sciendo

CIVIL AND ENVIRONMENTAL ENGINEERING REPORTS

E-ISSN 2450-8594

CEER 2022; 32 (3): 0019-0035
DOI: 10.2478/ceer-2022-0027
Original Research Article

THE INFLUENCE OF VARIOUS TYPES OF BRACING ON FORCE DISTRIBUTION IN BRACED BARREL VAULTS

Jędrzej CHRUST[1], Marcin CHYBIŃSKI[1], Piotr GARASZ[1],
Zdzisław KURZAWA[2], Wojciech PIÓRECKI[1], Łukasz POLUS[1],
Weronika STAJKOWSKA[1]
[1]Institute of Building Engineering, Faculty of Civil and Transport Engineering,
Poznan University of Technology, Poland
[2]The President Stanislaw Wojciechowski Calisia University, Poland

Abstract

In this paper the authors analysed four single-layered braced barrel vaults with different types of bracing. Each braced barrel vault covered the area of 20 m × 28 m and was made from steel hollow sections. The static-strength analyses of the structures were conducted using the Autodesk Robot Structural Analysis program, taking into account self-weight, glass cover, snow load and wind load. In the case of wind load, wind pressure perpendicular to the longitudinal wall was determined in accordance with the EN 1991-1-4 standard. However, the standard does not specify how to calculate wind pressure perpendicular to a gable wall for braced barrel vaults. For this reason, two variants suggested by the authors of this article were analysed for this direction of the wind. The influence of the type of bracing on force distribution in a braced barrel vault and on material consumption was analysed. The impact of the gable wall boundary conditions on force distribution in the braced barrel vault was also evaluated. Both the bracing type and the boundary conditions had an impact on the force distribution in the analysed braced barrel vault.

Keywords: braced barrel vault, bracing, large span structures, steel structures

---

[1] Corresponding author: Institute of Building Engineering, Faculty of Civil and Transport Engineering, Poznan University of Technology, Piotrowo 5 Street, 60-965 Poznan, Poland, marcin.chybinski@put.poznan.pl, +48-61-665-2477

1. INTRODUCTION

Space structures are often inspired by nature. Natural forms provide for a minimum use of materials and an exceptional rigidity [1]. Geodesic domes [2], spherical shells [3, 4] or braced barrel vaults are example of space structures.

Barrel vaults have been used in civil engineering since antiquity. Ancient Romans used braced barrel vault roofs over temples, public baths or audience halls [5]. Stones and bricks were the first materials used in barrel vaults. Today, braced barrel vaults are made of steel, aluminium or timber. Metal braced barrel vaults have many advantages:

- significant stiffness and lightness [6],
- they cover large and column-free spaces [1, 7],
- the damage of single bars does not lead to the destruction of the entire structure [8],
- aesthetic aspects [8],
- small dimensions of bars, which are easy to protect against corrosion in production plants [8],
- low height of the structure [8],
- easy assembly of the structure without welding [8].

One of the most famous researchers who investigated barrel vaults was Zygmunt Stanisław Makowski. He founded the Space Structures Research Centre in 1963 [9]. Steel barrel vaults may be single- or double-layered. Double-layered systems may be used for covering larger spans [10]. Connections in double-layered vaults are often hinged, while in single-layered vaults they are often rigid [11]. The geometry of the structures may be created in advanced programs, e.g., Formian [12], Rhino/Grasshoper [13]. The consumption of steel in structures depends on the support conditions, the vault height to span ratio and the bracing type [14]. Long barrel vaults need edge and internal stiffening spaced every 20 m [15, 16]. Force distribution depends, inter alia, on the braced barrel vault length [5, 7]. Bars are mainly subjected to normal forces. The values of moments and shear forces are often low [7]. Compression forces are most important in designing braced barrel vault bars. Designers should take into account member buckling, joint buckling, and general buckling of the whole structure or its part [1, 5, 7]. Today this type of structures are often used to create roofs over assembly halls, sport centres, aircraft hangars, auditoria, shopping centres, museums or railway stations [1, 7].

Steel bars most often have to be fire protected to satisfy the required fire resistance [17]. The costs of fire protection may be relatively high when the standard ISO curve is used to describe gas temperature. However, the temperature analysis presented in [18, 19] demonstrated that in case of big barrel vault structures, the use of the standard ISO curve may be unreasonable. Furthermore, the results from the said analysis showed that barrel vault steel structures have great potential to redistribute forces during a fire.

The bar members are connected using node connectors. The following node connector systems were developed for space frame structures: SDC, Novum Structures Free-Form, MERO, Octatube, Tuball, Nodus, Triodetic, Wuppermann-Tezet, Weimar, Oktaplatte, Unistrud, Varitec, Tridimatec, Tubacord, Tragsystem, Piramitec, Space Deck, Space Grid, Nenk, Tridilose, Obayachi Truss, Unibad, GEAI, Gyro, S-KSB and other.

The barrel vaults may have different types of bracing [20]. In this paper, the authors analysed four bracing types (see Fig. 1.)

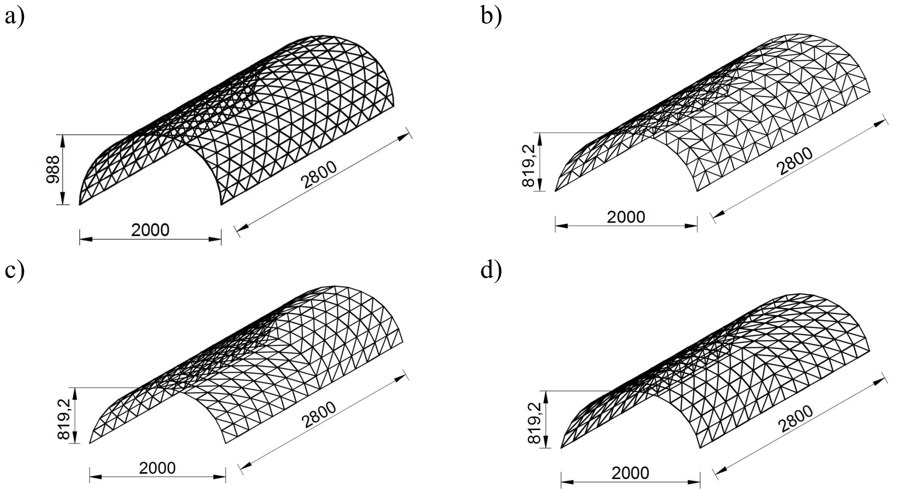


Fig. 1. Types of bracing analysed in this paper: a) Föppl type, b) Warren type, c) Pratt type 1, d) Pratt type 2

The three main goals of this paper are presented below. The first goal is to analyse the influence of various types of bracing on force distribution in a braced barrel vault. The second goal is to analyse the impact of various types of bracing on the consumption of material. The last goal is to evaluate the impact of gable wall boundary conditions on the distribution and value of the normal force.

