodesic curve and the area below that curve is calculated as the ellipsoidal area. In the Gillissen method, the area is calculated depending on Albers equal area projection by dividing a part of great circle by secants. Since the methods proposed by Danielsen and Sjöberg are based on series expansion formula, its effect is decreased while the area is growing. The area is calculated by using a large elliptical arc in the Tseng method and spherical triangles of parcel in the Karney method. Finally, in the Freire and Vasconcellos method, the area is calculated based on points that form each edge of the parcel on the ellipsoid surface.

MATLAB R2015a software is used to analyse the methods, and ITRF96 datum GRS80 ellipsoid is used in the processes. The secant length is determined as 50 m in Gillissen and Freire & Vasconcellos methods. In addition, Vincenty method (Vincenty, 1975) is used to solve the geodetic basic problem on the ellipsoid reference surface. A number of factors should be taken into consideration when comparing the methods with each other. These factors are accuracy values, utilisation limit, special cases and processing speeds. The oblate ellipsoid area is used to examine the accuracy values and special conditions. This area can be calculated with Equation (1):

$$F_{\text{ellipsoid}} = 2\pi a^2 \left(1 + \frac{\left(1 - e^2\right)}{e} \operatorname{arctanh}(e)\right) \tag{1}$$

where a and e are the large semi-axis and the first eccentricity, respectively. Two points ($\lambda_{P_1} = 0°, \varphi_{P_1} = 0°, \lambda_{P_2} = 0°$ and $\varphi_{P_2} = 90° E$) with latitude and longitude values are selected on the ellipsoid reference surface. The polar triangle (P_1, P_2, P_N) is produced by using these two points and the north celestial pole (P_N). The polar triangle area is equal to one-eighth of the area value calculated using Equation (1) ($F_e = 63,758,202,714,811.400 m^2$). Thus, the methods are examined according to the accuracy values, the special cases of latitude and longitude and the processing speeds (Table 1).

As can be seen in Table 1, Danielsen, Karney and Sjöberg methods can calculate the ellipsoid reference surface area completely. On the other hand, the positions of the points in the pole triangle are pole and equator. Therefore, Freire, Gillissen and Tseng methods do not work. The Kimerling method does not give correct results, as it calculates the sphere surface using its ellipsoid geographical coordinates. Danielsen, Karney and Sjöberg methods give correct results for all special cases such as point positions at the equator or pole and the polygonal edge on latitude or longitude. In addition, the calculated area gives accuracy results for one-eighth of the ellipsoid area.

In this context, Danielsen, Karney and Sjöberg methods can be used, which are studied for area calculation with geographical coordinates in the ellipsoid. If one of these methods should be preferred, Karney method can be preferred. In the Danielsen method, as the polygon edge length increases, the area decreases sensitivity. In addition, the Sjöberg method is not preferred because it is longer than the Karney method in terms of processing time.

Karney Method

An auxiliary sphere in which the azimuth values are preserved is determined on the ellipsoidal reference surface (Figure 1). The ellipsoidal area is calculated using the Gauss–Bonnet theorem.

The F area in Figure 1 is calculated with the MATLAB code from Equations (2) and (3). Also, a, b, e and e' are ellipsoid parameters. The solution of I_1 integral in Equation (5) can be calculated using math computer software or the serial coefficients produced by Karney method. A_0, A_1 and A_2 azimuths and the spherical geodesic line length are calculated by using spherical trigonometry in Equations (3–14):

Table 1. Examination of methods

Methods	Areas F_{method} [m²]	Difference ($F_{method} - F_e$) [m²]	$\Delta\lambda = 0°$	$\Delta\varphi = 0°$	$\lambda = (0°;90°)$	$\varphi = (0°;90°)$	t
Danielsen	**63,758,202,714,811.400**	**0.000**	√	√	√	√	**0.000**
Freire	-	-	√	√	-	-	7.906
Gillissen	-	-	√	√	-	-	0.125
Karney	**63,758,202,714,811.400**	**0.000**	√	√	√	√	**0.031**
Kimerling	63,900,265,931,354.400	142,063,216,543.047	√	√	-	-	0.000
Sjoberg	**63,758,202,714,811.400**	**0.000**	√	√	√	√	**0.281**
Tseng	-	-	-	√	-	-	0.000

√ Means it can be processed for special cases and - means no work
t – processing speed [milliseconds]

