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# AN ANALYSIS OF THE EFFICIENCY OF DESIGN METHODS

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***Abstract:***

*The objective of the paper is to analyze the efficiency of design methods proposed in different codes according to the load carrying capacity of concrete structures. In particular, three standard recommendations have been considered: the ACI 318 shear design model, the equation by Eurocode 2 and a new design procedure from Model Code 2010. In the paper the flexural concrete members, which are reinforced longitudinally but without transverse reinforcement, are considered. The design values of the load carrying capacity of such members obtained from these three codes were compared with experimental results. An advanced statistical analysis has been applied to predict of dependent variable (load carrying capacity of tested beams) and to perform a comparative analysis. On the basis of the analysis conclusions have been drawn according to the fit between the design methods and the test data.*

***Keywords:***

*reinforced concrete members, load carrying capacity, statistical analysis, standard code, efficiency of design methods*

## INTRODUCTION

Concrete is usually described as a quasi-brittle material. For most of structural engineering applications, concrete needs to be reinforced because its tensile strength is only around one tenth of its compressive strength. In flexural members two types of reinforcements are usually used: longitudinal and transverse reinforcement. Longitudinal steel bars are responsible for bending capacity and transverse reinforcement is responsible for shear capacity. However, there are still some types of concrete members in which the transverse reinforcement is not used, for example one-way slabs, footings or retaining walls. The load carrying capacity for such members is usually tested on beams during four point bending tests. The static scheme of such tests and the distribution of internal forces: bending moments $M_x$ and shear forces $V_x$ are presented in Figure 1.

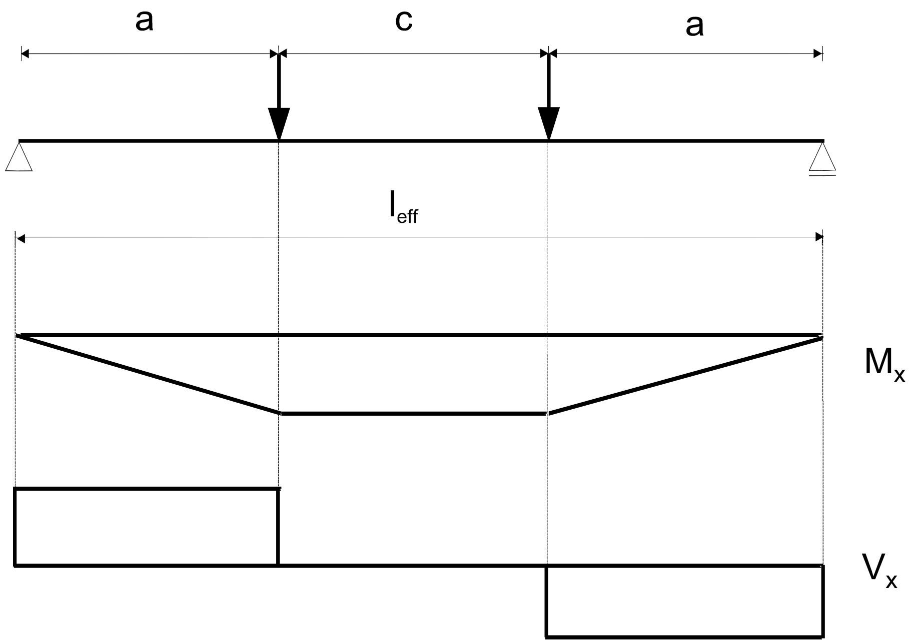


**Fig. 1. T**he scheme of tested beams

*Source: Own study*

In the case of beams without stirrups, shear failure is caused by the propagation of inclined cracks in the support zone of the member. A diagonal failure takes place when the principal tensile stress in concrete reaches the tensile strength. The distribution of principal stress in a support zone of a flexural beam is presented in Figure 2. The example of a possible diagonal failure in the member reinforced longitudinally and without transverse reinforcement is presented in Figure 3.

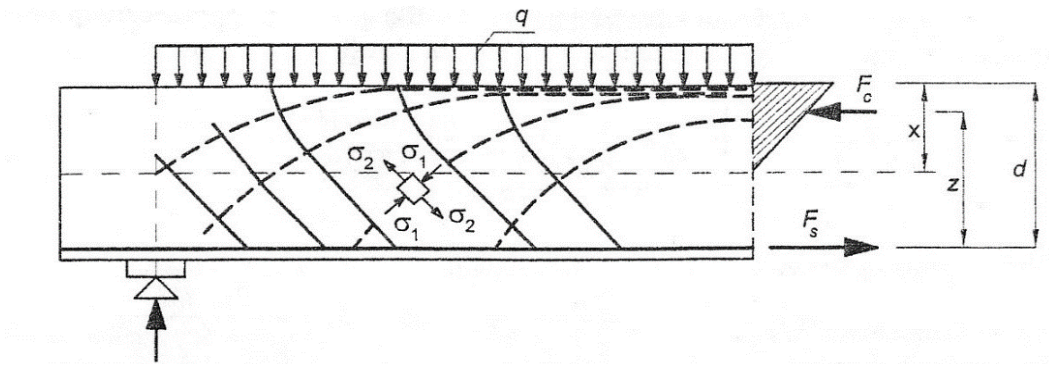


**Fig. 2.** Principal stress distribution in the support zone of a flexural beam

*Source: Own study*


**Fig. 3.** Diagonal failure in a beam without transverse reinforcement

*Source: Own study*

The main design condition for members without transverse reinforcement failed in shear which should be fulfilled is Vx ≤ VRdc. Vx is the maximum shear force caused by loading and VRdc is the shear capacity of the member. Standard recommendations provide rules for calculating VRdc but the design procedure in different codes varies significantly.