2. THE OBJECTS OF ANALYSES

The authors analysed four barrel-shaped vaults. One was 9.88 m high (see Fig. 1a) [21] and the remaining three were 8.192 m high (see Fig. 1b–d) [22–24]. Each vault covered the area of 20 m × 28 m. The Föppl type bracing consisted of equilateral triangles with a side length of 200 cm. The arc length of this bracing was 31.177 m. The Warren, Pratt 1 and Pratt 2 bracing types consisted of isosceles triangles (200 × 200 × 283 cm). The arc length of these bracing systems was 28 m. For this reason, the Föppl type bracing was higher than the remaining types of bracing. It also provided for a higher material consumption. Despite the differences in bracing heights, the force distribution in the Föppl type bracing can be compared with the force distribution in the remaining types of bracing provided that they all have the same support conditions. The single-layered construction was made from steel hollow sections which were fixed to each other using semi-rigid connections with high strength bolts [25]. The members were arranged on a cylindrical surface and the basic curve was a segment of a circle. The structure was simply-supported on every edge in every edge node. The structure served both as a wall and a roof. The gable walls were made of reinforced concrete and they were self-supporting. For a structure supported in all edges, the plate analogy can be used. The geometry of the structure was created using the AutoCAD program, and the static-strength analysis was conducted using the Autodesk Robot Structural Analysis program. In the program, the connections were modelled as rigid despite the fact that they were defined as semi-rigid in [25]. This simplification was made because the values of the moment were relatively small (i.e., below 5 kNm). The models used in the program are presented in Fig. 2.

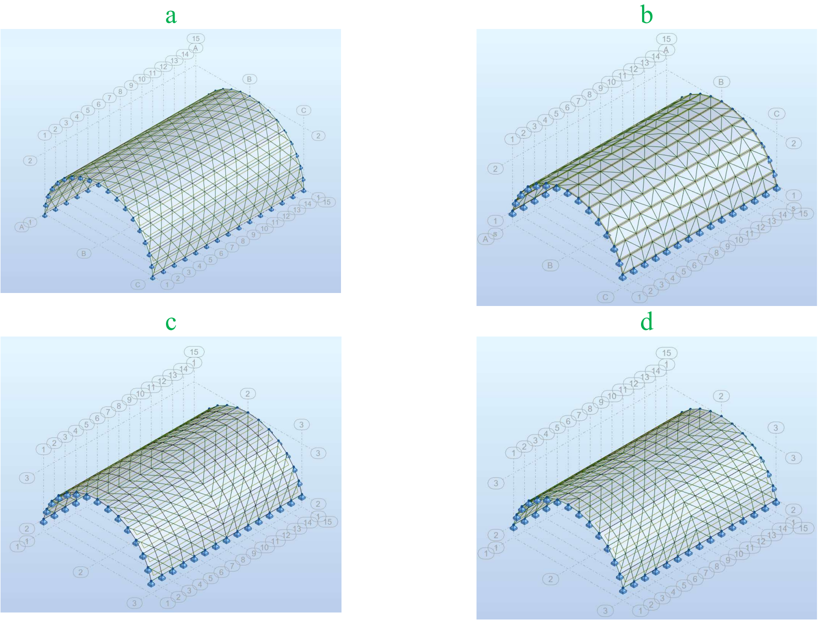
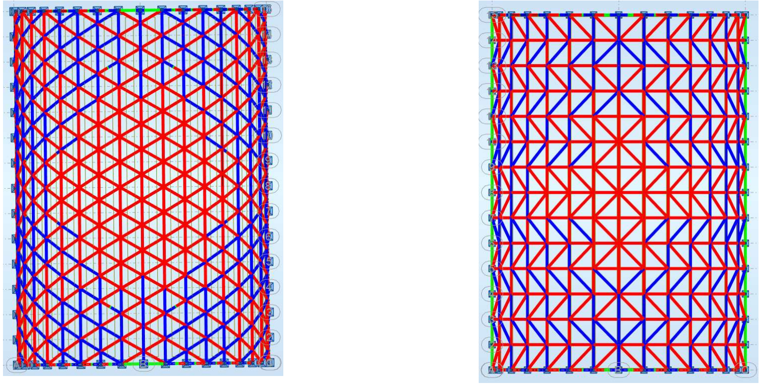


Fig. 2. Braced barrel vaults modelled in the Robot program: a) Föppl type, b) Warren type, c) Pratt type 1, d) Pratt type 2

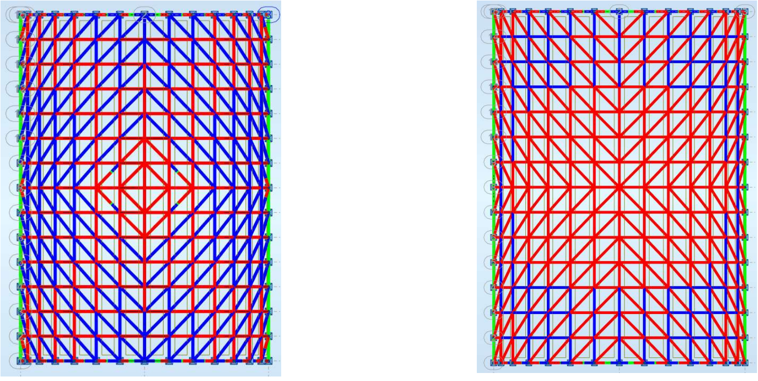
3. FORCE DISTRIBUTION ANALYSIS

A force distribution analysis was conducted assuming that each bar had an identical cross-section, i.e., 120 × 80 × 5 mm, and that the vertical uniformly distributed load was equal to 1.0 kN/m^2. This load was used to compare force distribution in each braced barrel vault. Loading the structure at the initial stage with a symmetrical load, easy to model, makes it possible to evaluate the correctness of the model (symmetrical distribution of forces) and to perform a preliminary evaluation of the behavior of the structure (which bars are the most loaded and which bars could fail by buckling). Figure 3 presents which bars were subjected to compression and which were subjected to tension in the analysed braced barrel vaults.