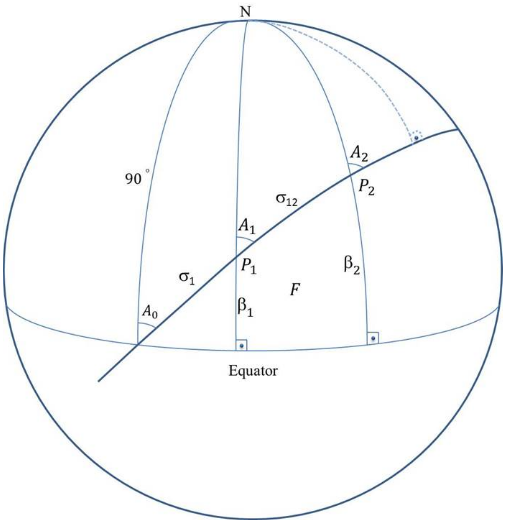

Figure 1. Auxiliary sphere

$$F = R^2(A_2 - A_1) + (F_{(\sigma_2)} - F_{(\sigma_1)}) \tag{2}$$

$$R^2 = \frac{a^2}{2} + \frac{b^2}{2}\frac{\tan h^{-1} e}{e} \tag{3}$$

$$F_{(\sigma_i)} = e^2 a^2 \cos A_0 \sin A_0 I_1(\sigma_i) \quad i = 1,2 \tag{4}$$

$$I_1(\sigma_i) = -\int_{\pi/2}^{\sigma_i} \frac{t\left(e'^2\right) - t(k^2 \sin^2 \sigma_i)}{e'^2 - k^2 \sin^2 \sigma_i} \frac{\sin \sigma_i}{2} d\sigma \tag{5}$$

$$t(x) = x + \sqrt{x^{-1} + 1} \sin h^{-1} \sqrt{x} \tag{6}$$

$$k^2 = e'^2 \cos^2 A_0 \tag{7}$$

$$A_1 = ph(\cos\beta_1 \sin\beta_2 - \sin\beta_1 \cos\beta_2 \cos\Delta\lambda + i\cos\beta_2 \sin\Delta\lambda) \tag{8}$$

$$A_2 = ph(-\sin\beta_1 \cos\beta_2 + \cos\beta_1 \sin\beta_2 \cos\Delta\lambda + i\cos\beta_1 \sin\Delta\lambda) \tag{9}$$

$$\Delta\lambda = (L_2 - L_1)/\sqrt{1 - e^2\left(\frac{\cos\beta_1 + \cos\beta_2}{2}\right)^2} \tag{10}$$

$$A_0 = ph(\cos A_1 + i\sin A_1 \sin\beta_1 + i\sin A_1 \cos\beta_1) \tag{11}$$

$$\sigma_j = ph\left(\cos A_j \cos\beta_j + i\sin\beta_j\right) \quad j = 1,2 \tag{12}$$

$$|x + iy| = \sqrt{x^2 + y^2} \tag{13}$$

$$ph(x + iy) = \arctan(\frac{y}{x}) \tag{14}$$

Equal Area Projections

Maps can be produced using equal area projections to minimise area deformation, instead of Karney solution for area calculation on the ellipsoid reference surface. Large and medium-scaled topographic maps (1000–250,000) are produced using UTM or Lambert Conformal Conic (LCC) projection in the world. Area corrections $(F - f)$ can be minimised with the selection of equal area projections. For calculating the area deformation, area correction formulas $(F - f)$ calculated with the help of coordinates produced from the maps are presented to the users by using UTM and LCC systems. Area correction formulas $(F - f)$ and area calculation on the ellipsoidal reference surface are not available in CAD and GIS applications. Therefore, instead of conformal projections, one of the projections that equal area projections should be selected for high accuracy calculation of a parcel area.

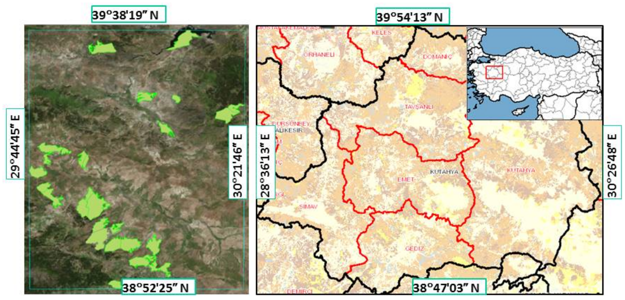

Figure 2. Satellite imagery of forest parcels (left) and geographical boundaries of the Kütahya Regional Directorate of forestry and forestry operation directorates in ITRF96 datum (right)

In this study, first, 15 forest parcels that varied between 300 and 3000 ha were selected within the boundaries of Kütahya Forest Regional Directorate (Figure 2). The area values of the 15 forest parcels selected in the study area were calculated with the equal area projections presented in Table 2 using GIS software. Thus, the most suitable equal area projection was determined by calculating the area deformations of the all projections (ITRF96 datum, 3° of longitude in width).

Areas of forest parcels were calculated by using the projections given in Table 2. At the same time, the areas of these forest parcels were calculated by the Karney method. The areas of the forest parcel calculated by the projections (Table 2) were compared with the areas calculated by the Karney method (Table 3). The distortion values are randomly distributed depending on the shape and size of the parcel and the distance of the parcel from the projection centre.

In Table 3, area corrections are calculated for the all equal area projections. According to Table 3 and Figure 3, two types of projections (Albers Equal Area Conic [AEAC] and Lambert Azimuthal Equal Area [LAEA]) with the least area corrections were determined. One of these projections can be preferred. AEAC projection was selected in this study in terms of the ease of calculation and processing time.

