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Measurement Results Based Approximation of the Nonlinear Flux-Current Characteristic of an Electromechanical Actuator

Abstract

A Hamiltonian model of an electromechanical actuator requires approximation of its flux-current characteristic. It can be made using a simplicial approximation, which requires the sets of corresponding points in the spaces of currents and fluxes. These points can be chosen from the trajectories that describe the behaviour of the electromechanical system in these spaces. In the paper, a method of obtaining these trajectories by measurements is shown and four original methods of choosing points lying on them to construct the required points sets are presented. The sets are constructed so that they contain a possibly small number of points, but the approximation which is based on them is precise. The presented methods are used to approximate a flux-current characteristic of the prototype synchronous reluctance machine. The simulation results, computed using the models based on these approximations, are also presented and discussed. The best results are obtained by methods using the computed value of the magnetic field coenergy in the modelled machine.

Keywords: simplicial approximation, mathematical modelling, electrical measurements, flux-current characteristic.

1. Introduction

A Hamiltonian model of an electromechanical actuator is a mathematical model described by the set of differential equations in canonical form, with the flux linkages and rotor angular momentum chosen as a state variables. Defining a matrix of dynamic inductances is not required in such a model. It is therefore well suited to describe an electromechanical actuator with highly saturated magnetic circuit (e.g. Synchronous Reluctance Machine, SynRM), as in this case the aforementioned matrix can be complicated [1].

Instead of the dynamic inductance matrix, a Hamiltonian model requires the approximation of the flux-current characteristic of an electromechanical actuator. A flux-current characteristic (referred to as Ψ - i function later in the paper) is a vector function of the vector of magnetic flux linkages of the actuator windings and it returns the vector of current values in these windings. The spaces of both arguments and values of the Ψ - i function are multidimensional.

A simplicial approximation can be chosen as an approximation method of the Ψ - i function [2]. In this approach, the sets of corresponding points in the spaces of currents and fluxes, followed by the partition (triangulation) of both spaces based on these sets, are required. In the regions between the fixed points Ψ - i function is approximated using the affine function. The whole Ψ - i function is therefore approximated using a piecewise affine function, which is commonly used to describe nonlinear systems [3]. To make the model compact, the sets of chosen points should contain a small number of elements, but the approximation based on these sets should be precise.

In the paper, methods of obtaining such sets of points in the spaces of currents and fluxes are presented. The methods are suited to measurement results, thus allowing for an identification of the mathematical model of the existing electromechanical actuator. Presented methods are then used to construct the approximation of the Ψ - i function of a prototype SynRM machine. Obtained approximations are compared and discussed. The comparison of the behaviour of the prototype SynRM machine and the Hamiltonian models using the obtained approximations are also presented. After the analysis of the results the best method is chosen.

2. Mathematical description of an electromechanical actuator during the measurement procedure

In the Hamiltonian model of an electromechanical actuator, the generalized flux linkages and the rotor angular momentum are the state variables. The model is formulated for a three-phase actuator with wye-connected phase windings. Phase currents are unambiguously defined by two current values, e. g. phase A current i_A and phase B current i_B . The generalized flux linkages $\Psi_{AC} = \Psi_A - \Psi_C$ and $\Psi_{BC} = \Psi_B - \Psi_C$, which are the linear combinations of flux linkages Ψ_A , Ψ_B and Ψ_C of phases A, B and C, respectively, are taken as a state variables. The generalized flux linkages are referred to as fluxes later in the paper. The state space dimension is thus reduced to two, which strongly simplifies the model. Additionally, because the rotor of the electromechanical actuator is blocked during the flux measurement procedure, only the electrical part of the model can be taken into account. As a result the equation describing the electromechanical actuator is as follows:

$$\frac{d}{dt} \Psi = \mathbf{u} - \mathbf{R}i(\Psi) \quad (1)$$

where: $\Psi = [\Psi_{AC}, \Psi_{BC}]^T$ – generalized flux linkages vector, $\mathbf{u} = [u_{AC}, u_{BC}]^T$ – phase-to-phase voltages vector, $\mathbf{R} = [r_A + r_C, r_C; r_C, r_B + r_C]$ – resistance matrix, where r_A, r_B, r_C – resistances of A, B, C phase, $i(\Psi) = [i_A(\Psi_{AC}, \Psi_{BC}), i_B(\Psi_{AC}, \Psi_{BC})]^T$ – phase currents vector.