Föppl type

Warren type



Pratt type 1

Pratt type 2

Range (+ compression, – tension)

🟦 < –0.01 kN

🟩 –0.01 kN to 0.01 kN

🟥 > 0.01 kN

Fig. 3. Bars subjected to compression or tension in braced barrel vaults

As evident from Fig. 3, the type of bracing had an impact on the sign convention (positive or negative) of the axial force. The braced barrel vault with Pratt type 2 bracing had the highest number of compressed bars which could have failed by buckling. For this reason, the Pratt type 2 bracing is not recommended and can be easily replaced with the Pratt type 1 bracing. The most loaded bars are demonstrated in Fig. 4. They are located at the ends of the braced barrel vaults.

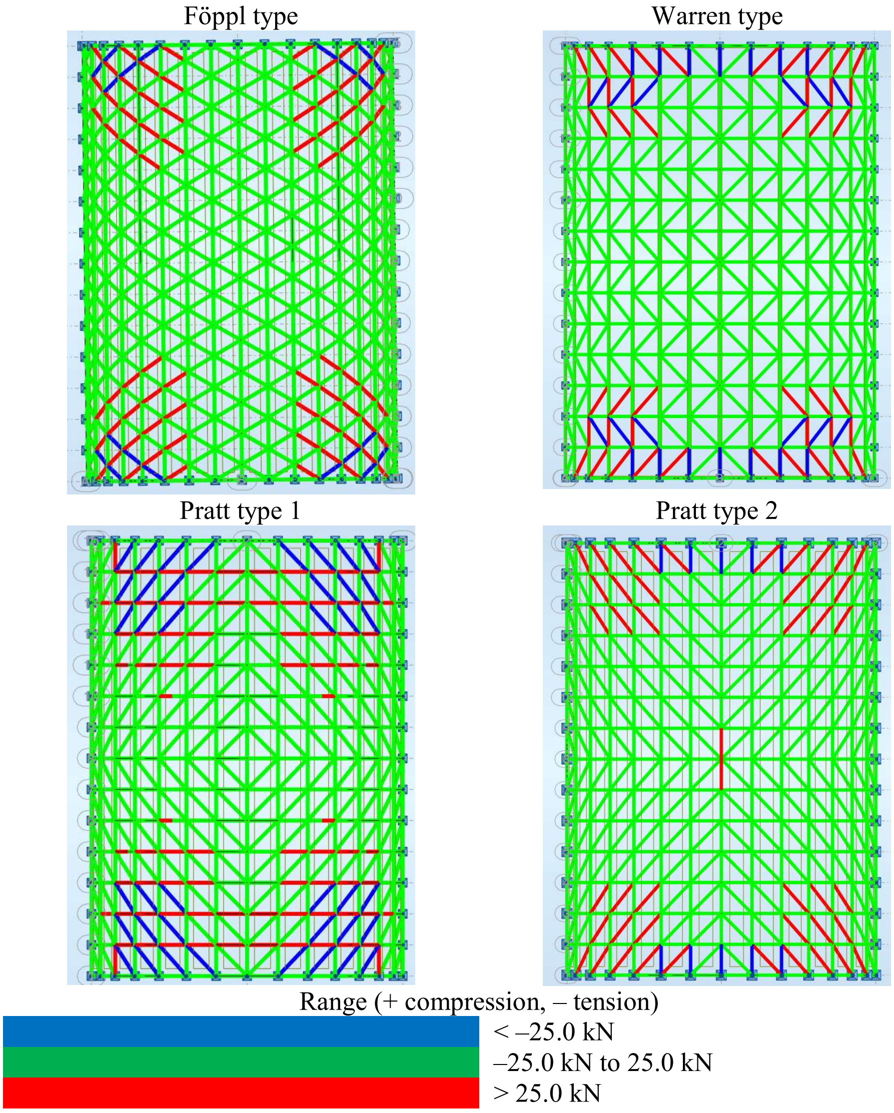

Föppl type
Warren type
Pratt type 1
Pratt type 2

Fig. 4. The most loaded bars in braced barrel vaults

4. STRUCTURE WEIGHT ANALYSIS

The authors compared material consumption for the construction of each braced barrel vault. All loads were taken into account, i.e., self-weight, glass cover, snow load and wind load. The self-weight of the steel bars was automatically taken into account in the Robot program. The weight of the glass cover was 0.25 kN/m^2. The snow load was determined in accordance with EN 1991-1-3 [26] (see Fig. 5). In case of wind load, wind pressure perpendicular to the longitudinal wall was determined in accordance with EN 1991-1-4 [27] (variant 0, see Fig. 6). However, the EN 1991-1-4 standard does not specify how to calculate wind pressure perpendicular to a gable wall. For this reason, two variants for wind pressure perpendicular to a gable wall were analysed. In the first variant, the structure was divided into two parts: the roof part (for the slope below 60º) and the wall part (for the slope above 60º) (see Fig. 6). Variant 1 was divided into two subvariants (1a and 1b). The external pressure coefficients were determined for the two extreme values of the slope, i.e., 5º (variant 1a) and 75º (variant 1b) from EN 1991-1-4. In the second variant, the entire structure was treated as the roof. The external pressure coefficients were calculated for each part of the roof individually, taking into account the slope and the wind area. In this variant the value of the external pressure was not constant in one zone as its value depended on the slope. The internal pressure was also taken into account by using the internal pressure coefficients of –0.3 and 0.2.

The 6.10a and 6.10b combinations from EN 1990 [28] were used in ultimate limit state calculations. The 6.14 combination from EN 1990 [28] was used in serviceability limit state calculations.