In the paper, design methods provided in different standards are presented. The dimensioning rules from the American Concrete Institute Design Code ACI 318 1, the European Standard Eurocode 2 4 and the International Federation for Structural Concrete fib Model Code 2010 7 according to design shear resistance of longitudinally reinforced concrete beams without shear reinforcement are considered. In order to examine the efficiency of design methods for shear capacity given in the codes, a statistical analysis was performed. In this analysis the results of calculated shear strength were compared with the test data. The statistical analysis allowed to draw conclusions related to the method which best fits the experiments.

## 1. SHEAR DESIGN METHODS

It has been observed that different models are used to describe shear failure and varied methods are provided in building standards concerning concrete structures to determine shear strength of reinforced concrete members. In particular three codes are considered in the analysis: ACI 318 1, Eurocode 2 4, and Model Code 2010 7.As the tensile strength of concrete is the main parameter which influences the shear ca-pacity of concrete beams it should be included in shear design models. Most codes, however, evaluate shear strength while assuming an empirical tensile-compressive rela-tionship. For example, in Eurocode 2 the tensile strength $f_{ct}$ is proportional to $\sqrt[3]{f_c}$ ($f_c$ – compressive concrete strength), in ACI 318 the tensile strength is expressed as $f_{ct} = 0.556\sqrt{f_c}$ and in Model Code 2010 $f_{ct}$ is a function of $\sqrt{f_c}$ .The basic rules from the codes according to shear strength of longitudinally reinforced concrete members in bending are presented in Table 1 in which the following symbols are used:

- $f_c$ – characteristic compressive strength of concrete in MPa;

- $b_w$ – width of the cross section in the tension area in mm;
- $d$ – effective depth of the cross section in mm;
- $\rho_l$ – ratio of longitudinal reinforcement;
- $\gamma_c$ – safety coefficient for concrete;
- $V_u$ – shear force in cross section considered in N;
- $M_u$ – bending moment in cross section considered in Nmm;
- $E_s$ – elastic modulus of reinforcing bars;
- $A_s$ – cross section of reinforcing bars;
- $d_g$ - aggregate diameter in mm;
- $z$ – effective shear depth in mm;
- $\varepsilon_x$ – average longitudinal strain at the mid-depth of the member;
- $z$ – effective shear depth in mm;
- $\varepsilon_x$ – average longitudinal strain at the mid-depth of the member.

**Table 1.** Standard rules for shear capacity

| **Standard** | **Shear capacity $V_{Rd,c}$ in N** |
|---|---|
| Eurocode 2 [1] | $V_{Rd,c} = \left(C_{Rd,c} k (100 \rho_l f_c)^{\frac{1}{3}}\right) b_w d$ (1)<br>$V_{Rd,c} \geq v_{\min} b_w d$ ; $v_{\min} = 0.035 k^{\frac{3}{2}} f_c^{\frac{1}{2}}$<br>$C_{Rd,c} = \frac{0.18}{\gamma_c}$ ; $k = 1 + \sqrt{\frac{200}{d}} \leq 2$ ; $\rho_l < 0.02$ |
| ACI 318 [4] | $V_{Rd,c} = \left(0{,}16\sqrt{f_c} + 17\rho_l \frac{V_{Ed} d}{M_{Ed}}\right) b_w d$ (2)<br>$V_{Rd,c} \leq 0{,}29\sqrt{f_c} b_w d$ ; $\sqrt{f_c} \leq 8.3\,\text{MPa}$ ; $\frac{V_{Ed} d}{M_{Ed}} \leq 1.0$ |
| Model Code 2010 [7] | $V_{Rd,c} = k_v \frac{\sqrt{f_c}}{\gamma_c} z b_w$ (3)<br>$k_v = \frac{0{,}4}{(1 + 1500\varepsilon_x)} \cdot \frac{1300}{(1000 + k_{dg} z)}$ ; $\varepsilon_x = \frac{M_{Ed}/z + V_{Ed}}{2 E_s A_s}$ ;<br>$z = 0.9d$ ; $k_{dg} = \frac{32}{16 + d_g} \geq 0.75$ ; $\sqrt{f_c} \leq 8\ \text{MPa}$ |

*Source: Own study*

## 2. STATISTICAL ANALYSIS

In the statistical analysis of the efficiency of design methods, the design shear strength was confronted with experimental results. The analysis was carried out for members made of normal strength concrete of compressive strength from 10 to 40 MPa. The comparison was based on the database of two different experiments: the test performed by Desai 3 and some tests from the experimental investigation performed by Perera and Mutsuyoshi 8. Concrete strength was the only changing parameter in the experiments. All beams had the same cross section 0.2 x 0.3 m and were tested in similar loading conditions (three or four point bending test). During the experiment, the tested members failed suddenly in shear soon after the appearance of diagonal cracks. The character of failure confirmed  that the shear failure was due to principle tensile stress. The obtained ultimate shear forces at failure are presented in Figure 4 according to concrete strength.