Although in general the flux-current characteristic is dependent on rotor angular position, it can be neglected during the measurement procedure and included later in the model by the transformation to dq reference frame or by the construction of separated approximation of the Ψ - i function for each angular position of the rotor from a predetermined set.

3. Simplicial approximation

In equation (1) the $i(\Psi)$ term stands for the Ψ - i function which is required to be approximated. A simplicial approximation is chosen as a method of approximation. In this method, the Ψ - i function is approximated by a piecewise affine function. Such an approach enables the use of known developed methods of linear systems description and optimized control for the nonlinear systems.

The simplicial approximation requires:

- 1) The sets of corresponding points in the space of currents and the space of fluxes. In the considered case, both spaces are two-dimensional.
- 2) The partition (triangulation) of the sets, which specifies subregions defined by three points from each of the defined set, so that in any subregion there is no other point from that set. Such a subregion is called simplex. In the two-dimensional space simplexes are triangles.

The piecewise affine function used to approximate the Ψ - i function is a homeomorphism, hence both $i = i(\Psi)$ and $\Psi = \Psi(i)$ functions can be considered equally in the process of the currents and fluxes sets construction.

The approximation precision is generally better in the regions where simplexes are smaller in size, that is where the points sets are more dense (sets contain more points). On the other hand, the simplicial approximation is precise in the regions where

the approximated function is close to affine even when the simplexes in this region are big in size.

In Figure 1 a surface representation of one component of the exemplary i - Ψ function, that is $i_B = i_B(\Psi_{AC}, \Psi_{BC})$, is shown. The regions where this function is close to affine, corresponding to the affine parts of magnetizing curve, can be seen. The function is also highly nonlinear in relatively small regions of the considered domain. For the precise approximation of the i - Ψ function, based on the sets with a possibly small number of elements, the created sets should therefore be more dense only in these highly nonlinear regions. The paper focuses on the methods of construction of the sets of currents and fluxes values that have such a feature.

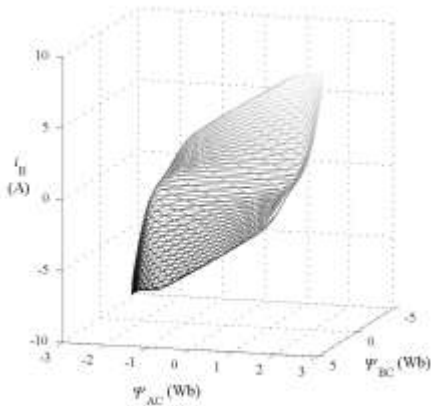


Fig. 1. Surface representation of one component of the exemplary nonlinear i - Ψ function

4. Current and flux linkage measurements

Current and flux linkage measurements are performed with the use of the modified current decay method, described in [4]. The procedure is as follows:

- 1) Settlement of the desired current values in the phases of the electromechanical actuator. During the test, the rotor of the machine is blocked. As the measurement method is viable for every angular position of the rotor and the purpose of this paper is to present the methods of processing of the obtained data, the angular position of the rotor during the measurements was set arbitrarily.
- 2) Short-circuit of the phases and registration of the decaying currents time series.
- 3) Computation of the generalized flux linkages time series by the integration of the equation (1) taking $u = 0$.

During the procedure it is important to precisely state the resistances matrix \mathbf{R} because the possible errors in resistance values highly influence the computed flux time series.

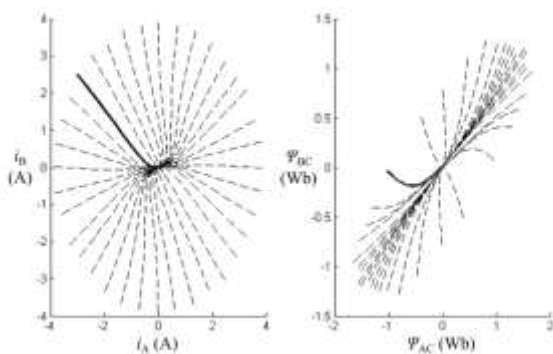


Fig. 2. Sets of trajectories obtained by measurements

As a result of the presented procedure, the decaying time series of the electrical variables describing the electromechanical actuator are obtained. Based on them, computation of the magnetic coenergy time series can be made:

$$E_{cm}(t) = E_{cm}^0 + \int_0^t \left[\Psi_{AC}(\tau) \frac{di_A(\tau)}{d\tau} + \Psi_{BC}(\tau) \frac{di_B(\tau)}{d\tau} \right] d\tau \quad (2)$$

where t – time, $E_{cm}^0 = -\int_0^T \left[\Psi_{AC}(\tau) \frac{di_A(\tau)}{d\tau} + \Psi_{BC}(\tau) \frac{di_B(\tau)}{d\tau} \right] d\tau$ – initial coenergy value, where T – time value after which it is assumed that all the currents and fluxes in the electromechanical actuator are equal to 0.