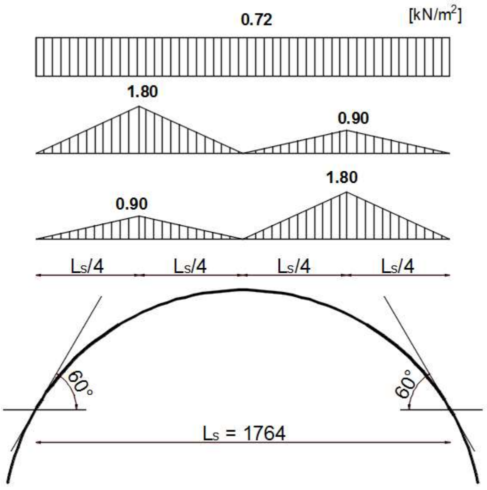


Fig. 5. Snow load

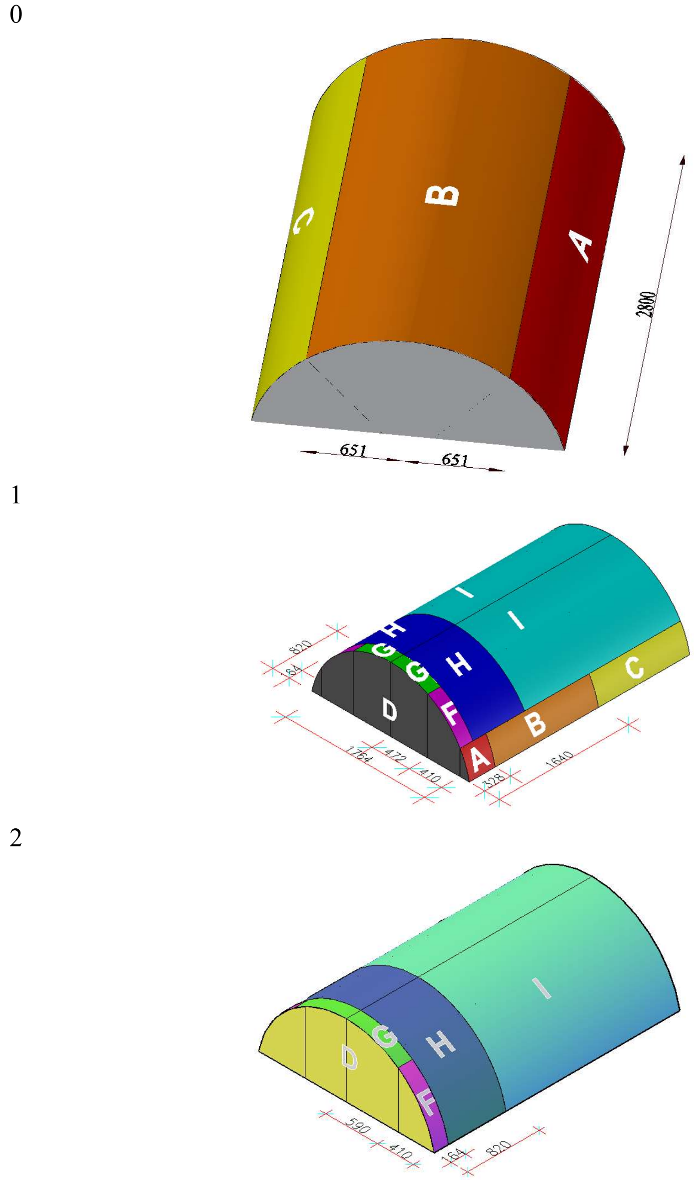
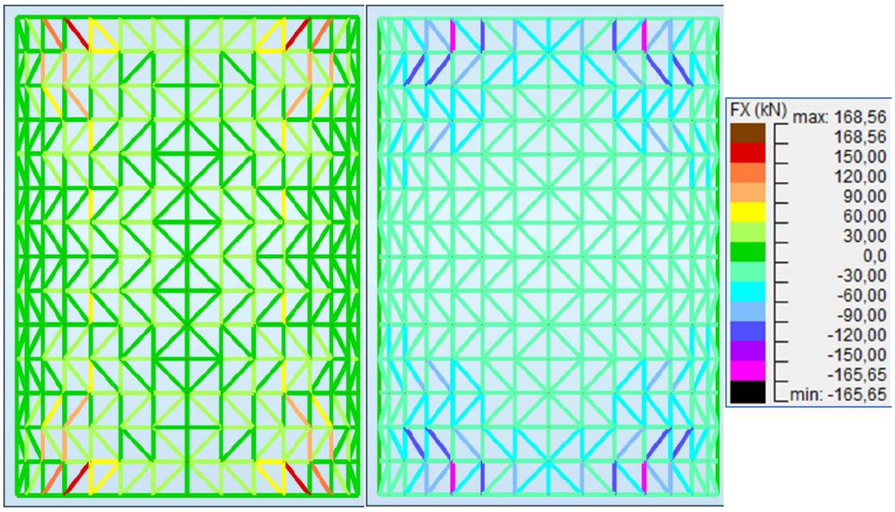


Fig. 6. Wind load variants: 0 – external wind pressure perpendicular to the longitudinal wall, 1 – external wind pressure perpendicular to the gable wall (the structure is divided into the roof and the wall parts), 2 – external wind pressure perpendicular to the gable wall (the structure is analysed as a roof)

The bars of the analysed structures were subjected mainly to axial loads. The minimum and maximum force values in the braced barrel vault with the Warren type bracing are presented in Fig. 7a–c.