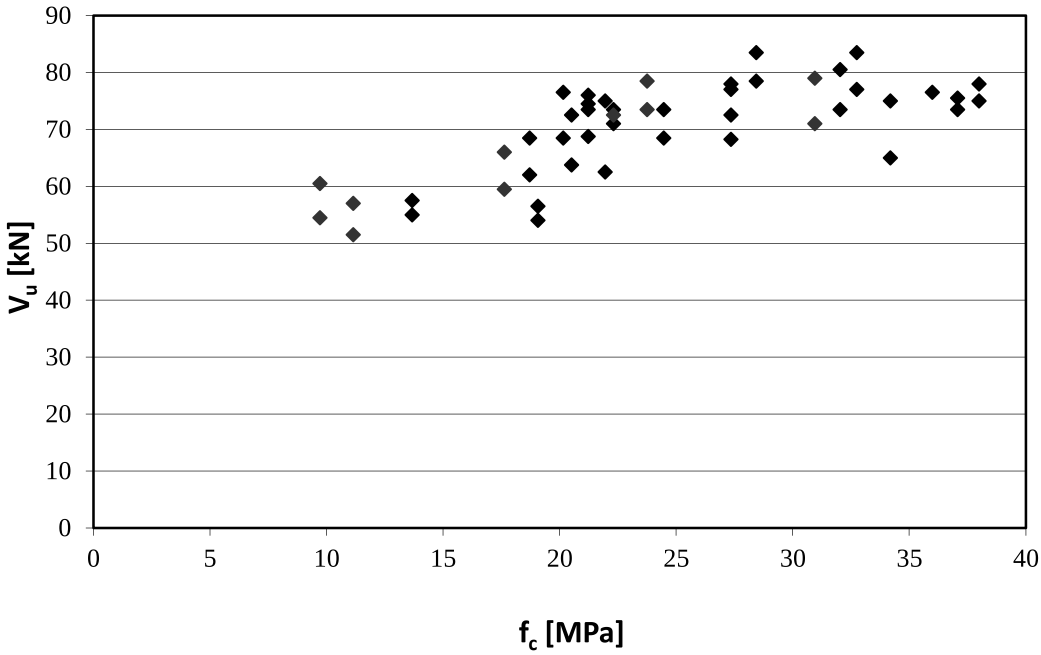


**Fig. 4.** Shear strength versus concrete compressive strength – on the basis of tests performed by Desai and Pereta-Mutsuyoshi

*Source: Own study*

The statistical analysis was carried out in two steps: the prediction of dependent variable vu and the comparison of the obtained regression equation with the design formulas. The StatSoft's Statistica programme was used for prognostic calculations. Two methods were applied: the Multiple Regression MR 6,10,11 and the Generalized Additive Models GAM 5. The partial autocorrelation function and the autocorrelation function2 of the residual number of different models were used in the statistical analysis.

## 2.1. REGRESSION ANALYSIS

During the regression analysis, the ultimate shear stress vu was the dependent variable and the concrete compressive strength fc, as the independent variable, was taken in the form of different functions, for example fc, $\sqrt{f_c}$, $\sqrt[3]{f_c}$. The best fit between the regression function and the test data vu was examined using two methods: the Multiple Regression Method and the Generalized Additive Method. The suggestion that a more advanced method should probably be applied appeared after examining the distribution character of the following variables: compressive concrete strength – fc, ultimate shear stress from test – vu = Vu/bwd, calculated shear stress on the basis of ACI 318 – vuACI, calculated shear stress on the basis of Eurocode 2 – vuEC2, calculated shear stress on the basis of the Model Code 2010 – vuMC. The Shapiro-Wilk normality test was applied and it showed that two variables vu (W=0.939 < Wcritical = 0.945) and vuMC (W=0.938 < Wcritical = 0.945 ) did not have a normal distribution (Wcritical = 0.945 for number of observation N = 45 and level of confidence α = 0.05).

A mean absolute percentage error MAPE was also calculated from the formula:

$$MAPE = \frac{1}{T-n} \sum_{i=T-n}^{T} \frac{\left|Y_i - Y_{ip}\right|}{Y_i} \tag{4}$$

where:

- T – calculation and forecast periods total number;
- n – forecast periods number;
- Yi – actual value of the variable in the period I;
- Yip – predicted value of the variable in the period i.

First, the Multiple Regression Method MR1 was applied. In this method the dependent variable vu was analyzed and the independent variable of compression strength was taken as the function $\sqrt[3]{f_c}$, like the correlation between fct and fc in Eurocode 2. The obtained results are presented below as: regression equation coefficients (Table 2), regression equation (Eq. 5), line plot of variables vu and applied models (Figure 5), partial autocorrelation function and autocorrelation function of the residual number (Figure 6), the mean absolute percentage error.