The obtained time series of currents and flux linkages lead the trajectories in the corresponding spaces. The set of trajectories obtained by measurements for a prototype SynRM machine is shown in Figure 2. In this figure a pair of corresponding trajectories in the spaces of currents and fluxes is excelled.

All measurement data presented in the paper were obtained using laboratory stand described in [4]. The decaying time series of currents were measured using LEM current transducers (LA 25-NP). Signals from transducers were gathered using voltage probes TPP0250 connected to the Tektronix MDO3012 oscilloscope.

5. Construction methods of current and flux sets

From a pair of trajectories obtained in a single measurement (i.e. two trajectories: a single trajectory in the space of currents and a single trajectory in the space of fluxes, both corresponding to each other), a few points can be chosen to include them into the constructed sets for the approximation of the Ψ - i function.

The general idea of construction of these sets is to choose points from a given set of pairs of trajectories. The set of these pairs should contain trajectories obtained for initial conditions (treated as points in the state space) lying in various regions of the spaces of currents and fluxes, e.g. initial conditions for phase current could be uniformly spread on a circle in the space of currents. Such an approach ensures obtaining the information about the Ψ - i function in every region of the spaces of currents and fluxes and puts an emphasis on the methods of choosing points lying on a single pair of trajectories.

There are four methods of this selection, divided into two categories, described in paragraphs 5.1 and 5.2. Methods are enumerated from I to IV for further reference.

5.1. Methods based on geometric features of trajectories

Method I uses the trajectory length function. The task of choosing points lying on a given pair of trajectories is reduced to the problem of choosing set of points Ω_s for the optimal (concerning the mean square error) piecewise affine approximation of one variable function: the trajectory length in the space of fluxes s_Ψ as a function of the trajectory length in the space of currents s_i . In the paper, both lengths are measured from the origins of both spaces.

In the beginning, the length function $s_\Psi = s_\Psi(s_i)$ is defined by the points corresponding to all the points of the sampled flux and current waveforms. Next, the Ω_s set is produced by elimination of the points from the base set, followed by shifting the remaining points to ensure that the piecewise affine function based on the actual set is optimal. Such a set is then taken as the input to the next iteration of the procedure until it contains a small number of points established arbitrarily at the beginning. During the procedure, a single point is eliminated from the Ω_s set if it lies close to the line determined by the points adjacent to it (i.e. would be already well approximated if the piecewise affine approximation was based on these adjacent points). This condition is checked subsequently for all the points in the actual Ω_s set. The shifting of points is partially based on the Groff algorithm [5].

In Figure 3, the function $s_{\Psi} = s_{\Psi}(s_i)$ (line 1) is presented against its piecewise affine approximation (line 2) based on the set of 5 points, obtained with the use of the proposed method. Presented function corresponds to the pair of trajectories shown in Figure 2. After the procedure, the points from the spaces of currents and fluxes which have to be included into the sets approximating the Ψ - i function are computed using the one-to-one $s_i(t)$ and $s_{\Psi}(\Psi)$ functions.

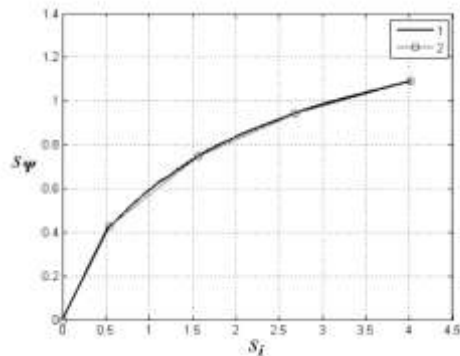


Fig. 3. Function $s_{\Psi} = s_{\Psi}(s_i)$ for an exemplary pair of trajectories