Normal force F_x



Shear force F_y

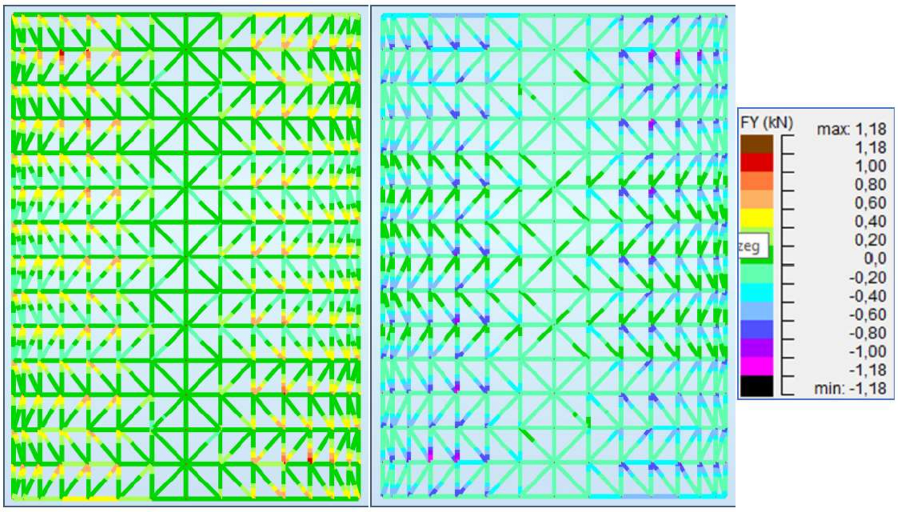
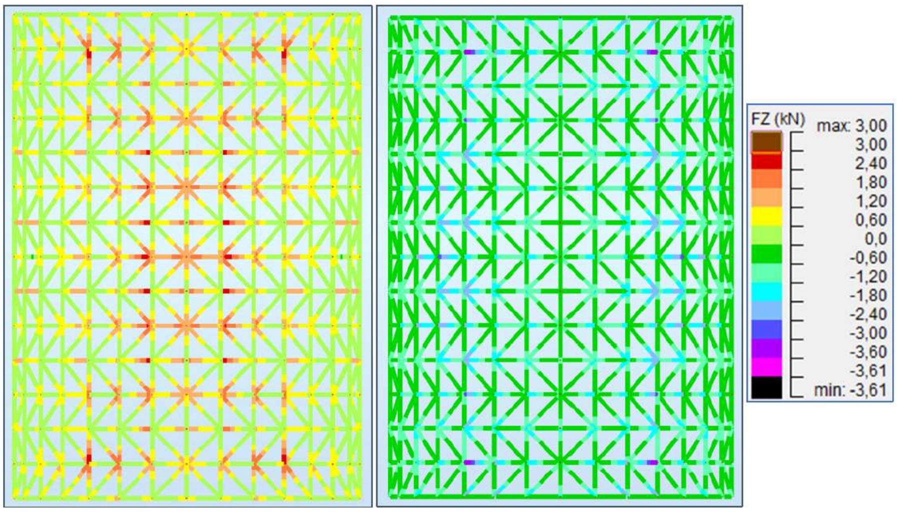


Fig. 7a. Forces in the braced barrel vault with the Warren type bracing: normal force F_x and shear force F_y

Shear force F_z



Torsional moment M_x

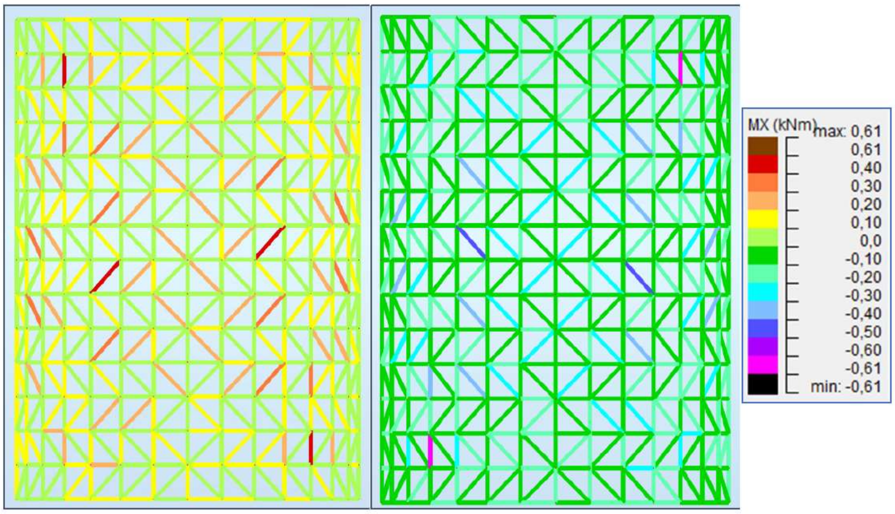
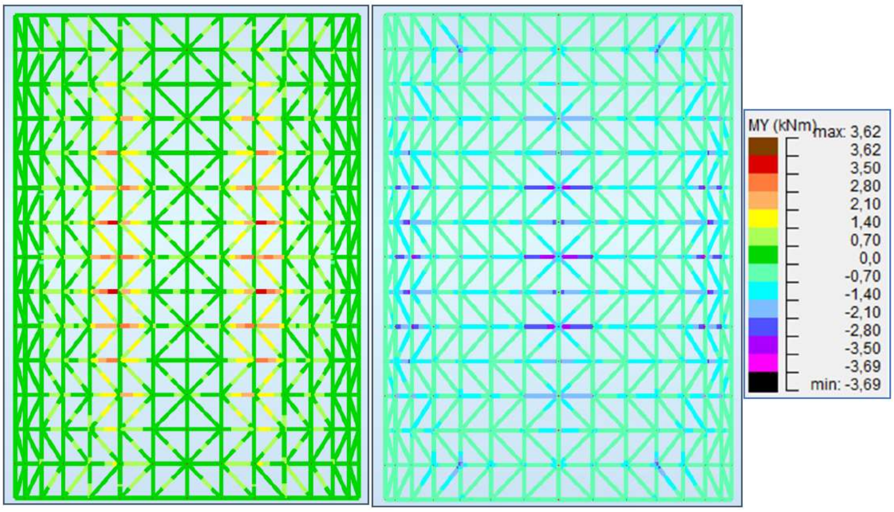


Fig. 7b. Forces in the braced barrel vault with the Warren type bracing: shear force F_z and torsional moment M_x