**Table 2.** Regression summary for dependent variable: vu ( $\sqrt[3]{f_c}$ )

| ***N* = 45** | **Regression Summary for Dependent Variable: $v_u$<br>$R$= 0.99914664 $R^2$= 0.99829401 Adjusted $R^2$= 0.99825433<br>F(1.43)=25162, *p* < 0.0000 Std. Error of estimate: 0.00663** | | | | | |
|---|---|---|---|---|---|---|
| | ***b**** | **Std.Err. of *b**** | ***b*** | **Std.Err. of *b*** | **t(43)** | ***p*-value** |
| Intercept | | | -2.50783 | 0.024120 | -103.974 | 0.00000 |

| $N$ = 45 | **Regression Summary for Dependent Variable: $v_u$ $R$= 0.99914664 $R^2$= 0.99829401 Adjusted $R^2$= 0.99825433 F(1.43)=25162, $p$ < 0.0000 Std. Error of estimate: 0.00663** | | | | | |
|---|---|---|---|---|---|---|
| | $b*$ | Std.Err. of $b*$ | $b$ | Std.Err. of $b$ | t(43) | $p$-value |
| $\sqrt[3]{f_c}$ | 0.999147 | 0.006299 | 3.49515 | 0.022034 | 158.626 | 0.00000 |

b* - multiple correlation coefficient
b – regression coefficient

*Source: Own study*

$$MR1(v_u;\sqrt[3]{f_c}) = -2.50783 + 3.49515\sqrt[3]{f_c} \quad (5)$$

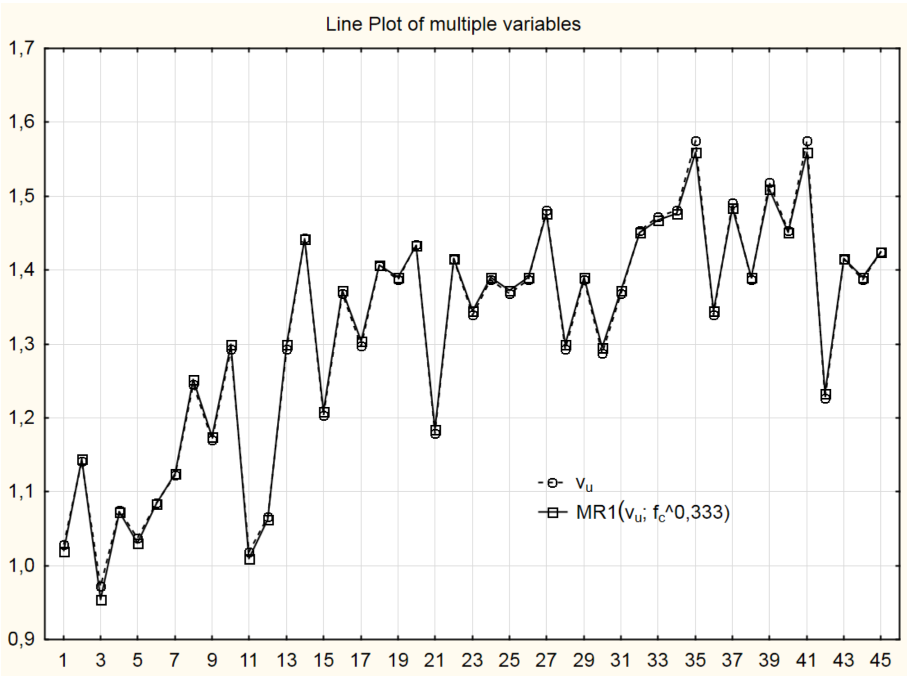


**Fig. 5.** Line plot of variables vu and MR1(vu; $\sqrt[3]{f_c}$ )-a very good fit

*Source: Own study*

The equation (5) is not a regression equation because the residual number RMR1($v_u$; $\sqrt[3]{f_c}$ ) is not a white noise. A very good line plot fit of variables $v_u$ and MR1, a very small mean absolute percentage error *MAPE* = 0.408556 % (perfect fit) and a very high adjusted $R^2$ = 0.99825433 are not sufficient factors to consider the equation MR1 as a regression equation.

The same procedure was applied when the dependent variable $v_u$ was analyzed and the independent variable of compression strength was taken as the function $\sqrt{f_c}$ like