Method II is based on curvature computations. In this method the curvature κ_{AC} of the $(i_A, i_B, \Psi_{AC}(i_A, i_B))$ curve and the curvature κ_{BC} of the $(i_A, i_B, \Psi_{BC}(i_A, i_B))$ curve are computed [6], followed by the “mean” curvature κ computation:

$$\kappa = \sqrt{\kappa_{AC}^2 + \kappa_{BC}^2} \quad (3)$$

The curvatures are computed after the parameterization of the current and flux time series by trajectory length s_i . Next, the points are chosen from the given pair of trajectories to reflect its mean curvature, i.e. more often from the fragments, where computed κ value is big. The points from the analyzed pair of trajectories that should be included into the approximating sets are determined through the analysis of the $\kappa = \kappa(s_i)$ function, followed by selection of the parameter s_i values and using the one-to-one $s_i(t)$ and $s_i(\Psi)$ functions, as stated for method I. In Figure 4 the plot function $\kappa = \kappa(s_i)$ is presented (line 1). The function values are computed for the pair of trajectories shown in Figure 2. The point of highest mean curvature value is marked (mark 2), along with other chosen points (marks 3). Presented set of chosen points contains 20 points to better illustrate the method.

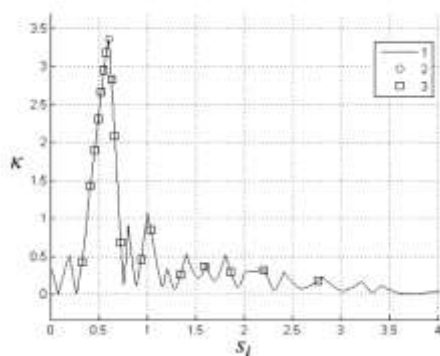


Fig. 4. Mean curvature $\kappa = \kappa(s_i)$ for the exemplary pair of trajectories

Because the curvature computation requires the computation of the derivatives of the parameterized current and flux waveforms, before the computation they are approximated using 3rd order spline function and the computations are performed using the obtained spline formulas. The fragments of the trajectories that

are placed close to the origins of both spaces are also neglected to decrease the influence of noise on the computation results.

5.2. Methods using the computed magnetic field coenergy

Method III is based on checking of the viability of the coenergy computation assuming affine Ψ - i function. In such a case the coenergy change ΔE_{cm} along a given fragment of a pair of trajectories depends only on its beginning points Ψ_{start}, i_{start} and its ending points Ψ_{end}, i_{end} in both spaces. The value of this change is given by the formula:

$$\Delta E_{cm} = \frac{1}{2}(\Psi_{end} + \Psi_{start}) \cdot (i_{end} - i_{start}) \quad (4)$$

To determine the range of application of the formula (4), the relative error is computed as a difference between coenergy change along the fragment, calculated using formulas (2) and (4), divided by the coenergy value in the beginning point of the fragment.

In this method a given pair of trajectories is divided into fragments so that the relative error in each fragment is smaller than the assumed threshold. The computation is started at the beginning points of each trajectory from a given pair (i.e. points for $t = 0$), which sets the beginning of the first fragment. Next, consecutive (with growing time) points from the trajectories are chosen as the possible end of the actual fragment, followed by the relative error calculation in a fragment determined in this manner. If the error is smaller than the threshold, next points are chosen as the possible ending points; if not, the currently considered ending points are set for the actual fragment, while also becoming the starting points for the next fragment. The procedure is repeated until the last points of the given pair of trajectories are chosen as a possible ending points. All the points that fix the boundaries of the chosen fragments, with the exception of the last points in the pair of trajectories, are included into sets used for the Ψ - i function approximation. Instead of the last points of each pair of trajectories, the origins of both spaces are included into the constructed sets.

The last method, method IV, is a hybrid one. First, points are chosen from the representative pair of trajectories with the use of one of the introduced methods I-III. Then, the coenergy values corresponding to the chosen points are computed. The representative pair of trajectories should cross the regions where the Ψ - i function is highly nonlinear. From every other pair of trajectories, the points are chosen so that their corresponding coenergy value is close to the coenergy values computed for the points selected from the representative pair of trajectories. In Figure 5 a sector of the space of currents is shown. The representative trajectory (line 1) and the points chosen to be included into the constructed points set (marks 2) are marked. For the calculation in the paper the method I was chosen as a method of selecting points from the representative pair of trajectories.

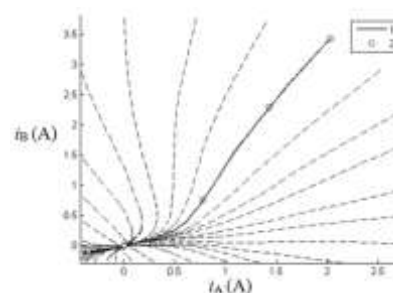


Fig. 5. Points selection from the representative trajectory in the space of currents

6. Measurement object and the methodology of research

To examine the developed methods, they were used to construct the points sets, which were then used to approximate the Ψ - i function of the prototype SynRM machine.