Bending moment M_y



Bending moment M_z

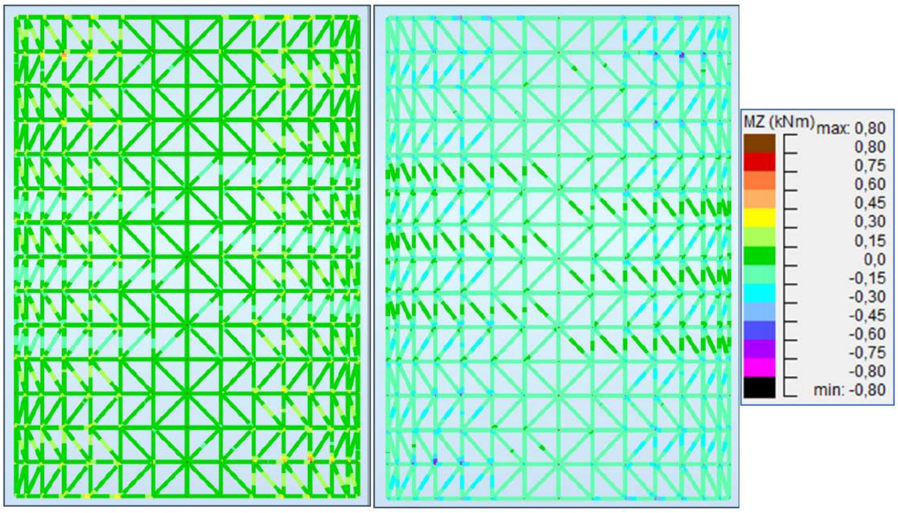


Fig. 7c. Forces the braced barrel vault with the Warren type bracing: bending moment M_y and bending moment M_z

The buckling resistance of the bars was evaluated according to the relevant buckling mode (see Fig. 8). The length of the corresponding buckling mode was assumed as the critical length about the y-axis.

The cross-sections of the bars in the analysed braced barrel vaults are presented in Fig. 9.

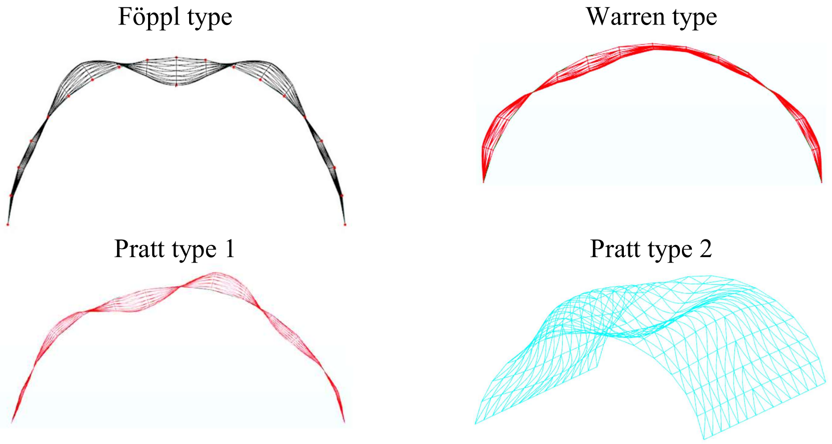


Fig. 8. The buckling mode of braced barrel vaults

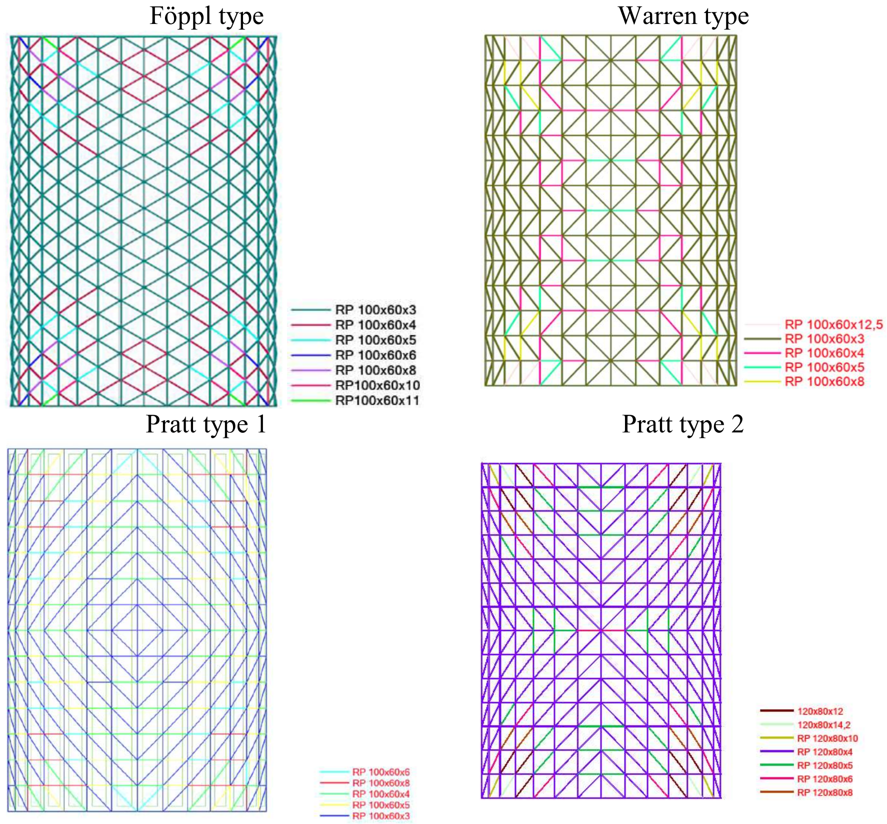


Fig. 9. The cross-sections of the bars in the analysed braced barrel vaults

The cross-sections were determined taking into account the ultimate limit state of the bars, the utilization of the bars, the assumption of using bars of cross-sections differing only in wall thickness to enable their connection at the nodes, and the available assortment of rectangular hollow sections. For each braced barrel vault the authors sought the optimal solution, taking into account material consumption. For example, the structure with the Warren type bracing made of 120 × 60 mm or 100 × 60 mm rectangular hollow sections was analysed.

Figure 10 presents steel consumption in each braced barrel vault. The weight of the structures did include the weight of their nodes.