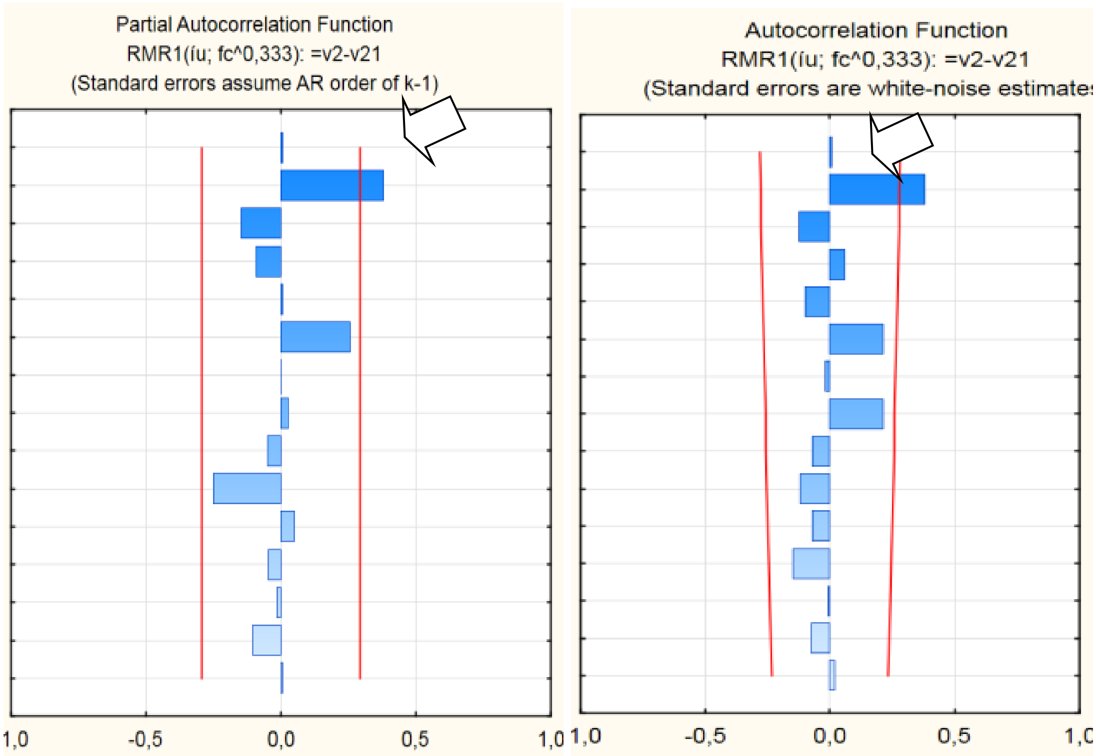


**Fig. 6.** The residual number RMR1(vu; $\sqrt[3]{f_c}$ ) of partial autocorrelation function and autocorrelation function RMR1(vu; $\sqrt[3]{f_c}$ ) is not white noise,
MAPE = 0.408556 % for N = 45

*Source: Own study*

the corellation between $f_{ct}$ and $f_c$ in ACI 318 and Model Code 2010. The obtained results are presented below in the following order: regression equation coefficients (Table 3), regression equation (Eq. 6), line plot of variables $v_u$ and applied models (Figure 7), partial autocorrelation function and autocorrelation function of the residual number (Figure 8), the mean absolute percentage error.

**Table 3.** Regression summary for dependent variable: vu ( $\sqrt{f_c}$ )

| ***N* = 45** | **Regression Summary for Dependent Variable: $v_u$<br>$R$= 0.74561578 $R^2$= 0.55594289 Adjusted $R^2$= 0.54561598<br>F(1.43)= 53.834 $p$ < 0.00000 Std. Error of estimate: 0.10700** | | | | | |
|---|---|---|---|---|---|---|
| | ***b**** | *** Std.Err. of *b*** | ***b*** | **Std.Err. of *b*** | **t(43)** | ***p*** |
| Intercept | | | 0.573762 | 0.102275 | 5.609997 | 0.000001 |
| $\sqrt{f_c}$ | 0.745616 | 0.101621 | 0.155049 | 0.021132 | 7.337192 | 0.000000 |