The prototype SynRM machine is three-phase, four-pole and is built based on the stator of an induction motor produced in series. The nominal current for this induction motor is 2.2 A. The resistances of the phase windings were determined once before the main measurements and rounded to be $r_A = r_B = r_C = 13 \Omega$. It is assumed that the resistance values are invariant throughout the measurements.

During the measurements the machine was supplied by two controlled DC power supplies. Phase windings were short-circuited with the use of IGBT transistors. The current time series were measured using LEM current transducer and registered using an oscilloscope.

In practice, during the measurement and computation a possible symmetry of the Ψ - i function can be exploited. Because a Ψ - i function of a SynRM machine is symmetric about the origin of both spaces, it is sufficient to analyze it only in one half of the state space. The presented results were obtained using this symmetry by measuring and analyzing only those pairs of trajectories, which contained the trajectories lying in the 1st and 2nd quarters of the space of currents. The set of trajectories that was used during the construction of the sets of points is thus a half of the set presented in Figure 2. The current initial conditions of the analyzed set of pairs of trajectories were set to be placed uniformly, every 10° , on a circle with the radius of 4 A, starting from the point (4 A, 0 A).

First, the approximation based on the constructed sets is developed and analyzed. For simplicity, approximation based on the sets of points constructed with the use of method N will be later in the paper referred to as method N approximation, and the sets themselves will be referred to as method N sets.

The parameters values for the methods were as follows:

- method I and II: the number of points chosen from one pair of trajectories – 4,
- method III: the threshold value of the relative error – 2.5%
- method IV: the number of points chosen from the representative pair of trajectories – 4.

The parameters are set so that the number of points in the constructed sets is similar regardless of the method used and the approximations based on these sets are accurate.

The constructed sets were triangulated using Delaunay algorithm [2] and the *DelaunayTri* class from the MATLAB environment. First, the triangulation of the set of points in the space of currents was made, followed by the triangulation of the set of points in the space of fluxes, which was made with the edges of the triangulation of the set of currents taken as constraints (i.e. points joined with an edge in the space of currents were also joined in the space of fluxes). The triangulation uniqueness was also checked and possible adjustments in the triangulation were made. Furthermore, the triangulations of methods III and IV sets were made with additional constraints to join the points that have the same (or similar) coenergy value corresponding to them. All actions above ensure the structure of the triangulation of the points sets in both spaces is the same.

To estimate the approximations precision two error values were computed: the relative error in every single simplex of the triangulated sets and the mean relative error. The error in a single simplex is the change in the coenergy value calculated along the simplex boundary, divided by the mean coenergy in the vertices of the simplex [7]. The mean relative error is weighted arithmetic mean of the error values in every simplex, with weights being the sizes (areas) of the simplexes in the triangulated set of points in the space of currents.

In the second part of the analysis a 12 initial conditions in the space of currents were randomly chosen and the decaying time series of current and flux starting from them were measured and computed. The initial conditions were only chosen to lie in the 1st and 2nd quarter of the currents space. Those measured time series were compared with the time series computed with the use of simplified Hamiltonian models (implementing equation (1)) based on the approximations using all previously constructed sets of points. The phase resistances in the model were set as stated in paragraph 6. In the model the simplicial approximation was performed using *baryToCart* and *pointLocation* methods of the *DelaunayTri* class.

To compare the time series i_A^m , i_B^m , Ψ_{AC}^m and Ψ_{BC}^m obtained by measurements with time series i_A^c , i_B^c , Ψ_{AC}^c and Ψ_{BC}^c computed using the model, the mean square error was calculated. Exemplary calculations, made for current i_A follow the formula:

$$\varepsilon_{iA} = \sqrt{\frac{\int_0^T (i_A^c(t) - i_A^m(t))^2 dt}{\int_0^T (i_A^m(t))^2 dt}} \quad (5)$$

where T is as stated in the equation (2). To compare the time series of the whole vector of current or flux, the square root of the weighted average is computed, with weights being the denominator of the ratio in the equation (5). Exemplary computation of the error for the vector of currents, are made with the use of formula:

$$\varepsilon_i = \sqrt{\frac{\varepsilon_{iA}^2 w_A + \varepsilon_{iB}^2 w_B}{w_A + w_B}} \quad (6)$$

where $w_A = \int_0^T (i_A^m(t))^2 dt$, $w_B = \int_0^T (i_B^m(t))^2 dt$ – weights.