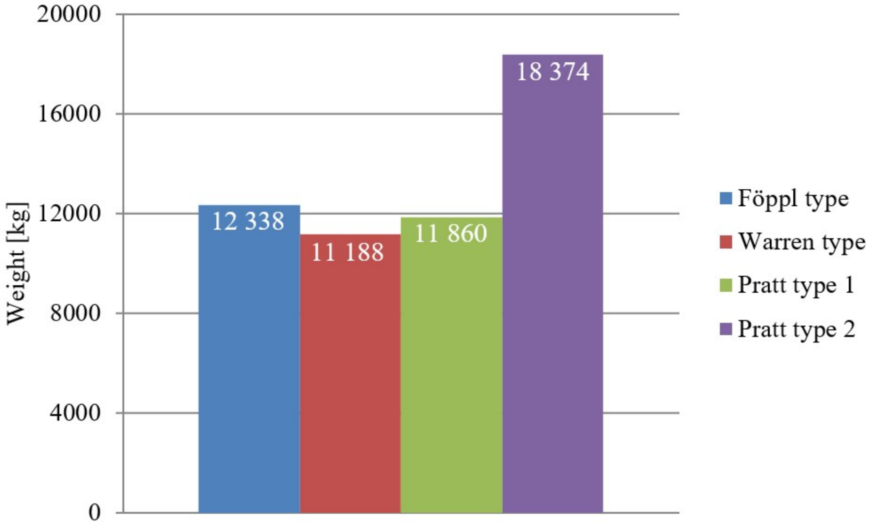


Fig. 10. The comparison of steel consumption in the analysed braced barrel vaults

The type of bracing has a major impact on steel consumption. The braced barrel vault with Pratt type 2 bracing had the highest number of bars subjected to compression. For this reason, the consumption of steel for this type of bracing was the highest. Each braced barrel vault covered the area of 20 m × 28 m. However, the braced barrel vault with the Föppl type bracing was higher than the remaining vaults and, as a consequence, it was not the lightest.

5. THE IMPACT OF THE GABLE WALL BOUNDARY CONDITIONS

In the final stage of the study, the impact of the support conditions on force distribution was analysed. In variant a, displacements in x, y and z directions were fixed in each support. In variant b, displacements in x, y and z directions were fixed in the supports in the longitudinal walls. For the gable walls of the braced barrel vault analysed in variant b, only displacements in *x* and *z* directions were fixed (see Fig. 11). When displacement in the longitudinal direction (*y*) is not fixed, the thermal expansion of the structure is not blocked and the braced barrel vault is free to expand. This is particularly important since temperature changes can lead to structure failure [29, 30].

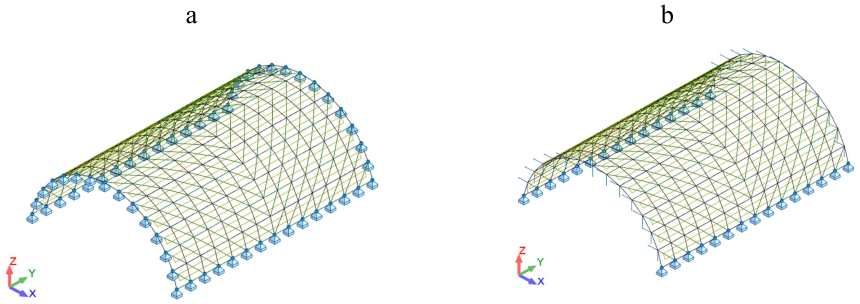


Fig. 11. The support conditions of the braced barrel vault with the Pratt type 1 bracing: a – the structure with all displacements fixed in the supports, b – the structure with displacement in *y* direction not fixed in the gable wall supports

The axial force distribution in the analysed structures is presented in Figure 12.

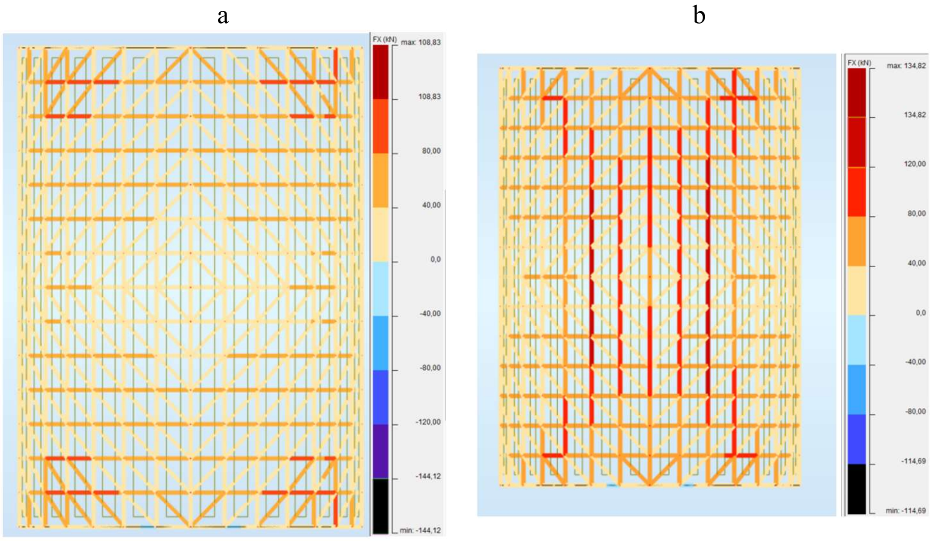


Fig. 12. Axial force distribution in the braced barrel vault with the Pratt type 1 bracing: a – the structure with all displacements fixed in the supports, b – the structure with displacement in *y* direction not fixed in the gable wall supports

In variant b (with displacement in *y* direction not fixed in the gable wall supports), the values of axial compression forces were higher than in variant a (with displacements in all directions fixed in the gable wall supports). What is more, the location of the highest values of axial compression forces in the structure was different for these two variants of boundary conditions. In variant a, the braced barrel vault had the maximum value of axial force at the edge of the structure, while in variant b it had the maximum axial force in the centre of the structure.

6. CONCLUSIONS

The type of bracing had an impact on the sign convention (positive or negative) of the axial force. The braced barrel vault with the Pratt type 2 bracing had diagonal bars mainly subjected to compression. For this reason, the Pratt type 2 bracing is not recommended and can be easily replaced with Pratt type 1 bracing. The type of bracing also had a major impact on steel consumption. The braced barrel vault with the Pratt type 2 bracing had the highest number of compressed bars that could have failed by buckling, which provided for the highest steel consumption.

The most loaded bars in each braced barrel vault were located at the ends of the braced barrel vaults. It was due to the support conditions, i.e., the structure was simply-supported at every edge in every edge node.