The area distortions have been also examined by employing Karney method in UTM. Thus, it is investigated whether the area reduction formula $(F - f)$ is sufficient in the UTM system. The area

Table 2. Projections and parameters used in the study

Map projections	L_0 Central longitude	B_0 Central latitude	B_1 Standard parallel	B_2 Standard parallel
UTM (3° of longitude in width)	30°E	0° (Equator)	-	-
AEAC	30°E	30.05°N	34.45°N	34.85°N
EAC	30°E	30.05°N	-	-
BEAC	30°E	0° (Equator)	-	-
BEA	30°E	0° (Equator)	-	-
LAEA	30°E	30.05°N	-	-
SEA	30°E	0° (Equator)	-	-

AEAC – Albers Equal Area Conic, BEA – Bonne Equal Area, BEAC – Behrmann Equal Area Cylindrical, EAC – Equal Area Cylindrical, LAEA – Lambert Azimuthal Equal Area, SEA – Sinusoidal Equal Area, UTM – Universal Transverse Mercator

Table 3. Area distortions of equal area projections

	Area calculation [m^2]							
Parcel number	Conformal map projection	Ellipsoid geographical coordinates	Area distortion (Karney- equal area map projections)					
	UTM (3°)	(Karney method)	AEAC	EAC	BEAC	BEA	LAEA	SEA
P1	28,570,421.774	28,570,400.287	9.440	90.265	90.247	1.854	5.603	6.880
P2	24,240,491.726	24,240,477.498	48.789	495.779	496.021	37.717	42.016	177.158
P3	19,454,360.215	19,454,337.971	6.217	64.581	64.516	7.223	6.395	32.869
P4	18,201,997.722	18,201,879.790	13.816	135.762	135.712	25.911	18.612	114.817
P5	16,730,817.085	16,730,812.149	3.664	32.262	32.260	4.938	4.220	20.196
P6	15,951,105.646	15,951,099.137	17.226	156.574	156.824	15.139	15.271	66.613
P7	14,809,228.918	14,808,973.667	4.482	44.089	44.112	5.986	5.158	27.267
P8	12,920,294.789	12,920,293.910	23.631	249.194	248.966	37.360	29.865	174.311
P9	11,067,018.225	11,066,945.786	6.986	58.247	58.261	4.470	5.488	16.470
P10	10,550,351.283	10,550,250.549	1.113	18.177	18.236	7.909	4.080	39.129
P11	9,498,509.712	9,498,505.310	28.252	294.518	294.698	28.161	27.681	133.011
P12	7,246,694.410	7,246,670.634	0.035	5.766	5.679	0.175	0.103	3.432
P13	6,575,866.610	6,575,812.176	9.264	93.417	93.364	6.931	7.786	32.686
P14	5,774,651.499	5,774,533.261	14.287	136.143	136.038	16.496	14.751	72.715
P15	3,673,397.840	3,673,391.983	1.317	13.831	13.764	0.586	0.922	3.230
$\sum$			**188.519**	**1888.605**	**1888.698**	**200.856**	**187.951**	**920.784**

AEAC – Albers Equal Area Conic, BEA – Bonne Equal Area, BEAC – Behrmann Equal Area Cylindrical, EAC – Equal Area Cylindrical, LAEA – Lambert Azimuthal Equal Area, SEA – Sinusoidal Equal Area, UTM – Universal Transverse Mercator

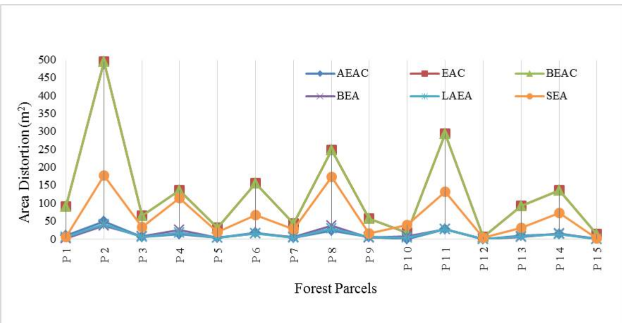

Figure 3. Comparison of the area distortion for equal area projections

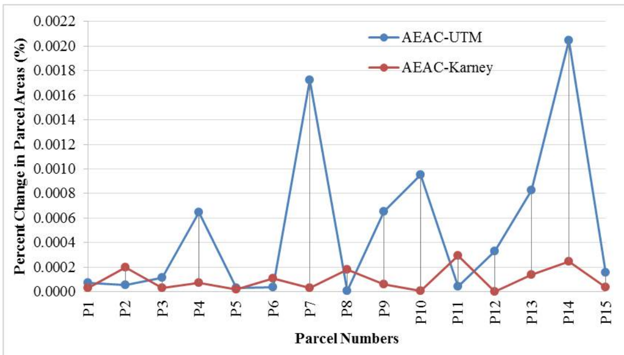

Figure 4. Comparison of UTM and Karney method

on the ellipsoidal reference surface can be calculated by adding the reduction formula ($F - f$) to the area calculated from the map. Area distortion formulas ($F - f$) are defined from the surface of the sphere and the non-concave shape in UTM. In conformal transformation, the deformation value is directly related to the size of the area and its distance to the y-axis (Grossmann, 1976):