The overall average error for the whole set of testing time series is also computed as the square root of the weighted average, with weights for the error of k -th testing time series being the sum of weights in the denominator of the ratio in equation (6). The formula used to calculate the average error of the set of testing time series in the space of currents ε_i^{avg} is:

$$\varepsilon_i^{avg} = \sqrt{\frac{\sum_{k=1}^n [(\varepsilon_i)_k]^2 ((w_A)_k + (w_B)_k)]}{\sum_{k=1}^n [(w_A)_k + (w_B)_k]}} \quad (7)$$

where $(\varepsilon_i)_k$ – the error for k -th time series of vector of currents, $(w_A)_k$, $(w_B)_k$ – weights used to calculate the $(\varepsilon_i)_k$ error using formula (6), n – number of testing time series, $n = 12$.

The computation of the errors ε_{iB} , $\varepsilon_{\Psi_{AC}}$ and $\varepsilon_{\Psi_{BC}}$ of time series i_B^c , Ψ_{AC}^c and Ψ_{BC}^c , respectively, as well as the computation of the error of the whole vector of fluxes ε_Ψ and the average error of the set of testing time series in the space of fluxes ε_Ψ^{avg} are carried out according to equations (5), (6) and (7).

7. Features of the approximations based on the presented methods

In Table 1 features of the simplicial approximations based on the points sets constructed with the use of the presented methods are shown.

Tab. 1. Features of the simplicial approximations based on various points sets

Method	Number of points in generated sets	Number of simplexes	Max. relative error, %	Mean relative error, %	Number of simplexes with the relative error >5%
I	109	180	33.6	3.74	24
II	108	178	30.0	6.84	36
III	91	144	19.1	3.23	26
IV	121	204	12.5	2.67	20

It is vital to construct the approximation so that the error value is lower than a certain threshold in every simplex, but not necessarily kept very close to 0. In particular, such a requirement would result in the construction of the sets that would be very dense in the regions of the spaces of currents and fluxes where the Ψ - i function is very nonlinear. Because in the Hamiltonian model several effects are neglected (e.g. ambiguities of the Ψ - i function caused by the magnetic hysteresis), the model itself would not be entirely precise even if the Ψ - i function approximation was ideal. As a result, setting a very small error threshold value would highly enlarge the model degree of complexity without the increase in the model precision. It is assumed that the reasonable threshold value of the relative error is 5%.

The analysis of Table 1 leads to the conclusion that the simplicial approximations based on the sets constructed with the use of methods based on the coenergy values are more precise than the methods I and II approximations (method I is precise on average, but has big maximum errors). The best precision is achieved by the method IV, but the triangulated sets of points constructed with the use of it have the most points and simplexes. Judging the approximation precision in respect to the number of vertices and simplexes in the triangulated sets, the method III is considered the best.

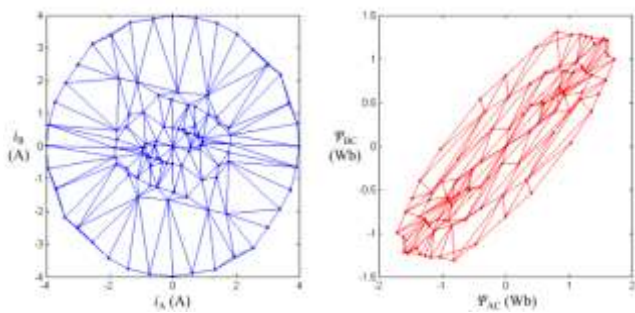


Fig. 6. Triangulated points sets, obtained using method I

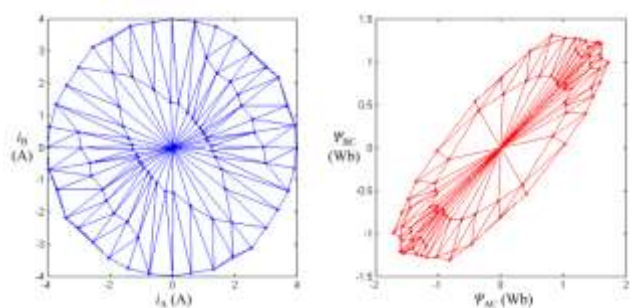


Fig. 7. Triangulated points sets, obtained using method III

The obtained results can be explained by analysis of the form of the triangulated sets. In Figure 5 the method I sets and in Figure 6 the method III sets are shown. In Figure 6 the concentration of points near the point (0, 0) of the space of currents can be observed, although the Ψ - i function is predicted to be close to affine in this region. The triangulated set in the space of fluxes is also very irregular and the triangulation contains a big number of long and thin simplexes that are not aligned with the direction of the linear rise of the Ψ - i function. This can decrease the approximation precision. On the other hand the method III sets are regular and no points are present in the close contiguity of the origins of both spaces. Thin simplexes are also present, but their long edges usually pass through the regions, where the Ψ - i function is close to affine, so the approximation precision is not affected.