*Source: Own study*

$$MR2(v_u;\sqrt{f_c}) = 0.573762 + 0.155049\sqrt{f_c} \tag{6}$$

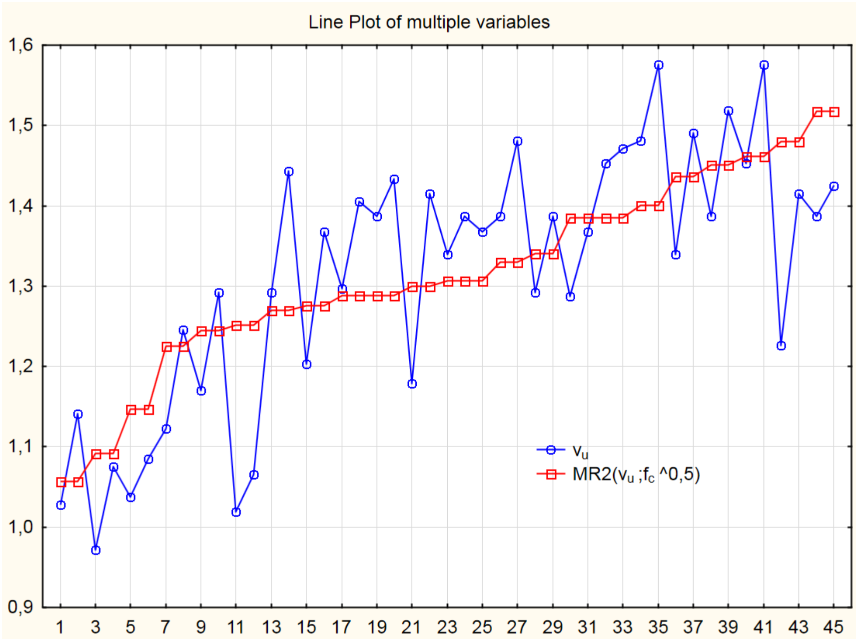


**Fig. 7.** Line plot of variables $v_u$ and MR2($v_u$; $\sqrt{f_c}$ ) – a middle fit

*Source: Own study*

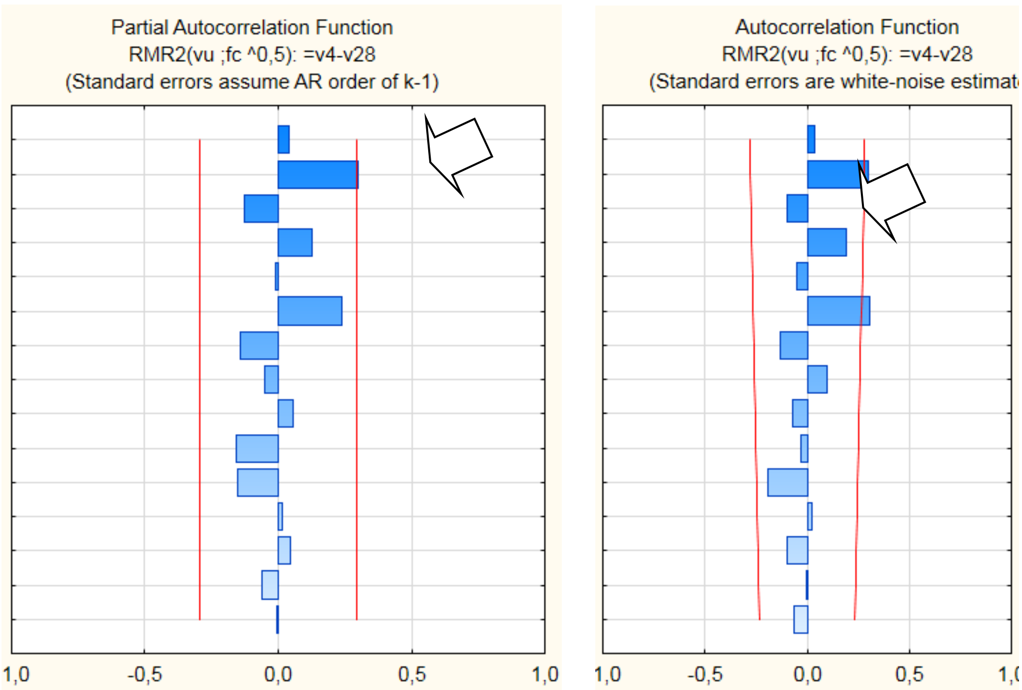


**Fig. 8.** The residual number RMR2(vu; $\sqrt{f_c}$ ) of partial autocorrelation function and autocorrelation function RMR2 (vu; $\sqrt{f_c}$ ) is not white noise, MAPE = 6.929256 % for N = 45

*Source: Own study*

The equation (6) is not a regression equation because the residual number RMR2($v_u$; $\sqrt{f_c}$ ) is not a white noise. A middle line plot fit of variables $v_u$ and MR2, a middle mean absolute percentage error *MAPE* = 6.929256 % and a low adjusted $R^2$ = 0.54561598 are the factors which do not allow to consider the equation MR2 as the regression equation.

In the next step the Generalized Additive Method GAM was used. In this method the dependent variable $v_u$ was analyzed with regard to different functions of compressive strength: $\sqrt{f_c}$ , $\sqrt[3]{f_c}$ , $f_c^2$ , $f_c^3$ , $f_c^4$ , $f_c^5$ , $f_c^6$ , $f_c^7$ , $f_c^8$ , $f_c^9$ , $f_c^{10}$ . The obtained results are presented below as: regression equation coefficients (Table 4), regression equation (Eq. 7), line plot of variables $v_u$ and applied models (Figure 9), the partial autocorrelation function and autocorrelation function of the residual number of models (Figure 10), the mean absolute percentage error.

**Table 4.** The list of regression equation coefficients for Gamma distribution

| | **Fit summary Response: $v_u$ Distribution: Gamma; link function: Log** | | |
|---|---|---|---|
| | **Variable index** | **Degr. of freedom** | **GAM coef.** |
| Intcpt | 0 | 1.000000 | 0.00000 |
| $\sqrt{f_c}$ | 1 | 4.002432 | -1.91852 |
| $\sqrt[3]{f_c}$ | 2 | 4.001182 | 2.83856 |
| $f_c^2$ | 3 | 4.001834 | 0.13130 |
| $f_c^3$ | 4 | 3.999337 | -0.02204 |
| $f_c^4$ | 5 | 3.999161 | 0.00207 |
| $f_c^5$ | 6 | 4.000788 | -0.00012 |
| $f_c^6$ | 7 | 4.000867 | 0.00000 |
| $f_c^7$ | 8 | 4.003390 | -0.00000 |
| $f_c^8$ | 9 | 4.000112 | 0.00000 |
| $f_c^9$ | 10 | 4.001686 | -0.00000 |
| $f_c^{10}$ | 11 | 4.001100 | -0.00000 |

*Source: Own study*

$$GAM1(v_u;\sqrt{f_c},\sqrt[3]{f_c},f_c^2,f_c^3,f_c^4,f_c^5)=e^{\left(-1.92\sqrt{f_c}+2.84\sqrt[3]{f_c}+0.13f_c^2-0.022f_c^3+0.002f_c^4-0.00012f_c^5\right)} \quad (7)$$