$$F - f = -\frac{f}{R^2} y_m^2 \tag{15}$$

where F, f, R, y_m represent the area on the ellipsoid reference surface, the area calculated from the map, the Gaussian mean radius and the average distance from the y-axis (prime meridian) of the parcel, respectively. Area reduction value ($F - f$) of application parcels in UTM was calculated using Equation (15) (y_m = 13 km, R = 6370 km). Area reduction values (F – f) of the parcels are given in Table 4.

The area reduction values calculated from Equation (4) in UTM were compared with the results obtained from the Karney method. In the light of the results, it has been observed that the results obtained from the Karney method are more reliable (Figure 4).

Therefore, in order to determine the area distortion in large areas, it is necessary to calculate the area with geographical coordinates on the ellipsoid reference surface or to transform to equal area projections.

Table 4. Comparison of UTM, area reduction ($F - f$), Karney method and AEAC projection

Parcel number	Area calculation [m²]				Area distortion [m²]		
	Conform map projection		Ellipsoid geographical coordinates	Equal area projection	Area reduction formula	Equal area projection	Conform map projection
	UTM [3°] ($f_{conformal}$)	(F − f) (4) formula	F (Karney)	AEAC (f_{equal_area})	Karney−[f−(F − f)]	Karney−AEAC	Karney−UTM
P1	28,570,421.774	118.994	28,570,400.287	28,570,390.847	140.481	9.440	21.487
P2	24,240,491.726	100.960	24,240,477.498	24,240,428.709	86.732	48.789	14.228
P3	19,454,360.215	81.026	19,454,337.971	19,454,331.754	58.782	6.217	22.244
P4	18,201,997.722	75.810	18,201,879.790	18,201,893.606	42.122	13.816	117.932
P5	16,730,817.085	69.683	16,730,812.149	16,730,815.813	64.747	3.664	4.936
P6	15,951,105.646	66.435	15,951,099.137	15,951,081.911	59.926	17.226	6.509
P7	14,809,228.918	61.679	14,808,973.667	14,808,969.185	193.572	4.482	255.251
P8	12,920,294.789	53.812	12,920,293.910	12,920,317.541	52.933	23.631	0.879
P9	11,067,018.225	46.093	11,066,945.786	11,066,952.772	26.346	6.986	72.439
P10	10,550,351.283	43.941	10,550,250.549	10,550,251.662	56.793	1.113	100.734
P11	9,498,509.712	39.561	9,498,505.310	9,498,533.562	35.159	28.252	4.402
P12	7,246,694.410	30.182	7,246,670.634	7,246,670.599	6.406	0.035	23.776
P13	6,575,866.610	27.388	6,575,812.176	6,575,802.912	27.046	9.264	54.434
P14	5,774,651.499	24.051	5,774,533.261	5,774,547.548	94.187	14.287	118.238
P15	3,673,397.840	15.299	3,673,391.983	3,673,390.666	9.442	1.317	5.857

AEAC – Albers Equal Area Conic, UTM – Universal Transverse Mercator

The area difference that was calculated by using the Karney method and AEAC projection varies from 1 to 50 m². When comparing the parcel areas on the reference surface calculated using Equation (15) in UTM and AEAC projection, it has been found that there are differences from approximately 10 to 200 m² for each parcel (Figure 4).

If AEAC projection was chosen instead of UTM, a total of 800 m² distortion was observed in the total of all area distortion values in the parcels.

Area Corrections Caused by Map Scale

One of the important criteria in the area calculation is how the corner coordinates of the parcel are produced. In this context, the scale of the map where the corner coordinates of a parcel are produced is directly related to the area calculation of the parcel. Area calculations of the parcels, which cannot be measured in the terrain, are calculated on the scaled map. For this, the corner coordinates of the parcel are produced on a scaled map by digitisation technique. In this context, the larger the scale of the map produced in coordinate data, the higher the sensitivity values of the data produced on the map.

The area calculation performed using the parcel corner coordinates produced from the scaled map is formulated below:

$$2f = \sum_{i=1}^{n} x_i(y_{i+1} - y_{i-1}) \tag{16}$$

$$2f = \sum_{i=1}^{n} y_i(x_{i+1} - x_{i-1}) \tag{17}$$

where n, f and (x,y) denote the total number of corners of the parcel, the area of the parcel on the map and the corner coordinates of the parcel. The parcel area is calculated using Equations (16) and (17). Then the root mean square error (RMSE) of the parcel area is calculated using following equations:

$$M_F = \frac{m_p}{2\sqrt{2}} \sqrt{\sum_{i=1}^{n} S^2_{i+1,i-1}} \tag{18}$$

$$S^2_{i+1,i-1} = (x_{i+1} - x_{i-1})^2 + (y_{i-1} - y_{i+1})^2 \tag{19}$$

$$f_{scale} = f \pm M_F \tag{20}$$

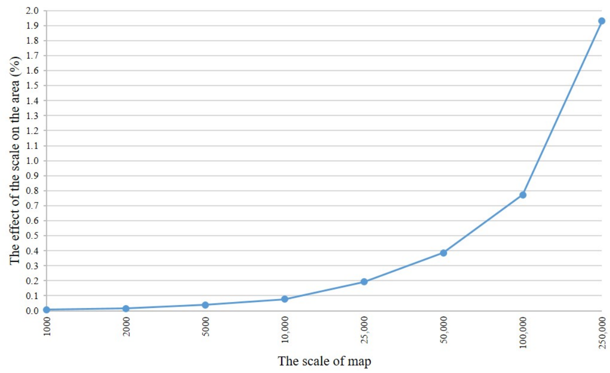

Figure 5. The effect of digitisation technique on the area in the scale map

where m_p and S represent the RMSE of the corner points and the square of the diagonal lengths of the parcel (Bogaert et al., 2005; Navratil and Feucht, 2008; Hejmanowska and Woźniak, 2009). Equations (18–20) are not available in GIS software. But users can add Equations (18–20) to the program with the help of the interface.

It is seen in Table 5 that as the scale of the map gets smaller and the area gets bigger, the value of the area correction due to the scale increases as a result of digitisation technique. For example, an area of 2500 ha, which is calculated using the corner coordinates produced from a 25,000-scale map, has been calculated as an area of 2.5 ha. When the parcels with large areas (forests, pastures, etc.) are taken into consideration, it is clearly seen that the area differences resulting from the use of medium-sized maps have high values. The areas calculated from the data obtained from the scaled maps using the digitisation technique are given in Table 5.

On the other hand, smooth square parcels are used for the application in Table 5. When using 15 forest parcels selected within the study area instead of these parcels, the results obtained are shown in Table 6.

When these forest parcels are examined, it is seen that the effect of digitisation technique used in scaled maps increases as the scale of the maps gets smaller. Therefore, large-scale map must be used for the area calculation with the digitisation technique in the parcels with large areas (Figure 5).

Table 5. The effect of scale-related accuracy of measurement on the area (for regular polygon parcels)

Scale of the map $m_p = m_x = m_y$ [m]			1000 0.2	2000 0.4	5000 1	10,000 2	25,000 5	50,000 10	100,000 20	250,000 50
$\sum S^2$[ha]	Edge [m]	Area	Root mean square error of the M_F area depending on the scale [m^2]							
0.08	10	100 m^2	2	4	10	20	50	100	200	500
8	100	1 ha	20	40	100	200	500	1000	2000	5000
32	200	4 ha	40	80	200	400	1000	2000	4000	10,000
200	500	25 ha	100	200	500	1000	2500	5000	10,000	25,000
450	750	55 ha	150	300	750	1500	3750	7500	15,000	37,500
800	1000	100 ha	200	400	1000	2000	5000	10,000	20,000	50,000
3200	2000	400 ha	400	800	2000	4000	10,000	20,000	40,000	100,000
20,000	5000	2500 ha	1000	2000	5000	10,000	25,000	50,000	100,000	250,000
80,000	10,000	100 km^2	2000	4000	10,000	20,000	50,000	100,000	200,000	500,000
180,000	15,000	225 km^2	3000	6000	15,000	30,000	75,000	150,000	300,000	750,000
320,000	20,000	400 km^2	4000	8000	20,000	40,000	100,000	200,000	400,000	1,000,000
720,000	30,000	900 km^2	6000	12,000	30,000	60,000	150,000	300,000	600,000	1,500,000

Table 6. The effect of scale-related accuracy of measurement on the area (forest parcels) in AEAC projection