8. The behaviour of the models based on the obtained approximations

In Table 2 the maximum, minimum and average error values computed for the testing time series of currents and fluxes are presented.

Tab. 2. Error values computed for the time series obtained with the use of simplicial approximations based on different points sets

Errors (%)	Method of construction of the points sets used by simplicial approximation			
	I	II	III	IV
$\min(\varepsilon_i)$	3.31	2.23	5.55	3.32
$\max(\varepsilon_i)$	10.36	12.77	14.85	15.65
ε_i^{avg}	4.45	5.57	5.99	5.36
$\min(\varepsilon_\Psi)$	3.58	4.18	3.38	2.95
$\max(\varepsilon_\Psi)$	24.40	26.01	19.76	22.34
ε_Ψ^{avg}	6.30	6.79	5.27	6.31

The average error values are generally similar and acceptable. Considering only the time series of currents it can be concluded that methods I and II of the points set construction are better, which is surprising in the light of the analysis presented in Section 7. It is also worth noting that for certain time series the computed errors exceed the value of 10% (currents) or even 20% (fluxes), independently of the method used.

The trajectories corresponding to the time series computed using the model are shown in Figures 7 and 8 against the trajectories obtained by measurements (line 1). For a better clarity, only the trajectories obtained using the models with method I approximation (lowest average current error, line 2) and method III approximation (lowest average flux error, line 3) are presented. The figures show only the fragments of both spaces where the computed errors are the highest. It can be seen that in the space of fluxes (Fig. 8) the measured and computed trajectories have very similar shape, but are translated against each other. The main reason for such a situation is a bad approximation of the initial condition in the space of fluxes, caused probably by the differences in the values of phase resistances during the measurements made for the construction of the sets of points and for testing the approximations. This result shows that to obtain reliable fluxes values, a phase resistances has to be precisely defined, possibly during every measurement taken for a single pair of trajectories. It can be seen in Figure 7 that the trajectories of currents obtained with the use of method I approximation contain points of sudden change of the trajectory direction. Such a behaviour is observed when the trajectory enters the simplexes (usually thin and long) where the coefficients of the approximating affine function are much different than those in the adjacent simplex. The shape of these trajectories can also be visually different than that of the measured ones.

The beginning fragments of the time plots of currents and fluxes, for which the computed ε_i and ε_Ψ errors are the highest (for the method I approximation (line 2) and method III approximation (line 3)) are shown in Figures 9 and 10 against the measured time series (line 1). The computed error in the space of currents is higher for the method III and equals 14.85% compared to 8.31% for method I, whereas the computed error in the space of fluxes is higher for method I and equals 24.40% compared to 19.76% for method III. The presented figures generally confirm the earlier observations: the flux time plots are similar to each other, but translated, and the time plots of currents obtained using the method III are smoother and resemble the measured time series more, but the computed error for them is higher compared to the time plots obtained with the use of the method I.

In the paper the approximation method of the nonlinear flux-current characteristic of an electromechanical actuator is described. The method uses simplicial approximation based on the sets of points which are constructed using measurement results. The result of a single measurement is a pair of trajectories,

that lead through the spaces of currents and fluxes. The series of such measurement results is the set of pairs of trajectories, that span the interesting region in both spaces. A few points, lying on every pair from this set, are chosen and incorporated into the constructed sets. The paper focuses on the methods of selection of these points. Four methods are presented: two are based on the geometric features of trajectories treated as curves and two use the computed coenergy value along the pair of trajectories. In the paper, the sets constructed with the use of discussed methods for a prototype SynRM machine are presented, followed by the analysis of the simplicial approximations based on them and a simplified Hamiltonian model that uses these approximations.