Last but not least, the gable wall boundary conditions had an impact on the value and distribution of the axial force. The structure with displacements in the longitudinal direction not fixed in the gable wall supports had higher values of axial compression forces and the maximum axial compression force was located in its centre part. The structure with displacements in all directions fixed in the supports showed lower values of axial compression forces and the maximum axial compression force was located at its edge.

REFERENCES

1. Ramaswamy, GS, Eekhout, M and Suresh, GR 2002. Analysis, design and construction of steel space frames. London: Thomas Telford.
2. Gwóźdź, M 2007. Problemy projektowe współczesnych konstrukcji aluminiowych. *Czasopismo Techniczne* **Z4-A**, 281-286.
3. Kołodziej, S and Marcinowski, J 2018. Experimental and numerical analyses of the buckling of steel, pressurized, spherical shells. *Advances in Structural Engineering* **21(16)**, 2416-2432, https://doi.org/10.1177/1369433218774371.
4. Błażejewski, P 2022. Development of a procedure for the determination of the buckling resistance of steel spherical shells according to EC 1993-1-6. *Materials* **15**, article 25, https://doi.org/10.3390/ma15010025.

5. Makowski, ZS 1985. Analysis, design and construction of braced barrel vaults. London: Elsevier.
6. Szmit, R and Sędrowska, E 2017. Analiza wybranych dwuwarstwowych stalowych przekryć walcowych [Structural analysis of some double-layer steel barrel vaults]. *Materiały Budowlane* **542**(**10**), 38-40, https://doi.org/10.15199/33.2017.10.12.
7. Kurzawa, Z 2011. Stalowe konstrukcje prętowe. Część II: Struktury przestrzenne, przekrycia cięgnowe, maszty i wieże. Poznań: Wydawnictwo Politechniki Poznańskiej.
8. Kowal, Z 1975. Wybrane działy z konstrukcji metalowych. Część III: Zbiorniki, płyty fałdowe i warstwowe, struktury prętowe, budynki wysokie. Wrocław: Politechnika Wrocławska.
9. Parke, G and Behnejad, SA 2014. Zygmunt Stanisław Makowski: A pioneer of space structures. *Architectus* **4(40)**, 33-40, https://doi.org/10.5277/arc140403.
10. Grzywiński, M 2015. Optimization of double-layer braced barrel vaults. *Transactions of the VŠB – Technical University of Ostrava, Civil Engineering Series*, **15(2)**, article 06. https://doi.org/10.1515/tvsb-2015-0006.
11. Szmit, R 2021. Projektowanie i analiza stalowych jednowarstwowych przekryć walcowych. *Przegląd budowlany* **7-8**, 73-76.
12. Rebielak, J 2015. Design of roof cover structures by help of numerical models defined in formian. *Journal of Civil Engineering and Architecture* **9**, 245-256. https://doi.org/10.17265/1934-7359/2015.03.001.
13. Banachowicz, M and Januszkiewicz, K 2019. Curvilinear glass technologies in the historical urban environment. A new glass city-event in public space. *SETSCI Conference Proceedings* **4(7)**, 215-224. https://doi.org/10.36287/setsci.4.7.054.
14. Kevadiya, SM and Bhavsar, Z 2020. Parametric study of braced barrel vault structure to cover large span. *International Research Journal of Engineering and Technology* **7(6)**, 4464-4470.
15. Łubiński, M and Żółtowski, W 2004. *Konstrukcje metalowe, Część II.* Warszawa: Arkady.
16. Polus, Ł 2012, *Pasaż handlowy przykryty kopułą walcową, praca magisterska.* Master Thesis. Poznan University of Technology, Poland.
17. Chybinski, M, Polus, L and Kloj, M 2019. Fire resistance of the rafter used in the steel portal frame. *IOP Conference Series: Materials Science and Engineering* **471**, article 052082. https://doi.org/10.1088/1757-899X/471/5/052082.
18. Malendowski, M, Glema, A, Kurzawa, Z and Polus, L 2015. Mechanical Response under Natural Fire of Barrel Shape Shell Structure. *Journal of Structural Fire Engineering* **6(1)**, 59-66. https://doi.org/10.1260/2040-2317.6.1.59.

19. Polus, L, Szumigala, M, Kurzawa, Z and Malendowski, M 2013. *CFD modelling of the braced barrel vault made from rectangular hollow sections in natural fire*. Proceedings of International Conference Application of Structural Fire Engineering, 19-20 April 2013, Prague, Czech Republic.
20. Chodor, L. 2016. *Przekrycia hal i galerii*. In: XXXI Ogólnopolskie Warsztaty Pracy Projektanta Konstrukcji. Naprawy i wzmocnienia konstrukcji budowlanych. Konstrukcje Metalowe, posadzki przemysłowe, lekka obudowa, rusztowania, Szczyrk, 24-27 lutego 2016 r., Polski Związek Inżynierów i Techników Budownictwa, Oddział w Katowicach, 25-202.
21. Chrust, J 2022. *A braced barrel vault roof over a car showroom parking lot*. Master Thesis. Poznan University of Technology, Poland.
22. Piórecki, W 2022. *A braced barrel vault roof over a market*. Master Thesis. Poznan University of Technology, Poland.
23. Stajkowska, W 2022. *A braced barrel vault roof over a restaurant winter garden*. Master Thesis. Poznan University of Technology, Poland.
24. Garasz, P 2022. *A braced barrel vault roof over a culture passage in a park*. Master Thesis. Poznan University of Technology, Poland.
25. Free Form System, Novum Structural System, data sheet.
26. EN 1991-1-3, Eurocode 1: Actions on structures, Part 1-3: General actions – Snow Loads, Brussels.
27. EN 1991-1-4, Eurocode 1: Actions on structures, Part 1-4: General actions – Wind actions, Brussels.
28. EN 1990, Eurocode 0: Basis of Structural Design, Brussels.
29. Marcinowski, J 1997. Large deflections of shells subjected to an external load and temperature changes. *International Journal of Solids and Structures* **34(6)**, 755-768.
30. Lytvyn, O, Sakharov jr., V, Marcinowski, J and Sakharov, V 2020. Destrukcyjny wpływ temperatury na eksploatację przekryć stalowych dużych rozpiętości. *Builder* **04(273)**. https://doi.org/10.5604/01.3001.0013.8797.

Editor received the manuscript: 13.06.2022