Scale of the map mp = mx = my [m]		1000 0.2	2000 0.4	5000 1	10,000 2	25,000 5	50,000 10	100,000 20	250,000 50
$\sum S^2$[m^2]	Parcels	Root mean square error of the area depending on the scale [m^2]							
33,022,713.72	P1	812.683	1625.367	4063.417	8126.834	20,317.084	40,634.169	81,268.338	203,170.845
43,907,616.54	P2	937.098	1874.196	4685.489	9370.978	23,427.446	46,854.891	93,709.782	234,274.456
25,256,098.93	P3	710.719	1421.439	3553.597	7107.193	17,767.983	35,535.967	71,071.934	177,679.835
49,422,346.14	P4	994.207	1988.413	4971.033	9942.067	24,855.167	49,710.334	99,420.668	248,551.670
30,571,936.51	P5	781.945	1563.891	3909.727	7819.455	19,548.637	39,097.274	78,194.548	195,486.369
37,908,657.94	P6	870.731	1741.463	4353.657	8707.314	21,768.285	43,536.570	87,073.139	217,682.848
42,306,252.37	P7	919.851	1839.701	4599.253	9198.506	22,996.264	45,992.528	91,985.056	229,962.639
50,596,221.46	P8	1005.945	2011.889	5029.723	10,059.445	25,148.614	50,297.227	100,594.455	251,486.136
15,412,135.51	P9	555.196	1110.392	2775.981	5551.961	13,879.903	27,759.805	55,519.610	138,799.025
34,040,110.57	P10	825.107	1650.215	4125.537	8251.074	20,627.685	41,255.370	82,510.739	206,276.848
36,106,507.87	P11	849.782	1699.565	4248.912	8497.824	21,244.560	42,489.121	84,978.242	212,445.604
10,698,760	P12	462.575	925.149	2312.873	4625.745	11,564.363	23,128.727	46,257.453	115,643.634
16,397,451.31	P13	572.668	1145.337	2863.342	5726.683	14,316.708	28,633.417	57,266.834	143,167.085
25,863,964.9	P14	719.221	1438.443	3596.107	7192.213	17,980.533	35,961.066	71,922.131	179,805.328
47,247,724.79	P15	972.088	1944.175	4860.438	9720.877	24,302.192	48,604.385	97,208.770	243,021.925

AEAC – Albers Equal Area Conic

Area Corrections Caused by Elevation and Slope Factors

Today, area calculations are made on the surface of the ellipsoid or projection plane with a conform transformation from the ellipsoid. The ellipsoid selected is the reference ellipsoid determined in the geodetic datum. GRS80 is used in ITRF datum and Hayford ellipsoid in ED50 datum. The ellipsoid selected as reference is determined by the geoid. Ellipsoidal height (h) is measured in Global Navigation Satellite System (GNSS) measurements. The height type shown on the maps is the geoidal height. The transformation between these two types of heights is enabled by geoid undulation (N). The effect of the height factor in the calculation of a parcel area is determined using Equation (21):

$$F_h = F + F\left(1 + \frac{2h_{avg}}{R}\right) \tag{21}$$

where F_h, F, h_{avg} denote the area of the parcel with a height from the ellipsoid, the area of the parcel on the surface of the ellipsoid and the average of the heights of the parcel corner points, respectively (Koçak, 1985). The slope factor is not taken into account in this calculation. Area correction of sample parcels with different heights and area values are shown in Table 7.

As seen in Table 7 and Figure 6, area deformations increase as the area and height values increase. When using 15 forest parcels selected within the study area instead of these parcels (for regular polygon parcels), the results obtained are shown in Table 8.

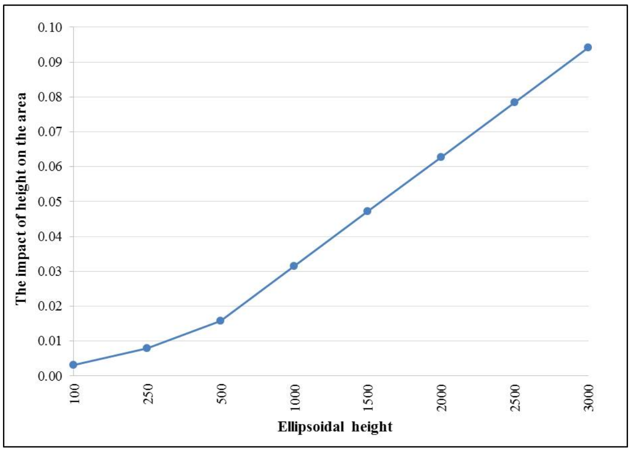

Figure 6. The effect of ellipsoidal height on area (graphical display)

In the light of the results obtained in Table 8, it is observed that the area correction values increase as the average ellipsoidal height values of the parcels increase. Equation (18) that is formulated above is not available in GIS software. In applications that require high precision, GIS users should consider the height factor when using large areas of parcels.

In the calculation of the parcel areas, besides the height factor

Table 7. The effect of ellipsoidal height on area (for regular polygon parcels)

Area values	Average ellipsoidal height of the parcel [m]							
	100	250	500	1000	1500	2000	2500	3000
	Area corrections [m^2]							
100 m^2	0.003	0.008	0.016	0.031	0.047	0.063	0.078	0.094
1 ha	0.314	0.785	1.570	3.140	4.710	6.279	7.849	9.419
4 ha	1.256	3.140	6.279	12.559	18.838	25.118	31.397	37.677
52 ha	7.849	19.623	39.246	78.493	117.739	156.986	196.232	235.479
55 ha	17.268	43.171	86.342	172.684	259.027	345.369	431.711	518.053
100 ha	31.397	78.493	156.986	313.972	470.958	627.943	784.929	941.915
400 ha	125.589	313.972	627.943	1255.887	1883.830	2511.774	3139.717	3767.661
2500 ha	784.929	1962.323	3924.647	7849.294	11,773.940	15,698.587	19,623.234	23,547.881
100 km^2	3139.717	7849.294	15,698.587	31,397.174	47,095.761	62,794.349	78,492.936	94,191.523
225 km^2	7064.364	17,660.911	35,321.821	70,643.642	105,965.463	141,287.284	176,609.105	211,930.926
400 km^2	12,558.870	31,397.174	62,794.349	125,588.697	188,383.046	251,177.394	313,971.743	376,766.091
900 km^2	28,257.457	70,643.642	141,287.284	282,574.568	423,861.852	565,149.137	706,436.421	847,723.705