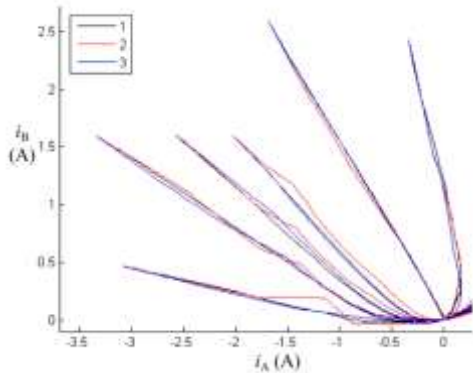


Fig. 8. Trajectories in the space of currents, obtained by measurements (1) and computed using the mathematical model (2 and 3)

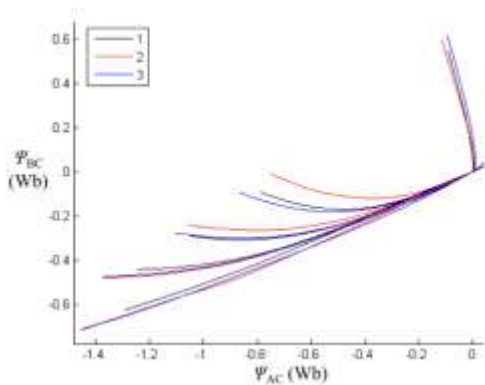


Fig. 9. Trajectories in the space of fluxes, obtained by measurements (1) and computed using the mathematical model (2 and 3)

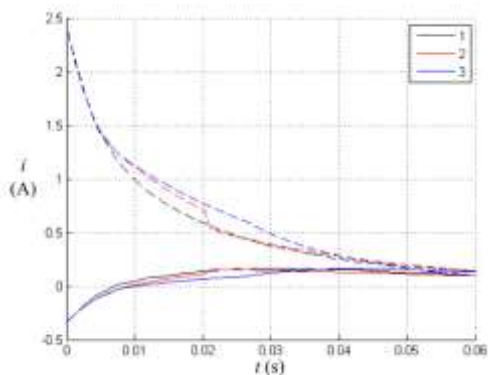


Fig. 10. Currents i_A (solid lines) and i_B (dashed lines) obtained by measurements (colour 1) and using the mathematical model (colours 2 and 3)

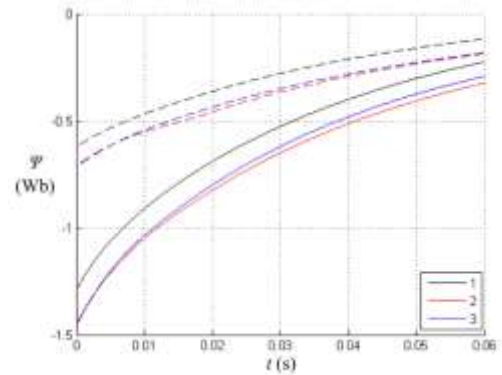


Fig. 11. Fluxes ψ_{AC} (solid lines) and ψ_{BC} (dashed lines) obtained by measurements (colour 1) and using the mathematical model (colours 2 and 3)

9. Conclusions

The approximations based on the sets constructed with the use of the proposed methods are reliable in respect of the approximation error. The mean error, maximum error and the number of simplexes, in which the computed error is big, are lower for the methods using coenergy of the magnetic field. The sets of points that are constructed with these methods are also more regular. Although method I, III and IV approximations are similar in precision, the method III approximation contains the lowest number of simplexes and thus it can be chosen as the best.

The results of the analysis of the Hamiltonian model based on approximation using sets constructed with the use of the presented methods show, that the mean error between the measured and computed time series of currents and fluxes is satisfactory, although for certain initial conditions measured and computed time series can differ significantly even when the general shape is similar. Such a situation is caused by the differences between the values of phase resistances during the construction of the sets of points and tests, which brings a high error into the flux computation, followed by the errors in determining the initial condition in the model. Therefore, during all the flux measurements an emphasis should be put on the precise determination of these resistances. When using the method I or II of constructing of the sets of points, the points of sudden change of the trajectory direction in the space of currents can be observed. Such a behaviour is caused by a sudden change in the parameters of the approximating affine function, determined in adjacent simplexes. A method of finding and eliminating (by the triangulation refinement) this kind of adjacent simplexes should be used when using the methods I or II. This can be considered as a disadvantage of using these methods, as such a situation was not observed when using the methods based on coenergy value.

All the presented methods relies on an arbitrarily determined sets of trajectories in the spaces of currents and fluxes. A possible way of improving the constructed sets of points would be to propose a method for constructing these sets of trajectories. Such a method should be iterative and, based on the approximation in actual step, give an initial condition for a trajectory that would lead through a region of both spaces, where the approximation precision is not satisfactory. Finding such a method will be the scope of further research.

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