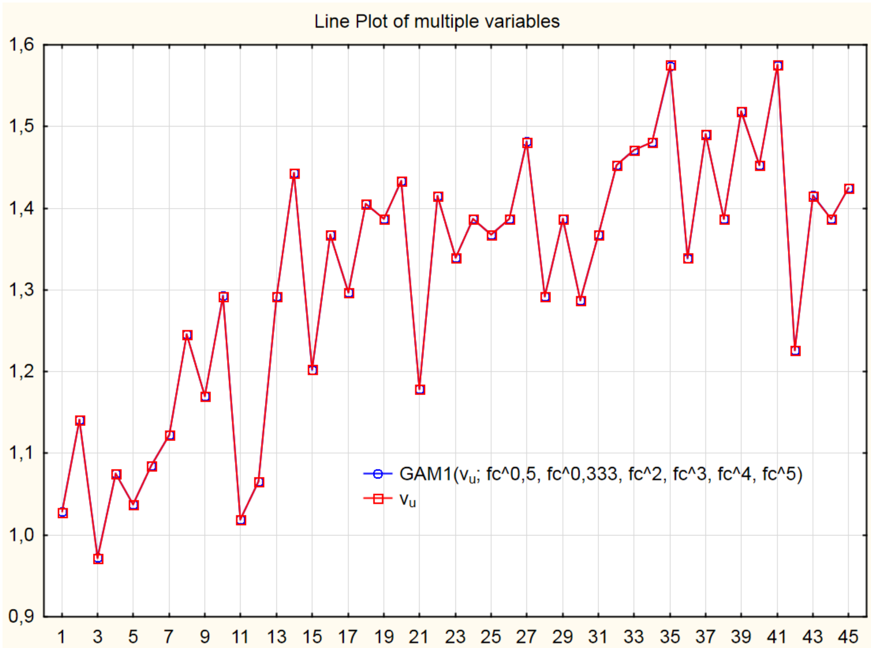


**Fig. 9.** Comparison of values GAM1 with variables vu and prognostic GAM1(vu; $\sqrt{f_c}$, $\sqrt[3]{f_c}$, $f_c^2$ $f_c^3$, $f_c^4$, $f_c^5$) – excellent fit

*Source: Own study*

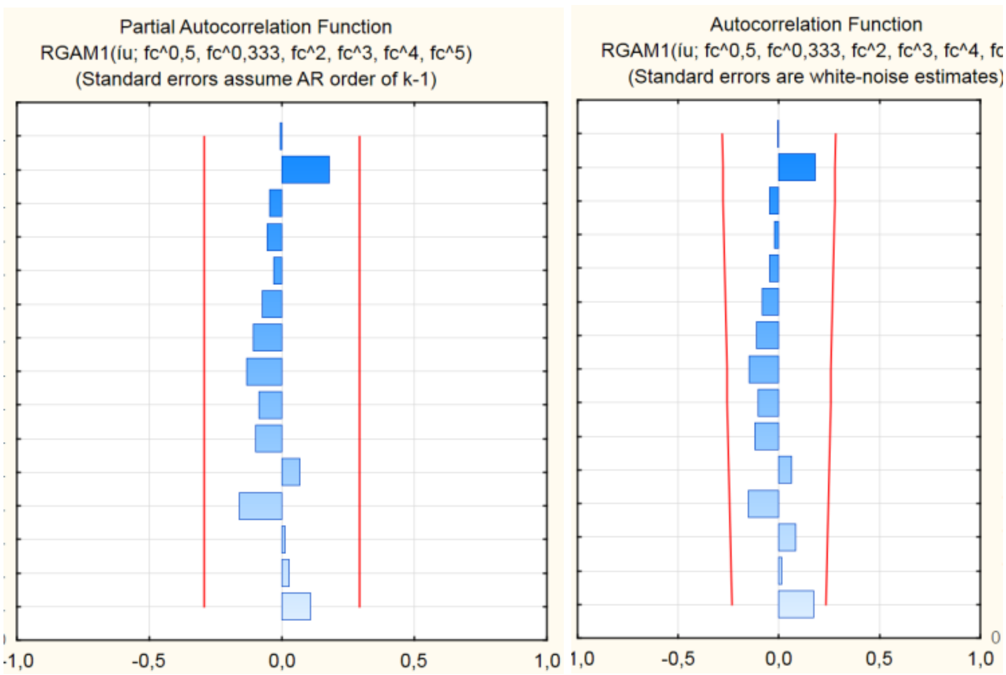


**Fig. 10.** The residual number RGAM1(vu; $\sqrt{f_c}$,$\sqrt[3]{f_c}$,$f_c^2$,$f_c^3$,$f_c^4$,$f_c^5$) of partial autocorrelation function and autocorrelation function RGAM1 is white noise, MAPE = 0.018715 % for N=45

*Source: Own study*

The equation (7) is a regression equation. The residual number RGAM1 is a white noise. An excellent line plot fit of variables vu and GAM1, a very small mean absolute percentage error MAPE= 0,018715 % (very good fit) were obtained. This equation was applied at the next step of the statistical analysis in which the test data were compared with the design values.