Table 8. The effect of ellipsoidal height on area (for forest parcels)

Forest parcels	Area [m^2]	Average ellipsoidal height of the parcels [m]							
		100	250	500	1000	1500	2000	2500	3000
		Area corrections [m^2]							
P1	28,570,391	897.030	2242.574	4485.148	8970.295	13,455.443	17,940.591	22,425.738	26,910.886
P2	24,240,429	761.081	1902.702	3805.405	7610.810	11,416.214	15,221.619	19,027.024	22,832.429
P3	19,454,332	610.811	1527.028	3054.055	6108.110	9162.166	12,216.221	15,270.276	18,324.331
P4	18,201,894	571.488	1428.720	2857.440	5714.880	8572.320	11,429.761	14,287.201	17,144.641
P5	16,730,816	525.300	1313.251	2626.502	5253.003	7879.505	10,506.007	13,132.508	15,759.010
P6	15,951,082	500.819	1252.047	2504.094	5008.189	7512.283	10,016.378	12,520.472	15,024.567
P7	14,808,969	464.960	1162.399	2324.799	4649.598	6974.397	9299.196	11,623.995	13,948.794
P8	12,920,318	405.661	1014.154	2028.307	4056.615	6084.922	8113.229	10,141.537	12,169.844
P9	11,066,953	347.471	868.678	1737.355	3474.710	5212.066	6949.421	8686.776	10,424.131
P10	10,550,252	331.248	828.120	1656.240	3312.481	4968.721	6624.962	8281.202	9937.443
P11	9,498,533.6	298.227	745.568	1491.136	2982.271	4473.407	5964.542	7455.678	8946.813
P12	7,246,670.6	227.525	568.812	1137.625	2275.250	3412.875	4550.500	5688.124	6825.749
P13	6,575,802.9	206.462	516.154	1032.308	2064.616	3096.924	4129.233	5161.541	6193.849
P14	5,774,547.5	181.304	453.261	906.522	1813.045	2719.567	3626.090	4532.612	5439.134
P15	3,673,390.7	115.334	288.335	576.670	1153.341	1730.011	2306.682	2883.352	3460.023

of the terrain, the slope of the terrain is an important factor to be taken into consideration. The effect of the slope factor on the area of the parcel with a certain height from the ellipsoidal reference surface is presented in Equation22:

$$F_S = F_h + F_h\sqrt{1 + (\tan a)^2} \quad (22)$$

where F_h, F, $\tan a$ denote the area of the parcel with a height from the ellipsoid, the area of the parcel on the surface of the ellipsoid and the average slope of terrain, respectively. The changes caused by the effect of the slope factor on the area of the parcel are presented in Figure 7.

Area values calculated using scaled maps are not exact area values. In order to compute the exact area value of a parcel, different types of area corrections must be added to the area calculated from the scaled map. The impact of these area corrections on the area calculated on the scaled map is shown in Tables 5 and 6 as a percentage.

As seen in Table 9, area corrections calculated based on projection and scale factors have negative values and area corrections calculated based on other factors have positive values. The biggest correction to be added to the parcel area is the area correction caused by the slope factor.

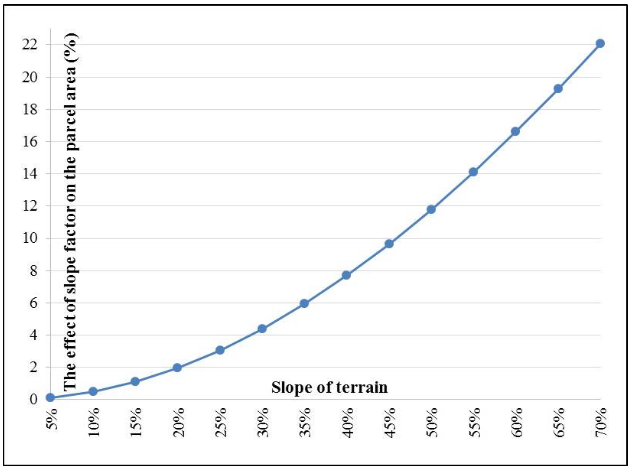

Figure 7. The effect of slope factor on the parcel area

Table 9. Rate of total area corrections

Area corrections	Formulas	Test range	-/+	Rate of correction (%)
Area in the map (UTM, 3°)	$f_{conformal}$	100 m^2–900 km^2 for regular polygon parcels		
		100 m^2–900 km^2 for forest parcels		
Area in AEAC projection	$f_{equal\ area}$		-	0.0001%–0.0020%
Scale of the map	f_{scale}	1:1000-1:250,000	-	0.0077%–1.9292%
Area (Karney method) in ellipsoid	$F \approx f_{equal\ area}$			
Ellipsoid height	F_h	100–3000 m	+	0.0031%–0.0942%
Slope of terrain	F_s	5%–70%	+	0.1249%–22.0656%

AEAC – Albers Equal Area Conic, UTM – Universal Transverse Mercator

3 Results and Discussion

In this study, the area corrections and their sizes to be added to the area value calculated from the scale map in GIS applications are examined.