## 2.2. Comparative statistical analysis

The comparative statistical analysis considered determining the efficiency of design methods given in different codes according to ultimate shear stress. To study if there is any significant difference between the obtained test results vu = Vc/bwd and the calculated values: vuACI, vuEC2, vuMC, the obtained regression function was compared to the theoretical ones plotted for design formulas from different codes. First, the t-test for independent samples was performed [9]. The conclusion was that the variables were independent and that they could be compared. This comparison is presented in Figure 11.

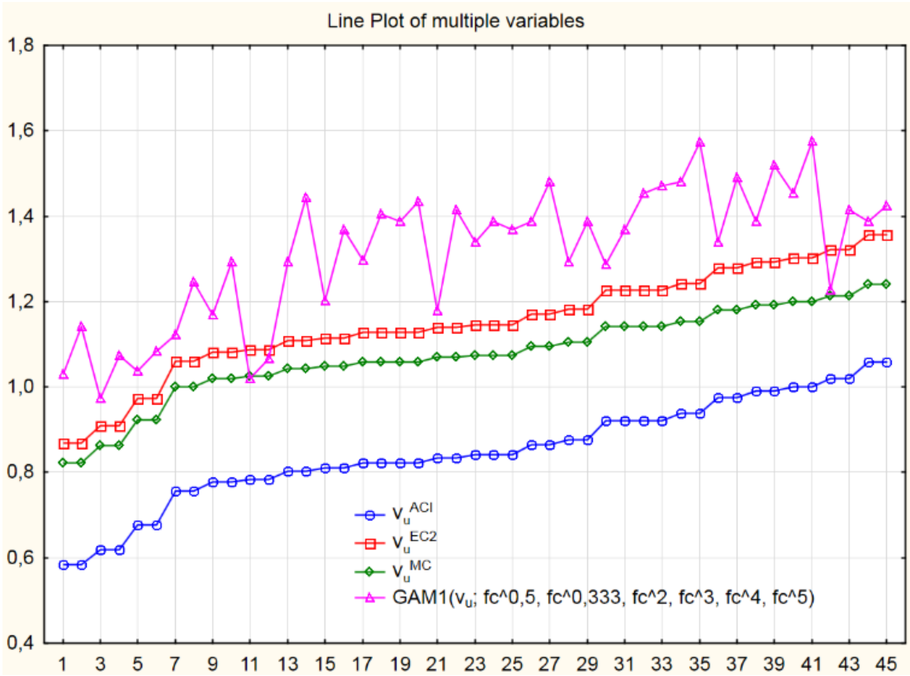


**Fig. 11.** Comparison of prognostic values GAM1 with theoretical ones: vuACI, vuEC2, vuMC

*Source: Own study*

The mean absolute percentage error MAPE was calculated from equation (4) to estimate the difference between the observed experimental data and the theoretical values. When comparing the regression equation (Eq. 7) with the formulas from different codes the following values of the mean absolute percentage error MAPE were obtained:

- for Eurocode 2: MAPE = 12.7 %;
- for Model Code 2010: MAPE = 17.7 %;
- for ACI 318: MAPE = 35.0 %.

The comparison of the obtained regression equation (Eq. 7) with the design formulas from the considered codes shows that the smallest value of the mean absolute percentage error MAPE was obtained for the formula from Eurocode 2. The value of MAPE was 5 % higher for the formula from Model Code 2010 and almost three times greater for the formula from ACI 318 comparing to the error MAPE obtained for Eurocode 2. It has to be pointed out that in the performed analysis, the design values of shear capacity were calculated on the basis of the standard recommendations without taking into account the margin of safety. The safety is considered in Eurocode 2 and Model Code 2010 in the same way using the partial factor method (the recommended value of safety coefficient for concrete is $\gamma_c$ =1.5). In ACI 318 the global safety coefficients are used to secure the safety of structure. Assuming that the safety factors fulfill the adequate protection against the failure of structural members, a better design recommendation is obtaining a smaller difference between the observed experimental data and the theoretical values.

## CONCLUSION

Predicting a diagonal failure of concrete members without stirrups is not researched in depth. The main parameter which influences such a kind of failure is the tensile strength of concrete. The tensile strength is commonly expressed by the compressive strength of concrete and due to the simplicity of design methods, tensile-compression relations. The performed statistical study has shown that although the regression equation for selected experimental observations cannot be successfully built on the basis of these relations, but better fit with ex-perimental data has been obtained in the case of the relation. Such a relation is used in Eurocode 2. When considering the economical aspect of design, the Eurocode 2 formula seems to be the best one, as it is the nearest to the regression function and the tolerable risk of failure is provided by the safety coefficient. The simple shear equation specified in the ACI Building Code for shear strength of reinforced concrete members not containing stirrups has been found to be strongly conservative.

The analysis presented in the paper has shown that the advanced statistical analysis of the efficiency of design methods can be useful for examining design procedures of buildings and engineering work.

Following the philosophy of standard regulations for designing buildings and engineering work, the design model should be sufficiently safe, simple and true. The statistical analysis performed in the paper allowed to chose a close to reality method for structural shear design of reinforced concrete members. It must be noted that to keep the risk of structural failure at a tolerable level, the adequate safety margin is additionally provided in the design codes by applying the capacity-reduction factors.

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