There are many factors that affect the value of an area. These are selection of map projection, the scale of the map, the ellipsoidal height and the slope of the terrain. GIS users often neglect these area corrections in GIS applications or cannot predict whether area correction values are within the accuracy limits. In addition, since the services provided by users of GIS-based software are limited, such software is insufficient in calculating the area correction values.

In this study, GIS users are presented with the area calculation method (Karney method) on the ellipsoid surface that is not available in GIS software. In addition, it was observed that the GIS software was insufficient in the area correction calculation of the parcels with large area values in the UTM system. In addition, the issue of which of the equal area projection types will be used in GIS applications where the area value is important has been examined with the help of the GIS software. At the end of the test, AEAC projection type has been proposed.

Area information of a parcel is calculated using different methods. One of these methods is the area calculation using the digitised technique from the scale map. When using this technique, it is absolutely necessary to calculate the area correction based on scale. In this study, the area corrections to be added to the areas of large forest parcels according to the size of the map scale (1:1000–1:250,000) were calculated and served to the users.

Slope and elevation parameters are other corrections that affect the parcel area calculated from the scaled map. In this context, area corrections to be added to large forest parcels were calculated from the elevation (100–3000 m) and slope (5%–70%) parameters according to the selected value ranges.

The percentages of the change in all the area corrections are shown in Table 9 according to the parcel areas. In GIS applications where a parcel's area information is used, the required precision and accuracy values are an important point. In this context, it is necessary to produce data according to the request and needs of the user. For this reason, in the applications where the area information is important, the area corrections to be added to the area due to different parameters should be taken into consideration.

Selection of projection and slope of the terrain parameters are available for users in GIS software. However, due to the map scale and the height of the terrain, area corrections to be added to the parcel area are not available in GIS software. In this study, these area corrections obtained by using Equations (18–21) can be added to GIS software by users.

Today, there are many parcel area-based applications such as real estate valuation, land tax, farming support and cost–benefit analysis in GIS applications. In this context, it is necessary to calculate the area of the parcel precisely in order for such applications to give correct results. Therefore, in order to obtain the correct result in determining the exact area calculation of the parcel, it is necessary to take into account the selection of projection, scale of the map, elevation of the map and slope of the terrain factors.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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Availability of data and materials

Data sharing is not applicable to this article as no new data were created or analysed in this study.

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Appendix

MATLAB code of Karney algorithm:

```
function[areakrn,cpukrn] = Karneyf(B,L)
global a f b e2 e12 n
%(a,f,b,e2,e12);GRS80 ellipsoid parameters
% n; corner point number of the polygon
%(B,L); ellipsoid geographical coordinates
startTime=cputime;
B(n+1)=B(1);
L(n+1)=L(1);
RQ2=a^2/2+(b^2*atanh(sqrt(e2))/(2*sqrt(e2)));
areakrn=0;
for j=1:n
f1=B(j);L1=L(j);f2=B(j+1);L2=L(j+1);dl=L2-L1;
B1=atan((1-f)*tan(f1));
B2=atan((1-f)*tan(f2));
a1=atan2(cos(B2)*sin(dl),(cos(B1)*sin(B2)-sin(B1)
    *cos(B2)*cos(dl)));
a2=atan2(cos(B1)*sin(dl),(-sin(B1)*cos(B2)+cos(B1
    )*sin(B2)*cos(dl)));
w12=dl/ sqrt(1-e2*((cos(B1)+cos(B2))/2)^2);
a12=((a2-a1));
a0=atan2(sin(a1)*cos(B1),sqrt(cos(a1)^2+(sin(a1)*
    sin(B1))^2));
sigma1=atan2(sin(B1),cos(a1)*cos(B1));
sigma2=atan2(sin(B2),cos(a2)*cos(B2));
k=sqrt(e12)*cos(a0);
Fx = @(x)(-((Tx(e12)-Tx(k^2.*sin(x).^2))./(e12-k
    ^2.*sin(x).^2)).*sin(x)./2);
DI4sg1=quadgk(Fx,pi/2,sigma1);
DI4sg2=quadgk(Fx,pi/2,sigma2);
Fsg1=RQ2*a1+e2*a^2*cos(a0)*sin(a0)*DI4sg1;
Fsg2=RQ2*a2+e2*a^2*cos(a0)*sin(a0)*DI4sg2;
parea=Fsg2-Fsg1;
areakrn=areakrn+parea;
cpukrn=cputime-startTime;
end
```